Creating a Practical Model Using Real Options to Evaluate Large-Scale Real Estate Development Projects

by

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ABSTRACT

Real Options analysis has only begun to be recognized as way to evaluate real estate and is considered "beyond the cutting edge" of financial analysis.

Several academic papers have looked at ways that real estate can be analyzed using real options; however a universally practical financial model using real options has not successfully been achieved. There are several reasons why real options analysis has not quickly come to the forefront of financial analysis. The first obstacle is that real options analysis can be quite rigorous and mathematically complex, making it difficult to be easily adopted by the everyday analyst.

Presently, the most common method of analyzing real estate is using Discounted Cash Flow, which is relatively systematic and can be universally understood by most persons in the finance world. However, real options theory is not nearly as intuitive, even to the most sophisticated financial persons. There is no tried and true, universally recognized methodology for real options analysis of real estate, at least not yet. Discounted Cash Flow does a very good job analyzing most real estate. However, complex, multi-phased, or very speculative developments justify significantly more sophisticated analysis methods, such as real options.

Real options is relatively new to real estate, and awaits daring pioneers who are willing to create intuitive, thorough, and transparent models that could be used by future real estate analysts before real options analysis will ever become a mainstream method for analyzing real estate.

With this in mind, this thesis intends to present a practical, comprehensive, and intuitively transparent financial model using Microsoft Excel for analyzing real estate development projects. This thesis will hopefully serve as a basis for future models, and will aid in others' understanding of the advantages and drawbacks of such analysis and how to properly utilize it as a tool for real-world projects. It is also the intent of this model to be utilized and further refined by future students in the Real Estate Development Studio course at MIT and by real-world real estate practitioners.

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Overview of Methodologies Involved

Introduction

In recent years, academics have explored ways of using real options analysis to value real estate. Real options analysis has only begun to be recognized as way to evaluate real estate.

Several academic papers have looked at ways that real estate can be analyzed using real options; however a universally practical financial model using real options has not successfully been achieved. There are several reasons why real options analysis has not quickly come to the forefront of financial analysis. The first obstacle is that real options analysis can be quite rigorous and mathematically complex. This complexity of the analysis makes it difficult to be easily adopted by the everyday analyst. Presently, the most common method of analyzing real estate is with a Discounted Cash Flow model, which is relatively systematic and can be universally understood by most persons in the finance world. However, real options theory is not quite as intuitive, even to the most sophisticated financial persons. There is no tried and true, universally recognized methodology for real options analysis of real estate, at least not yet. Secondly, a rigorous, real options analysis is not necessary for most real estate projects. Discounted cash flow does a very good job analyzing most real estate. It is only the complex, large, multi-phased, or very speculative endeavors that justify a significantly more sophisticated analysis, such as real options. Finally, real options is relatively new, and therefore awaits some daring pioneers who are willing to create an intuitive, thorough, and transparent model that could be used by future real estate analysts before real options analysis will ever become a mainstream method for analyzing real estate.

With this in mind, it is my intention to create a practical, comprehensive, and intuitively transparent financial model for analyzing real estate development projects. This model will hopefully be a model that will serve as a model for future similar models, and will aid in others' understanding in the value of such analysis and how to properly utilize it as a tool for real world projects. It is my goal to create a model that can be utilized and further refined by future students in the Real Estate Development Studio course at MIT and other real estate practitioners.

Background

As a base, I have integrated several modeling techniques that have already been created by others. These include the discounted cash flow model used in the development studio, David Geltner's models used in the Real Estate Finance and Investments courses, and other pro-forma type real options or discounted cash-flow models used in the real world by developers. I will take relevant parts of these models, then refine and combine them in a comprehensive manner that will form a new model that uses real options methods. I will need to test the model with empirical information, perhaps from real-life projects and the development studio. Also, rigorous Monte Carlo Simulation must be performed to validate the results, most likely using TreeAge® software. I will enlist the help of interested classmates to give feedback into the usability and transparency of the model. The classmates may also provide real world data that can be used to empirically test the model.

Some Important Concepts

Decision Tree Analysis

In typical Discounted Cash Flow analysis, net present value is calculated by discounting all possible foreseeable cash flows to time zero. However, this assumes that all originally contemplated decisions are executed, and the values of the decisions behave in a steady-state manner. The discount rates are increased to accommodate the uncertainty of the future. In real life, particularly in real estate development, decisions need only be executed if the decision is optimal given the market fluctuation at the time of the decision. Thus, discounted cash flow does not properly account for the flexibility of developer to make decisions as new information is revealed. Likewise, in many cases the high discount rate used in development projects over-compensates for the overall risk of the projects. Of course, the longer-term and more complex a project is, the more discounted cash flow analysis and reality diverge. Examples of such flexibilities are the decision to wait to build, change the product or size of the project, and decision to liquidate the property at some future time. These flexibilities are more accurately represented using decision tree analysis as opposed to discounted cash flow analysis.

The use of decision tree analysis for net present value decisions was first suggested by J. Magee in 1964. Most commonly, decision tree analysis is used to depict the value of certain strategic decisions based upon consideration of different alternative scenarios. In simplicity, the expected value of an undertaking at the present is the sum of the values of each possible scenario, each multiplied by the probability of such outcome. In theory, there would be infinitely many scenarios, but to keep analysis clean and manageable, a finite (and usually

small) number of scenarios are taken into consideration. This expected value can be calculated using the following expression:

$$EV = \sum s \ Pr_s * EV_s$$

Where Pr.s is the probability of each individual scenario, and EV.s is the expected value of the outcome of each scenario.

A typical decision tree is depicted as a set of nodes branching out to form a tree-shaped structure. According to de Neufville (1990), a decision tree is composed of three basic nodes:

- Decision nodes (square) – instances where decisions are contemplated and made

- Chance nodes (circle) – instances where outcomes are determined by events or states of nature. Nodes have probability of each event happening, with the sum of the probabilities at each node equal to one.

- Terminal nodes (triangle) – instances of completion or abandonment are accompanied by the terminal value of the path.

Probabilities are assigned at chance nodes and terminal payoffs are assigned to the terminal node of each branch. Thus, values can be assigned to each node of the decision tree using the above equation. It can be easily seen that a decision tree could get very large and cumbersome the larger and more complex a project becomes. Decision tree analysis does not replace discounted cash flow analysis, but acts as an excellent complimentary method of analysis and could even be integrated within the same analysis.

Simplified Analysis of Development Projects – "Canonical" Method

As we have discussed, investments in real estate development projects have unique aspects that differentiate them from investments in existing real estate. According to Geltner, Miller (2001) the three main distinct features of development investments are:

1. ***Time-to-build:*** *In development projects the investment cash outflow is spread out in time, instead of occurring all at once up front. This gives development investments inherent "operational leverage", even if no financial leverage is employed.*

2. ***Construction loans:*** *Use of debt financing is almost universal in the construction phase of the typical development project, and this debt typically covers all of the construction cost.*

3. ***Phased risk regimes:*** *Because of the operational leverage noted above, development investment typically involves very different levels of investment risk between the construction (or development) phase, the absorption (or lease-up) phase, and the long-term (stabilized or permanent) phase when the completed project is fully operational.*

These three factors need to be appropriately considered when using the NPV rule for development projects.

For development projects, the risk is greater than for stabilized properties, but it is not initially clear how to properly evaluate the OCC of this risk as it relates to the OCC of a stabilized property. Geltner and Miller (2001) proposed the use of a "canonical" development cash flow pattern in which cash flows at only two points in time: *(i) "time zero" (the present) when the irreversible commitment to the development project must be made and the cost of the land is effectively incurred; and (ii) "time T" when the construction is essentially completed and the developer obtains the net difference between the gross value of the built property as of time T minus the construction costs compounded up to time T.*

Considering the operational leverage during the development phase, a development phase OCC can be calculated using the following function synthesizing a long investment in

the built property and a short investment in the construction costs during the construction

phase:

$$\frac{V_T - K_T}{(1 + E[r_C])^T} = \frac{V_T}{(1 + E[r_V])^T} - \frac{K_T}{(1 + E[r_D])^T}$$

where:

V_T	=	Gross value of the completed building(s) as of time T.
K_T	=	Total construction costs compounded to time T.
$E[r_V]$	=	OCC of the completed building(s).
$E[r_D]$	=	OCC of the construction costs.
$E[r_C]$	=	OCC of the development phase investment.
T	=	The time required for construction

This formula can be manipulated to solve for the OCC of the development phase, $E[r_C]$:

$$E[r_C] = \left[\frac{(V_T - K_T)(1 + E[r_V])^T (1 + E[r_D])^T}{(1 + E[r_D])^T V_T - (1 + E[r_V])^T K_T} \right]^{(1/T)} - 1$$

We can use this development phase OCC to calculate a NPV of the project at any time

previous to completion by discounting the net value of the completed project ($V_T - K_T$) at an

OCC of $E[r_C]$.

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Real Options Analysis

Real Options analysis is a method of evaluating physical or real assets using the theories and methods used to evaluate financial options. In 1977, S.C. Myers first suggested the concept of Real Options to analyze corporate borrowings. According to Copeland and Antikarov (2001), a Real Option is "the right, but not the obligation, to take an action (e.g., deferring, expanding, contracting, or abandoning) at a predetermined cost called the exercise price, for a predetermined period of time – the life of the option." This definition is similar to options theory as it applies to financial options, which also uses six variables to account for the option. According to Copland and Antikarov & Leslie and Michaels (1997), these variables are:

- Value of the underlying risky asset / Stock Price (S);

- Exercise price / Strike Price (X);

- Time to expiration (T);

- Standard deviation of the value of the underlying asset / Uncertainty (σ);

- Risk free rate (r), and;

- Dividends rate (δ)

The options structure used for analysis must appropriately model the real world situation accordingly. A simple example of this is the synthesis of the right to build a new structure as similar to that of a call option with the construction cost as the strike price.

This approach to real options can be used directly with conventional discounted cash flow analysis. Mun (2002) suggests such an approach using what he calls the expanded net present value (eNPV) where:

$NPV = Benefits – Costs$

Options Value = Benefits of Options – Costs of Acquiring Options

eNPV = NPV + Options Value

The benefit of using a real options approach can easily be seen since the option value must always be non-negative, thus eNPV >= NPV is always true. A project without any flexibility would have an option value of zero and would not require any real options analysis. However, the more flexibility a project has, the larger its option value. Thus, such phenomena demonstrate the increased relevance of using real options analysis.

Several different methods are used to analyze options and are candidates for use in the model created in this thesis. A common method of evaluating financial options is with the use of closed form solutions such as the Black-Scholes formula. The Black-Scholes model uses formulas to determine the value of European options on dividend-paying stocks, which can be applied to particular real estate development situations:

$C.0 = S_0 * e^{(-q*T)} * N(d_1) – X * e^{(-r*T)} * N(d_2)$

Where,

$d_1 = [\ \ln(S_0/X) + (r – y + \sigma^2/2)*T\ ] / \sigma *\backslash T$, and

$d_2 = [\ \ln(S_0/X) + (r – y - \sigma^2/2)*T\ ] / \sigma *\backslash T$

However, the Samuelson-McKean formula is a closed form solution method that was introduced by Geltner and Miller (2001) as the "Black-Scholes formula for real estate." This method treats the development decision as a perpetual call option since real estate is typically an option that can held indefinitely. Such a perpetual call option cannot be appropriately calculated using the Black-Scholes method. The Samuelson-McKean formula is expressed as:

$$C_0 = (V^* - K_0)\left(\frac{V_0}{V^*}\right)^{\eta}$$

Where

$V^* = K^*\eta/(\eta - 1)$

$\eta = \{y - r + \sigma^2 + [r - y - \sigma^2/2\,)^2]^{1/2}\} / \sigma^2$

and

V = Current Value of underlying asset

K = Strike Price (construction cost)

σ = Volatility of underlying asset

y = Dividend payout ratio (Cap rate)

r = Risk-free rate

η = Option Elasticity

V^* = Critical Value of underlying asset at and above which it is optimal to immediately exercise the option

This ready-made formula provides a consistent and concise method of evaluating real options and can be used with relatively simple land development projects to determine the approximate land value.

A more general method of evaluation utilizes partial differential equations to give an open-form equation for options analysis. Utilizing appropriate boundary conditions on the open-form partial differential equations results in a closed-form formula such as Black-Scholes formula. Open formed solutions could be used to derive other methods of Real Options analysis that may be appropriate for particular circumstances.

Using Binomial Trees for Real Options Analysis

Perhaps the most intuitive and popular method for real options analysis is the use of binomial trees. Binomial tree method is advantageous in that it is easily modified and provides an intuitive presentation and organization of the analysis. A binomial tree is set up similarly to a decision tree. Each branch represents an up or down movement of the market from the previous node. The value of each node can be calculated based on the downstream branches that occur after the node and probability of their occurrences. The up or down movement of the value at each subsequent node is determined according to their volatility. A certainty-equivalent model is used to analyze this binomial world. The equations used for such a binomial tree are:

$$V_0 = \frac{E_0[V_T]}{\left(1+r_f+E[RP_V]\right)^T} = \frac{CEQ[V_T]}{\left(1+r_f\right)^T}, \Rightarrow \quad CEQ[V_T] = \left(\frac{\left(1+r_f\right)}{\left(1+r_f+E[RP_V]\right)}\right)^T E_0[V_T]$$

$$p = \frac{\left(1+r_V\right)-d}{u-d} = \frac{\left(1+r_V\right)-1/\left(1+\sigma\sqrt{T/n}\right)}{\left(1+\sigma\sqrt{T/n}\right)-1/\left(1+\sigma\sqrt{T/n}\right)}$$

where

r_V = Expected total return rate on the underlying asset (built property).

- y_V = Payout rate (dividend yield or net rent yield).

- r_f = Riskfree interest rate

- σ = Annual volatility of underlying asset (instantaneous rate)

- V_t = Value of the underlying asset at time (period) t, ex dividend (i.e., net of current cash payout, i.e., the value of the asset itself based only on forward-looking cash flows beyond time t). The asset is assumed to pay out cash at a rate of yV every period:

  $E[CFt+1] = (1+rV)Vt(1 - 1/(1+yV)).$**

Then, working from the terminal nodes towards the starting nodes, we can calculate the

option payoffs:

$$C = \frac{E[C] - \left(C_{period+1"up"}\$ - C_{period+1"down"}\$\right)\frac{E[r_V] - r_f}{"u" - "d"}}{1 + r_f}$$

where:

$$E[C] = p * C_{period+1"up"}\$ + (1 - p) * C_{period+1"down"}\$$$

$C_{period+1"up"}$ represents the "up" scenario of the following period;

$C_{period+1"down"}$ represents the "down" scenario of the following period,

$u = 1 + \sigma\sqrt{T/n}$

$d = 1/u$

Another benefit of binomial trees is that it can be integrated into a decision tree

analysis often used in real estate development projects. For a more in-depth explanation of

the use of binomial-trees, refer to chapter 28 of Gelter and, Miller's *Commercial Real Estate*

Analysis and Investments.

Alternatively, real options can be evaluated using Monte Carlo simulation. However,

such an approach is much more difficult to model, less flexible, and less universally intuitive

method compared to a binomial tree approach.

Conceptual Considerations Underlying the Real Options Model

It should be noted that there are several drawbacks to real options analysis in the real estate market. To use real options in a practical manner, it is important to understand the underlying assumptions involved and ways in which real options analysis for real estate varies from those assumptions. One of the central assumptions of options theory is that a market is efficient and behaves in random walks. Such behaviors can be modeled with stochastic models such as Geometric Brownian Motion. However, it is widely known that real estate is relatively inefficient and cyclical. This can be dealt with in the binomial model in a straightforward manner. However, the current version of the spreadsheet does not take these factors into account.

The other major assumption involved in options analysis is the assumption that there are no arbitrage opportunities in the market. This assumptions hold pretty well for the very large financial markets. The option model is traditionally derived via an arbitrage argument. However, it can be derived just as well from classical certainty-equivalence present value discounting, which is based only on the equilibrium or normative concept that the ex ante risk premium per unit of risk faced by investors must be equal across the relevant asset markets (for the option, the underlying asset, and bonds).

The other difficulties lye in the errors in estimating the asset values, market conditions, and volatility of the market. One cannot simply look at the REIT market to determine this data, since the REIT market does not necessarily behave as the specific location and product type would.

Approach to Real World Development Project

Maximizing flexibility is the key to unlocking the option value of large-scale real estate development projects. Although purely precise values of projects cannot be fully calculated using the methods discussed here, it is the most rigorous type of analysis appropriate for large scale developments. In the case of many large scale development projects, Discounted Cash Flow procedures often grossly undervalue real estate development projects with prolonged construction periods and staged phasing. Real options analysis provides a rigorous framework with which to describe a developer's intuition, whether it be accurate or misguided.

Kang (2004) recommends the following procedure for valuing large-scale development projects:

1. *Identify all the risks related to the development project, and determine the major source of identified risks.*

2. *Perform a Discounted Cash Flow valuation incorporating the expected future cash flows and the risks identified, as if there is no flexibility in the project. A rigorous Discounted Cash Flow analysis is critical because it is used as a bass of the later analysis.*

3. *Research market data for quantifiable risks, such as the volatility of underlying asset returns. If there is no reliable data available, a best subjective judgment has to be made.*

4. *Determine the structure of the option as to the identified risks. It is critical to know which options are valuable since real world projects would involve numerous options.*

5. *Once market based risks are identified and necessary input data are assumed, a Real Options model can be used to value a project's flexibility. The binomial tree approach is recommended because it is the most intuitive options valuation model and is easily customizable.* [If a development project can stand alone independently, the Samelson-McKean formula will give precise results for a perpetual option.]

6. *For project specific risks, the Decision Tree Analysis can be used. In most cases, there would be few data available for this type of risks. Hence, a degree of subjective judgment has to be used. The Decision Tree can help developers to understand interrelationship between different kinds of flexibilities and uncertainties. When appropriate, DTA can be used in conjunction with Real Options. For instance, the payoffs in a Decision Tree can be calculated with the Real Options model by varying input variable.*

7. *Thorough sensitivity analysis must be performed. The single value estimate from the proposed model is not reliable enough to base critical decisions. Sensitivity analysis would provide developers a range of value, and more importantly it would clearly show the relationship between input variable and the value of the resulting flexibility.*

8. *Once the relationship between the input assumptions and the value of the flexibilities becomes clearer, developers should look for the opportunities to maximize the value by influencing the options structure within the contracts, through negotiation with other parties, etc. This opportunity is a unique*

advantage to the real world options as opposed to financial options and should be

taken advantage of.

This procedure provides a very thorough analysis for large-scale development projects and may be too time consuming to be used for smaller development projects. Instead, smaller projects may be more efficiently evaluated using the Samuelson-McKean equation or canonical formula.

Section II
Introducing the Financial Model

The financial model is set up using the format borrowed from the Excel workbook that has been used by the Development Studio. Students have found this workbook to be a relatively user-friendly and flexible format. Nonetheless, there are certain flaws and redundancies in the calculations that were addressed in the new model. Furthermore, some modifications were made to clarify some of the aspects of the workbook that had caused some confusion in the past.

This base workbook included much of the same inputs that still exist, with some necessary additions to facilitate the real options analysis. With some minor changes, the discounted cash flow calculations are similar to the original worksheets. Some additions were made to the worksheet to necessitate calculations based on David Geltner's "canonical" method and certain data that is used in the real options worksheets. The remaining worksheets, additions, revisions, documentation, and diagrams are a product of this thesis. Much of the binomial tree methodology is adapted from the New Songdo City case study from Professor Geltner's Fall Real Estate Finance course.

The workbook is now organized into 5 main categories of worksheets, which have tabs that are color coded as shown below. The 5 categories are Inputs (white), Outputs (yellow), Discounted Cash Flow Calculations (orange), Real Options Calculations (blue), and Miscellaneous (red). To help the user understand the inner-workings of the workbook, the calculations of the different worksheets are explained in the pages that follow.
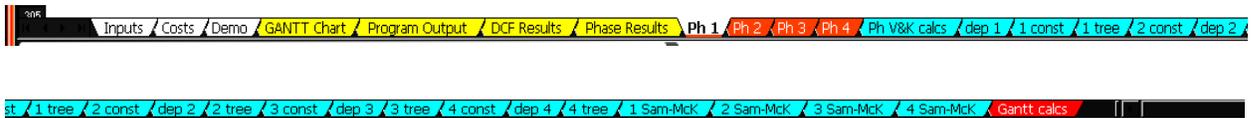
Inputs

Universal Assumptions

Universal Assumptions	
Period length (*T/n*) in yrs =	1/4
Expected Inflation (nominal annual) =	3.50%
Riskfree interest rate (rf) (nominal annual) =	5.50%
Volatility (sigma) =	15%

In this table, the user must input assumptions of critical variables that will be used throughout the workbook:

- Period length (T/n) in yrs – this input determines the length of the periods used to analyze the project. For example, quarterly analysis would require an input of ".25", or "1" for annual analysis.

- Expected Inflation and Riskfree interest rate (rf) – the user must input the expectations for these variable as nominal annual percentages.

- Volatility – a critical factor in real options analysis. Although it is difficult to rigorously determine this factor, studies estimate individual property annual volatility of approximately 15% for properties in the US.

Development Program

The Development Program input table is used to define the different uses within the proposed development project and the assumptions based upon market analysis for each use.

- The workbook allows for 11 different uses, 3 parking situations, costs for various additional expenses and demolition.

- For each of these uses, the size of each use is input in gross square feet.

- The efficiency of each use is input as percentages for most of the uses, except for hotels, which is input in gross square feet per room (gsf/room), and parking, which is input in square feet per space.

- Annual operating expense is input in annual dollars per net square foot for rental uses and as percentage of gross sales for for-sale uses and parking. The percent of the operating expenses that are fixed costs is input in the next column.

- Annual capital reserve funding is input as a percent of annual income (CI/V %), and 1% is set as a default value.

- The assumed percentage of stabilized occupancy is input denoting what percentage of the net square floor area is expected to be leased-up once the property is considered to be stabilized over time.

- For each use, the user must input the expected annual growth of market rents for each use.

- The annual absorption is input as a percent of net available square feet is expected to be absorbed annually, and this percent is used to calculate the annual absorption during the lease-up period following construction.

- Based on market analysis, the user must input the market cap rate, or yield for each use.

- The user should input an annual percentage for depreciation – usually 1-2% based on market and life-cycle analysis.

- A drop-down menu is used to input whether a use is speculative or build-to-suit. If a use is speculative, a spec. premium is added to the discount rate for that use.

- If a use is considered "speculative", a return premium can be input to describe the additional risk taken as a speculative development.

| Development Program	Asumptions																
Uses:	Gross SF	Efficiency	Net SF	Annual Gross Rent / Sales		Annual Op. Exp.		OPEX %Fixed	Annual Capital Reserve Funding CIV (%)	Stabilized Occupancy	Real Annual Rent Growth rate	Annual Absorption in % of NSF	Annual Absorption (sf)	Market Cap rate	Annual Depreciation (%)	Speculative or build-to-suit?	Spec premium
Office	0	0%	0	$0.00	per nsf	$0.00	per nsf	30%	1%	0%	0.0%	0%	0	0.0%	1.50%	speculative	1.50%
Lab/R&D	0	0%	0	$0.00	per nsf	$0.00	per nsf	30%	1%	0%	0.0%	50%	0	8.0%	1.50%	speculative	1.50%
Retail	75,000	90%	67,500	$60.00	per nsf	$0.00	per nsf	30%	1%	90%	2.0%	60%	40,500	8.0%	1.50%	speculative	3.50%
For Sale Hsg - Luxury	0	0%	0	$0.00	per sf sales	0.0%	of gross	30%	1%	0%	0.0%	0%	0	5.0%	0.00%	build-to-suit	0.00%
For Sale Hsg - Market	0	0%	0	$0.00	per sf sales	0.0%	of gross	30%	1%	0%	0.0%	100%	0	5.0%	0.00%	build-to-suit	0.00%
For Sale Hsg - Afford.	0	0%	0	$0.00	per sf sales	0.0%	of gross	30%	1%	0%	0.0%	0%	0	5.0%	0.00%	build-to-suit	0.00%
Theatre	60,000	100%	60,000	$30.00	per nsf	$0.00	per nsf	30%	1%	100%	2.0%	100%	60,000	10.0%	0.00%	build-to-suit	0.00%
Food Court Restaurants	25,000	30%	7,500	$160.00	per nsf	$0.00	per nsf	30%	1%	90%	2.0%	80%	6,000	8.0%	2.00%	speculative	2.00%
Themed Restaurants	50,000	90%	45,000	$50.00	per nsf	$0.00	per nsf	30%	1%	100%	2.0%	80%	36,000	8.0%	2.00%	speculative	2.00%
MIT Museum	15,000	90%	13,500	$21.00	per nsf	$0.00	per nsf	30%	1%	100%	2.0%	100%	13,500	6.5%	0.00%	build-to-suit	0.00%
Themed Retail	30,000	90%	27,000	$33.00	per nsf	$0.00	per nsf	30%	1%	95%	2.0%	85%	22,950	7.5%	0.50%	speculative	0.50%
Parking -1	30,000	350	86	$3,300.00	per space	25%	of gross	100%	1%	95%	4.0%	70%	60	8.0%	0.50%	speculative	0.50%
Parking -2	60,000	350	171	$3,300.00	per space	25%	of gross	100%	1%	95%	4.0%	70%	120	8.0%	0.50%	speculative	0.50%
Surface Parking	0	300	0	$0.00	per space	0%	of gross	0%	1%	0%	0.0%		0	0.0%		speculative	
Driveways	1,500	100%	1,500	$0.00		0%		0.00001		0%	0%	100%	0%				0
Peripheral/Buffer	12,000	100%	12,000	$0.00		0%		0.00001		0%	0%	0%	0%	0		0	0.00001
Public Open Space	15,000	100%	15,000	$0.00		0%		0.00001		0%	0%	0%	0%	0		0	0.00001
Semi-Public & Private	-	100%	-	$2.00		2%		100		100%	0%	0%	0%	0		0	0
Demolition	130,725	100%	130,725														

Construction Phases

For each use in the development program, the user must allocate the use to a particular phase of the development, estimate the construction time for each phase, and determine the type of construction for each use:

- For each use, the user must allocate the use into the appropriate phases. The percentage of the total square footage of each use must be allocated to the appropriate phases corresponding to the intended mix of uses in the phases, totaling 100%.

- For each use for each phase, a construction type must be selected from the choices in the drop-down menus. The user must also select what type of parking construction will be used for each type of parking.

- At the top of the Construction Phases table, the user must input the period after the beginning and end of construction. The starting period must be at least "1" for the soft cost to be calculated correctly. The real options calculations of this workbook permit a maximum construction period of 10 periods. This limitation will become apparent when we discuss the construction delay trees.

Construction Phases									
Phase 1		Phase 2		Phase 3		Phase 4		Total
Starting	Ending	Starting	Ending	Starting	Ending	Starting	Ending	Total
1	6	5	10	13	19	19	25	
% Built	Construction	% Built	Construction	% Built	Construction	% Built	Construction	
0%	1-4 s/c	0%	1-4 s/c	0%	1-4 s/c	0%	1-4 s/c	0%
50%	4-8 s/c	0%	4-8 s/c	25%	1-4 s/c	25%	1-4 s/c	100%
25%	1-4 s/c	25%	1-4 s/c	0%	1-4 s/c	50%	1-4 s/c	100%
75%	8+ s/c	25%	8+ s/c	0%	4-8 s/c	0%	4-8 s/c	100%
75%	Rehab	25%	Rehab	0%	4-8 s/c	0%	4-8 s/c	100%
75%	4-8 s/c	25%	4-8 s/c	0%	4-8 s/c	0%	4-8 s/c	100%
100%	Cineplex	0%	Cineplex	0%	Cineplex	0%	Cineplex	100%
25%	1-4 s/c	25%	1-4 s/c	25%	8+ s/c	25%	8+ s/c	100%
25%	1-4 s/c	25%	1-4 s/c	25%	4-8 s/c	25%	4-8 s/c	100%
75%	1-4 s/c	25%	4-8 s/c	0%	4-8 s/c	0%	4-8 s/c	100%
34%	1-4 s/c	33%	1-4 s/c	33%	1 s	0%	1 s	100%
100%	Struc. BG 1	0%	Struc. BG 1	0%	Struc. AG	0%	Struc. AG	100%
100%	Struc. AG	0%	Struc. BG 1	0%	Struc. BG 1	0%	Struc. BG 1	100%
0%	AG Cobble.	0%	AG Brick	0%	AG Brick	0%	Struc. BG 2	0%
100%		0%						100%
100%		0%						100%
100%	$435,000.00	0%						100%
100%		0%						100%
100%		0%						100%

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The GANTT Chart output as seen below can be used to visualize and verify the construction phasing input in this table.
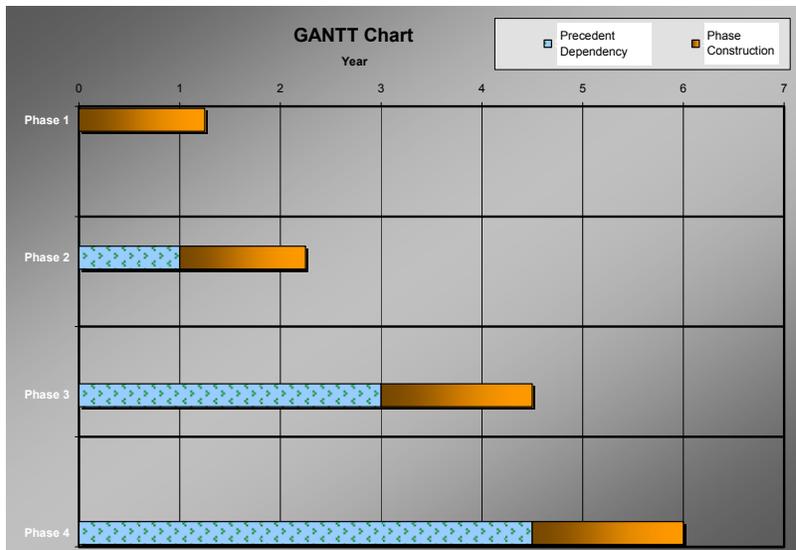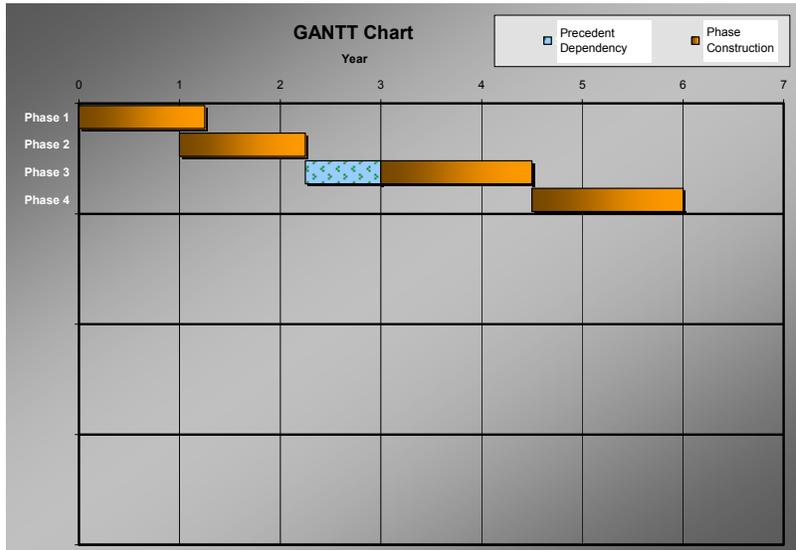
Phase Interaction Input Table

Phase Interaction Input Table	

Phase	Dependent on Phase:
1	0
2	1
3	2
4	3

This table is used to describe the interaction among different phases, which will determine the behavior of the real options analysis. The value of this table will become more apparent once we explore the real options calculations and calculations of compound options. In this table, the user describes the interaction between phases by inputting the number of the phase each phase is dependent upon. If a phase is independent, "0" is input. There must be at least one independent phase. For example, if each phase is dependent on the previously numbered phase, the table should be input as in the table above. Thus, Phase 4 depends upon the completion of Phase 3, which depends on the completion of Phase 2, which is dependent upon completion of Phase 1, which is independent. Any combination of dependencies is valid as long is there are no circular dependencies. A circular dependency would occur if a phase, Phase A, is dependent on a Phase B, however Phase B is dependent on Phase A. This type of input will cause faulty results for the real options analysis. It is also possible for all the phases to be independent, which, by the way, should yield the highest option value for the land. In this case a "0" is input for each phase.

Phase Interaction
GANTT Chart Preview

Series2   Series3

Phase 1
Phase 2
Phase 3
Phase 4
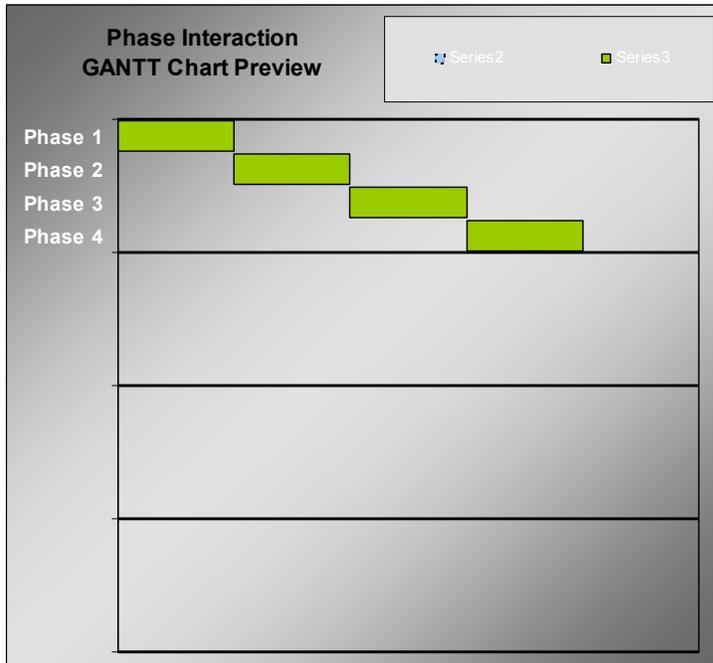
As a graphical aid, a Phase Interaction Diagram displays the dependencies of the phases. This diagram should be used to double check the inputs. Phases should align with it's dependent phases in the diagrams. Each chain of dependency will align together following the appropriate independent phase in the chain. Chains following Phase 1 as the independent phase will align in the top box. Chains following Phase 2 in the second box, Phase 3 chains in the third and Phase 4 chains in the bottom box. In the diagram above, the chain begins with Phase 1 as the independent phase, Phase 2 dependent upon Phase 1, followed by Phase 3 and Phase 4 respectively.

Construction Costs

The Cost worksheet contains all of the inputs pertaining to development and construction costs for the various uses and building types. The user must input the expected costs for site improvements, parking, building construction, tenant fit up, soft costs and construction financing according to the current market costs.

- Site Costs – costs for common site features such as driveways, landscaping, public open space, private open space, and demolition are to be input.

- Parking Construction Costs (per car):

    o Struc. BG 1 – Parking garage one level below ground

    o Struc. BG 2 – Parking garage two levels below ground

    o Struc. AG – Above ground parking garage

    o AG Bitum. – At grade, surface parking with bituminous (asphalt) pavement

    o AG Conc. – At grade, surface parking with concrete pavement

    o AG Cobble – At grade, surface parking with cobble stone pavement

    o AG Brick – At grade, surface parking with brick pavement

- Building Construction Costs are input for the different types of construction (per gross square foot). These are just shell costs for new construct, which includes the base building, elevators, water, sewer, and fire protection. Commonly used types of construction are already provided, however additional construction types can be added or changed by the user if necessary:

    o 1 s – single story steel-framed construction (commonly big box or industrial)

    o 1-4 w/s – 1-4 story wood or metal stud framed construction

    o 1-4 s/c – 1-4 story steel and/or concrete construction

    o 4-8 s/c – 4-8 story steel and/or concrete construction

- 8+ s/c – 8 or more stories of steel and/or concrete construction

- Rehab – Rehabilitation or adaptive-reuse of existing structures

- Cineplex – Construction of new Cineplex-style movie theatres

• Tenant Fit Up (per net square foot) – for each use, the user must provide the assumed fit-up costs anticipated before a tenant can occupy the space. This includes mechanical and electrical costs, as well as any other tenant improvements the current market conditions dictate.

• Development Soft Costs – the user must input what the relative costs are of various soft costs the developer will incur throughout the development process such as architectural, engineering, and legal fees, leasing fees, sales commissions, mitigation fees, and overhead.

• Construction Financing – at this point these inputs are not active in the worksheet. They are here in the case the user wants to incorporate these costs in a more thorough analysis.

Demolition Costs

The Demo worksheet tallies the building areas on the sites that are to be demolished. The current setup of this worksheet is to accommodate one particular development site. Most likely, the user may need to make adjustments to this worksheet to work with the specific development site being analyzed.

Cash Flow Projections

Each phase in the development has its own worksheet for discounted cash flow calculations. The tabs of these worksheets are orange. For the discounted cash flow calculations, the uses remained segregated to show the relative values of each use. However,

the canonical calculations prohibit the segregation of uses and use a weighted average of the

uses. Cash flows are projected over thirty periods.

Data from Inputs	

Construction Projections	
+	
Tenant Fit up	
+	
Soft Costs	
=	
Total Dev. Costs	

Absorption	
x Rent	
x % Occupancy	
= EGI	
-	
Operating Expenses	
=	
NOI	
-	
Leasing Costs As % of NOI	
-	
Capital Reserve As % of NOI	
=	
PBTCF	
+	
Rerversion (NOI/cap rate)	
=	
Total PBTCF	

- The assumed parameters that come from the inputs and are necessary for this worksheet are in the top-left corner of the worksheet.

- Construction – for each use, the total construction cost (hard costs) is calculated. The total construction cost is spread over the entire construction period for the phase, and magnified according to the expected inflation. **It should be noted that this is a simplification and if more precise timing of construction costs can be projected, these projections should be used.**

- Tenant Fit Up – total fit up costs are calculated and assumed to be paid out during the last period of construction.

- Development Soft Costs – for each type of fee and overhead, total soft cost is calculated for the entire phase based upon the total phase construction costs, multiplied by the fractions that are input in the Costs worksheets. These fees are assumed to be paid out in the phase prior to construction. Construction period property taxes are allocated to each period of construction.

- Total Development Costs – the total construction costs are calculated for each period and allocated to the different uses according to each use's pro-rata share of construction cost. Again, this may be a simplification requiring the user to make more thorough projections for each use.

- Absorption – following the completion of construction, each use begins its lease-up phase. Absorption is calculated for each use on a square foot per period basis. Each use begins with one period of absorption following construction and absorbs another increment leasable space until the total square footage is achieved. Fully absorbed levels continue through all periods afterward. The total number of periods required to fully absorb the space is tabulated also.

- Expected Gross Income – in each period, the absorbed square footage for each use is multiplied by the rent per period and projected percentage of stabilized occupancy and magnified according to the projected rent growth.

- Operating expenses – to project operating expenses, potential total rent and respective percentage of fixed versus variable expenses are tabulated for each use and projected throughout the life of the asset, taking into account the project vacancy in the space.

- Net Operating Income (NOI) – to calculate net operating income, the worksheet simply subtracts operating expenses from expected gross income.

- Leasing Costs – leasing and sales commission inputs are obtained from the Costs worksheet as a percentage of annual rent roll for retail, office, and lab or percentage of gross rent or sales for residential uses. These percentages are annualized based on the assumption that retail, office, and lab leases are for five years and residential leases for 1 year. These percentages are multiplied by NOI for each projected year.

- Capital Reserve Funding – capital reserve funding, as a percentage of NOI of each use is acquired from the Development Program and multiplied by NOI for each projected year

- Property-level, Before Tax Cash Flow – operating expenses, leasing costs, and capital reserve funding are subtracted from Expected Gross Income for each use and year.

- Reversion – In the top left corner of the worksheet, a cell calculates the period in which all uses are stabilized. In this period, a reversion value is estimated by dividing the NOI from the following year by a periodic cap rate obtained from the Development Program and adjusted according to the length of periods in the workbook.

- Total PBTCF – the reversion estimates and PBTCF leading up to that reversion period for each use are combined.

- IRR Calculation – PBTCF, development costs, land acquisition costs are totaled up for each period. An internal rate of return is calculated and annualized for the phase.

The discounted cash flow method we have used is a rigorous and accurate way of finding the value of a project. However, does it accurately account for the risk of a

development project? Since the discounted cash flow methods discount the construction costs and cash flows separately according to their respected OCCs, phases that are delayed or have long construction periods are burdened greatly. The construction costs are discounted at a lower OCC over a long period, while cash flows are discounted with much higher rates over the same period. Using this discounted cash flow method, we derive the pre-phase and construction period OCC as an output, but not as an input.   Often, as a result of this difference in discounting, the NPV of a project could end up being very small, or even negative.

Canonical Calculations

The idea in the next approach is to remove the worst affect of the failure to account for flexibility by discounting each development phase as a net value element. Thus, the OCC of each simple phase is derived, and this OCC is used to discount the projected net value of that phase back to the overall beginning point in time for the program. As an analogy, the discounted cash flow method brings costs and benefits separated. This canonical method "bundles" the costs and benefits together as a "package" at the projected completion year. This "package" is can be discounted by a newly calculated OCC.

This worksheet analyzes the cash flows and costs in a way that enables the user to utilize the canonical analysis to evaluate the OCC of the pre-stabilized development project. The parameters necessary to do these calculations are compiled and fed into the canonical equation to solve for the development phase as follows:

OCC of Development Phase

$$\frac{V_T - K_T}{(1 + E[r_C])^T} = \frac{V_T}{(1 + E[r_V])^T} - \frac{K_T}{(1 + E[r_D])^T}$$

$$E[r_C] = \left[\frac{(V_T - K_T)(1 + E[r_V])^T (1 + E[r_D])^T}{(1 + E[r_D])^T V_T - (1 + E[r_V])^T K_T} \right]^{(1/T)} - 1$$

V_T	36,019,946
K_T	12,785,547
$E[r_V]$	2.25%
$E[r_D]$	1.27%
T	6
$E[r_C]$ =	2.82%
$E[r_C]$ (annual) =	11.77%

At Time 0:
NPV $11,919,015.25
NPV from previous $ 5,490,875.89

At Start of Construction:
NPV $19,663,402.56

- Canonical V_T, value at time T - property cash flows are discounted by the stabilized OCC for each use to the final period of construction (time T) and summed for all uses.

- Canonical K_T, costs at time T – construction and development costs are discounted forward by the construction OCC to the same final period of construction (time T) as property cash flows and summed for each use.

- Using V_T, K_T, weighted average OCC for all uses' costs and cash flows $E[r_V]$ and $E[r_D]$, and the number of periods of construction (T) are fed into the canonical equation as shown in the above example from the workbook.

- This gives us an OCC for the development project $E[r_C]$, which can be converted into an annual rate.

- To calculate the present value of the project at time 0 and at the beginning of the project, the difference in V_T and K_T is discounted to the appropriate time using the $E[r_C]$ that was just calculated.

In many cases, this canonical approach will give a significantly different result than the previously discussed discounted cash flow methodology. In projects with long construction periods and/or delayed construction starts, discrepancies between the two approaches become magnified.

It should be noted that this method, like the previous method, makes the assumption that the developer is committed to the development at the initiation of investment. The difference with the previous method is here we allow (require) each phase to be treated as a development project with benefits and costs inseparably connected such that only the net difference between the two is tradable. As we will see with the real options analysis, this assumption of commitment does not show us the value of our flexibility to abandon a phase if we chose.

Real Options Calculations

Don't let the large number of worksheets full of complicated binomial trees scare you away. There's a lot going on there, but if you take the time to understand it in bite-size pieces you will come to appreciate the power of real options analysis. One worksheet synthesizes information from the cash flow projections for use in the real options calculations, as well as two main types of worksheets performing the fundamental real options calculations for each phase, and two types of supporting worksheets for each phase. As we will see, the workbook will use results from the worksheet that analyzes the option value based on the Samuelson-McKean formula if a phase does not have any other phases dependent on it (typically the last phase in a development or a phase which is completely independent from other phases). If a phase does not meet the criteria for Samuelson-McKean analysis, the workbook will use the option values calculated in the worksheets that use the binomial tree method. Other worksheets perform ancillary calculations necessary to properly evaluate the compound nature of the options. Let us explore these worksheets in more depth.

Setting Up the Real Options Worksheets – Implied V & K Values

This worksheet manipulates the costs and cash flows for all phases to feed necessary data into the real options worksheets for each individual phase. Here, the worksheet will calculate critical time 0, "implied" V and K values into the real options worksheet. We will soon see why it is necessary to use "implied" values as opposed to actual values, and why we call them "implied". The phase calculations are aligned vertically below the universal assumptions.

- At the top left corner of the worksheet are the assumptions based on inputs which are universal for all phases.

- Various inputs from the cash flow worksheets are tabulated to the left of the V and K calculations for each phase. In these tables, the critical values obtained from the cash flow worksheets are the beginning and ending phases of construction, annual and periodic V growth rates, K discount rates and V discount rates.

- To the right of these tables are calculations to determinate the "implied" V and K values. Total PBTCF and Development Costs are referenced from the cash flow worksheet for each phase.

- In the row labeled "K val @ commitment date", the development costs are discounted using the K discount rate for the phase to the commitment date of the phase, or the period prior to the "start at" period.

- In the row labeled "V val @ commitment date", the property-level cash flows are discounted using the V discount rate for the phase to the commitment date of the phase, or the period prior to the "start at" period. This V discount rate is a weighted average cost of capital (WACC) for all of the uses in the phase.

- Now that we have the V and K values at the commitment date of the, phase we discount each of these values to time 0 using V and K's respective **growth** rates. **This**

value we call the "implied" V and K values at time zero, since they are not actual present values in an investment sense, but V and K values as if the asset exists in time 0.

It is these Implied V and K Values at time zero that we need to begin our real options analysis in the other worksheet.

Binomial Tree Analysis

        The first type of worksheet performing fundamental real options calculations is the binomial tree worksheets. These sheets are named "1 tree", "2 tree", "2 tree", and "4 tree" representing the phase that the binomial tree calculations are analyzing. In these sheets we utilize the Certainty Equivalence formulas to populate the binomial trees that help us calculate the option value of the phase.

Phase Assumptions		Resulting (output):	
Period length (*T/n*) in yrs =	1/4		
Riskfree interest rate (rf) =	5.50%	rf/period=	1.38%
Underl Asset Total Return (rV) =	9.96%	rV/period =	2.49%
Underl Asset Cash yield (yV) =	7.66%	yV/period=	1.92%
K growth rate (gK) =	3.50%	K gro/period=	0.88%
V growth rate (gV) =	2.27%	gV/period=	0.57%
Volatility (sigma) =	15%	sig/period=	7.50%
V(initial) =	48,521,164	yK/period=	0.50%
K(initial) =	41,181,705	"p" real prob=	0.6535
Option Price (land+fee+convctr, fixed in adv)	0	"u" =	1.0750
		"d" =	0.9302
*Note: All input rates nominal annual rates.		**Option Val=**	**7,339,459**

- At the top left corner of the worksheet are the assumptions which are applicable for each particular phase worksheet. On the left side of the table are the annual assumptions that are referenced from other worksheets. To the right of these assumptions are the parameters in terms of the period length we are using throughout the workbook.

    o Period length, Riskfree interest rate, and volatility are obtained from the universal assumptions

    o The Underlying Asset Total Return (r_V), Cash Yield (y_V), and V growth rate (g_V) are based on a weighted average from the phase's cash flow worksheet

    o V (initial) and K (initial) are obtained fro the Phase V&K calculations worksheet and are the "implied" time 0 values calculated by the worksheet.

- o Since we are using the Certainty Equivalence methodology we need to calculate "p", which is the real probability of an "up" scenario. (Thus, the probability of the "down" scenario is 1-p.) This is based on the equation:

$$p = \frac{(1+r_V)-d}{u-d} = \frac{(1+r_V)-1/(1+\sigma\sqrt{T/n})}{(1+\sigma\sqrt{T/n})-1/(1+\sigma\sqrt{T/n})}$$

- o "u" and "d" represent value factors for the up and down scenarios that would occur after one period. The value of the up factor will be above 1 and the down factor will be less than 1:

$$u = 1+\sigma\sqrt{T/n} \qquad \text{and } d = 1/u$$

- o The final result of the worksheet, the option value is found in the lower portion of this box and is highlighted yellow.

- Below the Assumptions Table is the Value tree. The first cell, which represents time 0,in the tree references the initial V we calculated in the Initial V & K worksheet. The cell directly to the right calculates an "up" value based on the volatility assumption applied to the time 0 value. Each cell in the top row of the tree the same calculation is applied to the subsequent period progressing to the right. One cell to the right and down from the initial value calculates a "down" value based on the volatility assumption applied to the time 0 value. Each cell in the tree performs this "down" calculation on the value of the cell diagonal (above and to the left) to it. These same calculations progress throughout the tree to period 40.

Period ("j"):	0	1	2	3
Expected Values of V:		48,791,204	49,062,748	49,335,803
"down" moves ("i"):	V tree (net of payout, "ex dividend" values):			
0	48,521,164	51,179,665	53,983,827	56,941,630
1		44,287,433	46,713,966	49,273,449
2			40,423,118	42,637,923
3				36,895,985

- To the right of the V Tree are two trees used to further demonstrate the results of the V Tree calculations. The Real Probabilities Tree tabulates the probabilities of each value in the V Tree occurring. We expect this to resemble a normal distribution. The probabilities are multiplied by the values in the V Tree to populate the Expected

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Values tree. The sum of each column in the Expected Values tree will give us the value of the development projected in each period, which will equal the expected value of the development with the expected growth rate.

- Below the V Tree is the K Tree. The K tree is very similar to the V tree but since we do not consider the volatility of construction costs, the values do not go up or down. Instead, from period to period, there is steady growth in expected construction costs.

Period ("j"):	0	1	2	3
"down" moves ("i"): *K Value Tree:*				
0	41,181,705	41,542,045	41,905,538	42,272,212
1		41,542,045	41,905,538	42,272,212
2			41,905,538	42,272,212
3				42,272,212

- Below the V and K trees is the Phase Option Value tree, where the actual real options calculations are taking place. In each cell of this tree the formula calculates the maximum value of either executing the phase (beginning construction) or the option value of waiting another period.

Period ("j"):	0	1	2	3
"down" moves ("i"): *Phase Option Value:*				
0	7,339,459	9,637,620	12,078,289	14,669,418
1		3,502,590	4,955,184	7,001,238
2			1,638,920	2,330,808
3				750,275

  o The value of executing the phase is a relatively straightforward calculation. Here the cell subtracts the construction cost from the corresponding cell in the K tree and the land price from the assumptions from the expected value from the corresponding cell in the V tree. The value of dependent phases on a d is added to this amount as well. This dependency value is obtained from a corresponding cell in the dependency tree, which is in the dependency workbook for the phase. We will explore this work sheet in coming pages.

  o The value of the option is calculated using the equations for calculating a European Option, expiring in one period. The value of the option is computed using the certainty equivalence formula:

$$C = \cfrac{E[C] - \left(C_{period+1"up"}\$ - C_{period+1"down"}\$\right)\cfrac{E[r_V] - r_f}{"u"-"d"}}{1 + r_f}$$

where:

$$E[C] = p * C_{period+1"up"}\$ + (1-p) * C_{period+1"down"}\$$$

$C_{period+1"up"}$ is the cell to the right, representing the "up" scenario of the following period;

$C_{period+1"down"}$ is the cell to the right and down, representing the "down" scenario of the following period, and;

"u" and "d" are calculated in the assumptions table.

- Below the Option value tree are two trees that serve to describe the results of the options calculation. The Phase Optimal Exercise tree summarizes the results of the option calculations, telling us whether the cells in the option value tree are executing the option or giving the value of the option to wait another period. The second tree tells us the opportunity cost of capital for waiting one period.

Period ("j"):	0	1	2	3	4	5	6	7	8
"down" moves ("i"): *Phase Optimal exercise:*									
0	exer	exer	exer	exer	exer	exer	exer	exer	exer
1		exer	exer	exer	exer	exer	exer	exer	exer
2			exer	exer	exer	exer	exer	exer	exer
3				hold	exer	exer	exer	exer	exer
4					hold	hold	hold	exer	exer
5						hold	hold	hold	hold
6							hold	hold	hold
7								hold	hold
8									hold

Period ("j"):	0	1	2	3	4	5	6
"down" moves ("i"): *1-Period Option Opportunity Cost of Capital:*							
0	4.77%	4.49%	4.26%	4.07%	3.91%	3.77%	3.66%
1		5.66%	5.18%	4.82%	4.53%	4.29%	4.10%
2			7.32%	6.42%	5.75%	5.26%	4.88%
3				8.56%	8.14%	7.46%	6.57%
4					8.94%	8.82%	8.70%
5						9.47%	9.33%
6							10.15%

Binomial Tree Worksheet

Assumptions

Expected Values

V Initial

V tree

K Initial

K tree

Real Probabilities

Expected Values Tree

Option Value

Phase Option Values

Construction Delay Worksheet

To precedent phase

From dependent phase(s)

Dependency Worksheet

Phase Optimal Exercise?

1 period OCC

Samuelson-McKean Analysis

The second type of worksheet performing fundamental real options calculations is the Samuelson-McKean worksheets. These sheets are named "1 Sam-McK", "2 Sam-McK", "2 Sam-McK", and "4 Sam-McK" representing the phase that the Samuelson-McKean calculations are analyzing. In these worksheets we utilize the Samuelson-McKean formula for valuing perpetual options. Many of the calculations in this worksheet are similar or identical to those of the binomial tree worksheets.

Phase Assumptions		Resulting (output):	
Period length (*T/n*) in yrs =	1/4	η =	6.33
Riskfree interest rate (rf) =	5.50%	rf/period=	1.38%
Underl Asset Total Return (rV) =	9.96%	rV/period =	2.49%
Underl Asset Cash yield (yV) =	7.66%	yV/period=	1.92%
K growth rate (gK) =	3.50%	K gro/per=	0.88%
V growth rate (gV) =	2.27%	gV/period=	0.57%
Volatility (sigma) =	15%	sig/period=	7.50%
V(initial) =	48,521,164	yK/period=	0.50%
K(initial) =	41,181,705	"p" real prob=	0.6535
Option Price (land+fee+convctr, fixed in adv)	0	"u" =	1.0750
		"d" =	0.9302
*Note: All input rates nominal annual rates.		Option Val=	7,347,615

- The assumptions table in the Samuelson-McKean worksheets are similar to the assumptions of the binomial tree worksheets, with the exception of the calculation of η (eta), which is the option elasticity calculated by the equation:

$$\eta = \{y_V - y_K + \sigma_V^2/2 + [(y_K - y_V - \sigma_V^2/2)^2 + 2y_K\sigma_V^2]^{1/2}\}/\sigma_V^2$$

- The V, K, Real Probability, and Expected Value trees are identical to those of the binomial tree worksheets.

- One tree that is not found in the binomial tree worksheet is the Hurdle Value Tree. Here, we calculate a critical value necessary to calculate the option value using the Samuelson-McKean method, applying the following formula to the K values:

$$V^* = K^*\eta/(\eta - 1)$$

- Similar to the binomial-tree worksheet, below the Hurdle Value Tree Phase Option Value tree, where the actual real options calculations are taking place. In each cell of this tree the formula calculates the value of either executing the phase (beginning construction) or the option value of waiting **indefinitely**.

  - The value of executing the phase is a relatively straightforward calculation. Here the cell subtracts the construction cost from the corresponding cell in the K tree and the land price from the assumptions from the expected value from the corresponding cell in the V tree. The value of dependent phases on a d is added to this amount as well. This dependency value is obtained from a corresponding cell in the dependency tree, which is in the dependency workbook for the phase. We will explore the dependency worksheets in coming pages.

  - The value of the option is calculated using the equations for calculating a Perpetual Option. The value of the option is computed using the Samuelson-McKean formula:

  $$C_0 = (V^* - K_0)\left(\frac{V_0}{V^*}\right)^{\eta}$$

    where V* is the hurdle value obtained from the Hurdle Value Tree, V_0 is the value from the V Tree, and K_0 is obtained from the K Tree from the corresponding cells within the trees.

  You may be wondering why we need to populate a whole tree, if the Samuelson-McKean formula allows us to calculate an option value that is not calculated based upon the values from the next period. Yes, to calculate the value of the phase independently, we would only need to apply the Samuelson-McKean at time 0. However, as we will see, if the phase is dependent on a previous phase, we need to have this tree populated to feed the preceding phase.

Assumptions

Samuelson-McKean Worksheet

Expected Values

V Initial

V tree

Real Probabilities

Expected Values Tree

K Initial

K tree

Hurdle Values

Construction Delay Worksheet

Option Value

Phase Option Values

To precedent phase

Phase Optimal Exercise?

1 period OCC
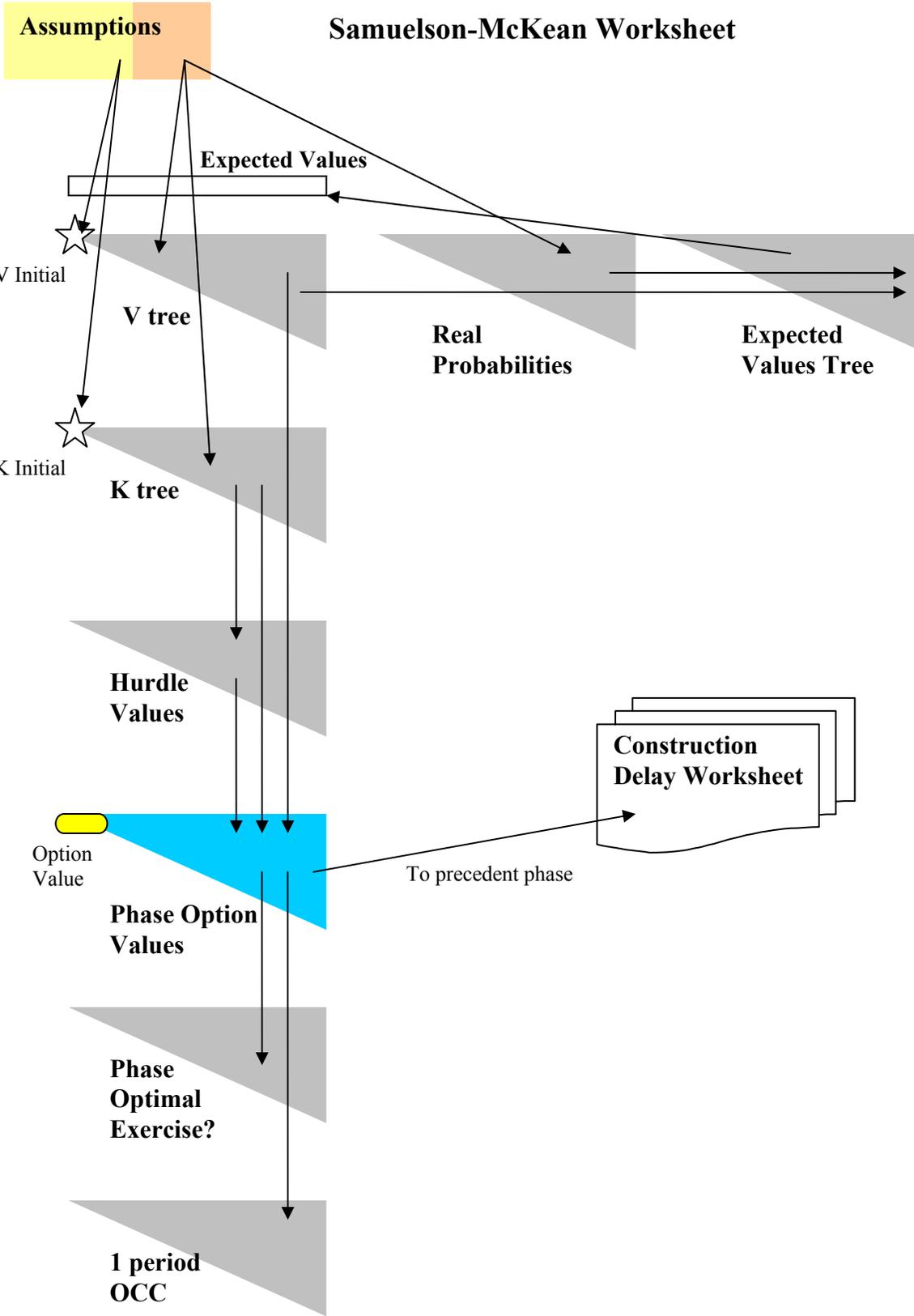
Construction Delay Worksheet

The Construction Delay Worksheet does not perform any options calculations directly, per se. However, the worksheet performs a critical task necessary for proper interaction between dependent and precedent phases. These sheets are named "1 const", "2 const", "2 const", and "4 const" representing the phase that the construction delay calculations are analyzing. If the particular phase is dependent upon the completion of a precedent phase, it is necessary to appropriately feed the values from one phase's Option Value Trees into the precedent phase's Option Value Trees. To properly achieve this, we must calculate a value tree that reflects the options values as if they were delayed by the length of the precedent phase's construction. The worksheet repeatedly calculates delays of one period at a time, up to 10 periods. (Now we know - this is the reason the input length of construction for a phase must be no more than 10) The delayed tree that corresponds to the length of the precedent phase's construction is then ready to be referenced by the precedent phase.

This worksheet has the added function of acting as a "switch" that determines whether the binomial results or Samuelson-McKean are appropriate for this phase.

Binomial Tree Worksheet

Samuelson-McKean Worksheet

Phase Dependency Worksheet

Assumptions
Dep. Const. Period = **n**

Construction Delay Worksheet

Check Independency

To precedent phase

Phase Option Value

Resulting Delayed Tree

1 Period Delay

$\neq$ **n**

0

2 Periods Delay

$\neq$ **n**

0

n Periods Delay

= **n**

10 Periods Delay

$\neq$ **n**

0
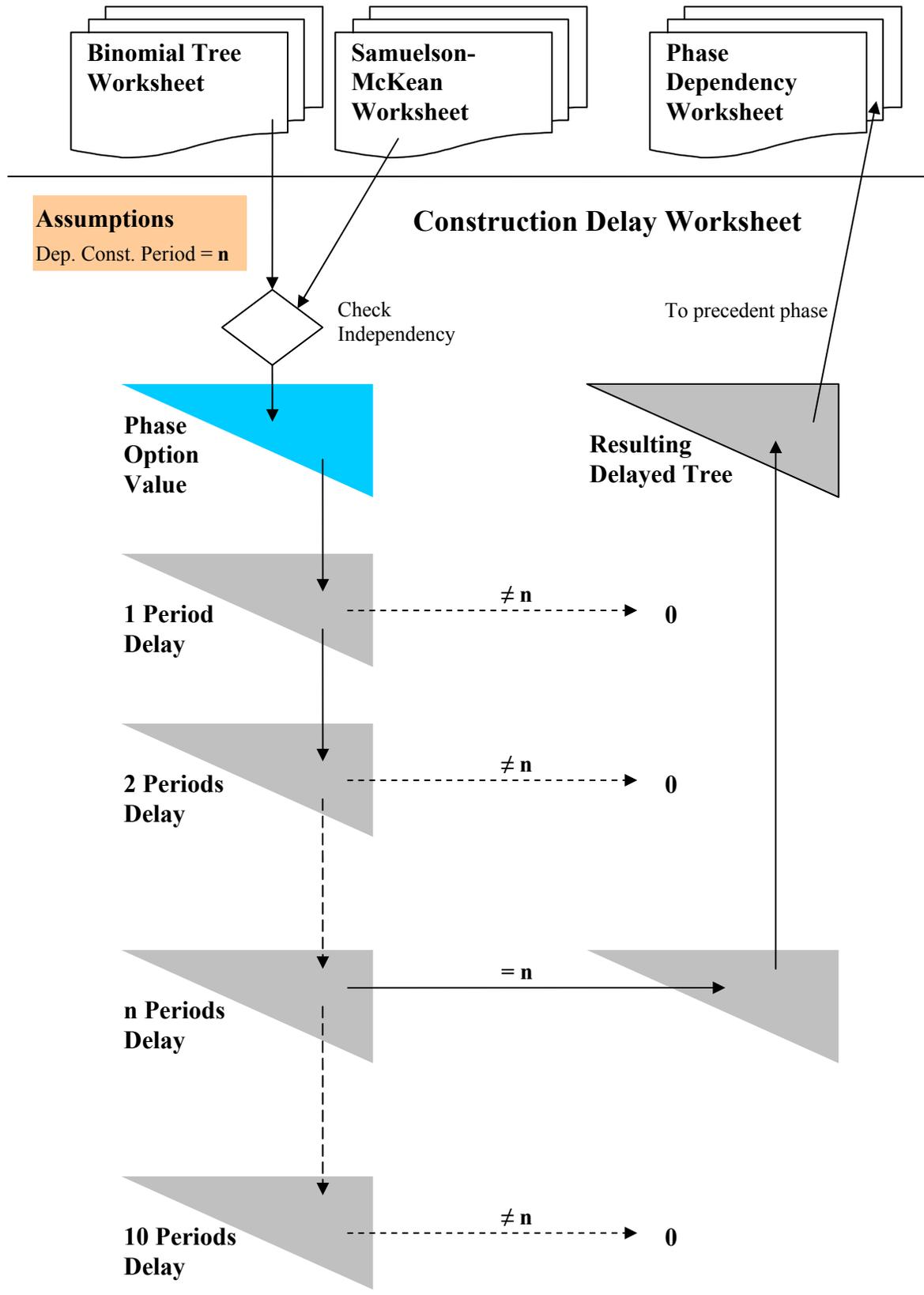
Phase Dependency Worksheet

The Phase Dependency Worksheets tally the value trees of all the phases that are dependent upon a phase's completion to be fed into the precedent phase's binomial Option Value Tree. If a phase is dependent on a precedent phase, the delayed value tree of the dependent phase is obtained from the corresponding Construction Delay Worksheet. The sum of all the dependent trees is calculated for reference into the phase's Option Value Tree in the Binomial Tree Worksheet.

Phase Dependency Worksheet

Construction Delay Worksheets

From Phase A

+

From Phase B

+

From Phase C

=

Total

Check Dependency:
If not dependent,
tree = 0

Binomial Tree Worksheet
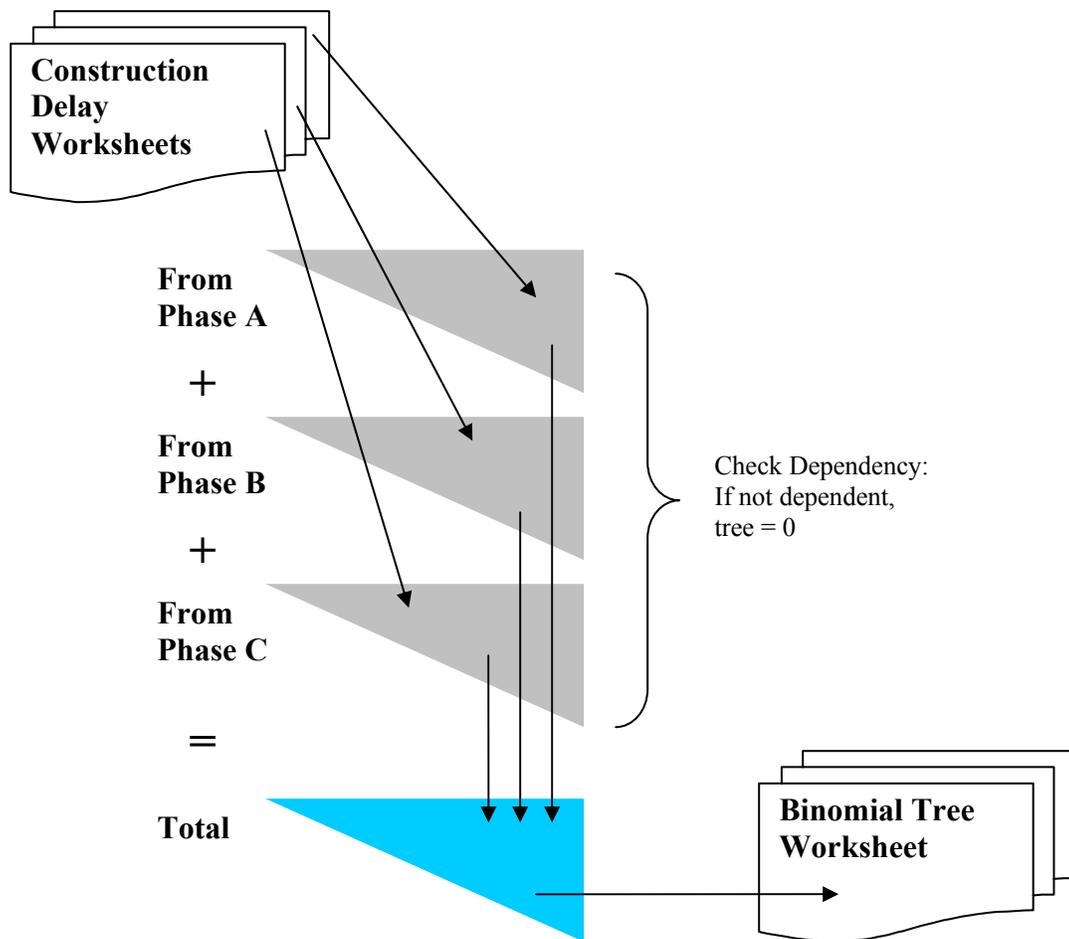
Outputs

Phase Results

The overall results of the workbook are summarized in this worksheet, which includes tabular and graphical representation of the present values of the developable projects. The table gives the present value of each phase for the three types of analysis performed by this workbook; Discounted Cash Flow, the Canonical Method, and Real Options Calculations. Accompanying this table is a column graph showing this comparison. It is very interesting and instructive to observe the relationship among the results of the different methods. Just as interesting, is the variation of the value of the real options analysis as the dependency varies, showing that there is certain value in the flexibility of phased development projects. This first example shows the results of a development in which all the phases are compounded, where each phase cannot be started until the preceding phase is completed:

Incremental Phase Value at Time 0			
Phase	Discounted Cash Flow	Canonical	Real Options
1	4,509,449	6,480,079	35,494,974
2	9,991,692	12,000,506	0
3	2,687,451	4,594,847	0
4	5,490,876	11,919,015	0
Total	22,679,468	34,994,447	35,494,974

51
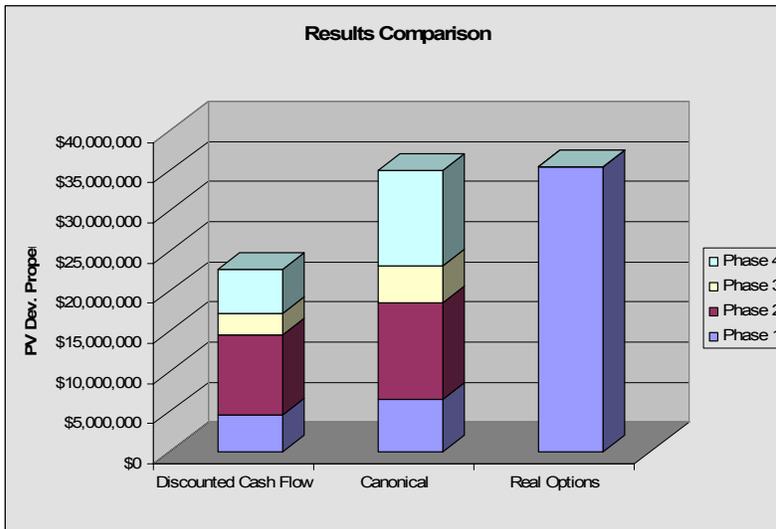
As we see from the results of the first example, when we treat all the phases as a sequence of dependent phases, we achieve results surprisingly very close to the results of the Canonical calculations. This result helps give us confidence in the ability of the Canonical model to approximate the results of committed development projects.

Now, let's compare the results of the sequential development project with the results of the same project, but all the phases are independent of each other in regards to sequencing of phases:

Phase	Discounted Cash Flow	Canonical	Real Options
		Incremental Phase Value at Time 0	
1	4,509,449	6,480,079	7,347,615
2	9,991,692	12,000,506	13,425,628
3	2,687,451	4,594,847	6,244,381
4	5,490,876	11,919,015	19,158,195
Total	**22,679,468**	**34,994,447**	**46,175,819**
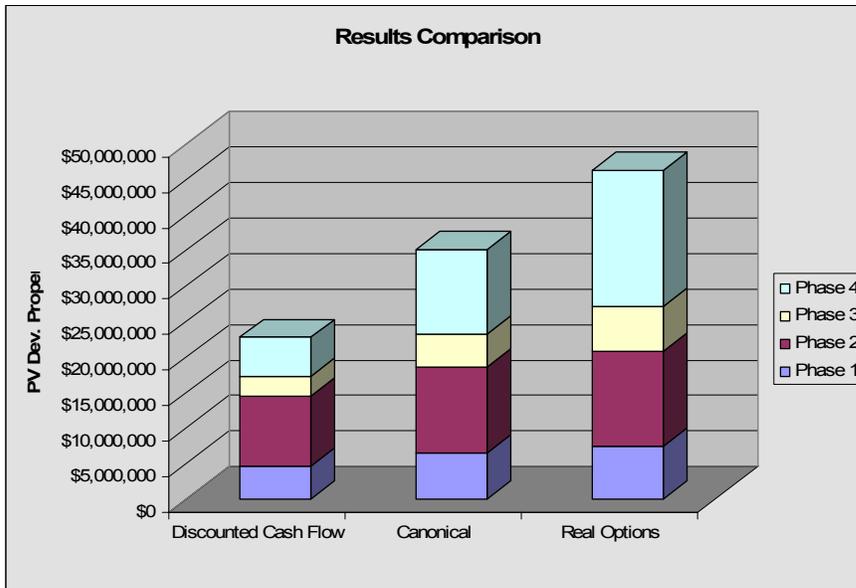
It should be apparent at first glance that the value of flexibility in the development phasing is very significant. In this example, there is a 31% increase in the value of the development project when flexibilities are incorporated into the program. Another very

important thing to note is that there is not much difference in the results of the earlier phases, but the option value of the later phases are greatly magnified compared to the canonical model.

Program Output

The Program Output Worksheet gives a general overview of the uses and site assembly for the development project. The land selection table gives a summary of when particular parcels are to be acquired, the size of the parcels, and the estimated acquisition price of the land. This Assembly output was set up for a specific development site, and would not necessarily be pertinent to all sites.

The Development Program table summarizes the uses and assumptions regarding these uses. These assumptions include the size of each use, rent, absorption, cape rates, and phasing for each use and site improvement.

Discounted Cash Flow Results

  The Discounted Cash Flow Results worksheet features a table which summarizes the present value of the property, development cost, and land price distributed through the uses. Property values and development costs are values obtained from the discounted cash flow worksheets, while land prices are obtained from the land assembly calculations table. These values are tabulated for each phase and totaled to give a net present value of each use.

  Although option values are not utilized in this table, the table is valuable in terms of defining an optimum mix of uses in the program. The uses with the higher net present values are obviously more beneficial to develop, which may encourage the developer to make this use larger to maximize the overall value of the development.

Section III

Conclusions and Recommendations

Conclusions

Although, the concepts involved with the topic of real options analysis is complex and intricate, the intent of this project was successfully achieved. The intent was to create and document a real options model that is practical, transparent, and instructive. Together with this document, the model should meet all these expectation for future users. It will take a serious amount of time for the user to familiarize herself with the concepts and complex calculations, but through utilization of the model and documentation the user should achieve a very good understanding and intuitive feel for the concepts involved. Most importantly, the user will have a valuable took to help her learn how to optimize a complex development project.

Troubleshooting

- If a phase does not stabilize within 30 periods, the worksheet for that phase will not give any results. Check to see if the phase after construction plus the lease-up period is less than 30 periods.

- If you do not get results for real options analysis, check to make sure that there are no circular dependencies in the Phase Interaction Input Table. Check to see that each phase depends on a phase that is not dependent on it. Also, make sure there is at least 1 independent phase, denoted with "0".

Recommendations for Future Revisions

Although this model is quite thorough, it will always remain a work-in-progress. As understanding of the concepts and tools used in the workbook evolve, there will certainly be some adaptation and improvements to the model. At the time of writing this thesis, several opportunities for future improvements of the model and documentation come to mind (perhaps some students from the Class of 2006 or later will be willing to take on some of these challenges):

- Fix any bugs that are found by future students or professors.

- Add more phases – the New Songdo City case study from David Geltner's fall Real Estate Finance and Investments course has six phases. If the workbook were altered to allow for 6 phases, it could be utilized by the students in this assignment.

- Add functionality to enable decision-tree input or output capabilities. There may be some Excel plug-ins that would make this possible to integrate decision tree functionality.

- Improve the worksheet to account for the cyclicality of real estate. As noted in this thesis, real estate is not an efficient market such as stocks and bonds, and is subject to cyclicality. However, the methods utilized in this worksheet treat the options as random-walks.

- There is infinite potential to invent graphical outputs to help visualize real options analysis and results.

- Graphical outputs for cash flows, workloads, etc.

- The inputs for the mix of uses are very simplistic and inflexible. It may be desirable to enhance the inputs to be more sophisticated. An example of this would be enabling the user to input very specific mixes of dwelling unit types, sizes, and prices.

- Konstantinos Kalligeros, a PhD student at MIT, has created workbooks that enable the user to evaluate the option to switch uses. Such a feature may greatly enhance the functionality of the worksheet.

- Create outputs showing results of zoning analysis – FAR, etc.

- As it is now, the workbook is greater than 8 MB. This is not unmanageable, however, saving and emailing the file is often a lengthy process. Someon may come up with methods that help minimize the size of the file..

- As mentioned in the methodology, it is crucial to perform sensitivity analysis when analyzing a project using real option. Methods of performing such sensitivity analysis should be explored and incorporated into the workbook.

- Currently, the workbook does not incorporate construction cost contingencies in it's analysis. The incorporation of these contingencies should be explored further.

- Currently the workbook only allows a dependent phase to begin at the completion of the precedent phase. Often times, it may be practical for dependent phases to begin before the completion of a precedent phase. For example, in the New Songdo City case study, a dependent phase can be purchased and begun after one year of construction has been completed by the precedent phase.

- The workbook does not take into account costs of holding a property undeveloped. The costs could be integrated into the workbook. The most prevalent example of such a cost would be property taxes. If a developer acquires a property, but does not

"exercise" development, she would still be responsible to pay the property taxes related to the property.

- There are many features that could be added to improve this document. A tutorial that walks a user through the input and analysis of a real-world project would be a very valuable feature.

- Documentation of the results of several types of development mixes would be instructive to the user of the workbook to understand different strategies that could optimize the real options value of a development scheme.

- Case studies for use in the Real Estate Finance and Investments courses, or the Development studio course could be written to utilize this model a tool.

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