Agency Conflicts in Financial Contracting with Applications to Venture Capital and CDO Markets

by

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Abstract

In these papers I examine efficient financial contracting when incentive problems play a significant role. In the first chapter (joint with Z. Fluck and S. Myers) we focus on the venture capital industry. We build a two-stage model capturing moral hazard, effort provision, and hold-up problems between entrepreneurs and investors. Across multiple financing scenarios we solve numerically for optimal decision policies and NPV, finding significant value losses from first-best. A commitment to competitive syndicate financing increases effort and NPV and benefits all parties. However, syndicate financing raises potential information problems, and the fixed-fraction participation rule of Admati-Pfleiderer (1994) fails with endogenous effort. We find that debt financing is often less efficient than equity financing, for while it improves effort incentives it worsens hold-up and debt overhang problems in later-stage financing.

In the next chapter I turn to the collateralized debt obligation or "CDO" market. CDOs are closed-end, actively-managed, highly leveraged bond funds whose managers typically receive subordinated compensation packages. I develop a model of manager trading behavior and quantify under-investment and asset substitution problems, calibrating to market parameters. Compared to prior studies, I find similar value losses to senior investors and significantly higher increases in debt default risk and spread costs. However, for even extremely conservative effort assumptions, the ex-ante benefit of greater effort incentives outweighs risk-shifting costs, rationalizing observed contracts. I also analyze the ability of various payout policies and trading covenants to curtail risk-shifting. Excess interest diversions, contingent trading limits, and coverage test "haircuts" of lower-priced assets are effective measures and increase allowable leverage and equity returns.

In the final chapter I examine the empirical relationship between CDO trading, manager compensation, and fund performance from 2001-2004. Using a large panel data set, I find a statistically significant relationship between trades which add volatility to the portfolio and the level of subordinated manager compensation. Worse deal performance increases risk-shifting behavior so long as subordinate investors are still in-the-money. Tendencies to group trades and the effect of managerial reputation are also considered.

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## Contents

1 Venture Capital Contracting and Syndication: An Experiment in Computational Corporate Finance (joint with Z. Fluck and S. Myers) 11

1.1 Introduction .......................................................... 11
   1.1.1 Venture capital contracting ...................................... 13
   1.1.2 Preview of the model and results ................................ 16

1.2 Model Setup and the First-Best Case .................................. 21
   1.2.1 First-best ..................................................... 22

1.3 Monopoly financing and staged investment ................................ 25
   1.3.1 Effort and investment at date 1. .................................. 26
   1.3.2 Effort and exercise at date 0 .................................... 27
   1.3.3 Monopoly financing without staged investment ..................... 28

1.4 Syndication .......................................................... 29
   1.4.1 Effort and investment at date 1 .................................. 31
   1.4.2 Renegotiation at date 1 .......................................... 32
   1.4.3 Effort and exercise at date 0 .................................... 33
   1.4.4 The fully competitive case ...................................... 34
   1.4.5 Syndication with asymmetric information ......................... 35

1.5 Summary of Numerical Results ......................................... 38

1.6 Debt financing ....................................................... 41

1.7 Conclusions ........................................................... 45

1.8 Appendix .................................................................. 47
   1.8.1 Appendix 1. Solving the optimization problems. ............... 47
2 Manager Incentives in Collateralized Debt Obligations: A Theoretical Study 78

2.1 Introduction .................................................. 78
  2.1.1 Overview of CDO Market and History ......................... 79
  2.1.2 Preview of Model ........................................ 82
  2.1.3 Preview of Results ...................................... 83
  2.1.4 Related Literature .................................... 84

2.2 Model Detail (Two-Period) ...................................... 86
  2.2.1 Time \( t = 0 \) ........................................ 87
  2.2.2 Time \( t = 1 \) ........................................ 89
  2.2.3 Time \( t = 2 \) ........................................ 92
  2.2.4 Solution and Calibration ................................ 92

2.3 Model Results (Two-Period) .................................. 96

2.4 Interest Diversion Features in Expanded Three-Period Model ........ 105
  2.4.1 Model Set-up ........................................ 106
  2.4.2 Results ............................................. 107

2.5 CDO Structural Modifications .................................. 112
  2.5.1 Extension 1: OC Whole Interest Diversions .................. 113
  2.5.2 Extension 2: Trade-based Management Fees .................. 113
  2.5.3 Extension 3: OC Haircuts ................................ 115
  2.5.4 Extension 4: Portfolio Turnover Limits ..................... 116
  2.5.5 Extension 5: Collateral Purchases from Excess Interest .... 118
  2.5.6 Extension 6: Contingent Trading Prohibition ............... 119
  2.5.7 Discussion .......................................... 119

2.6 Conclusion .................................................... 120

2.7 Appendix 1: Calibration Detail ................................ 123
  2.7.1 Description .......................................... 123
  2.7.2 Default Parameters ................................... 125
2.7.3 Effort Parameters and Returns ........................................... 126
2.8 Appendix 2: Expanded (Three-Period) Model Detail ....................... 127
  2.8.1 Time $t = 0$ ......................................................... 127
  2.8.2 Time $t = 1$ ......................................................... 127
  2.8.3 Time $t = 1.5$ ....................................................... 130
  2.8.4 Time $t = 2$ ......................................................... 132
  2.8.5 Solution ............................................................. 132
2.9 Tables and Figures .................................................................. 134

3 Bad Luck or Bad Incentives? An Empirical Investigation into CDO Manager

Trading Behavior ...................................................................... 153
  3.1 Introduction ....................................................................... 153
  3.2 Data Description .............................................................. 156
    3.2.1 Descriptive Information .............................................. 157
    3.2.2 Moody’s Data .......................................................... 158
    3.2.3 Intex Data .............................................................. 163
  3.3 Trade Data Analysis .......................................................... 168
  3.4 Regression Analysis .......................................................... 174
    3.4.1 Grouping Decisions ................................................... 177
    3.4.2 Discretionary Trading and Incentive Structure ............... 180
    3.4.3 Non-discretionary Volume ......................................... 189
  3.5 Conclusion ....................................................................... 191
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Chapter 1

Venture Capital Contracting and Syndication: An Experiment in Computational Corporate Finance (joint with Z. Fluck and S. Myers)

1.1 Introduction

This paper develops a model to study how entrepreneurs and venture-capital investors deal with effort provision, moral hazard, asymmetric information and hold-up problems when contracts are incomplete and investment proceeds in stages. How much value is lost in the entrepreneur-venture capital relationship relative to first-best value? How does the value lost depend on risk and the time-pattern of required investment? What determines whether a positive-NPV project can in fact be financed? What are the advantages and disadvantages of staged financing? Are there significant efficiency gains from syndication of later-stage financing?

We argue that these and related questions should not be analyzed one by one, but jointly in a common setting. Some features of venture-capital contracting may not solve a particular problem, but instead trade off one problem against another. For example, a study that focused just on the option-like advantages of staged investment could easily miss the costs of staging,
particularly the negative feedback to effort if venture-capital investors can hold up the entrepreneur by dictating financing terms in later stages. (We find many cases where hold-up costs outweigh the advantages of staged financing and full upfront financing actually increases value.)

A joint analysis of the problems inherent in the entrepreneur-venture capital relationship does not lead to closed-form solutions or simple theorems. Therefore we embarked on an experiment in computational corporate finance, which is the formal study of financing and investment problems that do not have closed-form solutions.\(^1\) We believe the time is ripe for a computational model of venture capital. Venture-capital institutions, contracts and procedures were well documented more than a decade ago. It was clear then that the agency and information problems encountered in ordinary financing decisions are especially acute in venture capital. The successes of venture capital have stimulated theoretical work on how these problems are mitigated. But most theoretical papers have zeroed in on only one problem or tradeoff and run the risk of missing the bigger picture.

Of course the breadth and richness of a computational model do not come free, and numerical results are never absolutely conclusive. One can never rule out the possibility that results would have been different with different inputs or modeling choices. But our model, though simplified, follows actual practice in venture capital. We have verified our main results over a wide range of inputs. We believe our results help to clarify why venture-capital investment works when it works and why it sometimes fails.

The structure of venture-capital financing is known from many sources, including Sahlman (1990), Lerner (1994), Fenn, Liang and Prowse (1995), Gompers (1995), Gompers and Lerner (1996, 2002), Hellman and Puri (2000, 2002) and Kaplan and Stromberg (2003). We will preview our model and results after a brief review of the features of venture-capital contracting that are most important to our paper. The review includes comments on related theoretical work.

\(^1\)Computational models are frequently used to understand the value of real and financial options, but their use on the financing side of corporate balance sheets is an infant industry. The short list of computational papers on financing includes Mello and Parsons (1992), Leland (1994, 1998), Boyd and Smith (1994), Parrino and Weisbach (1999), Robe (1999, 2001), Parrino, Poteshman and Weisbach (2002) and Ju, Parrino, Poteshman and Weisbach (2004). These papers explore the tradeoff theory of capital structure and the risk-shifting incentives created by debt financing.
1.1.1 Venture capital contracting

Venture capital brings together one or more entrepreneurs, who contribute ideas, plans, human capital and effort, and private investors, who contribute experience, expertise, contacts and most of the money. For simplicity, we will refer to one entrepreneur and to one initial venture-capital investor. Their joint participation creates a two-way incentive problem. The investor has to share financial payoffs with the entrepreneur in order to secure her commitment and effort. Thus the investor may not be willing to participate even if the startup has positive overall NPV. Second, the entrepreneur will underinvest in effort if she has to share her marginal value added with the investor. 

Sweat equity

The entrepreneur invests even when she puts up none of the financing. She contributes her effort and absorbs part of the firm’s business risk. The difference between her salary and her outside compensation is an opportunity cost. Specialization of her human capital to the new firm also creates an opportunity cost if the firm fails.

The entrepreneur receives shares in exchange for these investments. These shares may not vest immediately, and they are illiquid unless and until the firm is sold or goes public. The venture capitalist frequently requires the entrepreneur to sign a contract that precludes work for a competitor. The entrepreneur therefore has a strong incentive to stick with the firm and make it successful. In our model, the entrepreneur contributes no financial investment and is willing to continue so long as the present value of her shares exceeds her costs of effort.

Staged investment and financing

A startup is a compound call option. Financing and investment are made in stages. The stages match up with business milestones, such as a demonstration of technology or a successful

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2 This is an extreme version of Myers’s (1977) underinvestment problem.
3 This opportunity cost could perhaps be reduced if new ventures are developed as divisions of larger firms. See Gromb and Scharfstein (2003) and Gompers, Lerner and Scharfstein (2003).
4 Employees typically receive options that vest gradually as employment continues and the startup survives. But our entrepreneur is a founder, not an employee hired later. Founders typically receive shares, not options. The entrepreneur’s shares are fully vested, but additional shares may be granted later. See Kaplan and Strömberg (2003).
product introduction.

We assume that the entrepreneur and venture capitalist cannot write a complete contract to specify the terms of future financing. The terms are determined by bargaining as financing is raised stage by stage. If additional investors join in later stages, the bargain has to be acceptable to them as well as the entrepreneur and initial venture capitalist.

The option value added by staging is obvious, but staging may also serve other purposes. In Bergemann and Hege (1998) and Nöldeke and Schmidt (1998), staging allows the venture capitalist to learn the startup’s value and thereby induce the entrepreneur’s effort. In Neher (1996) and Landier (2002), the venture capitalist’s ability to deny financing at each stage forces the entrepreneur to exert higher effort and prevents her from diverting cash flows.

Venture capital investors usually buy convertible preferred shares. If the firm is shut down, the investors have a senior claim on any remaining assets. The shares convert to common stock if the firm is sold or taken public. Ordinary debt financing is rarely used, although we will consider whether debt could serve as an alternative source of financing.

Control

The venture capitalist does not have complete control of the new firm. For example, Kaplan and Strömberg (2003, Table 2) find that venture-capital investors rarely control a majority of the board of directors. But Kaplan and Strömberg also find that venture capitalists’ control increases when the firm’s progress is unsatisfactory.

Staged financing can give incumbent venture capitalists effective control over access to financing. Their refusal to participate in the second or later rounds of financing would send a strong negative signal to other potential investors and probably deter them from investing. In practice, the incumbents’ decision not to participate is usually a decision to shut down the firm.

Giving venture capitalists effective veto power over later-stage investment is in some respects efficient. The decision to shut down or continue cannot be left to the entrepreneur, who is usually

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6 The role of the monopolist financier was investigated in Rajan (1992), Petersen and Rajan (1994) and Cestone and White (2004).
happy to continue investing someone else’s money as long as there is any chance of success. The venture capitalist is better equipped to decide whether to exercise each stage of the compound call option.

Thus staged financing has a double benefit, at least for the venture capitalist. It can block the entrepreneur’s incentive to continue and it allows the venture capitalist to exploit the startup’s real-option value. But it is also costly if the venture capitalist can use the threat of shutdown to hold up the entrepreneur and dilute her stake. Anticipated dilution feeds back into the entrepreneur’s incentives and effort and reduces overall value. This is the holdup problem of staged financing. For a wide range of parameter values we find that the holdup problem is so severe that the venture capitalist is better off abandoning staged financing and providing all financing upfront. When later financing stages are syndicated on competitive terms, however, staged financing is always more efficient than full upfront financing. We will also show that the holdup problem cannot be solved simply by substituting debt for equity financing. When contracts are incomplete, stage by stage bargaining enables the incumbent venture capitalist to extract surplus regardless of the form of financing.

Most prior theory assumes that venture-capital investors retain residual rights of control. The venture capitalist’s rights to decide on investment (Aghion and Bolton (1992)) and replace the entrepreneur (Fluck (1998), Hellman (1998), Myers (2000), Fluck (2001)) play an important role in enforcing financial contracts between investors and entrepreneurs. The entrepreneur’s option to reacquire control and realize value in an initial public offering is a key incentive in Black and Gilson (1998), Myers (2000) and Aghion, Bolton and Tirole (2001).

Syndication of later-stage financing

Later-stage financing usually comes from a syndicate of incumbent and new venture-capital investors. We show how a commitment to syndicate can alleviate the holdup problem by assuring the entrepreneur more favorable terms in later rounds of financing. This encourages effort in all periods, which increases overall value.

Syndication of venture capital investments has been explained in several other ways. It is one way to gather additional information about a startup’s value – see, for example, Gompers and Lerner (2002, Ch. 9) and Sah and Stiglitz (1986). Wilson (1968) attributes syndication to
venture capitalists’ risk aversion. Syndication may also reflect tacit collusion: early investors syndicate later rounds of financing, and the syndication partners return the favor when they develop promising startups (Pichler and Wilhelm (2001)). In Cassamatta and Haritchabalet (2004), venture capitalists acquire different skills and experience and syndication pools their expertise. We offer a different rationale: syndication can protect the entrepreneur from ex post holdup by investors and thereby encourage effort.

Exit

Entrepreneurs can rely on venture capitalists to cash out of successful startups. Venture capital generally comes from limited-life partnerships, and the partners are not paid until the startups are sold or taken public. Myers (2000) shows that venture capitalists would cash out voluntarily in order to avoid the adverse incentives of long-term private ownership.

Chelma, Habib and Lyngquist (2002) consider how the various provisions of venture-capital contracts are designed to mitigate multiple agency and information problems. Their paper focuses on exit provisions and does not consider syndication.

1.1.2 Preview of the model and results

We aim to capture the most important features of venture capital. For simplicity we assume two stages of financing and investment at dates 0 and 1. If successful, the firm is sold or taken public at date 2, and the entrepreneur and the investors cash out. The entrepreneur and the investors are risk-neutral NPV maximizers, although the entrepreneur’s NPV is net of the costs of her effort.

We value the startup as a real option. The underlying asset is the potential market value of the firm, which we assume is lognormally distributed. But full realization of potential value requires maximum effort from the entrepreneur at dates 0 and 1. The entrepreneur’s effort is costly, so her optimal effort is less than the maximum and depends on her expected share of the value of the firm at date 2. The venture capitalist and the entrepreneur negotiate ownership percentages at date 0, but these percentages change at date 1 when additional financing is raised and invested.\footnote{This would be the case if the venture capitalist have decided that certain non-verifiable performance mile-}
We assume that the firm cannot start or continue without the entrepreneur. If financing cannot be arranged on terms that satisfy her participation constraints, no investment is made and the firm shuts down. The venture capitalist’s date-0 and date-1 participation constraints must also be met, since he will not invest if his NPV is negative.

The efficiency of venture-capital investment hinges on the nature and terms of financing. We compare six cases.

1. *First-best.* If the entrepreneur could finance the startup out of her own pocket, she would maximize overall value, net of the required financial investments and her costs of effort. First-best is our main benchmark for testing the efficiency of other cases.

2. *Fully competitive.* In this case, financing is available on competitive terms (NPV = 0) at both date 0 and date 1, which gives the highest possible value when the entrepreneur must raise capital from outside investors. We include this case as an alternative benchmark to first-best.

3. *Monopoly, staged investment.* Here the initial venture capitalist can dictate the terms of financing at dates 0 and 1 and can hold up the entrepreneur at date 1. The venture capitalist does not squeeze the last dollar from the entrepreneur’s stake, however. He squeezes just enough in each period to maximize the present value of his shares.

4. *Monopoly, no staging.* In this case, the venture capitalist commits all necessary funds at date 0 and lets the entrepreneur decide whether to continue at date 1. This means inefficient investment decisions at date 1, because the entrepreneur is usually better off continuing, even when the odds of success are low and overall NPV is negative. But effort increases at both date 0 and date 1, because the venture capitalist can no longer control the terms of later-stage financing. This case helps clarify the tradeoff between the real-option value of staged investment and the under-provision of effort because of the holdup problem.

5. *Syndication.* In this case a syndicate of additional investors joins the original venture capitalist at date 1. We assume that the syndicate financing comes on more competitive
terms than in the monopoly case, for simplicity we will focus on the fully competitive case (NPV = 0). Syndication mitigates the holdup problem, increasing the entrepreneur’s effort and overall NPV.

6. Debt. Venture capitalists rarely finance startups with ordinary debt, but we nevertheless consider debt financing briefly as an alternative. Debt financing effectively gives the entrepreneur a call option on the startup’s final value at date 2.

We assume that the initial venture capitalist and the entrepreneur are equally informed about potential value, although potential value is not verifiable and contractible. But financing terms in the syndication case depend on the information available to new investors. We start by assuming complete information, but also consider asymmetric information and explore whether the terms of incumbent venture capitalist’s participation in date-1 financing could reveal the incumbent’s inside information.

We solve the model for each financing case over a wide range of input parameters, including the potential value of the firm, the variance of this value, the amount and timing of required investment and the marginal costs and payoffs of effort. We report a representative subset of results in Table 1 and Figures 3 through 12. Results are especially sensitive to the marginal costs and payoffs of effort, so we vary these parameters over very wide ranges. Our main results include the following:

1. We find economically significant value losses, relative to first best, even when the dollar-equivalent cost of effort is a small fraction of required financial investment. Thus many startups with positive NPVs cannot be financed. Value losses decline as the marginal benefit of effort increases or the marginal cost declines.

2. Value losses are especially high in the monopoly case with staged investment, where the incumbent venture capitalist can dictate the terms of financing at date 1. For a wide range of parameter values, both the venture capitalist and entrepreneur are better off in the no-staging case with full upfront financing. That is, the costs of the no-staging case (inefficient investment at date 1) can be less than the value loss due to under-provision of effort in the monopoly, staged financing case.
3. Syndicate financing at date 1 increases effort and the date-0 NPVs of both the entrepreneur and the initial venture capitalist. The venture capitalist is better off than in the monopoly case, despite taking a smaller share of the venture. Moreover, staged financing with syndication always produces higher overall values than the no-staging case. The combination of staged financing and later-stage syndication dominates the alternative of giving the entrepreneur all the money upfront.

4. Syndicate financing is most effective when new investors are fully informed. The incumbent venture capitalist may be able to reveal his information through his participation in date-1 financing. However, the fixed-fraction participation rule derived by Admati and Pfleiderer (1994) does not achieve truthful information revelation in our model, because the terms of financing affect the entrepreneur’s effort. The fixed-fraction rule would lead the venture capitalist to over-report the startup’s value: the higher the price paid by new investors, the more the entrepreneur’s existing shares are worth, and the harder she works. The incumbent venture capitalist captures part of the gain from her extra effort. A modified fixed-fraction rule works in some cases, however. With the modified rule, the incumbent’s fractional participation increases as the reported value increases.

5. We expected venture-capital contracting to be more efficient for high-variance investments, but that is not generally true. Increasing the variance of potential value sometimes increases value losses, relative to first best, and sometimes reduces them, depending on effort parameters and the financing case assumed.

6. Debt financing does not solve the holdup problem, because the venture capitalist can still squeeze the entrepreneur by demanding a high interest rate on debt issued at date 1. Switching from equity to debt financing does add value in some cases, but not generally. Efficient use of syndicated debt financing will often require renegotiation between the entrepreneur and the incumbent venture capitalist – renegotiation after date-1 value is revealed but before the syndicate debt financing is raised. The opportunity to renegotiate may also be an opportunity for the incumbent to reassert monopoly power over the terms of later-stage financing. Opportunities for renegotiation also occur in the syndication case with equity, but with low probability and at low values of the startup firm.
We recognize that we have left out several aspects of venture capital that could influence our results. First, we ignore risk aversion. The venture capitalist and entrepreneur are assumed risk-neutral. This is reasonable for venture capitalists, who have access to financial markets. It is less reasonable for entrepreneurs, who can’t hedge or diversify payoffs without damaging incentives.

Second, we do not explicitly model the costs and value added of the venture capitalist’s effort. We are treating his effort as a cost sunk at startup and fixed afterwards. In effect, we assume that if the venture capitalist decides to invest, he will exert appropriate effort, and that the cost of this effort is rolled into the required investment.

Third, we assume that final payoffs to the entrepreneur and venture-capital investors depend only on the number of shares bargained for at dates 0 and 1. We do not explicitly model the more complex, contingent contracts observed in some cases by Kaplan and Strömberg (2003), and we do not attempt to derive the optimal financial contracts for our model setup. However, our results in Section 4 suggest that the use of contingent share awards may facilitate truthful revelation of information by the initial venture capitalist to members of a later-stage financing syndicate.

Finally, we do not model the search and screening processes that bring the entrepreneur and venture capitalist together in the first place. The costs and effectiveness of these processes could affect the terms of financing. For example, if an entrepreneur’s search for alternative financing would be cheap and quick, the initial venture capitalist’s bargaining power is reduced. Giving the entrepreneur the option to search for another initial investor would not change the structure of our model, however. It would simply tighten the entrepreneur’s participation constraint at

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8 Of course, the venture capitalist will seek an expected rate of return high enough to cover the market risks of the startup. The payoffs in our model can be interpreted as certainty equivalents.

9 Perhaps the entrepreneur’s risk aversion is cancelled out by optimism. See Landier and Thesmar (2003).

10 Kaplan and Strömberg (2003, Table 3) find contingent contracts in 73% of the financing rounds in their sample. The most common contingent contract depends on the founding entrepreneur staying with the firm, for example a contract requiring the entrepreneur’s shares to vest. Vesting is implicit in our model, because the entrepreneur gets nothing if the firm is shut down at date 1. Contingent contracts are also triggered by sale of securities, as in IPOs, or by default on a dividend or redemption payment. But solving the incentive and moral hazard problems in our model would require contracts contingent on effort or interim performance, which are non-verifiable in our model. Such contracts are rare in Kaplan and Strömberg’s sample.

11 Inderst and Muller (2003) present a model of costly search and screening, with bargaining and endogenous effort by both the entrepreneur and the venture capitalist. Their model does not consider staged investment and financing.
date 0 and thereby reduce the initial venture capitalist’s bargaining power.

The rest of this paper is organized as follows. Section 2 sets up our model and solves the first-best case. Section 3 covers monopoly financing with and without staging. Section 4 covers the syndication and fully competitive cases. Numerical results are summarized and interpreted in Section 5. Section 6 briefly considers debt financing. Section 7 sums up our conclusions and notes questions remaining open for further research.

1.2 Model Setup and the First-Best Case

The entrepreneur possesses a startup investment opportunity that requires investments $I_0$ and $I_1$ at dates 0 and 1. If both investments are made, the startup continues to date 2 and the final value of the firm is realized.

If the investment at date 1 is not made, or if the entrepreneur refuses to participate, the startup is shut down and liquidated. We assume for simplicity that liquidation value is zero. (It is typically small for high-tech startups.) This assumption simplifies our analysis of financing, because the venture capitalist’s preferred shares have value only if converted. Thus, we can treat these shares as if they were common in the first place.

The total payoff at date 2 is $P$, which is stochastic and depends on the entrepreneur’s effort at time 0 and time 1, $x_0$ and $x_1$, and on $V_2$, the potential value of the firm at date 2. Effort affects the payoff multiplicatively through the effort functions $f_0(x_0)$ and $f_1(x_1)$:

$$P = f_0 f_1 V_2$$  \hspace{1cm} (1.1)

Effort generates positive but decreasing returns, that is, $f(0) = 0$, $f > 0$ and $f'' < 0$. The entrepreneur bears the costs of her effort, $g_0(x_0)$ and $g_1(x_1)$. The effort cost function is strictly increasing and convex, that is, $g(0) \geq 0$, $g' > 0$ and $g'' > 0$.

The potential value $V_2$ is the sole source of uncertainty. We define the expected value $E_1(V_2)$ at date 1 as $V_1$ and expected value $E_0(V_2)$ at date 0 as $V_0$. The expected payoffs at dates 0 and
1, assuming that the firm will survive until date 2, are:

\[ E_1(P) = E_1(f_0 f_1 V_2) = f_0 f_1 E_1(V_2) = f_0 f_1 V_1 \]
\[ E_0(P) = E_0(E_1(P)) = f_0 E_0(f_1 V_1|V_0) \]

(1.2)

where \( E_0(f_1 V_1|V_0) \) is an integral that accounts for the dependence of \( f_1 \) on \( V_1 \). We assume risk-neutrality and a risk-free interest rate of zero. We use lognormal probability distributions for \( V_1 \) and \( V_2 \), with standard deviation \( \sigma \) per period.

Define the effort function \( f \) and the effort cost function \( g \) as

\[ f_t = 1 - e^{-\theta_f x_t} \]
\[ g_t = e^{\theta_g x_t} \]

(1.3)

for \( t = 0,1 \). The effort function \( f \) asymptotes to 1, so we interpret \( V_1 \) and \( V_2 \) as maximum attainable values as \( x \to \infty \). The degree of concavity and convexity of \( f \) and \( g \) depends on \( \theta_f \) and \( \theta_g \). The effort functions are plotted in Figure 1 for several values of \( \theta_f \) and \( \theta_g \).

The entrepreneur \( (M) \) goes to the initial venture capitalist \( (C) \) to raise startup financing. If he is willing to invest, then she and he negotiate the initial ownership shares \( \alpha_0^M \) and \( \alpha_0^C \). At date 1, \( V_1 \) is observed and there is another round of bargaining over the terms of financing. If the initial venture capitalist also supplies all financing at date 1, then he can dictate the terms and his share becomes \( \alpha_1^C \), with a corresponding adjustment in \( \alpha_1^M \). If the initial venture capitalist brings in a syndicate of new investors at date 1, then the syndicate receives an ownership share of \( \alpha_1^S \), and \( \alpha_1^M \) and \( \alpha_1^C \) adjust accordingly. The terms of financing are fixed after date 1.

1.2.1 First-best

In the first-best case, the entrepreneur supplies all of the money, \( I_0 + I_1 \), and owns the firm \( (\alpha_0^M = \alpha_1^M = 1) \). The entrepreneur maximizes NPV net of her costs of effort. If she decides to invest, she expends the optimal efforts \( x_0^* \) and \( x_1^* \). The timeline is:
The entrepreneur has a compound real call option. The exercise price at date 1 is endogenous, however, because it includes the cost of effort, and effort depends on the realized potential value \( f_0 V_1 \). Since we use the lognormal, our solutions will resemble the Black-Scholes formula, with extra terms capturing the cost of effort.

We now derive the first-best investment strategy, solving backwards. Details of this and subsequent derivations are in the Appendix. By date 1, the entrepreneur’s date-0 effort and investment are sunk. Her date-1 NPV is

\[
NPV_1^M = \max[0, \max(f_0 f_1(x_1)V_1 - g_1(x_1) - I_1)]
\]

The first-order condition for effort is \( f_0 V_1 = \frac{g_1(x_1)}{f_1} \), which determines optimal effort \( x_1^* \) and the benefit and cost of effort, \( f_1(x_1^*) \) and \( g_1(x_1^*) \).

Define the strike value \( \overline{V}_1 \) such that \( NPV_1^M(\overline{V}_1) = 0 \). The entrepreneur exercises her option to invest at date 1 when \( V_1 > \overline{V}_1 \) and \( NPV_1^M > 0 \). This strike value is similar to the strike price of a traded option, except that the strike value has to cover the cost of the entrepreneur’s effort \( g_1(x_1^*) \) as well as the investment \( I_1 \).

At date 0, the entrepreneur anticipates her choice of effort and continuation decision at date 1. She determines the effort level \( x_0^* \) that maximizes \( NPV_0^M \), the difference between the expected NPV at date 1 and the immediate investment \( I_0 \) and cost of effort \( x_0 \).

\[
NPV_0^M = \max[0, \max(E_0(NPV_1^M(x_0)) - g_0(x_0) - I_0)]
\]

\( E_0(NPV_1^M(x_0)) \) depends on \( x_0 \) in two ways. First, increasing effort at date 0 increases \( f_0 \), and thus increases the value of the startup when it is in the money at date 1. Second, increasing effort at date 0 decreases the strike value \( \overline{V}_1 \) for investment at date 1 and makes it more likely that the startup will continue.
This tradeoff is illustrated in Figure 2. The top payoff line is the date-1 NPV for a call option with no cost of effort. In this case the value would be $V_1$ and the strike price $I_1$. The lower payoff line shows the net NPV when the entrepreneur exerts less than the maximum effort at date 0 ($f_0 < 1$). $NPV_1$ is close to linear in $V_1$, but the slope and the level of $NPV_1$ are reduced by the cost of effort. We have added a lognormal distribution to show the probability weights assigned to these NPVs. The two horizontal lines are the date-0 financial investment $I_0$ and the full cost $I_0 + g_0$ of investment and effort.

We calculate $E_0(NPV_1)$ by integrating from $\bar{V}_1(x_0)$. Since $NPV_1^M(x_0, \bar{V}_1(x_0)) = 0$, the entrepreneur’s first-order condition reduces to

$$E_0(NPV_1^M(x_0^*)) = g_0(x_0^*).$$

From (??) we obtain

$$E_0(NPV_1^M(x_0^*)) = f_0' \left[ \int_{\bar{V}_1(x_0)}^{\infty} \Pi(V) V dV - \frac{1}{1+\theta_r} \left( \theta_r^{\frac{\theta_f}{\theta_f + \theta_g}} + \theta_r^{-\frac{1}{\theta_f + \theta_g}} \right) \int_{\bar{V}_1(x_0)}^{\infty} \Pi(V) V^{\frac{1}{\theta_f + \theta_g}} dV \right]$$

where $\Pi(V)$ is the lognormal density and $\theta_r = \theta_f / \theta_g$.

We solve for $x_0^*$ analytically, using properties of the lognormal distribution. Then we evaluate $NPV_0^M(x_0^*) = E_0(NPV_1^M(x_0^*)) - g_0 - I_0$. When $NPV_0^M(x_0^*) > 0$, the entrepreneur invests and the firm is up and running.

Table 1 includes examples of first-best numerical results. Start with the first two lines of Panel A, which report Black-Scholes and first-best results when potential value is $V_0 = E_0(V_2) = 150$ and required investments are $I_0, I_1 = 50, 50$. The standard deviation is $\sigma = 0.4$ per period. The effort parameters are $\theta_f = 1.8$ and $\theta_g = 0.6$, so the value added by effort is high relative to the cost. Thus the option to invest in the startup should be well in the money, even after the costs of effort are deducted.

If the costs of effort were zero, first-best NPV could be calculated from the Black-Scholes formula, with a date-1 strike price of $\bar{V} = 50$. But when the cost of effort is introduced, $\bar{V}$ increases and NPV declines. First-best NPV is 37.90, less than the Black-Scholes NPV by
12.13. The difference reflects the cost of effort and the increase in strike value to $V = 55.35$.

Panel B repeats the example with higher standard deviation of $\sigma = 0.8$. Panels C and D assume lower investment at date 0 and higher investment at date 1 ($I_0, I_1 = 10, 90$). NPV increases for higher standard deviations and when more investment can be deferred. The first-best initial effort decreases in these cases, though not dramatically. Panels E and F assume $\theta_f = 0.6$, so that effort is less effective, and also back-loaded investment (again, $I_0, I_1 = 10, 90$). First-best effort actually increases, compared to panels C and D, but NPV declines dramatically.

Figure 3 plots first-best NPV for a wide range of standard deviations and effort parameters. It turns out that only $\theta_r = \theta_f / \theta_g$, the ratio of the effort parameters, matters, so that ratio is used on the bottom-left axis. The ratio is $\theta_r = 3$ in Panels A to D of Table 1 and $\theta_r = 1.0$ in Panels E and F. In Figure 3, $\theta_r$ is varied from $1/11$ to 11. The startup becomes worthwhile, with first-best $NPV \geq 0$, for $\theta_r$ slightly below 1.0. NPV increases rapidly for higher values of $\theta_r$, then flattens out. NPV also increases with standard deviation, especially when most investment can be deferred to date 1.

### 1.3 Monopoly financing and staged investment

Now we explore the monopoly case in which the entrepreneur approaches the venture capitalist for financing and the initial venture capitalist can dictate terms of financing at both date 0 and date 1, subject to the entrepreneur’s participation constraints. The venture capitalist will not exploit all his bargaining power, however, because of the feedback to the entrepreneur’s effort. In some cases, the venture capitalist is better off if he gives up all his bargaining power and gives the entrepreneur all the financing upfront. We do not argue that the monopoly case is realistic, but it is a useful benchmark, and we believe that venture capitalists do have bargaining power, especially in early-stage financing, and receive at least some (quasi) rents.

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12 The effort parameters in panel A of Table 1 are $\theta_f = 1.8$ and $\theta_g = 0.6$. Date 0 effort is $x_0 = 2.4$, so $f_0 = 0.99$ and $g_0 = 4.59$. Of course the date 0 effort is sunk by date 1. From (??), $f'_1 = 0.978$ and $g'_1 = 3.58$. The breakeven value level $V = 55.35$ is determined by $0.99 \times 0.978 \times 55.35 / 3.58 = I_1 = 50$.

13 We do not include panels for equal investment ($I_0, I_1 = 50, 50$) and $\theta_f = 0.6$, because NPVs are negative in all cases where outside venture-capital financing is required. A startup with these parameters could not be financed.
By “terms of financing” we mean the fraction of common shares held by the entrepreneur and venture capitalist at dates 0 and 1. The entrepreneur’s fractional share at dates 1 and 2 is \( \alpha_1^M \), the complement of \( \alpha_1^C \). The entrepreneur’s share at date 0 is irrelevant in the monopoly case, because a monopolist venture capitalist can force the terms of financing at date 1 and is free to dilute shares awarded earlier. We do assume that the entrepreneur has clear property rights to her shares at date 2 and that these shares cannot be taken away or diluted between dates 1 and 2. The division of the final payoff \( P \) is enforceable once date-1 financing is completed.

1.3.1 Effort and investment at date 1.

Both the entrepreneur and the venture capitalist now have the option to participate at date 1. There are two derivative claims on one underlying asset. Both must be exercised in order for the project to proceed.

At date 1 the entrepreneur decides whether to exercise her option to continue, based on her strike price, the cost of optimal effort \( g_1(x_1^*) \). But first the venture capitalist sets \( \alpha_1^C \) and \( \alpha_1^M = 1 - \alpha_1^C \) and decides whether to put up the financial investment \( I_1 \). We can focus on the venture capitalist’s decision if we incorporate the entrepreneur’s response into the venture capitalist’s optimization problem.

The equation for the entrepreneur’s NPV is similar to Eq. (??), except that the second-period investment \( I_1 \) drops out and firm value is multiplied by the entrepreneur’s share \( \alpha_1^M \).

\[
NPV_1^M(\alpha_1^C) = \max[0, \max_{x_1} (\alpha_1^M f_0 f_1(x_1) V_1 - g_1(x_1))]
\]

The maximum share the venture capitalist can take is obtained by setting \( NPV_1^M(\alpha_1^C) = 0 \). This defines \( \alpha_1^C(\text{max}) \) and \( \alpha_1^M(\text{min}) \). When \( \alpha_1^M(\text{min}) \geq \alpha_1^M \), the entrepreneur will not participate.\(^{14}\) The venture capitalist chooses \( \alpha_1^C \) to maximize his date-1 NPV, subject to his and the entrepreneur’s participation constraints.

\[
NPV_1^C = \max \left[ 0, \max_{\alpha_1^C \in (0, \alpha_1^C(\text{max})]} (\alpha_1^C f_0 f_1(x_1^*) V_1 - I_1) \right]
\]

\(^{14}\)When \( f_0 \) is small or \( V_1 \) is low, \( \alpha_1^M(\text{min}) \) may be greater than 1, so that continuation is impossible even if the entrepreneur is given 100% ownership.
If the entrepreneur’s participation constraint is binding, the venture capitalist assigns \( \alpha^C_1(\text{max}) \). Otherwise, he assigns an interior value. But in most of our experiments, \( \overline{V}_1^C \), the venture capitalist’s strike value for the monopoly case, falls in the region \( \alpha^C_1 \in \left[0, \alpha^C_1(\text{max})\right] \) where the entrepreneur’s NPV is positive. In these cases the venture capitalist is better off by taking a share \( \alpha^C_1 < \alpha^C_1(\text{max}) \) in order to give the entrepreneur stronger incentives. Nevertheless, those incentives are weaker than first-best, because \( \alpha^{M*}(x_0) < 1 \), which decreases the expected payoff by reducing \( x_t^1 \).

Figure 4 plots values of \( \alpha^C_1 \) as a function of \( V_1 \) when \( I_0, I_1 = 50, 50; \sigma = 0.4; \theta_f = 1.8 \), and \( \theta_g = 0.6 \), the same parameters used in Panel A of Table 1. The optimal share \( \alpha^{C*}_1 \) is less than the maximum share \( \alpha^C_1(\text{max}) = 1 - \alpha^M_1(\text{min}) \) for all \( V_1 \geq \overline{V}_1^C \), the venture capitalist’s strike value at date 1. Thus the maximum share that the venture capitalist could extract is irrelevant. But notice that the venture capitalist’s optimum share increases as the project becomes more valuable, with a corresponding decline in the entrepreneur’s share. This implication of the monopoly case is contrary to the evidence in Kaplan and Stromberg (2003), who find that entrepreneurs gain an increasing fraction of payoffs as and if the firm succeeds. This suggests that in practice later-stage venture-capital financing is not provided on monopolistic terms.

Assuming the \( \alpha^C_1(\text{max}) \) constraint does not bind, we compute \( \overline{V}_1^C \) by looking for the pair \( \overline{V}_1^C, \alpha^{C*}(\overline{V}_1^C) \) that sets \( NPV_1^C \) equal to zero. Investment occurs if \( V_1 > \overline{V}_1^C \).

### 1.3.2 Effort and exercise at date 0.

In the first-best case, the entrepreneur anticipates \( x_t^1 \) and \( I_1 \) in her choice of \( x_0 \). In the monopoly case, the entrepreneur anticipates the venture capitalist’s decisions at date 1. She then evaluates whether \( NPV_0^C(x_0^*) \geq 0 \). As in the first-best case, higher effort at \( t = 0 \) lowers the threshold for investment at \( t = 1 \) (makes it more likely that both the entrepreneur’s and venture capitalist’s options are in the money) and increases the value of the project when the option is in the money.

The entrepreneur’s date 0 value is

\[
NPV_0^M = \max[0, \max_{x_0}(E_0(NPV_1^M(x_0)) - g_0(x_0))] \quad (1.6)
\]
with the first-order condition

\[ E_0(NPV_1^M(x_0^*))' = \Pi(V_1)NPV_1^M(f_1^*, V_1)V_1' + g'(x_0^*) \]  

(1.7)

Here there are no closed-form expressions. We solve the first-order condition and determine \( x_0^* \) numerically. Given \( x_0^* \), and assuming that the entrepreneur wants to go ahead (\( NPV_0^M(x_0^*) > 0 \)), the venture capitalist invests if:

\[ NPV_0^C = \max[0, E_0(NPV_1^C(x_0^*)) - I_0] > 0 \]  

(1.8)

Thus two options must be exercised at date 0 in order to launch the startup. The entrepreneur picks \( x_0^* \) to maximize the value of her option to continue at date 1, and then determines whether this value exceeds her current strike price, the immediate cost of effort \( g_0^* \). The venture capitalist values his option to invest \( I_1 \) at date 1, taking the entrepreneur’s immediate and future effort into account, and then decides whether to invest \( I_0 \).

Monopoly financing can be extremely inefficient. The venture capitalist’s ability to claim a large ownership fraction at date 1 reduces the entrepreneur’s effort at date 0 as well as date 1, reducing value and increasing the venture capitalist’s breakeven point \( V_1^C \). For example, compare the monopoly and first-best results in Panel A of Table 1. The entrepreneur’s initial effort falls by about 50 percent from the first-best level and the date-1 strike value \( V_1^C \) increases by almost 30 percent. The entrepreneur’s NPV drops by more than 90 percent. Overall NPV drops by more than half. Similar value losses occur in panels B to F. In Panel E, a startup with first-best NPV of 10.98 cannot be financed in the monopoly case. NPV would be negative for both the entrepreneur and the initial venture capitalist.

1.3.3 Monopoly financing without staged investment

The incentive problems of the monopoly case can sometimes overwhelm the option-like advantages of staged financing. All may be better off if the entire investment \( I_0 + I_1 \) is given to the entrepreneur upfront and she is granted full control thereafter.

In the monopoly, no-staging case, the entrepreneur and venture capitalist bargain only once at date 0 to determine their ownership shares \( \alpha^M \) and \( \alpha^C \), which are then fixed for dates 1 and
The entrepreneur calculates her NPV at date 1 just as in the monopoly case with staging, but her share of the firm $\alpha^M_1$ is predetermined. Also, she ignores the financial investment $I_1$ and continues at date 1 so long as her NPV exceeds her cost of effort, $g_1(x^*_1)$.

The venture capitalist retains monopoly power over financing at date 0, but loses all his bargaining power at date 1. He sets $\alpha^C$ to maximize his NPV at date 0, taking account of the effects on the entrepreneur’s effort at dates 0 and 1. His maximization problem is identical in appearance to Eq. (1.8) but the values of $x_0$, $x_1$ and $\alpha^C_1 = \alpha^C_0$ are different. The entrepreneur’s date-0 maximization problem closely resembles Eq. (1.6) except for the choices of $x_1$ and $\alpha^C_1 = \alpha^C_0$. If both parties’ participation constraints are met at date 0 ($NPV^C_0 \geq 0$ and $NPV^M_0 \geq 0$), the startup is launched.

The value loss from the holdup problem in the monopoly, staged financing case can be so severe that it can actually exceed the cost of inefficient continuation in the no-staging case. For example, note the improvement in the no-staging case in Panel A of Table 1. The NPV to the entrepreneur more than doubles, compared to the monopoly case with staged investment, and overall NPV increases from 16.47 to 26.89.

Figures 5 and 6 compare the monopoly NPVs with and without staging. Figure 5 assumes equal investment in both periods ($I_0$, $I_1 = 50$, 50). Here NPV is higher without staging, except at extremely high standard deviations. Figure 6 assumes back-loaded investment ($I_0$, $I_1 = 10$, 90), which adds to option value and the value of staging. In Figure 6, a monopolist venture capitalist would give up staging and provide 100% upfront financing only at relatively low standard deviations.

### 1.4 Syndication

The value losses in the monopoly case would be reduced if the entrepreneur could promise higher effort at date 1, or if the venture capitalist could promise to take a lower ownership fraction $\alpha^C_1$. Neither promise is credible, however, since effort and potential value are non-contractible. But suppose that the venture capitalist can commit (explicitly or implicitly) to bring in a syndicate of new investors to join him in the date-1 financing. Suppose further that

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$^{15}$We do assume that the entrepreneur cannot launch the firm at date 0 and then run off with the date-1 investment $I_1$. Venture-capital investors typically hold convertible preferred shares and have priority in liquidation.
the incumbent venture capitalist does not collude with syndicate members and allows them to dictate the terms of financing. We will show that these commitments are in the initial venture capitalist’s interest. Syndication alleviates the holdup problem and generates extra effort and value.

Syndication of later stage financing is common in practice. The initial venture capitalist approaches a group of other venture-capital investors that he has worked with in the past, or hopes to work with in the future, and offers participation in the financing. We interpret syndication as a mechanism that introduces competition into date-1 financing and restrains the initial venture capitalist’s temptation to hold up the entrepreneur. We do not know what NPV syndicates obtain in practice, but it is natural to explore NPV = 0 as a limiting case. (If the syndicate gets positive NPV, but still less than a monopolist venture capitalist could extract, there are still value gains relative to the monopoly case.) We start with the full-information case, where the investors who compete to join the syndicate have the same information as the incumbent venture capitalist.\footnote{We doubt that potential syndicate investors really have full information. If they did, then the entrepreneur could negotiate with these investors directly and possibly hold up the incumbent venture capitalist. This scenario seems implausible and we do not explore it in this paper.}

The results for syndicate financing differ from the monopoly case in at least two ways. First, the terms of financing shift in the entrepreneur’s favor. The new syndicate investors are forward-looking. They do not care about the value of the existing shares held by the incumbent venture capitalist and have no incentive to hold up the entrepreneur. The incumbent has no control over the terms of financing at date 1, so his ultimate ownership and payoff are determined by his initial share $\alpha_0^C$ and the performance of the startup. The syndicate accepts $NPV^S = 0$ and does not trade off extra NPV against reduced effort from the entrepreneur. Thus the syndicate’s ownership share $\alpha_1^S$ will generate lower $NPV^S$ than the NPV-maximizing share $\alpha_1^{C*}$ that a monopolist incumbent would set. The entrepreneur suffers less dilution, exerts more effort, and total value increases. This outcome benefits the initial venture capitalist at date 0. By delegating the terms of date-1 financing, he solves the incomplete contracting problem that causes the holdup problem in the monopoly case with staged financing.

Second, under zero-NPV date-1 financing, the initial venture capitalist effectively owns a call option with a zero exercise price and will always want the investment to proceed at date 1
if the project can generate enough value to cover the syndicate’s investment. With full information, it does not matter whether the initial venture capitalist participates in the syndicate, because the syndicate’s investment is zero-NPV. Of course the initial venture capitalist’s participation matters if the syndication terms are not fully competitive. The higher the NPV for the syndicate, the closer is the syndicate case to the monopoly case.

1.4.1 Effort and investment at date 1

For a given share \( \alpha_1^M \), the entrepreneur’s NPV and maximization problem at date 1 are the same as in the monopoly case. We obtain \( x_1^*, f_1^*, g_1^*, \) and \( NPV_1^M(\alpha_1^S) \) exactly as in Eq. (1.4), but with \( \alpha_1^M = \alpha_0^M(1 - \alpha_1^S) \). The share given to the outside syndicate, \( \alpha_1^S \), is determined by \( NPV_1^S = 0 \), that is, by \( I_1 = \alpha_1^S f_0 f_1(x_1) V_1 \). Investment at date 1 occurs for \( V_1 > \overline{V}_1^S \). We solve for \( \overline{V}_1^S \) by finding the value of \( \alpha_1^S \) that maximizes \( NPV_1 \), subject to the constraint that \( NPV^S = 0 \) for the syndicate. The solution is generally in the region where \( NPV_1^M(\alpha_1^S) > 0 \) at \( \overline{V}_1 \) and \( \alpha_1^M < \alpha_1^M(\min) \).

Figure 7 plots ownership shares against value at date 1 for the syndicate case when \( I_0, I_1 = 50, 50, \sigma = 0.4, \theta_f = 1.8 \) and \( \theta_g = 0.6 \), the same parameters used in Panel A of Table 1. The top two lines show the syndicate’s maximum and optimal shares if the new investors were given free rein to maximize their NPV. The maximum share is irrelevant, as in Figure 4. The syndicate’s actual share equals its optimal share at the strike value \( \overline{V}_1^S \) and then declines as \( V_1 \) increases. The shares held by the entrepreneur and incumbent venture capitalist therefore increase as performance improves, consistent with the evidence in Kaplan and Stromberg (2003) and contrary to the pattern in the monopoly case, as plotted in Figure 4.

One might expect the better financing terms from the syndicate to decrease the strike value \( \overline{V}_1 \) from the monopoly case. But \( \overline{V}_1 \) is actually higher in the syndicate case – for example, \( \overline{V}_1 \) is 84.9 in Figure 7 and 70.8 in Figure 4. This increase can be traced to the initial venture capitalist’s fixed original ownership share in the syndicate-financing case. When the original venture capitalist provides both rounds of financing, he picks the share at date 1 that is best for him at date 1. He may reduce his share to strengthen the entrepreneur’s incentives. Unlike the monopolist, the syndicate cannot reset the venture capitalist’s original share \( \alpha_0^C \) and is therefore faced with a free-riding incumbent. Thus the syndicate has a smaller value pie to
carve up, and a higher threshold for investment.

Thus the commitment to syndicate later-stage financing has two countervailing effects. The syndicate may require a higher threshold for investment, so that marginal projects will be rejected more often. On the other hand, syndication provides better incentives for the entrepreneur, so that low values of $f_0V_1$ are less likely. Our numerical analysis will show that the second effect outweighs the first and that shifting from monopoly to syndication always adds value.

1.4.2 Renegotiation at date 1

Of course a low realization of $V_1$ could trigger a renegotiation between the incumbent and the entrepreneur to reset the incumbent’s initial share $\alpha_0^C$ before syndicate financing is sought. The incumbent can transfer ownership to the entrepreneur, retaining $\alpha^C(R) < \alpha_0^C$, where $\alpha^C(R)$ denotes the incumbent’s renegotiated equity stake. The incumbent may be better off accepting a reduced ownership share to improve the chance of success for low values of $V_1$ or to increase continuation for $V_1 < V_1^S$. By accepting a lower ownership share, the incumbent improves effort incentives for the entrepreneur to the point where enough extra value is added to support syndicate financing at NPV = 0. Of course the incumbent will give up as little as possible. In the worst renegotiation case, where $V_1$ approaches a lower bound, the value of the incumbent’s shares approaches zero, just as in the monopoly case.

Renegotiation requires dilution of the incumbent venture capitalist’s ownership share. Dilution could happen in several ways. For example, the incumbent could provide bridge financing on terms favorable to the entrepreneur. Dilution could also occur in a "down round" – a round of financing where new investors buy in at a price per share lower than in previous rounds. But our model says that a down round should dilute the entrepreneur less than the incumbent venture capitalist. The entrepreneur could be given additional shares or options, for example.

While renegotiation adds value ex post by improving effort and preserving access to financing, the flexibility to reset shares at date 1 could introduce new problems. Suppose the initial venture capitalist sets $\alpha_0^C$ at a very high level, knowing that he can renegotiate down to the monopoly level at date 1, even when the realized value $V_1$ exceeds the syndicate strike value.
V.17 The entrepreneur would then cut back effort at dates 0 and 1 and reduce the value of the firm. This strategy amounts to a return to monopoly financing. It would reduce date-0 value to the venture capitalist as well as the entrepreneur.

Thus two conditions must hold in order for syndicate financing to work as we have described it. First, the initial venture capitalist has to commit at date 0 to syndicate at date 1. In practice this is not an explicit, formal commitment, but syndication is standard operating procedure. As part of the commitment, the initial venture capitalist has to limit his initial ownership share $\alpha_0^C$ to its level in the syndicate case, so that he cannot start with a higher value and bargain down to the monopoly share $\alpha_1^{C*}$ at date 1. The commitment is in the venture capitalist’s interest, because it increases his ex ante value relative to the monopoly case. Second, the terms of financing in later rounds should be reasonably competitive. In practice they may not be perfectly competitive, but we believe the terms are materially better for the entrepreneur than the monopoly terms would be.

It turns out that opportunities for renegotiation are rare in our numerical experiments for the syndicate case. Therefore, incorporating the benefits of renegotiation would add relatively little to NPV at date 0. For example, including renegotiation gains would increase the NPVs reported in Table 1 by about 2% of the required total investment of $I_0 + I_1 = 100$.18

1.4.3 Effort and exercise at date 0

At date 0, the venture capitalist sets $\alpha_0^C$ and the entrepreneur decides how much effort to exert. Given $\alpha_0^C$, the entrepreneur chooses $x_0$ to maximize:

$$NPV_0^M(\alpha_0^C) = \max_{x_0} \max_0 \left[ E_0(NPV_1^M(x_0(\alpha_0^C), \alpha_0^C) - g_0(x_0(\alpha_0^C))) \right]$$ (1.9)

17 The entrepreneur could retain the upside if she could bypass the incumbent venture capitalist and go directly to the syndicate for financing. In practice the incumbent could block this end run by refusing to participate in the syndicate. The syndicate would assume that the incumbent has inside information, and would interpret the refusal to participate as bad news sufficient to deter their investment.

18 We approximate renegotiation gains (holding $\alpha_0^{C*}$ and $x_0$ constant) by solving for (1) the value realization at which the venture capitalist will start to reduce his share; (2) the new strike value and (3) the integral of NPV changes over this range. Only a small portion of the renegotiation gains come from more efficient continuation decisions ($V_1^S(R) < V_1 < V_1^S$). Most of the gains can be attributed to better effort incentives (higher $x_1$) in the region where the project continues regardless ($V_1 > V_1^S$). These gains further increase the value advantages of syndicate financing over monopoly financing.
The entrepreneur anticipates the syndicate’s share $\alpha_1^S$ as a function of date-1 value $V_1$. For a given $\alpha_0^C$, date-1 syndicate financing will result in less dilution of her share than in the monopoly case, so she provides higher effort at $t = 0$ as well as at $t = 1$. We cannot express NPV or effort in closed form, so we compute them numerically.

The venture capitalist anticipates the entrepreneur’s reaction when he sets $\alpha_0^C$. He must restrict his search to $\alpha_0^C \in (0, \alpha_0^C(\text{max})]$, where $\alpha_0^C(\text{max})$ is determined by

$$E_0(NPV_1^M(x_0^*(\alpha_0^C(\text{max})), \alpha_0^C(\text{max})) - g_0(x_0^*(\alpha_0^C(\text{max})))) = 0$$

(1.10)

This constraint rarely binds, since at the margin there is almost always value added by leaving positive value to the entrepreneur. Thus $\alpha_0^C^*$ is determined by

$$\alpha_0^C^* = \arg \max_{\alpha_0^C \in (0, \alpha_0^C(\text{max})]} (E_0(NPV_1^C(x_0^*(\alpha_0^C), \alpha_0^C)) - I_0)$$

(1.11)

If $NPV_0^C \geq 0$, investment proceeds.

Typical results for syndicate financing are shown in Table 1. Effort and value increase across the board, despite increases in the strike value $\overline{V}_1^S$ from the monopoly case. We find that syndicate financing dominates monopoly financing with or without staged financing. Syndicate financing is better ex ante for the initial venture capitalist and also increases overall NPV. This is true for all parameter values, including values outside the range reported in Table 1. Yet there are still value losses relative to the first best.

1.4.4 The fully competitive case

Of course first best is never attainable when the entrepreneur has to seek outside financing. Table 1 shows an alternative benchmark, the fully competitive case, in which all venture capital investors, including the initial investor at date 1, receive $NPV = 0$. Fully competitive financing gives an upper bound on the overall NPV when the entrepreneur has no money and has to share her marginal value added with outside investors. Solution procedures for the fully competitive case are identical to the syndication case, except that $\alpha_0^C^*$ is set so that $NPV_0^C = 0$.

Figure 8 shows date-1 ownership shares for the fully competitive case in the same format as the syndication case in Figure 7. Two things stand out. First, the entrepreneur’s share is higher
and the initial venture capitalist’s lower than in Figure 7, because competitive financing at date 0 gives relatively more shares to the entrepreneur. Both shares of course increase with the realized value $V_1$. Second, the strike value $\overline{V}_1$ is lower in the competitive case, primarily because the entrepreneur’s initial effort is higher. Note also that the fully competitive NPVs, which go entirely to the entrepreneur, are less than in the first-best case, because the entrepreneur’s effort is lower. There is always some value loss when the entrepreneur has to share the marginal value added by her effort with outside investors.

1.4.5 Syndication with asymmetric information

So far we have assumed that the incoming syndicate investors and the incumbent venture capitalist are equally informed. Now we consider asymmetric information between the incumbent and new investors.

Both the incumbent and entrepreneur want the syndicate to perceive a high value $V_1$. The more optimistic the syndicate, the higher the ownership shares retained by the incumbent and entrepreneur. Reducing the syndicate’s share also increases the entrepreneur’s effort. Therefore, mere announcements of “great progress” or “high value” coming from the entrepreneur or incumbent are not credible.

Credibility may come from the incumbent’s fractional participation in date-1 financing. Suppose the incumbent invests $\beta I_1$ and the outside syndicate the rest. What participation fraction $\beta$ is consistent with truthful revelation of $V_1$? If we could hold the entrepreneur’s effort constant, we could rely on Admati and Pfleiderer’s (1994) proof that $\beta$ should be fixed at the incumbent investor’s ownership share at date 0, that is, at $\alpha_0^C$. This fixed-fraction rule would remove any incentive for the incumbent to over-report $V_1$. (The more he over-reports, the more he has to overpay for his new shares. When $\beta = \alpha_0^C$, the amount overpaid cancels out any gain in the value of his existing shares).\(^{19}\) The fixed-fraction rule would also insure optimal investment, since the incumbent’s share of date-1 investment exactly equals his share of the final payoff $V_2$. Admati and Pfleiderer also show that no other financing rule or procedure works in their setting.

\(^{19}\)The fixed-fraction rule would also remove any incentive to underreport. The more the incumbent underreports, the more he gains on the new shares. But the amount of profit made on the new shares is exactly offset by losses incurred on existing shares.
Fixed-fraction financing does not induce truthful information revelation in our model, although a modified fixed-fraction financing works in some cases. The problem is the effect of the terms of date-1 financing on the entrepreneur's effort. Suppose the incumbent investor takes a fraction $\beta = \alpha_0^C$ of date-1 financing and then reports a value $\bar{V}_1$ that is higher than the true value $V_1$. If the report is credible, the new shares are over-priced. The incumbent does not gain or lose from the mispricing, because $\beta = \alpha_0^C$, but the entrepreneur gains on his old shares at the syndicate's expense. Since the entrepreneur's ownership share is higher than it would be under a truthful report, she exerts more effort, firm value increases, and both the entrepreneur and incumbent are better off. Therefore the incumbent will over-report.

A modified fixed-fraction rule can work, however, provided that $\beta$ is set above $\alpha_0^C$ and effort is not too sensitive to changes in the entrepreneur's NPV at date 1. The required difference between $\beta$ and $\alpha_0^C$ depends on the responsiveness of the entrepreneur's effort to her ownership share. In many cases, a constant $\beta$ set a few percentage points above $\alpha_0^C$ removes the incentive to overreport over a wide range of $V_1$ realizations. But this rule may break down as a general revelation mechanism in at least three ways.

First, when $V_1$ is very low but exceeds $\bar{V}_1$, we find situations where the required $\beta$ exceeds 1. This would make sense only if the new syndicate investors could short the company, so we must constrain $\beta < 1$. This outcome is common in our numerical results, because the incumbent's initial share $\alpha_0^C$ is frequently above 0.85 or 0.90, and in some of these cases the entrepreneur's effort is very sensitive to the value of her stake in the firm. There is not much room for $\beta$ to increase between these starting points and a maximum level strictly less than 1. When $\beta$ hits the maximum, the modified fixed fraction rule fails to induce truthful revelation.\footnote{This failure is less frequent if the entrepreneur has some personal wealth and can co-invest with the venture capitalist at date 0. The co-investment reduces the venture capitalist's ownership share and provides more room for $\beta$ to increase to a maximum level strictly less than 1.}

Second, the modified fixed fraction rule also fails when $V_1$ falls just below $\bar{V}_1$. In this case the incumbent's incentive to over-report becomes very strong, and only extremely high $\beta$s can discipline the incumbent to tell the truth. This problem flows from the discontinuity of the entrepreneur's effort at the strike value $\bar{V}_1$. Here the limit of $\beta$ as $V_1$ approaches $\bar{V}_1$ from below is infinity and no fixed-fraction rule works. This problem can be solved, however, if the incumbent and the entrepreneur renegotiate their ownership shares when $V_1$ falls between...
the monopoly and syndicate strike values $V_1^C$ and $V_1^S$. If the incumbent venture capitalist renegotiates, the lower strike value removes the discontinuity of effort. As the incumbent’s share declines, it is easier to find a $\beta < 1$ that works. The required $\beta$ approaches 1.0 as $V_1$ approaches $V_1^C$ and the incumbent’s share approaches zero.

The third problem arises at high levels of $V_1$. Setting $\beta > \alpha_0^C$ gives the incumbent venture capitalist an incentive to under-report $V_1$. The incumbent would gain more from underpricing the new shares and buying them cheaply than he would lose from dilution of his existing stake. Revelation works only if this incentive is offset by the impact on the entrepreneur’s effort. But as $V_1$ and $\hat{V}_1$ increase, effort becomes higher and less sensitive to the terms of financing. As effort tops out, the incentive to under-report takes over. This could be prevented locally by allowing $\beta$ to decrease with $\hat{V}_1$, returning to $\beta = \alpha_0^C$ at very high values. But then the almost-fixed fraction rule fails to induce truthful information revelation, because each participation fraction $\beta$ would signal two values.

One possible solution, not fully explored here, is to introduce more complex contracts that allow signalling along two dimensions. For example, the incentive for the incumbent venture capitalist to under-report at high levels of $V_1$ could be offset by an incentive contract that grants the entrepreneur extra shares if the incumbent reports very high project value. With this additional provision in place, it should be possible to allow $\beta$ to decrease with $\hat{V}_1$, reaching $\beta = \alpha_0^C$ at high values of $V_1$. This could be one justification for contingent share awards to the entrepreneurs, as observed in Kaplan and Stromberg (2003). Alternatively, the entrepreneur could be granted a series of stock options with increasing exercise prices, so that the entrepreneur’s final ownership share increases at high values of $V_1$.

When the modified fixed fraction rule fails, the syndicate investors face the asymmetric information problem analyzed by Myers and Majluf (1984). In the special case of their model that is closest to our problem here, the firm has no assets in place (no value in liquidation), so it goes ahead with financing on terms fixed by the new investors’ knowledge of the average value of $V_1$. Syndicate investors would have to infer the average $V_1$ from conditions at date 0, the entrepreneur’s effort functions and the entrepreneur’s and incumbent’s decision rules, given the anticipated terms of date-1 financing. But the investors do not know the true value $V_1$, so their new financing is overpriced when $V_1$ is low and underpriced when $V_1$ is high. This leads to
more effort when $V_1$ is low and less when it is high, compared to the full-revelation case. But again there are problems. For example, if $V_1$ is high, the incumbent will be better off cancelling syndicate financing and providing the date-1 money directly. But if this is allowed, then the incumbent will have an incentive to claim a high value $V_1$ in order to reclaim monopoly power over the terms of financing. In addition, if the syndicate investors know less than the incumbent and the incumbent is free to finance the investment from his own pocket, then the incumbent will only raise syndicate financing if the syndicate is paying too much. Therefore a rational syndicate will not invest.\footnote{This is a variation of Myers and Majluf's (1984) pecking-order proofs.}

Even if the revelation mechanism fails, there may be other ways to convey $V_1$. The value of the incumbent investor’s reputation could generate truthful reports in a repeated game setting, for example. The syndicate usually includes other venture capitalists that the incumbent has worked with in the past and expects to work with in the future.

### 1.5 Summary of Numerical Results

Table 1 illustrates our main results. It shows surprisingly large value losses in most cases, relative to first-best. (For now ignore the debt-financing entries.) Value losses are greatest in the monopoly case where the initial venture capitalist provides all financing and dictates the terms of financing at date 1 as well as date 0. This does not imply that the venture capitalist extracts all value, leaving the entrepreneur with zero NPV. The venture capitalist wants to preserve the entrepreneur’s incentives to some extent. Nevertheless, the financing terms that maximize value for the venture capitalist usually leave the entrepreneur with small minority slice of a diminished pie.

The problem with staged financing is that a monopolistic venture capitalist cannot commit not to hold up the entrepreneur ex post. Thus NPV can be higher and the initial venture capitalist better off if staged financing is abandoned and all financing is committed at date 0. Complete upfront financing is superior for all effort parameters (all values of $\theta_r = \theta_f/\theta_g$) when option value is relatively low, as it is for most of the range of standard deviations in Figure 5. Figure 6 shows that when the option value is high, staged financing is more efficient, despite
the monopoly holdup problem.

Syndication of date-1 financing always makes both the entrepreneur and the initial venture capitalist better off as long as the syndicate’s financing terms are reasonably competitive. This key result of our paper is evident in Table 1 and also holds generally. We believe that our syndicate case, in which the initial venture capitalist can set financing terms at date 0 but not date 1, is a good match to actual venture capital contracting. Of course we observe syndication in practice, but that observation does not settle whether the terms of syndicate financing are competitive (NPV = 0) or monopolistic. Our analysis indicates that syndication terms are reasonably competitive. With monopoly financing terms at date 1, we find that the entrepreneur’s final ownership share is a decreasing function of firm value. With competitive terms, as in our syndication case, the entrepreneur’s share is an increasing function of value, consistent with practice (Kaplan and Stromberg (2003)).

The syndicate case is still inefficient, because it gives the initial venture capitalist the bargaining power to set financing terms at date 0. We believe that venture capitalists do have bargaining power and receive at least some (quasi) rents in early financing rounds. They have bargaining power because of experience and expertise, because of the fixed costs of setting up a venture capital partnership and because of the cost and delay that the entrepreneur would absorb in looking for another venture-capital investor. But there are obviously efficiency improvements if and as the terms of date-0 financing become more competitive. The fully competitive case shows the limit where the initial venture capitalist has no special bargaining power and just gets NPV = 0. Even the fully competitive case falls short of first best, however. The entrepreneur’s effort falls whenever outside financing has to be raised, because the entrepreneur bears the full cost of effort, but has to share the marginal value added by effort with the outside investors.

Figure 9 summarizes value losses for the monopoly, no-staging, syndication and fully competitive cases over a wide range of the effort parameter $\theta_r$. Value loss is defined as the difference between NPV at date 0 and first-best NPV. The four panels correspond to panels A to D in Figure 1, except for the variation in $\theta_r$. Figure 10 repeats Figure 9 for a more profitable startup with $V_0 = 200$.

The value losses plotted in Figure 9 increase rapidly with $\theta_r$ when $\theta_r$ is below 1.0, but
the losses are always less in the syndication case than in the monopoly cases. The losses in
the syndicate case still appear economically significant, however. The only situations in which
losses do not appear significant occur in the fully competitive case when $\theta_r$ is above 2.0. High
values for $\theta_r$ mean that effort generates value at relatively little cost, so that the entrepreneur
is willing to expend close to first-best effort in the fully competitive case, even though the
marginal benefit of effort is shared with outside investors.

Value losses in the monopoly, no staging case increase as standard deviation is increased from
$\sigma = 0.4$ to 0.8. This is as expected, since staged financing is more valuable as volatility increases.
But value losses may also increase with standard deviation in the monopoly and syndication
cases, at least for the region where $\theta_r$ is about 1.0 and higher. We found this surprising. Our
original intuition was that increased uncertainty would enhance the optionality of investment
and mitigate incentive problems. Instead it can make these problems worse, because more
uncertainty can lead to lower initial effort.\textsuperscript{22} Compare the bottom-left and bottom-right panels
in Figure 9, for example. The effects of volatility on effort and value can also be seen in panels
E and F of Table 1. In the syndicate case, the value loss in panel E, with $\sigma = 0.4$, is $10.98 - 1.55 = 9.43$. In panel F, with $\sigma = 0.8$, value loss is $27.72 - 17.5 = 10.22$. Initial effort falls from
$x_0 = 3.05$ in panel E to $x_0 = 2.8$ in panel F.

The value-loss patterns in Figure 9 are repeated in Figure 10, where $V_0 = 200$ rather than
150. Financing is feasible in Figure 10 at lower levels of the effort parameter $\theta_r$. Value losses
are lower for the fully competitive case, but actually increase for the monopoly and syndication
cases. This problem can be traced to the initial venture capitalist’s bargaining power at date
0. Consider the monopoly case. Since the marginal product of effort is higher when $V_0 = 200$,
the entrepreneur will put in more effort at date 1. This allows the initial venture capitalist to
tighten the screws and extract a greater ownership share, which in turn feeds back to lower
effort by the entrepreneur at date 0.

The effects of other parameters on our results are generally as expected. NPV increases
when the ratio $\theta_r = \theta_f/\theta_g$ increases. The strike value $V$ falls with $\theta_r$, increasing the probability

\textsuperscript{22}When overall NPV is near zero, the entrepreneur’s effort $x_0$ increases rapidly with $\sigma$. The more uncertainty,
the greater chance that the entrepreneur’s call option will be in the money and the greater the marginal reward
to effort. But as $\theta_r$ increases and NPV rises, effort eventually declines as $\sigma$ increases, because the marginal
impact of effort is less. The difference can be traced to the slope of the cumulative lognormal, which is lower at
the mean when $\sigma$ is high.
that the date-1 option to invest is in the money, and when the option is in the money it is worth more. Overall NPV increases when $\sigma$ increases (generating more uncertainty in $V_1$ and $V_2$) and when a greater fraction of investment can be deferred to date 1. These effects are natural for investments in real options.

1.6 Debt financing

So far we have considered only equity financing, following venture-capital practice. Is equity financing efficient for venture capitalists? We cannot answer this question without deriving optimal contracts, a task that we do not attempt in this paper. But it is interesting to consider the alternative of debt financing. We have interpreted syndicate financing as a device to secure the entrepreneur’s effort by protecting her from ex-post holdup. Could a switch to debt financing achieve the same or better result? In traditional agency models, debt financing calls forth maximum effort, because the entrepreneur retains the maximum fraction of the value added at the margin by her effort. Perhaps we have oversimplified venture-capital practice to the extent that debt dominates equity as a financing contract.

In this section we show that debt is not superior to equity. When effort and investment are made in stages, debt financing can actually amplify the hold-up problem and reduce the entrepreneur’s initial effort at date 0. We will show that debt financing could increase efficiency in some cases, however.

Suppose that the startup firm issues debt rather than equity to venture-capital investors. The face value of the debt equals the required investment. Of course, this debt faces a high probability of default, so the promised payoff at date 2, including interest, is well above face value. (Safe debt is nearly impossible, given the high variance of most startups and most entrepreneurs’ limited funds available for equity investment.) The promised debt payoff (face value plus interest) sets a strike value for $V_2$ below which the startup defaults and the investors receive all of the startup’s payoff $P$. Above this point the investors’ payoff is capped and the entrepreneur receives the residual. Thus debt financing converts the entrepreneur’s stake to a call option, and our discussion of debt financing also applies if the entrepreneur receives no shares but only options. The implicit call options created by debt financing would probably
be out of the money, however, because the promised payment to the venture capitalists would have to exceed total investment by enough to cover the risk of failure and default.

Now we revisit the monopoly case, holding all aspects of our model constant, except that at date 1 the venture capitalist and entrepreneur negotiate a promised debt payoff \( K_1 \) instead of ownership shares \( \alpha^C_1 \) and \( \alpha^M_1 \). (With debt, \( \alpha^M_1 = 1 \).) The entrepreneur’s NPV at date 1 is her expected residual payout at date 2, that is, \( E_0[\max(0, f_1 V_2 - K_1)] \). As before, the entrepreneur chooses effort to maximize \( NPV^M_1 \), and the venture capitalist chooses \( K^*_1 \) to maximize \( NPV^C_1 \). The startup continues if \( NPV^C_1 > I_1 \). At date 0, the entrepreneur anticipates \( K^*_1 \) and chooses initial effort accordingly. We solve numerically for the entrepreneur’s effort, the promised payoff \( K^*_1 \) and the venture capitalist’s and entrepreneur’s NPVs. If both parties’ participation constraints are met at date 0, the venture is launched.

Table 1 shows examples comparing debt versus equity financing in the monopoly case. It is immediately clear that debt is no panacea. When monopoly debt financing is feasible, it can enhance effort and NPV. In Panels B and D of Table 1, for example, initial effort is higher and NPVs increase in the monopoly (debt) cases. The resulting NPVs in these cases approach the fully competitive outcome. But in Panels A, C and F startups that could be financed by a monopolist venture capitalist with equity cannot be financed with debt. This breakdown occurs because debt makes the holdup problem worse. At date 1, the venture capitalist is able to set the promised debt payoff so high that the entrepreneur is left with with a very small slice of value. The entrepreneur’s effort at date 1 is increased, given \( K^*_1 \), because the entrepreneur holds an option and receives all value in excess of \( K^*_1 \). But the entrepreneur’s date-1 NPV is very small, and her effort at date 0 is drastically reduced. It appears that linear equity contracts between the entrepreneur and venture capitalist mitigate the hold-up problem at date 1 and generate more efficient effort at date 0.

In the syndicate financing case, the venture capitalist negotiates an initial debt level \( K^C_0 \) at date 0 in exchange for initial funding. At date 1, if \( V_1 \) is large enough, new, pari passu debt with face value \( K^S_1 \) is issued to a syndicate of investors for zero expected return. The incumbent venture capitalist and syndicate share in debt payouts at date 2 according to their shares of debt ownership \( \frac{K^C_0}{K^C_0 + K^C_1} \) and \( \frac{K^S_1}{K^S_1 + K^S_1} \). The entrepreneur makes the same decisions as in the monopolistic case, but faces the combined debt level \( K^C_0 + K^S_1 \). As before, the syndicate
provides date-1 financing on competitive terms.

Introducing debt contracts in the syndication case increases the entrepreneur’s initial effort, but also increases \( \bar{V} \), the threshold for project continuation. Higher effort increases NPV and higher \( \bar{V} \) reduces it. If the first (second) effect outweighs the second (first), then syndicated debt yields higher (lower) NPV. In Panels A to D and F of Table 1, overall NPV falls when financing is syndicated and debt is substituted for equity. Note the high values for \( \bar{V} \) in Panels A to D and F. Panel E is an exception, where syndicate financing is feasible with debt but not with equity.

Syndication in the debt financing case reduces the initial venture capitalist’s financial flexibility. Since the syndicate does not allow the initial venture capitalist to hold up the entrepreneur ex post, he sets the face value of his own date-0 debt higher in order to extract as much upside as possible. This creates a debt overhang, which translates in turn to higher strike values (higher \( \bar{V} \)s) and to lower NPV.

According to Table 1, the initial venture capitalist should abandon syndication and switch the monopoly debt financing if faced with the parameter values in panels B and D. Why is syndicate debt financing not better than monopoly debt financing? Syndicated debt should eliminate holdup problem at date 1. One answer is that Table 1 does not include potential gains from renegotiation at date 1 before syndicate financing is raised. We discussed renegotiation for the syndicated equity case, but left it out of our calculations. Opportunities for renegotiation are rare with equity financing, and syndicated equity is more efficient than monopoly equity even when potential gains from renegotiation are ignored. With syndicated debt, opportunities for renegotiation are much more common, because of the high strike values created by the initial venture capitalist’s debt holdings, and potential value gains much larger.

Other things equal, including the date-1 strike value \( \bar{V}_1 \), financing on competitive rather than monopolistic terms at date 1 must improve effort and ex ante NPV. But it is not clear whether renegotiation is a reliable mechanism to reduce the high strike values that we have calculated for the debt syndication case. Frequent opportunities for renegotiation may also mean frequent opportunities for the incumbent venture capitalist to reassert his monopoly power. The incumbent could force renegotiation even when the startup is performing well and \( V_1 > \bar{V}_1 \). The syndicate investors, who are unlikely to be fully informed about \( V_1 \), have no
particular reason to object to renegotiation, since their investment is zero-NPV in any case. These issues, which we leave as a topic for further research, are probably second-order with equity financing, because opportunities to add value by renegotiation are rare and only occur at low values of $V_1$, for example in down rounds. These issues may be much more serious with syndicated debt financing.

The syndication results in Table 1 may shed some light on why venture capitalists choose to hold equity when later-stage financing is syndicated. In Panels A through D and F, syndicate financing with equity rather than debt increases both overall NPV and the NPV to the original venture capitalist. Hence, if the choice of financing can be dictated by the venture capitalist at time 0, then he would maximize his NPV by offering equity financing to the entrepreneur. The entrepreneur does better with debt, but cannot bribe the venture capitalist to change to debt. Even if the entrepreneur had independent wealth to finance a bribe, no deal could be struck, because the change from equity to debt would decrease overall value. Panel E is the exception where a switch from equity to debt would make both the entrepreneur and the venture capitalist better off.\(^{23}\)

We summarize our findings on debt versus equity in venture capital as follows. First, debt does not eliminate the holdup problem when incumbent venture capitalists can dictate the terms of later-stage financing. Second, debt financing is not generally more efficient than equity. Even in the fully competitive case, switching from equity to debt sometimes adds value (in Panels C, E and F of Table 1) and sometimes reduces value (Panels A, B and D). Third, debt financing for high-risk startups is equivalent to compensating the entrepreneur exclusively with options rather than shares. We think that the most efficient contract will be a mixture of shares and options for the entrepreneur – or equivalently, a combination of an initial share award plus additional shares conditional on high realized value for the startup. Kaplan and Strömberg (2003) document such contingent payoffs in practice.\(^{24}\)

\(^{23}\)We have not explored differences between debt and equity over all possible parameter values. Thus we cannot rule out cases where a switch from equity to debt in the syndication case increases overall NPV at the venture capitalist's expense. In such cases the entrepreneur could compensate the venture capitalist for the switch, providing she has sufficient wealth. If her wealth is not sufficient, the alternative is to offer the venture capitalist some extra shares, options or warrants at date 1. But then we have a mix of debt and equity financing, which would affect effort and value. For the use of equity securities in debt renegotiation, see Kalay and Zender (1997) and Bhattacharya and Faure-Grimaud (2001).

\(^{24}\)Contingent payoffs are implicit in our model, at least in the syndication and the fully competitive cases,
Contingent compensation for the entrepreneur should also help reveal the startup’s value to potential syndicate investors. Recall our discussion of the modified fixed-fraction rule, in which the incumbent venture capitalist’s participation in date-1 financing can negate his incentive to over-report the value of the startup. We noted that that options or contingent share awards to the entrepreneur may be necessary to prevent a breakdown of revelation at high values of $V_1$.

An investigation of optimal contracts for venture capital will have to address (1) the mix of shares, options or other securities given to the entrepreneur and venture capitalists, (2) bargaining and renegotiation between incumbent venture capitalists and the entrepreneur before additional financing is raised and (3) information revelation. We believe that these three issues are interconnected. We leave them for further research.

1.7 Conclusions

As far as we know, this paper is the first to combine the main features of venture-capital contracting in a consistent formal model. As we expected, the model has no closed-form solution, except in the first-best case, so we embarked on an experiment in computational corporate finance. We show how multiple contractual provisions that are common in venture capital contracts affect the moral hazard, effort provision, asymmetric information and holdup problems in the entrepreneur-venture capital relationship. Venture capital contracting does not solve these problems individually but trades them off. For example, staged financing induces more efficient investment decisions in later stages but creates a potential holdup problem. A commitment to later stage syndication can alleviate the holdup problem between the entrepreneur and the initial venture capitalist but introduces information revelation problems between the incumbent venture capitalist and members of the later stage syndicate.

Venture capital financing comes with efficiency losses. We find significant underinvestment: many positive NPV-projects cannot be financed. For projects that can be financed, there can be large value losses due to under-provision of effort, even for relatively small effort costs. A commitment to syndicate financing in later stages reduces the entrepreneur’s underprovision of effort, increasing overall efficiency. Syndication increases the NPVs of both the entrepreneur

because the entrepreneur’s share of the firm increases as date-1 value increases. See Figures 7 and 8.
and the initial venture capitalist. The venture capitalist’s profits increase despite taking a smaller share than in the monopoly case.

Syndicate financing is most effective when the incumbent venture capitalist’s inside information is revealed through his participation in financing. However, the fixed-fraction participation rule derived by Admati and Pfleiderer (1994) does not work as a revelation mechanism in our model, because the terms of financing affect the entrepreneur’s effort. A modified fixed-fraction rule, in which the incumbent’s fractional participation increases as the reported value increases, can work in some cases. We suggest that a combination of the modified rule with additional contingent share awards to the entrepreneur should work generally, although a full analysis of asymmetric information and revelation will remain a topic for further research.

The startups that venture capitalists invest in are compound call options. Therefore we expected strong option-like behavior, for example a strong dependence of strike values and NPVs on the variance of final payoffs. But this expected behavior was attenuated or overridden by agency and incentive problems. We noted how increased uncertainty dampens the entrepreneur’s effort, for example. This feedback is not a result of risk aversion, because the entrepreneur is assumed risk-neutral. It arises because increased uncertainty reduces the marginal value added by effort when potential value is sufficiently high.

The option-like properties of venture capital investments are also attenuated because financing is feasible only for startups that are well in the money, from a purely financial point of view. They have to be well in the money to overcome incentive problems and costs of effort. That is why all the numerical results presented in this paper assume expected potential value of \( V_0 = 150 \) or 200, versus total investment of only 100. Even with these prospects, financing may not be feasible, even with competitive syndicate financing. Note the negative NPVs in panel E of Table 1, for example.

Of course numerical results are never conclusive. One can never rule out the possibility that results would have been different with different inputs or modeling choices. But we verified our results over a wide range of inputs. Our model, though simplified, follows actual practice in venture capital. Our only judgment calls were the choices of the lognormal distribution for the startup’s value and of exponential functions for the value added and cost of effort. We believe these assumptions are reasonable, but further research could explore alternatives.
1.8 Appendix

1.8.1 Appendix 1. Solving the optimization problems.

First-best.

**Date 1:** The entrepreneur’s date-1 NPV is

\[ NPV_1^M = \max[0, \max_{x_1}(E_1 P(x_1) - g_1(x_1) - I_1)] \]  \hspace{1cm} (1.12)

with first-order condition for effort \( f_0 V_1 = \frac{\partial}{\partial f_1} \). This implies the first-best effort level and function values

\[
\begin{align*}
x_1^* &= \frac{1}{\tilde{\theta}} \ln(\theta_r f_0 V_1) \\
f_1^* &= 1 - (\theta_r f_0 V_1)^{-\theta_f/\tilde{\theta}} \\
g_1^* &= (\theta_r f_0 V_1)^{\theta_g/\tilde{\theta}}
\end{align*}
\]

(1.13)

letting \( \tilde{\theta} = \theta_f + \theta_g, \theta_r = \theta_f/\theta_g \). Substituting \( f_1^* \) and \( g_1^* \) into (1.12) yields

\[
NPV_1^M = \max[0, f_0 V_1 - (f_0 V_1)^{\theta_g/\tilde{\theta}} - I_1]
\]

(1.14)

letting \( \tilde{\theta} = \theta_r^{\theta_f/\tilde{\theta}} + \theta_r^{\theta_g/\tilde{\theta}} \).

Investment proceeds when \( NPV_1^M \geq 0 \). Let \( \bar{V}_1 \) be such that \( NPV_1^M(\bar{V}_1) = 0 \). Then \( \bar{V}_1 \) is calculated numerically from

\[
I_1 = f_0 \bar{V}_1 - (f_0 \bar{V}_1)^{\theta_g/\tilde{\theta}}
\]

(1.15)

**Date 0:** The entrepreneur’s date-0 NPV is the difference between her expected date-1 NPV and the \( t = 0 \) costs of investment and effort:

\[
NPV_0^M = \max[0, \max_{x_0}(E_0 NPV_1^M(x_0) - g_0(x_0) - I_0)]
\]

(1.16)

The NPV expectation is taken by weighting each possible \( NPV_1^M \) realization (Eq. (1.14)) from
\[ V_1 \text{ onwards} \]

\[
E_0 NPV_1^M(x_0) = f_0 \int_{V_1(x_0)}^{\infty} \Pi(V) V dV - \theta f_0^{2\theta/\theta} \int_{V_1(x_0)}^{\infty} \Pi(V) V^{\theta_2/\theta} dV - I_1 \int_{V_1(x_0)}^{\infty} \Pi(V) dV \quad (1.17)
\]

where \( \Pi(V) \) is the lognormal density. Since \( NPV_1^M(x_0, V_1(x_0)) = 0 \), the entrepreneur's first-order condition for effort reduces to

\[
E_0(NPV_1^M(x_0^*))' = g_0'(x_0^*)
\]

with \( E_0(NPV_1^M(x_0))' \) taken from (1.17)

\[
E_0(NPV_1^M(x_0))' = f_0' \left[ \int_{V_1(x_0)}^{\infty} \Pi(V) V dV - \frac{\theta_2}{\theta} \int_{V_1(x_0)}^{\infty} \Pi(V) V^{\theta_2/\theta} dV \right] \quad (1.18)
\]

Change of variables on the lognormal establishes that

\[
\int_{a}^{\infty} \Pi(V) V^k dV = e^{\left(\frac{k^2 \sigma^2}{2} + \mu k\right)} N(0,1) \left( \frac{\mu - \ln a}{\sigma} + k\sigma \right) \quad (1.19)
\]

where \( \mu = E(\ln V) \), \( \sigma^2 = Var(\ln V) \), and \( N(0,1) \) is the standard normal cdf. Using this property the entrepreneur's first-order condition simplifies to

\[
\frac{1}{\theta_r} e^{\theta_2 x_0^*} = V_0 N(0,1) \left( \phi + \frac{1}{2} \frac{\sigma}{\sigma^2} \right) - e^{-\left(\frac{\theta_2^2 \sigma^2}{2\sigma^2}\right)} \int_{0}^{\infty} \Pi(V) V^{\theta_2/\theta} f_0^{\theta_2/\theta} N(0,1) \left( \phi + \sigma \frac{2\theta_2 - \theta}{2\theta} \right) \quad (1.20)
\]

letting \( \phi = \frac{1}{\sigma} \ln \frac{V_0}{V_1} \). We solve numerically for \( x_0^* \) since \( \phi = \phi(V_1(x_0^*)) \). A calculation whether \( NPV_0^M(x_0^*) \geq 0 \) determines whether the initial \( I_0 + g_0 \) investment will be made.

**Monopolist case.**

**Date 1:** The entrepreneur's date-1 effort and NPV are similar to Eqs. (1.12)-(1.14). Firm value is multiplied by the entrepreneur’s share of the project \( \alpha_1^M = 1 - \alpha_1^C \) and \( I_1 \) drops out of
her NPV term, yielding

\[
NPV_1^M(\alpha_1^C) = \max \left[ 0, \max_{x_1} \left( \alpha_1^M E_1 P(x_1) - g_1(x_1) \right) \right]
\]

(1.21)

\[
x_1^* = \frac{1}{\theta} \ln(\theta_r \alpha_1^M f_0 V_1)
\]

\[
f_1^* = 1 - (\theta_r \alpha_1^M f_0 V_1)^{-\theta_f/\theta}
\]

(1.22)

\[
g_1^* = (\theta_r \alpha_1^M f_0 V_1)^{\theta_s/\theta}
\]

\[
NPV_1^M(\alpha_1^C) = \max \left[ 0, \alpha_1^M f_0 V_1 - \bar{g}(\alpha_1^M f_0 V_1)^{\theta_s/\theta} \right]
\]

(1.23)

The venture capitalist chooses \(\alpha_1^C^*\) to maximize his date 1 NPV, equal to his share of the firm value decreased by time 1 investment costs. Plugging in \(f_1^*\) from (1.22) yields

\[
NPV_1^C = \max \left[ 0, \alpha_1^C E_1 P(x_1^*(\alpha_1^C)) - I_1 \right]
\]

(1.24)

\[
= \max \left[ 0, \max_{\alpha_1^C \in (0, \alpha_1^C(\text{max})]} \left( \alpha_1^C f_0 V_1 - \theta_r^{-\theta_f/\theta} (f_0 V_1)^{\theta_s/\theta} \alpha_1^C (1 - \alpha_1^C)^{-\theta_f/\theta} - I_1 \right) \right]
\]

The venture capitalist operates subject to the entrepreneur’s participation constraint of \(NPV_1^M(\alpha_1^C) \geq 0\). The maximum \(\alpha_1^C\) share he can take is obtained by setting the last term in (1.23) equal to zero. This yields

\[
\alpha_1^C(\text{max}) = 1 - \alpha_1^M(\text{min}) = 1 - \frac{c_{\theta_f/\theta}}{\theta_f f_0 V_1}
\]

(1.25)

the limit point for \(\alpha_1^C\) in (1.24). The unconstrained first-order condition for \(\alpha_1^C\) is:

\[
(\theta_r f_0 V_1)^{\theta_f/\theta} = (1 - \alpha_1^C^*)^{-\theta_f/\theta - 1} \left( 1 - \frac{c_{\theta_s/\theta}}{\theta f_0 V_1} \alpha_1^C^* \right)
\]

(1.26)

If \(\alpha_1^C^* \in [0, \alpha_1^C(\text{max})]\), the venture capitalist chooses the unconstrained value; if not, he assigns \(\alpha_1^C(\text{max})\).\(^{25}\)

\(^{25}\)This follows because \(NPV_1^C\) is concave in \(\alpha_1^C\). \(\alpha_1^M(\text{min})\) may be greater than 1 if \(f_0\) is small or the \(V_1\) realization is low. Liquidation would ensue.
However, in most parameterizations the constraint never binds. That is, $V_1$, the continuation point, is typically high enough that the entrepreneur’s participation constraint is always slack. For large $V_1$, $\alpha_1^{C^*}$ grows more slowly than $\alpha_1^C(\text{max})$, so it is only for small $V_1$ (which result from small $I_1$) that the constraint may bind. For realistic parameterizations, the entrepreneur enjoys positive $NPV$ everywhere past (and including) $V_1$.\textsuperscript{26}

Since the venture capitalist typically hits his participation constraint first, his $NPV$ determines $V_1$. We compute $V_1^C$ by looking for the pair $\left(\frac{\partial C}{\partial C^*}(V_1), \alpha_1^{C^*}(V_1)\right)$ that sets $NPV^C_1 = 0$. This pair solves, from Eq. (1.24),

$$I_1 = \alpha_1^{C^*}(V_1) f_0 V_1 - \left(f_0 V_1^C\right)^{\theta_f/\theta} \alpha_1^{C^*}(V_1^C) \left(\theta_f \left(1 - \alpha_1^{C^*}(V_1^C)\right)\right)^{-\theta_f/\theta}$$

(1.27)

and from Eq. (1.26)

$$\left(\theta_f f_0 V_1^C\right)^{\theta_f/\theta} \left(1 - \alpha_1^{C^*}(V_1^C)\right)^{-\theta_f/\theta - 1} \left(1 - \frac{\theta_f}{\theta} \alpha_1^{C^*}(V_1^C)\right)$$

(1.28)

Solving this system of equations implies

$$\frac{I_1}{\theta_f f_0} = \frac{\frac{\partial C}{\partial C^*}(V_1^C)}{\alpha_1^{C^*}(V_1^C)^2}$$

(1.29)

after first solving for $\alpha_1^{C^*}(V_1^C)$ numerically from

$$I_1 = \frac{\theta_f}{\theta} \left(\alpha_1^{C^*}(V_1^C)\right)^2 \left(1 - \frac{\theta_f}{\theta} \alpha_1^{C^*}(V_1^C)\right)^{\frac{1}{\theta_f}}$$

(1.30)

\textsuperscript{26}One can look at the entrepreneur’s positive $NPV$ intuitively or mathematically. Intuitively, the venture capitalist is better off when the entrepreneur has a higher incentive to provide effort. $\alpha_1^C(\text{max})$ is generally very high, and venture capitalist does better by taking a smaller than maximum share.

Mathematically, the continuation point $V_1$ is determined by the venture capitalist’s $NPV$ because of the additive investment cost function. Since $I_1$ does not affect $FOC_1^C$ or $FOC_1^M$, by altering $I_1$ one alters the venture capitalist’s profitability without changing the optimal $\alpha_1^{C^*}$ and the entrepreneur’s profitability. Reasonable $I_1$ is high enough to make the venture capitalist’s $NPV$ surpass 0 without causing the entrepreneur’s to fall below 0.
Investment occurs if $V_1 > \bar{V}_1^C$.

**Date 1:** The entrepreneur’s date 0 value is

$$NPV_0^M = \max[0, \max_{x_0}(E_0 NPV_1^M(x_0) - g_0(x_0))]$$  \hspace{1cm} \text{(1.31)}$$

with first-order condition

$$E_0(NPV_1^M(x_0^*))' = \Pi \left( \bar{V}_1^C \right) NPV_1^M \left( f_0^*, \bar{V}_1^C \right) \bar{V}_1^{C'} + g_0'(x_0^*)$$  \hspace{1cm} \text{(1.32)}$$

To compute $(NPV_1^M(x_0^*))'$, we need the formula for $\frac{\partial \alpha_1^{C^*}}{\partial x_0}$ or $\alpha_1^{C^*}$. Using the implicit function theorem on Eq. (1.26) yields

$$\alpha_1^{C^*} = \frac{f_0'(1 - \alpha_1^{C^*})(1 - \theta \frac{\theta}{\theta})}{f_0' \left( 2 - \theta \frac{\theta}{\theta} \alpha_1^{C^*} \right)}$$  \hspace{1cm} \text{(1.33)}$$

Deriving entrepreneur value (Eq. (1.23)) with respect to initial effort, taking its expectation over all possible $V_1$ realizations from $\bar{V}_1^C$ onwards, and using (1.33) yields the left-hand side term in (1.32):

$$E_0(NPV_1^M(x_0^*))' = f_0' \left[ \int_{\bar{V}_1(x_0)}^{\infty} \Pi(V) \frac{\alpha_1^{M*}}{\left( 2 - \theta \frac{\theta}{\theta} \alpha_1^{C^*} \right)} dV - \theta \frac{\theta}{\theta} \int_{\bar{V}_1(x_0)}^{\infty} \Pi(V) \frac{\alpha_1^{M*}}{\left( 2 - \theta \frac{\theta}{\theta} \alpha_1^{C^*} \right)} dV \right]$$  \hspace{1cm} \text{(1.34)}$$

The middle term in Eq. (1.32) is evaluated as follows. $NPV_1^M$ is found plugging $\bar{V}_1^C$ and $\alpha_1^{C^*} \left( \bar{V}_1^C \right)$ from Eqs (1.29) and (1.30) into Eq. (1.23). $\Pi \left( \bar{V}_1^C \right)$ is taken from the lognormal distribution. For $\bar{V}_1^{C'}$, the implicit function theorem on Eq. (1.29) and $\alpha_1^{C^*}$ from Eq. (1.33) yield

$$\bar{V}_1^{C'} = \frac{-f_0' \bar{V}_1^C}{f_0' \alpha_1^{C^*}}$$  \hspace{1cm} \text{(1.35)}$$

We solve numerically for $x_0^*$ from Eq. (1.32). We evaluate the integrals point-by-point, ending the summation at 6 times the standard deviation of $V_1$. Given $x_0^*$, and assuming that the entrepreneur wants to continue, i.e. $NPV_0^M(x_0^*) > 0$, the venture capitalist decides whether
to invest. His option is worth

\[ NPV_0^C = \max[0, E_0NPV_1^C(x_0^*) - I_0] \]  

(1.36)

with

\[
E_0NPV_1^C(x_0^*) = \begin{bmatrix}
\int_{V_1(x_0^*)}^\infty \Pi(V)\alpha_1^{C^*}VdV \\
-\theta_r^{\theta_f/\theta} \int_{V_1(x_0^*)}^{\infty} \Pi(V)\alpha_1^{C^*}(1 - \alpha_1^{C^*})^{-\theta_f/\theta}V^{\theta_f/\theta}dV - I_1 \int_{V_1(x_0^*)}^\infty \Pi(V)dV
\end{bmatrix}
\]

(1.37)

If \( NPV_0^C > 0 \) the venture capitalist will provide funding.

**Monopolist case, no staging.**

**Date 1:** The entrepreneur’s date-1 NPV and effort is determined as in Eqs. (1.21-1.22) except that \( \alpha_1^M \), the manager’s date 1 project share, has already been negotiated and set at date 0. Denoting this as \( \alpha_1^M = \alpha_0^{M^*} \), the manager’s date-1 \( NPV \) simplifies to

\[ NPV_1^M = \max \left[ 0, \alpha_0^{M^*}f_0V_1 - \tilde{\theta}(\alpha_0^{M^*}f_0V_1)^{\theta_f/\theta} \right] \]  

(1.38)

Because all funding \((I_0 + I_1)\) has been supplied upfront, the venture capitalist no longer has any decisions to make at \( t = 1 \). The entrepreneur now makes the continuation decision, so that \( V_1 \), the continuation point, is found by setting \( NPV_1^M = 0 \). This yields

\[ V_1^M = \frac{\theta^{\theta_f/\theta}f_0}{\alpha_0^{M^*}f_0} \]  

(1.39)

Letting \( \alpha_1^C = \alpha_0^{C^*} \), the venture capitalist’s \( NPV \) (1.24) simplifies to

\[ NPV_1^C = \begin{cases} 
0 & \text{if } V_1 < V_1^M \\
\alpha_0^{C^*}f_0V_1 - \alpha_0^{C^*}(f_0V_1)^{\theta_f/\theta} (\theta_r (1 - \alpha_0^{C^*}))^{-\theta_f/\theta} & \text{if } V_1 \geq V_1^M
\end{cases} \]  

(1.40)

**Date 0:** At date 0, the venture capitalist decides his ultimate project share \( \alpha_0^{C^*} \) and the
entrepreneur decides how much effort to exert. For a given $\alpha_0^C$, the entrepreneur’s value is

$$NPV_0^M(\alpha_0^C) = \max \{ 0, \max_{x_0} (E_0 NPV_1^M (x_0(\alpha_0^C), \alpha_0^C) - g_0 (x_0(\alpha_0^C))) \}$$  \hspace{1cm} (1.41)$$

The expectation of the entrepreneur’s date-1 NPV is found by weighting (1.38) over possible $V_1$ realizations. Again letting $\alpha_1^M = \alpha_0^M$ and using the strike value from (1.39), we obtain

$$E_0 NPV_1^M (x_0(\alpha_0^C), \alpha_0^C) = \alpha_0^M f_0 \int_{V_1^M} \Pi (V) \, dV - \tilde{\theta}(\alpha_0^M f_0, \theta) \int_{V_1^M} \Pi (V) V^\theta \, dV$$  \hspace{1cm} (1.42)$$

Change of variables on the lognormal (as in the first-best case) establishes that

$$E_0 NPV_1^M (x_0(\alpha_0^C), \alpha_0^C) = \alpha_0^M f_0 V_0 N(0,1) \left( \phi + \frac{1}{2} \sigma \right) - e \left( -\theta \frac{\sigma^2}{2} \right) \tilde{\theta}(\alpha_0^M f_0, \theta) \left( \frac{2\theta - \theta}{2\theta} \right)$$  \hspace{1cm} (1.43)$$

leading to the entrepreneur’s first-order condition for effort

$$\frac{1}{\tilde{\theta}} e^{\theta x^*_0} = \alpha_0^M V_0 N(0,1) \left( \phi + \frac{1}{2} \sigma \right) - e \left( -\theta \frac{\sigma^2}{2} \right) \tilde{\theta}(\alpha_0^M V_0, \theta) \left( \frac{2\theta - \theta}{2\theta} \right)$$  \hspace{1cm} (1.44)$$

letting $\phi = \frac{1}{\sigma} \ln \frac{V_0}{V_1^M}$. Again we solve numerically for $x^*_0$.

The venture capitalist anticipates $x^*_0(\alpha_0^C)$ when he sets $\alpha_0^C$. He must restrict his search to $\alpha_0^C \in (0, \alpha_0^C(\max)]$, where $\alpha_0^C(\max)$ is such that

$$E_0 NPV_1^M (x_0^*(\alpha_0^C(\max)), \alpha_0^C(\max)) - g_0 (x_0^*(\alpha_0^C(\max))) = 0$$  \hspace{1cm} (1.45)$$

This constraint rarely binds, and $\alpha_0^C*$ will be chosen to maximize

$$NPV_0^C = \max \left[ 0, \max_{\alpha_0^C(\max) \geq \alpha_0^C} E_0 NPV_1^C (x_0^*(\alpha_0^C), \alpha_0^C(\max)) - I_0 - I_1 \right]$$  \hspace{1cm} (1.46)$$

Taking the expectation of (1.40) and using $x^*_0$ from the entrepreneur’s maximization and the
strike value from (1.39), we obtain

\[
E_0 NPV_1^C(x_0^*(\alpha_0^C), \alpha_0^C) = \alpha_0^C f_0 \int_{\overline{V}_1^M}^{\infty} \Pi(V) V dV
\]

\[
= \alpha_0^C f_0 V_0 N_0(\alpha) \left( \phi + \frac{1}{2} \sigma \right)
\]

\[
(1.47)
\]

\[
- \alpha_0^C (1 - \alpha_0^C)^{-\theta_f/\theta} v_0(\alpha) \int_{\overline{V}_1^M}^{\infty} \Pi(V) V^{\theta_s/\theta} dV
\]

\[
= \alpha_0^C f_0 V_0 N_0(\alpha) \left( \phi + \frac{1}{2} \sigma \right)
\]

\[
(1.48)
\]

again using change of variables and letting \( \phi = \frac{1}{\sigma} \ln \frac{V_0}{\overline{V}_1^M} \). We solve for \( \alpha_0^C \) numerically, since \( f_0 = f_0(x_0^*(\alpha_0^C)) \). If \( NPV_0^C \geq 0 \) investment will proceed.

**Syndicate case.**

**Date 1:** At date 1, for a given share \( \alpha_1^M \), the entrepreneur’s NPV and maximization problem are the same as before. We obtain \( x_1^*, f_1^*, g_1^* \), and \( NPV_1^M(\alpha_1^S) \) exactly as in Eq.s (1.21)-(1.23), but where \( \alpha_1^M = \alpha_0^M(1 - \alpha_1^S) \). The share given to the outside syndicate, \( \alpha_1^S \), is determined by setting \( NPV_1^S \) equal to zero, which yields

\[
I_1 = \alpha_1^S f_0 f_1^*(\alpha_1^S) V_1
\]

\[
(1.49)
\]

\[
= \alpha_1^S f_0 V_1 - \alpha_1^S (1 - \alpha_1^S)^{-\theta_f/\theta} (f_0 V_0) V_1^{\theta_s/\theta} (\theta_r \alpha_0^M)^{-\theta_f/\theta}
\]

\[
(1.50)
\]

after substituting \( f_1^* \). This is solved numerically for \( \alpha_1^S \).

We solve for \( \overline{V}_1 \) using the same procedure as in the monopolistic venture capitalist scenario. As previously, \( \overline{V}_1 \) may imply that \( NPV_1^M(\overline{V}_1) > 0 \) or that \( NPV_1^M(\overline{V}_1) = 0 \). Again, for realistic values of \( I_1 \) the entrepreneur’s participation constraint is always slack. Therefore we identify \( \overline{V}_1 \) by finding the point where the maximum value to the syndicate is zero. At this point the value of \( \alpha_1^S \) implied by the syndicate’s zero-profit condition (1.49) is equal to the value implied by the syndicate’s hypothetical maximization problem. The syndicate’s hypothetical
FOC$_1^S$ is:\footnote{In general there will be two solutions to the zero-profit equation, $\alpha_1^S$ and $\alpha_1^S$. $\alpha_1^S$ is the maximum share which can be given to the syndicate and still return $NPV^S = 0$; $\alpha_1^S$ is the minimum share. For $\alpha_1^S \in (\overline{\alpha}_1^S, \overline{\alpha}_1^S)$, the syndicate enjoys positive expected profits (indeed these two values can be thought of as bounds for the profit-maximizing share). When $V_1 = \overline{V}_1$, then $\alpha_1^S = \overline{\alpha}_1^S = \overline{\alpha}_1^S$. We use $\overline{\alpha}_1^S$ in all computations.}

\[
(\theta_r \alpha_0^M f_0 V_1)^{\theta_f/\theta} = (1 - \alpha_1^{S*})^{-\theta_f/\theta - 1} \left( 1 - \frac{\theta_g}{\theta} \alpha_1^{S*} \right) \tag{1.51}
\]

We solve for $\overline{V}_1$ by finding the pair $\left( \overline{V}_1^S, \alpha_1^S \left( \overline{V}_1 \right) \right)$ such that $\alpha_1^S \left( \overline{V}_1^S \right) = \alpha_1^{S*} \left( \overline{V}_1^S \right)$. We compute $\alpha_1^S \left( \overline{V}_1^S \right)$ numerically from

\[
I_1 \alpha_0^M = \frac{\theta_g}{\theta} \left( \alpha_1^S \left( \overline{V}_1^S \right) \right)^2 \left( 1 - \frac{\theta_g}{\theta} \alpha_1^S \left( \overline{V}_1^S \right) \right)^{1/\theta_g} \left( 1 - \alpha_1^S \left( \overline{V}_1^S \right) \right)^{1+\theta/\theta_g} \tag{1.52}
\]

Using FOC$_1^S$ (Eq. (1.51)) once again, we compute $\overline{V}_1^S$ as

\[
\overline{V}_1^S = \frac{I_1 \left( 1 - \frac{\theta_g}{\theta} \alpha_1^S \left( \overline{V}_1^S \right) \right)}{\frac{\theta_g}{\theta} f_0 \left( \alpha_1^S \left( \overline{V}_1^S \right) \right)^2} \tag{1.53}
\]

These two expressions are nearly identical to Eqs. (1.29) and (1.30), pertaining to the monopolistic venture capitalist’s breakeven point. The sole difference in the expressions is the presence of $\alpha_0^M$, the entrepreneur’s initial project share, on the left-hand side of Eq. (1.52). This lowers $\alpha_1^S \left( \overline{V}_1^S \right)$, implying a higher $\overline{V}_1^S$ than in the monopolistic venture capitalist scenario.

Investment at date 1 occurs for $V_1 > \overline{V}_1^S$.

\textbf{Date 0:} At date 0, the venture capitalist decides his project share $\alpha_0^C*$ and the entrepreneur decides how much effort to exert. For a given $\alpha_0^C$, the entrepreneur’s value is

\[
NPV_0^M (\alpha_0^C) = \max\{0, \max_{x_r} (E_0 NPV_1^M (x_0(\alpha_0^C), \alpha_0^C) - g_0 (x_0(\alpha_0^C))) \} \tag{1.54}
\]
Using the solutions for $f_1^*, g_1^*$, and $\alpha_i^S$ from Eqs. (1.22) and (1.49), we get

$$E_0NPV_{1M}(x_0(\alpha_0^C), \alpha_0^C) = \alpha_0^M f_0 \int_{\overline{V}_1(x_0(\alpha_0^C), \alpha_0^C)} \Pi(V) V (1 - \alpha_1^S)dV$$

$$-\tilde{\theta}(\alpha_0^M f_0)^{\theta_\theta/\theta_\theta} \int_{\overline{V}_1(x_0(\alpha_0^C), \alpha_0^C)} \Pi(V) (V (1 - \alpha_1^S))^{\theta_\theta/\theta_\theta}dV$$

We solve for $x_0^*(\alpha_0^C)$ numerically, exactly as in the monopolistic venture capitalist case, only using the lower $\alpha_1^S$ share in the entrepreneur's NPV term.

The venture capitalist anticipates $x_0^*(\alpha_0^C)$ when he sets $\alpha_0^C$. He must restrict his search to $\alpha_0^C \in (0, \alpha_0^C(\text{max})]$, where $\alpha_0^C(\text{max})$ is such that

$$E_0NPV_{1M}^*(x_0^*(\alpha_0^C(\text{max})), \alpha_0^C(\text{max})) - g_0(x_0^*(\alpha_0^C(\text{max}))) = 0$$

(1.56)

This constraint rarely binds, and $\alpha_0^C$ will be chosen to maximize

$$NPV_0^C = \max \left[ 0, \max_{\alpha_0^C \leq \alpha_0^C(\text{max})} E_0NPV_{1M}^*(x_0^*(\alpha_0^C), \alpha_0^C) - I_0 \right]$$

(1.57)

Using the solutions for $f_1^*, g_1^*$, and $\alpha_i^S$ from Eqs (1.22) and (1.49), and $x_0^*$ from the entrepreneur's maximization, we obtain

$$E_0NPV_{1C}(x_0^*(\alpha_0^C), \alpha_0^C) = \alpha_0^C f_0 \int_{\overline{V}_1(x_0^*(\alpha_0^C), \alpha_0^C)} \Pi(V) V (1 - \alpha_1^S)dV$$

$$-\alpha_0^C (1 - \alpha_0^C)^{\theta_f/\theta_f} f_0^{\theta_\theta/\theta_\theta} \int_{\overline{V}_1(x_0^*(\alpha_0^C), \alpha_0^C)} \Pi(V) (V (1 - \alpha_1^S))^{\theta_\theta/\theta_\theta}dV$$

(1.58)

We solve for $\alpha_0^C$ numerically, again evaluating the integrals point-by-point, ending the summation at 6 standard deviations of $V_1$. If $NPV_0^C \geq 0$ investment will proceed.
1.8.2 Appendix 2. Syndicate Financing: Asymmetric Information.

The Case of no effort at $t = 1$.

Consider a project where there is no date 1 effort, only the $I_1$ cost. In expectation the project is worth (letting $f_0 = 1$ for simplicity) $P = V_1 - I_1$. The original venture capitalist’s exposure is

$$NPV_1^C = \alpha_1^C V_1 - \beta I_1$$ \hspace{1cm} (1.59)

where

$$\alpha_1^C = \alpha_0^C + \alpha_1^S (\beta - \alpha_0^C)$$ \hspace{1cm} (1.60)

If the truth is reported in equilibrium, then

$$\alpha_1^S = \frac{I_1}{V_1} = \frac{I_1}{\hat{V}_1} = \hat{\alpha}_1^S$$ \hspace{1cm} (1.61)

To make the capitalist indifferent between telling the truth and lying about $V_1$, we take the derivative of Eq. (1.59) with respect to $\hat{V}_1$ and set it equal to zero which yields

$$V_1 \frac{\partial \alpha_1^C}{\partial \hat{V}_1} = I_1 \frac{\partial \beta}{\partial \hat{V}_1}$$ \hspace{1cm} (1.62)

or, using $\hat{\alpha}_1^S \hat{V}_1 = I_1$,

$$V_1(\beta - \alpha_0^C) \frac{-I_1}{\hat{V}_1^2} + I_1 \frac{\partial \beta}{\partial \hat{V}_1} = I_1 \frac{\partial \beta}{\partial \hat{V}_1}$$ \hspace{1cm} (1.63)

The only solution is $\beta = \alpha_0^C$. Similarly, we can derive the solution when $I_1$ can vary (for instance if $I_1$ influences the probability of favorable $V_2$ realizations).

The Case of $t = 1$ effort by the entrepreneur.

When we have endogenous effort,

$$NPV_1^C = \alpha_1^C f_1 V_1 - \beta I_1$$ \hspace{1cm} (1.64)
and
\[ \alpha_1^S = \frac{I_1}{f_1 V_1} = \frac{I_1}{f_1 \hat{V}_1} = \hat{\alpha}_1^S \]  
(1.65)

Differentiating by \( \hat{V}_1 \), we obtain
\[ V_1 \left[ f_1 \frac{\partial \alpha_1^C}{\partial \hat{V}_1} + \alpha_1^C \frac{\partial f_1}{\partial \hat{V}_1} \right] = I_1 \frac{\partial \beta}{\partial \hat{V}_1} \]  
(1.66)

or
\[ V_1 f_1 \frac{\partial f_1}{\partial \hat{V}_1} + \left( \frac{1}{f_1} \frac{\partial f_1}{\partial \hat{V}_1} + \frac{1}{V_1} \right) \left( \beta - \alpha_0^C \right) + I_1 \frac{\partial \beta}{\partial \hat{V}_1} + V_1 \alpha_1^C \frac{\partial f_1}{\partial \hat{V}_1} = I_1 \frac{\partial \beta}{\partial \hat{V}_1} \]  
(1.67)

Enforcing \( \hat{V}_1 = V_1 \) and using \( \alpha_1^S f_1 V_1 = I_1 \), this reduces to
\[ (\beta - \alpha_0^C) \frac{I_1}{V_1} = \frac{\partial f_1}{\partial \hat{V}_1} \left( V_1 \alpha_1^C - V_1 (\beta - \alpha_0^C) \alpha_1^S \right) = V_1 \alpha_0^C \frac{\partial f_1}{\partial \hat{V}_1} \]  
(1.68)

that is
\[ \frac{\alpha_1^C - \alpha_0^C}{\alpha_0^C} = \frac{V_1 \frac{\partial f_1}{\partial \hat{V}_1}}{f_1 \frac{\partial f_1}{\partial \hat{V}_1}} \]  
(1.69)

First we note that if the effort did not change with \( V_1 \), the right-hand side would equal zero requiring again that \( \alpha_1^C = \alpha_0^C \) or \( \beta = \alpha_0^C \). However when \( \frac{\partial f_1}{\partial \hat{V}_1} > 0 \), \( \alpha_1^C \) must be greater than \( \alpha_0^C \) in order to prevent overreporting of \( \hat{V}_1 \). The gap between \( \alpha_1^C \) and \( \alpha_0^C \) is responsible for the under-reporting motive, because for high \( V_1 \) realizations the venture capitalist can increase his share of the project, relative to \( \alpha_0^C \), on an \( NPV > 0 \) basis.
1.8.3 Appendix 3. Debt Financing

Monopolist debt financing.

**Date-2:** The final payouts in the debt financing case are options on final firm value:

\[
NPV_2^M = \max(0, f_0 f_1 V_2 - K_1)
\]

\[
NPV_2^C = f_0 f_1 V_2 - \max(0, f_0 f_1 V_2 - K_1)
\]

Let \(\overline{V}_2\) identify the point at which the entrepreneur begins receiving cashflows or

\[
\overline{V}_2 = \frac{K_1}{f_0 f_1}
\]

**Date-1:** The entrepreneur’s date-1 effort and NPV must be solved for numerically with debt financing. The entrepreneur’s NPV is the probability weighted expectation of her \(t = 2\) payouts past \(\overline{V}_2\). That is

\[
NPV_1^M(K_1, x_1) = f_0 \int_{\overline{V}_2(x_1)}^{\infty} \Pi(V) f_1(x_1) V dV - K_1 \int_{\overline{V}_2(x_1)}^{\infty} \Pi(V) dV - g_1(x_1)
\]

Change of variables on the lognormal establishes that

\[
NPV_1^M(K_1) = \max[0, \max_{x_1} (f_0 f_1 V_1 N(0,1) \left( \frac{\phi + \sigma}{2} \right) - K_1 N(0,1) \left( \frac{\phi - \sigma}{2} \right) - g_1(x_1))]
\]

where \(N(0,1)\) is standard normal cdf and \(\phi = \frac{1}{\sigma} \ln \frac{\overline{V}_2}{V_2}\). We solve numerically for \(x_1^*\) since \(\phi = \phi(\overline{V}_2(x_1^*))\).

The venture capitalist chooses \(K_1^*\) to maximize his date 1 NPV, anticipating \(f_1^*(K_1)\) from the entrepreneur’s maximization. The venture capitalist’s NPV is derived similarly to the entrepreneur’s, yielding

\[
NPV_1^C = \max[0, \max_{K_1 \in (0, K_1(\text{max})]} (f_0 f_1 V_1 \left( 1 - N(0,1) \left( \frac{\phi + \sigma}{2} \right) \right) + K_1 N(0,1) \left( \frac{\phi - \sigma}{2} \right) - I_1)]
\]
As before, the venture capitalist operates subject to the entrepreneur’s participation constraint of $NPV_1^M(K_1) \geq 0$. The maximum $K_1$ level he can take is obtained numerically, as is $K_1^*$. If $K_1^* \in [0, K_1(\text{max})]$, the venture capitalist chooses the unconstrained value; if not, he assigns $K_1(\text{max})$.

Since we impose that $NPV_1^M(K_1) \geq 0$, we solve for the continuation point $\bar{V}_1^C$ by setting the venture capitalist’s maximized NPV from (1.74) to zero. Investment occurs if $V_1 > \bar{V}_1^C$.

**Date-0:** The entrepreneur’s date 0 value is

$$NPV_0^M = \max[0, \max_{x_0}(E_0 NPV_1^M(x_0) - g_0(x_0))] \quad (1.75)$$

where

$$E_0 NPV_1^M(x_0) = \int_{V_1^C}^{\infty} \Pi(V) f_1^* V N(0,1) \left( \phi + \frac{\sigma}{2} \right) dV$$

$$- \int_{\bar{V}_1^C}^{\infty} \Pi(V) K_1^* N(0,1) \left( \phi - \frac{\sigma}{2} \right) dV - \int_{\bar{V}_1^C}^{\infty} \Pi(V) g_1^* dV \quad (1.77)$$

Recall that each $V_1$ realization implies $K_1^*(V_1), f_1^*(K_1(V_1)), \phi(\bar{V}_2(K_1^*, f_1^*))$. We solve for $x_0^*$ numerically, evaluating the integrals point-by-point, ending the summation at 6 times the standard deviation of $V_1$.

Given $x_0^*$, and assuming that the entrepreneur wants to continue, i.e. $NPV_0^M(x_0^*) > 0$, the venture capitalist decides whether to invest. His option is worth

$$NPV_0^C = \max[0, E_0 NPV_1^C(x_0^*) - I_0] \quad (1.78)$$
with
\[
E_0 NPV_C^C(x_0) = \int_\mathcal{V}_1^c \Pi(V) f_1^V \left( 1 - N_{0,1} \left( \phi + \frac{\sigma}{2} \right) \right) dV
\]
\[+ \int_\mathcal{V}_1^c \Pi(V) K_1^C N_{0,1} \left( \phi - \frac{\sigma}{2} \right) dV - \int_\mathcal{V}_1^c \Pi(V) I_2 dV_0 \]
(1.79)

If \( NPV_0^C > 0 \) the venture capitalist will provide funding.

**Debt financing with date-1 syndication.**

**Date-2:** At \( t = 1 \) a syndicate of investors provides \( I_1 \), purchasing a pari passu debt issuance on competitive terms. Denoting this debt level as \( K_1^S \), and defining \( K \) as the total debt due or \( K = K_0^C + K_1^S \), final payouts are

\[
NPV_2^M = \max(0, f_0 f_1 V_2 - K)
\]
\[
NPV_2^C = \frac{K_0^C}{K} f_0 f_1 V_2 - \max(0, \frac{K_0^C}{K} f_0 f_1 V_2 - K_0^C)
\]
\[
NPV_2^S = \frac{K_1^S}{K} f_0 f_1 V_2 - \max(0, \frac{K_1^S}{K} f_0 f_1 V_2 - K_1^S)
\]
(1.80)

Let \( V_2^S \) identify the point at which the entrepreneur begins receiving cash flows or

\[
V_2^S = \frac{K}{f_0 f_1}
\]
(1.81)

**Date-1:**

At date 1, for a given combined debt level \( K \), the entrepreneur’s NPV and maximization problem are the same as before. We obtain \( x_1^*(K), f_1^*(K), g_1^*(K), \) and \( NPV_1^M(K) \) from Eq. (1.73), but using \( K = K_0^C + K_1^S \) rather than \( K_1^* \).

The face value of debt issued to the outside syndicate is determined by setting \( NPV_1^S \) equal to zero. Taking the expectation of his \( t = 2 \) payouts from (1.80) and using change of variables
yields

$$NPV_1^S(K_1^S) = \frac{K_1^S}{K} f_0 f_1 V_1 \left( 1 - N_{(0,1)} \left( \phi + \frac{1}{2} \sigma \right) \right) + K_1^S N_{(0,1)} \left( \phi - \frac{1}{2} \sigma \right) - I_1$$  \hspace{1cm} (1.82)

where $\phi = \frac{1}{2} \ln \frac{V_1}{V_2}$. This implies

$$I_1 = \frac{K_1^S}{K} f_0 f_1 V_1 \left( 1 - N_{(0,1)} \left( \phi + \frac{1}{2} \sigma \right) \right) + K_1^S N_{(0,1)} \left( \phi - \frac{1}{2} \sigma \right)$$  \hspace{1cm} (1.83)

Substituting $f_1^*(K)$ from the entrepreneur’s maximization, this is solved numerically for $K_1^S$.

We solve for $V_1$ using the same procedure as in the monopolistic venture capitalist scenario, finding the point where the maximum value to the syndicate is zero. There are two differences in the syndicate’s and monopolistic venture capitalist’s maximization problem, both stemming from the incumbent venture capitalist’s outstanding debt level. First, the debt default point, $\bar{V}_2$, is higher by $K_0^C$. Second, the presence of $K_1^S$, the syndicate’s share of total debt, reduces the syndicate’s share of firm value when debt defaults. Both imply a higher $\bar{V}_1^S$ than in the monopolistic venture capitalist scenario. Investment at date 1 occurs for $V_1 > \bar{V}_1^S$.

**Date-0:**

At date 0, the venture capitalist decides his debt level $K_0^C*$ and the entrepreneur decides how much effort to exert. For a given $K_0^C$, the entrepreneur’s value is

$$NPV_0^M(K_0^C) = \max \left[ 0, \max_{x_0} \left( E_0 NPV_1^M(x_0(K_0^C), K_0^C) - g_0(x_0(K_0^C)) \right) \right]$$  \hspace{1cm} (1.84)

where

$$E_0 NPV_1^M(x_0(K_0^C), K_0^C) = f_0 \int_{\bar{V}_1^S}^{\infty} \Pi(V) f_1^* V N_{(0,1)} \left( \phi + \frac{1}{2} \sigma \right) dV - \int_{\bar{V}_1^S}^{\infty} \Pi(V) K N_{(0,1)} \left( \phi - \frac{1}{2} \sigma \right) dV - \int_{\bar{V}_1^S}^{\infty} \Pi(V) g_1^* dV$$  \hspace{1cm} (1.85)

Each $V_1$ realization implies $K(V_1), f_1^*(K(V_1))$, and $\phi(\bar{V}_2(K, f_1^*))$. We solve for $x_0^*(K_0^C)$ numerically, exactly as in the monopolistic venture capitalist case, but using the combined debt level.
\( K = K^C + K^S \).

The venture capitalist anticipates \( x^*_0(K^C) \) when he sets \( K^C \). He restricts his search to \( K^C \in (0, K^C_{\text{max}}] \), where \( K^C_{\text{max}} \) is such that

\[
E_0NPV^M_1(x^*_0(K^C_{\text{max}}), K^C_{\text{max}}) - g_0(x^*_0(K^C_{\text{max}})) = 0
\] (1.86)

This constraint rarely binds, and \( K^C_0^* \) is chosen to maximize

\[
NPV^C_0 = \max[0, \max_{K^C_0 \geq K^C_{\text{max}}} E_0NPV^C_1(x^*_0(K^C_0), K^C_0) - I_0]
\] (1.87)

Using \( x^*_0 \) from the entrepreneur’s maximization, the venture capitalist’s expected \( t = 1 \) value is

\[
E_0NPV^C_1(x^*_0(K^C_0), K^C_0) = K^C_0 f_0 \int_{V^1_i}^{\infty} \Pi(V) f_1^* \frac{V}{K} \left( 1 - N_{(0,1)} \left( \phi + \frac{1}{2} \sigma \right) \right) dV \\
+ K^C_0 \int_{V^2_i}^{\infty} \Pi(V) N_{(0,1)} \left( \phi - \frac{1}{2} \sigma \right) dV
\] (1.88)

We solve for \( K^C_0^* \) numerically, again evaluating the integrals point-by-point, ending the summation at 6 standard deviations of \( V_1 \). If \( NPV^C_0 \geq 0 \) investment will proceed.
1.9 Tables and Figures

Table 1.1: Example of numerical results for nine cases: (1) No cost of effort (Black-Scholes), (2) First-best, (3) Monopoly (one investor provides all financing), (4) 100% Upfront financing, (5) Syndicate Financing at date 1, (6) Fully Competitive financing at dates 0 and 1, (7) Monopoly Debt Financing (investor sets date 1 debt level instead of equity share), (8) Syndicate Debt Financing at date 1 (initial investor sets debt level and syndicate sets pari-passu debt level at date 1) and (9) Fully Competitive Debt Financing at dates 0 and 1. The entrepreneur's initial ownership share or contracted debt level, where relevant, is $\alpha_0^M$ and $K_0$ and effort is $x_0$. $\overline{V}$ is the minimum value necessary for investment at date 1. NPV is the net present value at date 0, overall and for the entrepreneur (M) and initial investor (C). Potential value is $V_0 = E_0(V_2) = 150$. $I_0$ and $I_1$ denote fixed investment costs, and $\sigma$ is standard deviation per period of $V_t$. Effort parameters are $\theta_f$ (value-added) and $\theta_g$ (cost). Increases in $\theta_f$ and $\theta_g$ represent increases in marginal returns and costs to effort, respectively. Projects that yield negative net present value, at best, to at least one party will not be implemented and are indicated with a (*).

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### B. $I_0 = 50, I_1 = 50, \sigma = 0.8, \theta_f = 1.8, \theta_g = 0.6$

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### D. $I_0 = 10, I_1 = 90, \sigma = 0.8, \theta_f = 1.8, \theta_g = 0.6$

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65
E. $I_0 = 10, I_1 = 90, \sigma = 0.4, \theta_f = 0.6, \theta_g = 0.6$

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<th>Financing Scenario</th>
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<th>$x_0$</th>
<th>$\bar{V}$</th>
<th>$\text{NPV}^M$</th>
<th>$\text{NPV}^C$</th>
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<tr>
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F. $I_0 = 10, I_1 = 90, \sigma = 0.8, \theta_f = 0.6, \theta_g = 0.6$

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<th>$x_0$</th>
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<th>$\text{NPV}^M$</th>
<th>$\text{NPV}^C$</th>
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<td>27.72</td>
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<td>Monopoly</td>
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Figure 1-1: Effort functions. As effort increases, the return to effort approaches 1 (100% of potential value). The marginal cost of effort is positive and increasing. The return and cost curves depend on parameters $\theta_f$ and $\theta_g$, which are varied here between 0.2 and 2.2.
Figure 1-2: Date-1 Net present values are plotted against $V_1$, the date-1 realization of maximum potential value. Potential value is $V_0 = E_0(V_2) = 150$. The lognormal probability density is plotted for a standard deviation of 0.4. Date-1 investment is $I_1 = 50$. With no costs of effort, $NPV_1 = V_1 - I_1$. When effort is costly, the level and slope of $NPV_1$ decline because of reduced effort at date 0 (the optimal effort $x_0^*$ is too low to attain maximum potential value). The strike value $V_1$, which incorporates the cost of date-1 effort, increases from $I_1$ to $I_1 + g(x_1^*) = 70.84$. 

![Graph showing NPV(x) and V1 - I1 relationship](image)
Figure 1-3: Net Present Value at date 0 in the first-best scenario. $NPV$ is shown across a range of standard deviation ($\sigma \in [0.1, 1.2]$) and effort return ($\theta_f/\theta_g \in [1/11, 11]$) parameters. The dark surface presents date-0 and date-1 investment levels $I_0 = I_1 = 50$ while the light surface presents $I_0 = 10, I_1 = 90$. Potential value is $V_0 = E_0(V_2) = 150$. 

$NPV_0$ $\theta_f$ $\sigma$
Figure 1-4: A typical plot of ownership shares at date 1 for the entrepreneur (M) and venture capitalist (C) in the monopoly case in where one venture capitalist provides all stage-0 and stage-1 financing. Two curves are shown for the venture capitalist. The first $\alpha^{C}_{MAX}$ is the maximum share at which the entrepreneur will still participate. The second $\alpha^{C}_{FOC}$ is the optimal share, at which the entrepreneur is provided with positive NPV and the incentive for continued effort. The optimal share is lower than the maximum share in the region where $V_1$ exceeds the strike value $\overline{V}_1$. The entrepreneur’s minority share $\alpha^M$ declines as $V_1$ increases.
Figure 1-5: Net Present Value at date 0, plotted for the Monopoly and Monopoly, No Staging financing scenarios. $NPV$ is shown across a range of standard deviation ($\sigma \in [0.1, 1.2]$) and effort return ($\theta_f/\theta_g \in [1/11, 11]$) parameters. The dark surface presents non-staged $NPVs$ while the light surface presents staged $NPVs$. Figure 1-5 assumes date-0 and date-1 investment levels $I_0 = I_1 = 50$ and potential value of $V_0 = E_0(V_2) = 150$. 
Figure 1-6: Net Present Value at date 0, plotted for the Monopoly and Monopoly, No Staging financing scenarios. $NPV$ is shown across a range of standard deviation ($\sigma \in [0.1, 1.2]$) and effort return ($\theta_f/\theta_g \in [1/11, 11]$) parameters. The dark surface presents non-staged $NPVs$ while the light surface presents staged $NPVs$. Figure 1-6 assumes date-0 and date-1 investment levels $I_0 = 10$, $I_1 = 90$ and potential value of $V_0 = E_0(V_2) = 150$. 
Figure 1-7: A typical plot of ownership shares at date 1 for the entrepreneur (M), initial venture capitalist (C) and syndicate of new investors (S) for the syndicate case. The syndicate provides financing on competitive terms ($NPV = 0$) whenever $V_1$ exceeds the strike value $\overline{V}_1$, which is higher than in the monopoly case shown in Figure 1-4. The syndicate’s optimal share $\alpha^S_{FOC}$ is lower than the maximum share $\alpha^S_{MAX}$ at which the entrepreneur would still participate. The constrained share $\alpha^S$ declines as $V_1$ increases, while the ownership shares held by the entrepreneur and incumbent venture capitalist increase with $V_1$. 

\begin{center}
\includegraphics[width=\textwidth]{figure1-7.png}
\end{center}
Figure 1-8: A typical plot of ownership shares at date 1 for the entrepreneur (M), initial syndicate (S1) and a second syndicate of new investors (S2) for the wholly competitive case. The second syndicate provides date 1 financing on competitive terms \((NPV = 0)\) whenever \(V_1\) exceeds the strike value \(\overline{V}_1\), which is lower than in both the syndicate and monopoly cases shown previously. The constrained share \(\alpha^{S2}\) declines as \(V_1\) increases, while the ownership shares held by the entrepreneur and incumbent investors increase with \(V_1\). The share held by the entrepreneur \((\alpha^M)\) is more in parity with incumbent investors' shares, compared with the syndicate case.
Figure 1-9: Net Present Value Loss at date 0, relative to first best, expressed as a percentage of total required investment. Potential value is $V_0 = E_0(V_2) = 150$. The value loss depends on the ratio of the return and cost of effort parameters, $\theta_f/\theta_g$. Lost $NPV$ is plotted for the monopoly, syndicate, and fully competitive cases, and also for the monopoly case without staging, in which the monopolist provides upfront financing and sets $\alpha_C^0$ at time 0. Figure 1-9 presents 4 assumptions on date-0 and date-1 investments and lognormal standard deviation. In 1-9a and 1-9b, $I_0 = I_1 = 50$. In 1-9c and 1-9d, $I_0 = 10$ and $I_1 = 90$. In 1-9a and 1-9c, $\sigma = 0.4$ while in 1-9b and 1-9d, $\sigma = 0.8$. 

![NPV Loss Graphs](image)
Figure 1-10: Net Present Value Loss at date 0, relative to first best, expressed as a percentage of total required investment. Potential value is $V_0 = E_0(V_2) = 200$. The value loss depends on the ratio of the return and cost of effort parameters, $\theta_f/\theta_g$. Lost $NPV$ is plotted for the monopoly, syndicate, and fully competitive cases, and also for the monopoly case without staging, in which the monopolist provides upfront financing and sets $\alpha_0^C$ at time 0. Figure 1-10 presents 4 assumptions on date-0 and date-1 investments and lognormal standard deviation. In 1-10a and 1-10b, $I_0 = I_1 = 50$. In 1-10c and 1-10d, $I_0 = 10$ and $I_1 = 90$. In 1-10a and 1-10c, $\sigma = 0.4$ while in 1-10b and 1-10d, $\sigma = 0.8$. 
Chapter 2

Manager Incentives in Collateralized Debt Obligations: A Theoretical Study

2.1 Introduction

This paper investigates under-investment and asset substitution problems in the financial management arena, specifically in the collateralized debt obligation or "CDO" market. A fast-growing component of financial derivatives markets, CDOs are closed-end, actively-managed, tranched bond funds. CDOs are an excellent setting in which to study agency problems because, while conforming very closely to the stereotypical corporate finance example of a leveraged firm, the ex-ante riskiness of the "projects" can be assessed quantitatively from historical and current bond market structure. The latter is something virtually impossible in corporate finance calibrations.

As in more traditional contexts, CDO managers almost always own subordinate positions. High leverage, in combination with extremely bad collateral performance, has led to strident criticism of CDO structures and allegations of manager risk-shifting in recent years (as well as two lawsuits against investment banks, privately settled). In this paper I do two things: First, I demonstrate that over a wide range of parameterizations and effort assumptions, equity
contracts are indeed more efficient (ex-ante) than other combinations of debt and fee contracts. Second, however, I explore and weigh which of various proposed CDO structural modifications are most efficient at curtailing risk-shifting while still promoting effort.

For these purposes, I develop an integrated model of manager behavior, modeling an effort and risk-shifting choice based on corporate debt ratings and associated default rates. Using both mean historical default parameters and those observed recently, I numerically estimate the costs of manager actions. I differentiate the costs of financial and agency-induced stress and weigh against effort benefits. Compared to agency cost estimates in other computational finance papers, I find similar to higher ex-post value losses. Other agency costs such as changes in debt default probability and implied increases in equity opportunity costs are larger and more significant than in other contexts. However, I find that even very small value loss assumptions can deter risk-shifting in many cases, and that for even extremely conservative effort parameterizations, ex-ante effort benefits greatly outweigh risk-shifting costs, making subordinated manager contracts more efficient than the debt and fee contracts considered.

Next I alter the model and repeat this procedure for a variety of proposed CDO modifications. Proposed structural features and indenture restrictions, analogous to dividend payout policies and debt covenants in more traditional contexts, aim to allow the manager to add value while still curtailing asset substitution. For a wide range of parameterizations, I find that excess interest diversions, contingent trading limits, and coverage test "haircuts" of lower-priced assets add the most value.

The CDO market, witness to great controversy currently, will resume its rapid expansion once agency problems are adequately understood and addressed. This paper provides a framework in which to quantify manager behavior and tranche outcomes, allowing for more efficient structures, behavior, and pricing. I believe that the results herein are a good indicator of the strength of various manager motivations, and can be reliably compared to other asset markets, both financial and corporate.

2.1.1 Overview of CDO Market and History

CDOs are closed-end, tranched bond funds that attempt to create value for equity investors by exploiting ratings, liquidity, or funding arbitrages in financial markets. Due to recent poor
performance, CDO structures and managers have been subject to intense scrutiny, by everyone from disgruntled debt investors and rating agency analysts to the SEC. Market growth is threatened by bad press and increased administrative and structuring costs.

In the prototypical CDO structure, illustrated in Figure 2-2, the manager selects an initial portfolio of securities which collateralizes the issuance of senior and subordinate debt securities. Debt provides leverage and risk premia benefits to equityholders and is issued to the point at which the resulting higher default risk raises the debt credit spread enough to lower equity returns. The manager’s motivation lies in ongoing management fees, partial deal ownership, and the chance to bolster his reputation and client list. Investors benefit from the new, alternate supply of securities. Figure 2-3 illustrates CDOs’ dramatic issuance growth. The CDO sector first surpassed $10 billion in issuance in 1996 and grew nearly 30% annually between 1996 and 2002.

Unsurprisingly, these vehicles performed poorly as corporate default activity peaked over the 1998-2002 period. For example, the five-year investment-grade cohort was the worst on record, 32% worse than the next worse cohort of 1982-1986, and 328% worse than the average.¹ Resultant downgrade rates for CDOs are presented in Table 1. Through 2003, 63% of all CDO tranches backed by high-yield bonds had been downgraded at some point, and 41% of those backed by investment-grade bonds. Because of the option-like nature of CDO liabilities, performance is sensitive to underlying collateral volatility and likely suffered from an underestimation of corporate correlation.

However, there is widespread belief that equity-holding managers purposefully increased collateral volatility, exploiting weaknesses in untested structures to the detriment of senior investors. For instance, in an analysis of relative downgrade rates (see Table 2), Moody’s Investor Services finds that collateral held by corporate CDOs has fared worse than the overall corporate market, at each broad rating level. Moody’s finds both initial collateral selection and ex-post trading behavior to be at fault for the riskier collateral. In a 2002 case study of a typical 1998 high-yield bond CDO, Moody’s finds that had the CDO held market collateral, 13 of 27 debt downgrades would have been avoided. Further, had the manager pursued a more conservative trading strategy after the onset of stress, CDO investors could have avoided an

¹Source: Moody’s Investors Service (2004a).
additional 8 debt downgrades (realizing just 6 of 27). After related calculations for a wider set of CDOs, they conclude that "managers have introduced risk... beyond that justified" by practices which "deviate from the spirit of the indenture".²

Other market participants echo Moody's findings. A director of CDO research at a major investment bank detailed several trades by managers resulting in direct gains to equity of several millions of dollars. A CDO performance study by the same firm cites the "great variability in credit quality of assets with the same rating" as the opportunity for managerial mischief, and the nature of CDO equity as a "fast-pay investment very sensitive to the amount of excess spread flowing through the structure" as the motive. They conclude that investors and rating agencies considered neither the possibility to game the structure nor managerial / equity incentives to do so.³ Another investment bank cites the market’s lack of emphasis on the value of an established manager with proven track record (such managers have larger reputation concerns), noting that "An experienced and inexperienced manager will often be subject to identical subordination levels... rating agencies are slowly unveiling their process for rating managers."⁴

Just the suspicion that managers and equityholders were "gaming" the system is enough to jeopardize the growth of the CDO market as well as impose unnecessary costs on managers who would otherwise refrain from risk-shifting. Unfortunately, the risk-shifting problem cannot be completely avoided because it results directly from the tranched structure of the CDO vehicle. Tranching is necessary for two reasons. First, it creates low-risk assets, a primary reason for the large growth of CDOs and other structured finance assets.⁵ Second, it allows the manager to be given a cheap, effective effort incentive when he is capital constrained. That managers continue to own equity and receive subordinated fees in newly-issued CDOs attests to the importance of

²Source: Moody’s Investors Service (2002b), Moody’s Investors Service (2003i). Moody’s cites "aggressive manager practices" which include the "purchase of Caa / deeply discounted securities". They note that manager behavior was "problematic because not contemplated in initial ratings" and state that their rating approach will move to a "more realistic modeling of manager trading strategies and reinvestment."


⁴Source: Thompson, Reeves, Weaver, and Folkerts-Landau (2002).

⁵Risk appetite and / or regulatory constraints encourage market participants to segregate by tranche and pay a premium for highly rated assets. This motivates less risk-averse subordinate investors to create highly rated senior securities and keep the residual risk. For instance insurance companies and banks often require or receive favorable capital treatment for investing in Aaa-rated assets. Table 3 presents ratings stratifications in the corporate and structured finance markets. Most recently, approximately 50% of structured finance debt issuance was Aaa- or Aa-rated. This contrasts to barely 12% in the corporate market. Any such "risk-arbitrage" motivation for CDO issuance implies a non-zero equity return even in the absence of beneficial effort provision.
the effort incentive; the amount of energy and resources devoted to the design of new trading restrictions and cashflow diversion rules attests to the costs of risk-shifting. 6 7

2.1.2 Preview of Model

This model incorporates both the positive and negative effects of active management and prices / sizes each component of a simplified CDO structure. Equity investors initiate the deal, contract with the manager, and reap the gains from the project, measured in terms of their expected rate-of-return.

At date 0 the deal is structured and a debt tranche sold. The equity price is set to cover the cost of remaining collateral. The manager invests in debt and equity shares to the extent of his available funds.

Collateral defaults realize at dates 1 and 2. After the time 1 default realization, the manager provides costly effort, which allows him to identify and purchase under-valued bonds. These bonds exhibit positive excess returns as a lower-than-priced-in default rate is realized. Though market value increases gradually, par value increases immediately through the lower market price.

At date 1, the manager also has an option to switch the remaining collateral portfolio into more risky securities. Such a trade increases the portfolio default rate and increases par value commensurately. Total CDO value is diminished by a small transactions cost.

Collateral bonds mature at date 2. Tranches share in the final cashflow according to their seniority. Expectations of manager behavior and default rates determine date 0 tranche valuations and sale prices.

An important assumption of my model is that the market cannot distinguish between effort-driven and risk-shifting trades. Because an undervalued bond is identified as a low-quality bond, an effort-driven, "value-scouting" trade can be confused for a conventional "down-in-quality"

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6 Generally investors, not managers, insist on a managerial equity stake. Raising funds is a challenge for smaller firms and frequent issuers, and some must securitize their fee stream in order to fund the purchase (McDermott, Rajan, and Benzschawel (2004)).

or risk-shifting trade. Both types of trades involve the purchase of low-rated collateral and both result in increases in portfolio par value.

I do not focus on the possibility of "credit-risk" trades. These would be when the manager finds an over-valued bond in the portfolio and replaces it at zero market or par value loss. Economic value is increased by avoiding a higher default rate. While such trades are certainly an avenue by which a manager adds value, in my model they can be easily identified as value-adding trades. I focus on trades whose motives are ambiguous and cause most of the discussion in the CDO market.

2.1.3 Preview of Results

My primary results concern the effects of managerial ownership on effort provision and risk-shifting. Key results are that

1) Effort varies with respect to collateral performance and the tranche the manager favors. An equity-holding manager works when deal performance is good but underinvests when deal performance is bad. A debt-holding manager works primarily when deal performance is bad.

2) Only if the manager owns a higher equity than bond share will he consider risk-shifting. He will do so as the volatility gains to equity begin to outweigh the transactions cost loss. This is when leverage is high, the par value gain from trading is large, or transactions costs are low. Equity investors gain from ex-post risk-shifting activity while debt investors generally lose. Even debt may gain if a higher upside restores equity’s prospects and prompts higher effort from the manager.

3) For even small postulations of effort effectiveness, managerial equity ownership is a more efficient route to boost ex-ante CDO returns than debt or fee contracts. Higher probability is put on good default realizations which lead to the highest effort and least risk-shifting. Debt ownership avoids risk-shifting but leads to effort only in the least likely outcomes. Further, the equity tranche is small relative to total deal size, making it the least expensive incentive.

4) For severe default outcomes, or misspecified collateral default rates and volatility options, risk-shifting costs are generally modest compared with the financial stress. The bulk of bad performance results from higher default and loss rates and lower effort gains (with exceptions however). Risk-shifting has its largest effect on the probability of debt default, which would
necessitate lower supported leverage and increased equity opportunity costs.

5) CDO interest distributions increase risk-shifting activity when performance is bad. Performance criteria which redirect excess interest away from equityholders act as a deterrent. They increase payments to debtholders and decrease leverage when risk-shifting is most likely to occur. They lessen risk-shifting costs and support an increase in initial leverage but can have a negative effect on effort.

6) Structural modifications can be effective at eliminating risk-shifting while not curtailing effort. The most efficient are more punitive excess interest diversions, contingent trading limits, and coverage test "haircuts" of lower-priced assets.

2.1.4 Related Literature

For an excellent overview of features of the CDO market, see Goodman and Fabozzi (2002). Oldfield (2000), Calomiris and Mason (2003), and Donahoo and Shaffer (1991) focus on various "arbitrage" motivations for securitized issuance. An introduction to rating agency analysis can be found in Fitch Ratings (2003), Moody’s Investors Service (1996), Moody’s Investors Service (2004c), and Standard and Poor’s (2002a).

Canonical corporate finance results on agency problems apply here. Jensen and Meckling (1976) and Myers (1977) describe the effort-provision problems arising when the manager holds fractional or equity claims. Jensen and Meckling (1976) describes the asset-substitution problem, predicting that optimal capital structure will be determined by the trade-off between effort provision and risk-shifting problems, with bond covenants used to reduce the cost of risk-shifting. Both Dessi (2001) and Garvey (1995) find equity ownership to be consistent with maximal effort provision, while acknowledging the increase in asset substitution. Ou-Yang (2003) and Stoughton (1993) derive fixed fee contracts with a performance bonus as optimal contracts under various agency problems and risk-aversion assumptions.

The possibility of a manager tempted to boost short-term equity returns rather than long-term firm value is presented in Stein (1989). Such short-termism results here when the manager’s equity principal options become worthless and equity interest options remain valuable. Higher debt levels or more stringent covenants (modeled here as interest-diverting coverage tests) can alleviate these problems as in Hart and Moore (1998).
Another strain of capital structure analysis focuses on the signaling aspect of capital structure (Myers (1984), Myers and Majluf (1984), Leland and Pyle (1977)). This strain predicts higher-quality firms to have a higher debt ratio and a higher manager equity share. Papastaioudi (2004b) analyzes this idea in context of CDOs or other closed-end funds. Reputation concerns can prevent risk-shifting or increase effort provision as in Hirshleifer and Thakor (1992) and Diamond (1989). Inspections of agency problems in the asset management arena include Chevalier and Ellison (1997), Golec (1992), Heinkel and Stoughton (1994), and Huddart (1999).

Computational studies of the risk-shifting problem include Mello and Parsons (1992), Leland (1998), Parrino and Weisbach (1999), Ross (1997), and Ericsson (1997). Like Leland and Ross I model manager choice over asset volatility, finding a positive relationship between agency costs and both leverage and cashflow riskiness. The cost of risk-shifting is less in NPV-destruction (I model only a small transactions cost) but rather in its impact on capital structure (lower initial leverage and higher funding costs). Estimates of the ex-post costs of risk-shifting are modest in these studies, on the order of 1% - 2.5%. I find similar to higher costs to debt when using historical and observed default estimates. While decreases in leverage are similar to those found in Leland, I find much larger increases in debt yields, measured either in terms of ratings spreads or equity opportunity costs. Likewise, while Andrade and Kaplan (1998) finds no instances of risk-shifting in a study of firms, I predict situations of frequent or constant risk-shifting. To my knowledge the quantification of the impact of CDO covenants on asset substitution is new, although Ross models dividend policies as a choice variable (in my model corresponding to interest diversion features).

Modeling techniques for non-standard option valuation are discussed in Kwok (1999), while Duffie and Garleanu (2001) and Gibson (2004) calculate the effect of volatility specifically on CDO tranches. In the numerical analysis, deal structure is informed by Goodman and Fabozzi (2002) while default risk is calibrated according to various Moody’s Investor Services reports (e.g. Moody’s Investors Service (2004a)).

The structure of this paper is as follows: Section 2 presents, and Section 3 analyzes, the basic two-period CDO model. Section 4 presents interest distribution policies in a three-period expansion of the basic model. Section 5 discusses proposed structural modifications and Section 6 concludes.
2.2 Model Detail (Two-Period)

I start with a simple, stylized CDO model. There is no interest rate risk and no time-discounting.\(^8\) Cashflows are evaluated on a risk-neutral basis.

Typically CDOs issue sequential liabilities \((x_1 \ldots x_{Q-1})\) with respective coupon rates \((y_1 \ldots y_{Q-1})\). These tranches will be differently affected by management behavior depending upon how low in the capital structure, i.e. how equity-like, they are (as in Duffie and Garleanu (2001)). I focus on the combined debt tranche, \(X = \sum_{i=1}^{Q-1} x_i\), with weighted-average coupon \(Y = \sum_{i=1}^{Q-1} x_iy_i/X\). Residual risk belongs to the equity tranche, denoted by \(x_Q\). The manager participates in the deal via debt and equity shares \(\nu^X\) and \(\nu^Q\).

The debt size at issuance, \(X_0\), is limited by a default frequency target, drawn from the debt tranche’s desired rating level and assumptions about the riskiness of the beginning collateral pool. I follow rating agency criteria in proscribing no benefit to managerial effort. The size of \(X_0\) is crucial as it determines the leverage in the CDO.

Debt is collateralized by a portfolio of bonds with initial par value \(F_0\). There are two types of collateral, "good" (\(G\)) and "bad" (\(B\)) bonds, distinguished solely by their probability of default. The observed types \(\kappa\) determine market prices \(b^\kappa_i\), which are used to price trades.

Bonds make interest payments at the periodic rate of \(c\). This pays for debt interest at rate

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\(^8\) There are two complications resulting from interest rate risk. First, if defaults occur faster than expected, CDOs may become overheded with respect to their fixed-rate collateral and floating-rate liabilities. This accentuates loss in a declining rate environment but can be captured through a higher default rate. Second, the manager can exploit coupon differences in favor of equity. See Section 2.4, Prop. 7 for an discussion of this. Since deals possess unequal levels of interest rate risk but share default risk, I focus on the latter.
As in all debt markets, interest distributions are made on the basis of face value rather than market value.

I reduce the CDO time frame considerably. CDO tranches are long-dated instruments whose periodic interest payments and interest diversion features make them a complicated time-series of options on the collateral pool. For illustration I reduce to a 2-period model where the manager has one decision-making period.\(^9\)

In what follows superscript \(X\), \(Q\), \(C\), and \(M\) refer to the debt tranche, equity tranche, entire CDO, and CDO manager, respectively.

### 2.2.1 Time \(t = 0\)

At date 0, the deal is structured and brought to market backed entirely by generic, high-quality collateral of type \(\kappa = G\). The debt price is set off the assumption of a static collateral portfolio, that is no effort-based or risk-shifting trades, meaning debtholders will in actuality get a non-zero return. Let \(V_t^i\) represent the cumulative expected value stream to each CDO participant \(i\) at time \(t\) (after current cashflows have been distributed), \(B_0^i\) the initial dollar price of a tranche, and \(\Re^i\) the rate of appreciation, or for simplicity the expected return, to tranche \(i\). Then the tranche price and return to debt investors is

\[
B_0^X = \frac{V_{0,NT}^X}{X_0} \\
\Re^X = \frac{V_0^X}{V_{0,NT}^X} - 1
\]

where \(V_{0,NT}^X\) is the static pricing assumption valuation and \(V_0^X\) the valuation based on the manager’s actual behavior.\(^{10}\)

The amount equity must contribute is the difference between the capital raised from debthold-

\(^9\)In Section 2.4 I model an expanded 3-period model with two decision-making periods.

\(^{10}\)Deal structurers typically run cashflow analyses for their clients based on characteristics of the initial collateral portfolio. Rating agencies, while not providing guidance on prices, rate and size tranches based solely on initial collateral default assumptions. Of course, data is often available on the manager’s previous default performance relative to his peer group and rating agencies have recently begun to account for reinvestment risk. In numerical results I relax this assumption.
ers and the cost of collateral $b_0^G F_0$. This establishes the equity investment price and return

$$B_0^Q = (b_0^G F_0 - B_0^X X_0) / (F_0 - X_0)$$

$$\mathcal{R}^Q = V_0^Q / (b_0^G F_0 - B_0^X X_0) - 1$$

(2.2)

I assume the manager is capital-constrained and limits his purchases accordingly.

The market price $b_0^G$ of a collateral bond is determined by its default characteristics. For tractability I approximate the binomial distribution with the Poisson distribution. Poisson probabilities approach binomial probabilities as the number of bonds goes to infinity or the probability of default goes to zero. The Poisson probability for the number of defaults $d$ in a continuous period of length $t$, with default intensity $\Lambda \geq 0$, is

$$\Pi_{\Lambda t}^d = e^{-\Lambda t} (\Lambda t)^d / d!$$

(2.3)

Suppose there are a large number of bonds, with total face value of 1. Since bonds are identical and i.i.d. and participants are risk-neutral, the price of this portfolio is the unit price of an individual bond. Also, let each default wipe out $1 - \phi$ percent of current collateral par, so that the remaining par at time 1 is $\phi^d$. The price of the unit portfolio weights time 1 value by $\Pi_{\Lambda t}^d$, establishing the initial price of a type $\kappa$ bond to be

$$b_0^\kappa = \sum_{d=0}^{\infty} \Pi_{\Lambda t}^d \phi^d (b_1^\kappa + c) = (1 + c) e^{-2\Lambda_{\kappa} (1 - \phi)} + c e^{-\Lambda_{\kappa} (1 - \phi)}$$

(2.4)

using the time 1 collateral price $b_1^\kappa = (1 + c) e^{-\Lambda_{\kappa} (1 - \phi)}$.

This price formula uses what can be considered an expected loss or depreciation rate for the portfolio, $\Lambda_{\kappa} (1 - \phi)$, and exponentiates it to weight due cashflows by expected remaining portfolio par. Though a portfolio or individual bond is priced risk-neutrally, debt and equity investors are impacted by risk. $\Lambda_{\kappa}$ represents the default rate in the portfolio, while $\phi$ represents the loss given default. A more concentrated portfolio is proxied by a lower $\phi$ (assuming $\Lambda_{\kappa}$ is decreased to keep the expected loss rate constant). The lower $\phi$, the larger the tail risk in the portfolio, capturing a lack of diversification or systemic risk.

Date 0 tranche/cashflow valuations are simply the expectation of $t = 1$ cashflows and future
expected cashflows. Let $N_t^i$ represent the payments made to each participant $i$ at time $t$ (detailed in Section 2.2.2 below). Then

\begin{align*}
V_0^X &= E_0 N_1^X + E_0 V_1^X \\
V_0^Q &= E_0 N_1^Q + E_0 V_1^Q \\
V_0^M &= E_0 N_1^M + E_0 V_1^M
\end{align*}

(2.5)

### 2.2.2 Time $t = 1$

Between date 0 and 1 defaults realize. Original face value $F_0$ adjusts to reflect new defaults by

\begin{align*}
F_1 &= \phi^{d_1} F_0
\end{align*}

(2.6)

Interest cashflows are distributed to CDO participants and subsequently the manager decides on effort provision and risk-shifting.

**Interest Payment**

Each non-defaulted collateral asset makes a coupon payment at rate $c$. Available cashflow, $c\phi^{d_1} F_0$, is distributed amongst participants according to a set priority of payments (or "waterfall" in industry parlance). The debt tranche is due a coupon payment at rate $Y$. Since equityholders receive any residual interest, the interest distributions $N_t^i$ are expressed as options. The manager’s total time 1 payment consists of his shares of the debt and equity payments:

\begin{align*}
N_1^X &= c F_1 - \max(0, c F_1 - X_0 Y) \\
N_1^Q &= \max(0, c F_1 - X_0 Y) \\
N_1^M &= \nu^X N_1^X + \nu^Q N_1^Q
\end{align*}

(2.7)

If there is insufficient collateral interest to make the debt interest payment, equityholders receive no cash and the debt level is increased ("PIK’ed" or "Paid-In-Kind") to reflect the unpaid interest. New debt accrues interest at the same interest rate $Y$ and is carried forward to the next period according to

\begin{align*}
X_1 = X_0 (1 + Y) - N_1^X
\end{align*}

(2.8)
Effort and Risk-Shifting Decision

First I examine the manager’s effort choice. The manager’s effort level is unobservable and expressed through trading.\(^{11}\) While available collateral bonds fall into two distinct types, good and bad, these bonds may become misclassified. Misvaluation occurs if a bond is priced according to its identified type \(\kappa\) but the actual, underlying default process is not supportive of that type. Value-adding trades result from finding under-valued bonds in the market universe. An under-valued bond is identified as low-quality but is actually high-quality.

The manager chooses a continuous effort level \(\varepsilon\) with cost \(\Xi(\varepsilon)\). Effort takes effect through a function \(\xi(\varepsilon)\) having the following attributes: 1) it represents the fractional portion of the collateral which is traded for good but misclassified bonds, building par; 2) boundaries are \(\xi(0) = 0\) and \(\xi(\infty) = 1\) with \(\partial \xi / \partial \varepsilon > 0\); and 3) effort exhibits decreasing marginal returns or \(\partial^2 \xi / \partial \varepsilon^2 < 0\). As usual the cost function \(\Xi(\varepsilon)\) is increasing and convex in effort \((\partial \varepsilon / \partial \varepsilon > 0, \partial^2 \varepsilon / \partial \varepsilon^2 > 0)\).

The gain from expending effort is dependent upon the market value ratio \(\beta\) of the two types of collateral, as well as the transactions costs loss in trading \(1 - \Psi\). High-quality collateral is sold at the \(t = 1\) market price \(b^G_1\) and reinvested into misclassified bonds at the low-quality market price \(b^B_1\). However a transactions cost \(1 - \Psi\) is incurred making the par value gain on the exchanged portion of the portfolio, \(\xi(\varepsilon)F_1\), equal to \(\beta \Psi - 1.\)\(^{12}\) After effort par value increases to \(F'_1\), defined as
\[
F'_1 = \beta \Psi \xi(\varepsilon)F_1 + (1 - \xi(\varepsilon))F_1
\]
Collateral market prices \(b^G_1\) and \(b^B_1\) are determined by the default process in Eq. (2.3) establishing
\[
b^G_1 = \sum_{d=0}^{\infty} \Pi_{\alpha}^d \phi^d (1 + c) = (1 + c)e^{-\Lambda}(1 - \phi) \\
\beta = \frac{b^G_1}{b^B_1} = e^{(1 - \phi)(\Lambda_B - \Lambda_G)}
\]

The larger the gap in default expectations, that is the larger \(\Lambda_B - \Lambda_G\), the more the manager gains from effort provision. Likewise, the larger the transactions costs loss, that is the further

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\(^{11}\)From a practical standpoint, a portfolio manager can only affect returns through trading. Whatever research, surveillance, or analytics he does translates to gains only when acted upon.

\(^{12}\)Obviously if \(\beta\) is so negligible or \(\Psi\) costs are so prohibitive as to make \(\beta \Psi \leq 1\), there is no benefit from effort. I concentrate on the relevant instances where the reverse holds true.
Ψ from 1, the lower the manager’s incentive to provide effort. The maximum par the manager can deliver is \( \beta \Psi F_1 \), obtained when \( \xi(e) = 1 \), while \( F'_1 \) remains at \( F_1 \) if he provides zero effort.

After effort-driven trades the manager chooses whether to risk-shift. This is modeled as an option to switch the remaining portion of the collateral into truly bad collateral with default intensity \( \Lambda_B \). The par value gain is still given by the market value ratio \( \beta \). However, risk-shifting trades do not lead to any economic value gain, as the higher default rate counterbalances the par gain. If the manager chooses to remain invested in good bonds (NS) the collateral par and default rate remain at

\[
F'_{1,NS} = F'_1 = F_1 (1 + \xi(e)(\beta \Psi - 1)) \\
\Lambda_{1,NS} = \Lambda_G
\]

On the other hand, risk-shifting or switching in my terminology (SW) produces the maximum par value gain and an increase in portfolio default intensity to

\[
F'_{1,SW} = F_1 \left( \frac{\ln \beta + \ln(1 + \xi(e)(\Psi - 1)) - \ln(1 + \xi(e)(\beta \Psi - 1))}{1 - \phi} \right) + \Lambda_G
\]

\( \Lambda_{1,SW} = \Lambda_{SW} \)

\( \Lambda_{SW} \) is derived from a no-arbitrage equation, ensuring that the remaining collateral \((1 - \xi(e))F'_1\) is switched on a value-neutral basis. Specifically, expected cashflow for both the beginning portfolio \( F'_1 \) and the post-switch portfolio \( F'_{1,SW} \) are equivalent, when the remaining portfolio is switched with no transactions costs. In actuality, expected cashflow from \( F'_{1,SW} \) is lower due to the transactions cost loss of \((1 - \Psi)F'_1\). In the special case where effort is zero, \( \Lambda_{SW} \) reduces to \( \Lambda_B \) and expected portfolio par becomes \( E_1 F_{2,SW} = e^{-\Lambda_B (1 - \phi)} \Psi F_1 \).

The manager decides on his optimal effort level and switching behavior jointly. To identify

\[13\] The one observable aspect of the manager’s risk-shifting is the larger traded volume. This suggests one could limit risk-shifting by limiting the allowed amount of par-increasing trades. Although I explore this as an extension in Section 2.5, initially I model what is effectively a 100% par-increasing limit. This is useful because: 1) Cost estimates are based on the maximum traded volume, providing an upper bound and 2) Over a 6-year period, this translates into 16\(\frac{2}{3}\)% turnover annually, while typically CDOs allow 15%-25% annual discretionary trading. That is, effort can be reasonably considered as more of a continuous process than the binary approach I take here for simplicity.

\[14\] The expression for \( \Lambda_{SW} \) is an approximation chosen to keep the default distribution univariate. This approximation posits a switch to a bond of type \( \kappa = SW \), with default intensity \( \Lambda_{SW} \), rather than having a bi-modal portfolio composed of both high and low-default-intensity bonds.
his payout, the manager looks ahead to date 2 cashflows, identifies both \( \epsilon^*_\text{SW} \) and \( \epsilon^*_\text{NS} \), and evaluates his maximal switching policy. That is,

\[
V_1^{M*} = \max \left[ \max_{\epsilon} \left( E_1 N_{2,SW}^M - \Xi(\epsilon) \right), \max_{\epsilon} \left( E_1 N_{2,NS}^M - \Xi(\epsilon) \right) \right]
\]

\[\epsilon^* = \arg \max \left[ V_1^{M*} \right] \]

\[\kappa^* = \begin{cases} 
G & \text{if } \epsilon^* = \epsilon^*_\text{NS} \\
\text{SW} & \text{if } \epsilon^* = \epsilon^*_\text{SW}
\end{cases} \]

His effort level and switching decision are used in the calculation of date 1 valuations

\[
V_1^X(d_1) = E_1 N_2^X(\epsilon^* (d_1), \kappa^* (d_1), d_1)
\]

\[
V_1^Q(d_1) = E_1 N_2^Q(\epsilon^* (d_1), \kappa^* (d_1), d_1)
\]

\[
V_1^M(d_1) = \nu^X V_1^X(d_1) + \nu^Q V_1^Q(d_1) - \Xi(\epsilon^* (d_1))
\]

### 2.2.3 Time \( t = 2 \)

At date 2, non-defaulted bonds mature, paying back their par value and making their final coupon payment. The final principal amount of

\[
F_2 = \phi^d_2 F_{1,\kappa}
\]

generates cash at rate \( 1 + c \) which goes towards debt redemption and equity residual. Date 2 distributions are

\[
N_2^X = (1 + c) F_2 - \max (0, (1 + c) F_2 - X_1(1 + Y))
\]

\[
N_2^Q = \max (0, (1 + c) F_2 - X_1(1 + Y))
\]

\[
N_2^M = \nu^X N_2^X + \nu^Q N_2^Q
\]

### 2.2.4 Solution and Calibration

The standard approach of backwards induction is used to solve for expected cashflows, optimal manager behavior, and tranche rates-of-return. I take the initial CDO capital structure and intensity parameters and project each possible \( t = 1 \) default realization. The default realization determines intermediate cashflows and the manager’s joint effort and trading decision, which
then implies expected tranche payouts at \( t = 2 \). Tranche values at \( t = 0 \) are the probability-weighted average of the future expected cashflows.

Thus, at \( t = 0 \), given initial face value \( F_0 \), let \( d_1 \) identify the point at which the equity tranche begins to receive residual interest at \( t = 1 \). Above this default point, the debt tranche receives all interest cashflow; below, the debt tranche receives \( X_0Y \) while equity receives the residual. Setting \( cF_1 = X_0Y \) yields

\[
\frac{\ln \left( \frac{X_0Y}{cF_0} \right)}{\ln \phi} = d_1
\]

(2.17)

Then \( t = 1 \) cashflow expectations are

\[
E_0N_1^X = ce^{-\Lambda \phi(1-\phi)} F_0 - \sum_{d < d_1} \Pi^d_{\Lambda^G} \left [ c\phi^d F_0 - X_0Y \right ]
\]

\[
E_0N_1^Q = \sum_{d < d_1} \Pi^d_{\Lambda^G} \left [ c\phi^d F_0 - X_0Y \right ]
\]

\[
E_0N_1^M = \nu^X ce^{-\Lambda \phi(1-\phi)} F_0 + \left ( \nu^Q - \nu^X \right ) \sum_{d < d_1} \Pi^d_{\Lambda^G} \left [ c\phi^d F_0 - X_0Y \right ]
\]

(2.18)

Cashflows at \( t = 2 \) are determined similarly. At \( t = 1 \), given \( X_1 \) and \( F_{1,\kappa}' \), let \( d_2, \kappa \) identify the point at which the equity tranche finishes in-the-money. This is solved for as before yielding

\[
\frac{\ln \left( \frac{X_1(1+Y)}{(1+c)F_{1,\kappa}'} \right)}{\ln \phi} = d_2, \kappa
\]

(2.19)

Then time 2 cashflow expectations are

\[
E_1N_{2,\kappa}^X = \sum_{d \leq d_2, \kappa} \Pi^d_{\Lambda^\kappa} X_1(1+Y) + \sum_{d > d_2, \kappa} \Pi^d_{\Lambda^\kappa} (1+c) \phi^d F_{1,\kappa}'
\]

\[
E_1N_{2,\kappa}^Q = \sum_{d > d_2, \kappa} \Pi^d_{\Lambda^\kappa} \left [ (1+c)\phi^d F_{1,\kappa}' - X_1(1+Y) \right ]
\]

\[
E_1N_{2,\kappa}^M = \nu^X (1+c) e^{-\Lambda(1-\phi)} F_{1,\kappa}' + \left ( \nu^Q - \nu^X \right ) \sum_{d < d_2, \kappa} \Pi^d_{\Lambda^\kappa} \left [ (1+c)\phi^d F_{1,\kappa}' - X_1(1+Y) \right ]
\]

(2.20)

For both \( \kappa = NS \) and \( \kappa = SW \) the manager maximizes \( E_1N_{2,\kappa}^M \) over \( \epsilon_\kappa \). The first-order
The first-order condition for effort when the manager does not switch is

\[
\frac{\partial \mathcal{E}}{\partial \epsilon} (\epsilon_{NS}^*) = \left( \nu^X \sum_{d \geq d_{2,NS}} \Pi_{\phi \Lambda_G}^d + \nu^Q \sum_{d < d_{2,NS}} \Pi_{\phi \Lambda_G}^d \right) (1 + c) (\beta \Psi - 1) e^{-\Lambda G (1-\phi)} F_1 \frac{\partial \mathcal{E}}{\partial \epsilon} (\epsilon_{NS}^*)
\]

(2.21)

The first-order condition for effort when switching is

\[
\frac{\partial \mathcal{E}}{\partial \epsilon} (\epsilon_{SW}^*) = \frac{\partial \Lambda_{SW}}{\partial \xi} \frac{\partial \xi}{\partial \epsilon} (\epsilon_{SW}^*)
\]

\[
= \left[ e^{-\Lambda_{SW} (1-\phi)} \beta \Psi F_1 \left( \nu^X (1 + c) \sum_{d \geq d_{2,SW}} \frac{\Pi_{\phi \Lambda_{SW}}^d \left( \frac{d}{\Lambda_{SW}} - 1 \right) \prod_{d_{2,SW} = 1}^d \right) \right] + \sum_{d < d_{2,SW}} \Pi_{\phi \Lambda_{SW}}^d \left( \frac{d}{\Lambda_{SW}} - 1 \right) \left[ \nu^Q (1 + c) \beta \Psi \phi^d F_1 - (\nu^Q - \nu^X) X_1 (1 + Y) \right]
\]

using

\[
\frac{\partial \Pi_{\phi \Lambda_{\kappa}}^d}{\partial \Lambda_{\kappa}} = \Pi_{\phi \Lambda_{\kappa}}^d \left( \frac{d}{\Lambda_{\kappa}} - \phi \right)
\]

(2.23)

The manager’s gain from risk-shifting (holding effort constant, or \(\epsilon_{SW} = \epsilon_{NS} = \epsilon\)) is

\[
\Delta V^M_1 = \nu^X (1 + c) (\Psi - 1) (1 - \xi(\epsilon)) e^{-\Lambda_{SW} (1-\phi)} \beta F_1
\]

(2.24)

\[
+ (\nu^Q - \nu^X) \left[ \sum_{d=0}^{d_{2,SW}} \Pi_{\Lambda_{SW}}^d \left( (1 + c) \phi^d F_{1,SW} - X_1 (1 + Y) \right) - \sum_{d=0}^{d_{2,NS}} \Pi_{\Lambda_{G}}^d \left( (1 + c) \phi^d F_{1,NS} - X_1 (1 + Y) \right) \right]
\]

This is a good indicator of the manager’s switching policy, although technically he chooses for \(\epsilon_{NS}^* \geq \epsilon_{SW}^*\). The manager’s effort and switching policy is solved for each possible \(d_1\) realization. This determines \(t = 1\) values in Eq. (2.14) and hence \(t = 0\) valuations in Eq. (2.5).

I compute managerial effort choices, risk-shifting decisions, and ultimate tranche returns numerically. The majority of deal parameters are set by outside sources, mainly Moody’s

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\(^{15}\) The NS and SW FOCs are not universally correct, since the value functions are only stepwise concave. The summation term in the manager’s NPV is defined over integer default levels whereas effort raises \(F_{1,\kappa}\) and hence \(d_{2,\kappa}\) continuously. Usually marginal effort does not increase \(d_{2,\kappa}\) enough to surpass the next \(d_2\) integer. However when \(d_{2,\kappa}\) rises above the next highest integer value there will be a jump in marginal effort return, possibly generating multiple roots. These non-concavities gradually smooth out with the diminishing returns to higher effort.
Research Services and Goodman and Fabozzi (2002). Default and leverage parameters are calibrated for a six-year total deal time frame, using Moody's default statistics and CDO rating methodologies. Default intensities $\lambda_\alpha$ represent six-year cumulative default rates. Along with $\phi$, they are based off the six-year collateral loss rate for their targeted rating level, assumed recovery rate level, and choice of deal concentration level. A debt default limit of 0.25% is imposed, corresponding to the six-year cumulative default rate for Aaa-rated securities. The initial collateral level is scaled to $F_0 = 10$. These parameters establish the maximum supportable debt level $X_0$.

Managerial ownership shares are informed by current CDO market structure. It is common for managers to own up to 33% of the equity tranche. I explore debt ownership shares equivalent in terms of total deal ownership.

The main choice parameters are effort parameterization and level of transactions cost. Transactions costs are informed by common bid-ask spreads in asset markets and scaled to a small but reasonable cost of 0.50%. Clearly, the higher transactions costs, the lower the value added by effort and the higher the cost of would-be risk-shifting. Effort parameters are chosen to provide a reasonable level of effort and a range of first-best CDO excess returns. As discussed in Proposition 5, for even extremely conservative assumptions on effort effectiveness, the results in this paper hold true.

Results presented in the text encompass three different deal characteristics, four effort assumptions, and various mispricing scenarios. The deal characteristics are level of portfolio concentration, level of transactions cost, and volatility option. The baseline is a deal with 2.5% asset concentration (loss given default is approximately 1.5% factoring in recovery rates), a transactions cost loss of 0.50%, and an option to switch into Ba-rated collateral. Variations on this are no transactions cost loss, 5% asset concentration, and the option to switch into B-rated collateral. Deal parameters are ranged across four effort parameterizations ($\epsilon = 0$, weak, medium, and strong). I also consider certain mispricing possibilities: mainly the use of observed or amplified default rates from recent years, rather than mean historical rates, and for instance the unanticipated option to switch into C-rated collateral.

Appendix 1 presents further details and lists of parameterizations referenced in the text.
2.3 Model Results (Two-Period)

This section analyzes the efficiency implications of a positive equity ownership scheme (PEC). There are two conflicting agency problems, the manager’s disinclination towards effort and equity’s benefit from volatility. The compensation scheme which produces the highest expected equity return is necessarily a constrained maximizer and will be sub-optimal in some contingencies. The PEC is the cheapest contract and does the best job of promoting effort and minimizing risk-shifting in the most probable outcomes (good performance). The byproduct of this is that it may do the worst job in poor performance outcomes. I start with a cursory look at the first-best case in order to provide a frame of reference. Away from first-best the overall cost-benefit analysis to PEC is decided by measuring the equity rate-of-return.

**Proposition 1** In the first-best case, in which \( \nu^X = \nu^Q = 1 \), the manager provides optimal effort and never switches into low-quality collateral.

The no-switch result is easily established by comparing NPVs. Whenever \( \nu^X = \nu^Q = \nu \), the probability sum term in the manager’s NPV (Eq. (2.20)) drops out, leaving

\[
V_{1,\kappa}^M = \nu(1 + c)e^{-\Lambda(1 - \phi)}F_{1,\kappa}^t
\]  
\[(2.25)\]

Plugging in the \( F_{1,\kappa} \) and \( \Lambda_{\kappa} \) terms from Eqs. (2.11)-(2.12) establish an NPV ratio of (for \( \epsilon_{SW} = \epsilon_{NS} = \epsilon \))

\[
\frac{V_{1,\kappa}^M}{V_{1,SW}^M} = e^{(\Lambda_{SW} - \Lambda_{\kappa})(1 - \phi)} \frac{1 + \xi(\epsilon)(\beta\Psi - 1)}{\beta\Psi} = \frac{1 - \xi(\epsilon)(1 - \Psi)}{\Psi} > 1
\]  
\[(2.26)\]

For every possible effort level \( \epsilon \) his value from not switching is higher, by the transactions costs loss saved on the non-switched portion of the portfolio. Since the manager’s debt and equity interests balance out, he maximizes deal cashflow without regard to tranche.

Effort is provided according to first-best marginal benefits and costs. The first-best FOC is

\[
\frac{\partial \Xi}{\partial \epsilon} = (1 + c)e^{-\Lambda(1 - \phi)}F_1(\beta\Psi - 1)\frac{\partial \xi}{\partial \epsilon}
\]  
\[(2.27)\]
from simplifying Eq. (2.21). The expression \((1 + c) e^{-\Lambda_\kappa (1-\phi)} F_1\) is the expected cashflow from current asset par, while \(\beta \Psi - 1\) is the par increase on the units switched. Together the terms express the expected cash increase from the modified par. \(\frac{\partial \xi}{\partial \epsilon}\) is the marginal increase in collateral switched for increases in effort, while \(\frac{\partial \xi}{\partial \epsilon}\) is the marginal cost. Effort increases with better default performance \((d_1 \downarrow)\), lower transactions costs \((\Psi \rightarrow 1)\), and higher market value gains \((\beta \uparrow)\).

**Proposition 2** If the manager owns a proportionate slice of the deal \((\nu^X = \nu^Q = \nu)\), or if the equity tranche is completely in- or out-of-the-money, the manager provides effort proportionate to first-best and never switches into low-quality collateral.

When \(\nu^X = \nu^Q = \nu\), the manager will never switch, using the same logic as before. Effort is provided according to a FOC identical to (2.27) but reduced by his ownership share \(\nu\).

The same result is obtained when the deal becomes "riskless", i.e. the equity tranche is completely in- or out-of-the-money. For instance, if deal performance is so poor that debt becomes the residual claimant, the manager’s NPV reverts to his debt interest in the collateral

\[
V_{1,\kappa}^M = \nu^X (1 + c) e^{-\Lambda_\kappa (1-\phi)} F_{1,\kappa}^v
\]  

(2.28)

Likewise, as equityholders becomes more certain to receive payments, \(d_2, \rightarrow \infty\) and the manager begins to maximize deal value without regard to the debt level. The manager’s NPV consists then of his constant debt share and his share of the equity residual or

\[
V_{1,\kappa}^M = \nu^Q (1 + c) e^{-\Lambda_\kappa (1-\phi)} F_{1,\kappa}^v - (\nu^Q - \nu^X) X_1 (1 + Y)
\]  

(2.29)

Both of these NPV expressions imply the no-switching result and yield effort proportionate to first-best weighted by his debt share and equity share respectively. When there is no risk in the deal, there is no opportunity for the manager to benefit by redistributing risk amongst tranches.

To view effort reaction to partial ownership, see Figure 2-4. The top line shows first-best effort, while the lower straight line shows a \(\nu = 2.5\) ownership share.

**Proposition 3** When \(\nu^X \leq \nu^Q\), effort tilts toward default outcomes where the manager’s holdings receive the marginal impact. An equity-holding manager works when the equity tranche is
When the manager’s debt and equity shares differ, he is sensitive to which tranche gains the most from his actions. This intuition is evident in his NS-FOC for effort (Eq. (2.21)). It differs from first-best through the marginal return term $\nu^X \sum_{d \geq d_{2,NS}} \Pi^d_{\phi_{AG}} + \nu^Q \sum_{d < d_{2,NS}} \Pi^d_{\phi_{AG}}$. This term is the probability each tranche finishes as the residual claimant, weighted by the manager’s ownership shares $\nu^X$ and $\nu^Q$. Only for $d_{2}$ default realizations in which equity finishes out-of-the-money does the manager’s debt share provide an incentive; only when equity finishes in-the-money does the manager’s equity share provide an incentive. The worse the time 1 default realization, the more reliant the effort incentive is on $\nu^X$, and vice-versa.

Figure 2-4 illustrates effort response to performance in a no-switch setting. The top line is first-best ownership. The two kinked lines show 33.8% equity ownership (line decreasing to the right) and 2.7% debt ownership (line increasing to right). Each of these shares is equivalent to 2.5% deal ownership in this example. 100% equity ownership would replicate first-best effort when performance is good, representing 7.4% deal ownership. 100% debt ownership replicates first-best effort when performance is bad, representing 92.6% deal ownership.

Figure 2-5 illustrates each tranche’s effort gains for the same deal parameters as in Figure 2-4. Total CDO gain is equal to the sum of debt and equity gains decreased by the effort costs of the manager. In Figure 2-5-A, the manager owns a proportionate 2.5% of the deal. Marginal benefits accrues to equityholders when time 1 defaults are low and debtholders when defaults are high. In Figure 2-5-B, the manager owns 2.7% of the debt tranche. As the debt payout becomes less certain the manager provides increasing levels of effort, the benefits of which predominantly accrue to debtholders. Figure 2-5-C shows the converse, in which the manager owns 33.8% of the equity tranche. Effort declines rapidly as performance worsens and the vast majority of gains accrue to equity.

**Proposition 4** Only if $\nu^Q > \nu^X$ will the manager switch. Whether he does so depends on the risk in the deal and the strength of transactions costs. He is most likely to risk-shift when transactions costs are low ($\Psi \to 1$) or equity payouts in jeopardy ($X_1 \to F_1$ while $X_1 < \beta \Psi F_1$).

The manager will only pursue risk-shifting trades if he owns more equity than debt. This is because the debt tranche loses unequivocally from risk-shifting, from both extra volatility.
and the transactions cost loss. (For now I restrict effort to zero to focus on the risk-shifting incentive).

It is not immediately obvious that the risk-shifting incentive from traditional corporate finance literature would operate in this context. Standard agency models deal with a manager who owns a call option on firm equity. For a mean-constant volatility increase, he receives the greater upside of the firm but still has the same downside, and so is always risk-loving. But bond payouts are capped - as bond market participants like to remark, "there is no upside in bonds". Bond investors are displeased with a firm that suddenly becomes more risky and frequently insist on leverage covenants.

However, here the manager chooses to increase the riskiness of the CDO bond collateral, and receives a higher par value in exchange. Equity benefits from the greater upside of the portfolio value, not of the individual firms which constitute the collateral portfolio. Bond market investors might be content with more volatile firms if their face value (or spread) increased in compensation - exactly what happens in the CDO when the manager switches.\(^{16}\)

Another way to look at this is from the CDO debtholders’ perspective. Suppose there are no transactions costs (\(\Psi = 1\)). When the manager trades into low-quality collateral, the higher portfolio default rate \(A_B\) counteracts exactly the par value gain. Risk-neutral, pari passu investors are indifferent to this trade. For each possible default outcome they receive the full upside which compensates for the increased default risk. However debtholders only receive the upside of the par value gain when equity finishes out-of-the-money. Elsewhere they receive their capped debt level \(X_1\). So long as there is a positive possibility of equity finishing in-the-money, the debtholders are worse off. Debt’s loss is equity’s gain and whenever \(\nu^Q > \nu^X\) the manager’s equity position gains more than his debt position loses. He will always switch.\(^{17}\)

\(^{16}\)Actually, bond investors may benefit from greater firm volatility if the firm is performing badly. A call option on a firm’s bond is a "vertical spread". This position consists of a long call on the underlying firm at a low strike price and a short call at a higher strike price. The partial derivative of the call option value with respect to the volatility of underlying firm returns is known as the "vega" and is always positive. The net effect of volatility depends on the relative vega importance of the two calls. When the long call is at-the-money, vega is higher for the long call, and the portfolio gains. Volatility is more valuable, the more equity-like the bond is, that is, the further from full repayment it is.

\(^{17}\)Note the corollary that when a manager’s ownership is tilted towards debt (\(\nu^X > \nu^Q\)) he wants to reduce volatility by trading into higher-quality investments.
When $\Psi < 1$, transactions costs offset the manager’s gain from risk-shifting. Whether they are large enough to outweigh the gain depends on the strength of the manager’s tilt towards equity and the current leverage in the deal. The higher leverage is, the more attractive limited liability becomes, and the larger the share of transactions costs born by the debt tranche. For instance refer to the expression for the manager’s gain from switching Eq. (2.24). The first term, $-\nu^X (1 - \Psi)(1 + c)e^{-\Lambda_0(1-\phi)}F_1$, is negative, representing the transactions costs loss on the manager’s share of the debt tranche. As $\nu^X \to 0$, this term goes away. The extra $(\nu^Q - \nu^X)(1 - \Psi)$ transactions costs loss when equity finishes in-the-money is captured in the second, bracketed term.

The main advantage to the manager from switching is his limited downside, expressed in the second, bracketed term. The increase in collateral par value to $F_{1,SW}$ does not compensate for the fact that equity finishes out-of-the-money more frequently. The only gain is that $X_1$ is repaid with probability $\sum_{d=0}^{d_{2,SW}} \Pi_{A_d}^d$ rather than $\sum_{d=0}^{d_{2,NS}} \Pi_{A_d}^d$. When $X_1$ is high relative to $e^{-\Lambda_0(1-\phi)}F_1$, the bracketed term is likely to be positive. What determines whether the manager switches is whether $\nu^Q - \nu^X$ weights the gain enough to overcome the broad transactions costs loss.18

Interest arbitrage (that is, $c > Y$) increases the gains and losses from risk-shifting if it supports an increase in initial leverage $X_0$. Higher leverage and more default risk means higher benefits of limited liability to equity. Gains and losses grow with $c - Y$.

Figure 2-6 illustrates the magnitudes and frequencies of risk-shifting losses, for various parameter sets, assuming the manager owns equity and no debt ($\nu^X = 0$, $\nu^Q > 0$). These numbers are the expected time 2 value changes from the manager’s switching decisions, measured from a no-effort baseline. Points on the x-axis are where the gains to equity would be negative, so the manager remains in good collateral. Points above zero show positive gains to equity, while points below zero show losses to debt. The total CDO loss is the sum of debt and equity gains, $1 - \Psi$ of the discounted face value. Risk-shifting peaks in the middle range of default realizations. There performance jeopardizes but does not wipe out equity.

18Both Ross (1997) and Leland (1998) find solitary breakpoints at which the manager begins to maximize volatility. Parrino and Weisbach (1999) find a cutoff NPV loss which equityholders are willing to accept in exchange for higher volatility, and that this number increases as firm leverage increases.
Four parameter sets show different concentration levels, absence of transactions costs, and a worse bad type collateral. With the 2.5% concentration level, risk-shifting is attractive for time 1 defaults of $d_1 = 4...6$. With a 5% concentration level, risk-shifting occurs earlier ($d_1 = 2, 3$), is more probable, and causes greater gains and losses. As concentration increases, tail events become more likely, and the benefit of limited liability is more significant. This suggests that CDOs with less diversity in their holdings should have seen more disadvantageous managerial behavior. This also highlights the nature of CDO investments as a correlation play. Any increase in systemic risk or name-by-name correlation, whether deliberate or from underestimation, makes risk-shifting more prevalent than anticipated.

The third parameter set presents the 2.5% concentration level with $\Psi = 1$, i.e. no transactions costs. Risk-shifting begins immediately ($d_1 = 0$) and continues until the manager’s equity option is out-of-the-money at $d_1 = 8$. Tranche changes are larger by the reduced transactions costs. Here risk-shifting is a pure transfer between debtholders and equityholders. It is interesting to note that even a small NPV loss, here a transactions costs loss of 0.50%, can deter risk-shifting in most cases. The extra risk-shifting occurring at $d_1 = 0...3$ due to no transactions cost loss represents 99.76% of default outcomes by probability weight.

The fourth parameter set increases the volatility option of the manager. The bad type default intensity, $\Lambda_B$, is larger, reflecting a move to single-B rather than to double-B-rated collateral. The manager’s upside is now larger so that there are default situations where switching brings him in-the-money but wouldn’t have before. In this case the manager risk-shifts more frequently ($d_1 = 2...9$) and with larger costs.

This suggests that managers can exploit quality and price differences in collateral rated at the same level. Particularly in a market with rapidly deteriorating collateral, rating agencies may not keep pace in terms of making timely downgrades. Managers can "cherrypick" assets which are more volatile but remain highly enough rated to keep within any ratings-based trading restrictions. Such assets produce larger risk-shifting gains to equity, across a wider range of outcomes. This description of events meshes nicely with what occurred in the late 1990’s. Unprecedented high levels of defaults occurred in the early goings. Rating agencies were late following through with downgrades, evidenced by the continuing high downgrade levels in 2002 and 2003 even though the bulk of credit worsening and defaults had already occurred.
Risk-shifting gains and losses can be significant in absolute terms but are modest in percentage terms. Debtholders see ex-post percentage losses of between 1/2% - 2%, and equity often increases in value by between 33% and 100% (CDO loss is simply the transactions cost loss). Because of the low probability of realizing high levels of defaults, ex-ante losses are minute. Table 4-A presents various cost-of-risk-shifting measures, for effort restricted to zero, measured from a no-switch baseline. Expected losses range from about 2 to 7 basis points - not necessarily insignificant when the typical spread on a Aaa asset ranges between 25 and 50 basis points.

Risk-shifting has a significant impact on default probability and supported leverage levels. For instance, in 5 of the 6 cases (all but the 2.5% concentration level) risk-shifting inflates the debt default probability dramatically, so that the implied rating on the debt tranche falls to triple-B or below. Depending on credit yield curves, this easily represents over 200 basis points of extra spread cost, much larger than the 40 basis point estimate found in Leland (1998). To maintain the Aaa debt rating, leverage must decline by several percent. The opportunity cost to equityholders of having to supply additional funds is calculated as the product of equity’s rate-of-return without risk-shifting (for the Medium effort parameterization) and the percent change in funds supplied by debtholders due to decreased debt issuance and prices with risk-shifting. I find increases of between 20 and 250 basis points, depending on the parameterization-dependent $\delta^Q$ used.\(^{19}\)

**Proposition 5** The effort benefits of managerial equity ownership usually outweigh the risk-shifting costs for the CDO as a whole and each tranche. The equity tranche always gains and the net effect on the debt tranche is frequently positive. The PEC generates the highest return for equity investors.

These results depend on parameter selection, but hold true across a wide range of reasonable parameterizations. The two effects of equity ownership are value-adding effort when perfor-\(^{19}\)Note that though senior and subordinate debt tranches are combined into one representative debt tranche here, tranches are differently affected by their position in the capital structure. In general, subordinate tranches bear a larger portion of the cost of the manager’s risk-shifting (but also a larger portion of the gains from effort). However, a larger increase in default probability may have a smaller effect on implied ratings, extra spread cost, etc. due to credit / default curves’ exponential shape. Modigliani-Miller doesn’t hold here due to riskiness of debt and credit spreads reflecting risk-aversion costs.
mance is good and costly risk-shifting behavior when performance is bad. Generally the former situation has much more probability weight on it, resulting in an overall gain. Further, risk-shifting can occasionally result in gains. This stems from an increased effort incentive when the equity tranche would otherwise be out-of-the-money.

Figure 2-7 illustrates, for a typical parameter set, tranche value changes from managerial behavior, conditional on the time 1 default realization. Total CDO gains (solid lines) are the sum of equity and debt gains lowered by the cost of effort. The equity tranche (dotted lines) benefits across-the-board from both effort and risk-shifting. Debtholders (dashed lines) receive moderate effort gains when performance is poor, outweighing their risk-shifting losses. One default realization illustrates the possibility that debtholders can benefit from the risk-shifting option. For \( d_1 = 6 \) the manager only provides effort when switching resulting in value gains to each tranche.

In this particular case, ex-ante effort gains fully dominate risk-shifting losses. Whether they do so in other cases depends on the effort benefit and cost functions \( \xi(\epsilon) \) and \( \Xi(\epsilon) \), the strength of transactions costs, level of equity ownership, and collateral concentration. For instance, if effort is extremely costly or ineffective, then as \( \epsilon \to 0 \), debt and the CDO see a net loss. Conversely as \( \epsilon \to \infty \), debt, equity, and the CDO gain from effort and risk-shifting never occurs. Paradoxically, although a PEC leads to risk-shifting, as long as the manager holds equity it is advantageous to have as large a share as possible. Large \( v^Q \) shares lead to more effort and less frequent risk-shifting, since effort gains bolster performance. More collateral diversification leads to less frequent and less costly risk-shifting behavior. Lastly, as \( \Psi \to 1 \), risk-shifting becomes inevitable but the trading loss shrinks until the only effect is redistribution between tranches.

The PEC is the cheapest effort incentive because it translates into fairly small deal ownership even for large equity ownership. I.e. equity leverages return by leveraging the manager’s effort incentives. Table 5 shows return numbers for various compensation schemes and parameter sets, assuming that debt is priced and sized off a zero-effort, no risk-shifting baseline. The PEC is clearly dominant for CDO and equity returns, and frequently debt. Due to higher effort incentives...
provision debt investors may benefit more from a PEC than from a debt-share contract (Tables 5-A, 5-C). Debt’s excess return is most harmed under a weaker effort assumption (Table 5-B), when transactions costs are low, or when deal concentration is high (Table 5-D). When the par gain from switching is high, debt is again better off because the high gains from effort make poor deal performance and risk-shifting less likely (also Table 5-D).\textsuperscript{21}

It is important to consider whether the misspecification of pricing and parameter assumptions could significantly alter these estimates of risk-shifting costs. Investors allege sizable costs imposed by managers’ actions during recent years. Certainly, in high default outcomes, the cost of risk-shifting becomes more pronounced while the benefit of effort provision becomes less significant. A wrong belief that CDO trading limits and quality restrictions would prevent risk-shifting may also have worsened problems. My numeric results indicate that, for the most part, risk-shifting costs, while higher, are still small both absolutely and compared to the overall financial stress. For instance, Table 4-A shows two cases of misspecified parameters. The first is lower portfolio diversification (5% rather than 2.5% concentration) which could be brought about by underestimated correlation or intentional over-concentration by the manager. If debt is priced off an assumption of 2.5% deal concentration, ex-ante risk-shifting costs increase by slightly over 6/10 of a basis point. With a higher volatility option (the manager can choose to invest in triple-C-quality rather than single-B assets) risk-shifting costs to debt increase to a higher but still modest 3/4 of a percent. Again what increases most is the probability of debt default, over 2% and 19% respectively.

Likewise, in many cases unanticipated financial stress does not dramatically increase debtholder losses. Table 4-B presents risk-shifting numbers based off observed default and loss rates from the 1998-2003 period, but assuming debt was priced using lower, mean historical default rates. In some cases (2.5%, 5%, and Ψ = 1) risk-shifting costs increase but still remain in the 8 - 12 basis point range. As can be seen in Table 6, the fundamental economic losses dwarf the added

\textsuperscript{21}The effort assumptions required to make equity ownership superior to debt ownership are very conservative. For instance, consider the five basic deal scenarios presented in Table 5 (2.5% deal concentration, 5% deal concentration, no transactions costs loss, Single-B volatility option, and Triple-C volatility option). The minimum assumptions on effort strength required to generate positive and superior expected CDO returns (for $\nu^2 = 33.8\%$) translate into six-year cumulative first-best returns of just 0.14, 1.79, 0.0046, 6.13, and 39.42 basis points, respectively. For debtholders to be better off with equity ownership (assuming a static pricing assumption) the hurdles are much higher. However anytime CDO excess returns are positive while debt’s are negative, there is room for a transfer in the form of better initial debt pricing.
risk-shifting costs. For instance in the 2.5% concentration case with zero effort, debt return is 
-2.11%, decreasing to -2.19% on account of managerial activity.

Exceptions seem to occur mainly when the unforeseen stress interacts with a higher volatility option. In the Single-B and misspecified triple-C scenarios in Table 4-B, the maximum ex-post risk-shifting costs to debt are -2.15% and -4.52% of current debt value, respectively. These are within the range of estimates found in other papers but here are large relative to the fundamental stress. The ex-ante risk-shifting costs add over 80% and 190%, respectively, to debt’s negative return from bad default performance alone (See Table 6).

These results imply that managers do have opportunities for very costly, deleterious behavior, although it often takes effect through unnecessary downgrade risk. Further, even when risk-shifting cost estimates are minor, they can be a large component of debt’s negative returns, due to CDO leverage. Note that the higher agency costs and negative returns from observed default rates do not reverse the PEC’s desirability - the shock occurring from 1998-2003 should be viewed as an occurrence with very low probability weight. However, CDO investors are actively trying to diminish the possibility of any misspecification via various trading restrictions and structural improvements.

2.4 Interest Diversion Features in Expanded Three-Period Model

In the wake of recent CDO performance problems, various structural modifications have been proposed which aim to limit risk-shifting costs. To be able to evaluate these proposals, I begin with the most prevalent structural feature: interest diversion tests. Such tests exist currently in almost every CDO and form the basis for many of the proposed structural modifications. In this section I add such tests to an expanded three-period CDO model and analyze their cashflow and trading implications. In Section 2.5 I investigate effects of proposed modifications.

These interest diversion, or coverage tests, are par-based collateral tests which divert excess cashflow away from equityholders. As such they are analogous to dividend policies in Ross (1997). They are intended to protect debtholders when performance is poor by redirecting excess interest which would otherwise go to equityholders. Coverage tests are relevant only for
intermediate interest payments. To look at their effect on manager behavior it is necessary to add a decision-making period to the basic model, one in which the manager affects intermediate rather than final period CDO cashflows. For complete details of the three-period model setup, see Appendix 2.

Below I evaluate whether the manager’s gain from risk-shifting is larger with increased cashflow control and whether the presence of coverage tests makes him more or less likely to switch collateral types. Market participants have argued that managers shift into weaker collateral in order to boost par, prevent close tests from failing, and keep residual interest cashflow flowing to equityholders rather than paying down debt. I find that risk-shifting occurs more frequently with the extra period. However I also find that the coverage tests work as intended, as a deterrent to risk-shifting trades.

2.4.1 Model Set-up

The time period from \( t = 1 \) to \( t = 2 \) is subdivided into two half periods, and three things change. First, interest payments are also made at the \( t = 1.5 \) intermediate point. Collateral bonds make a coupon payment of \( \frac{1}{2}c \) at \( t = 1.5 \) which is used to pay intermediate interest distributions \( N_{1.5} \). Second, while the manager still provides effort at \( t = 1 \), he now has the option to risk-shift at \( t = 1.5 \) if he has not done so at \( t = 1 \). (He cannot switch back).\(^{22}\)

Third, the priority of payments at \( t = 1 \) and \( t = 1.5 \) is altered by the presence of coverage tests. Without coverage tests, excess interest is distributed to equityholders each period. However, the most common coverage test, the over-collateralization or \( OC \) test, mandates that whenever the collateral-to-debt ratio falls below a certain trigger level of \( \overline{OC} \), residual cashflow is directed towards debt paydowns, to the extent necessary to "cure" the test.

If the \( OC \) test is failing and the excess interest available is sufficient to "cure" the test, equityholders’ payout is reduced by the amount necessary but is still positive. When excess interest is insufficient, debtholders receive all residual interest. The points at which interest is diverted, as well as the resultant interest diversion amounts, are used in the manager’s NPV calculations. The higher \( \overline{OC} \), the more likely that interest is diverted to debtholders.\(^{23}\)

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\(^{22}\)Ross (1997) finds reversals are never optimal.

\(^{23}\)When \( \overline{OC} \) is as low as \( Y/c \), there is effectively no coverage test and only interest parameters are relevant.
As before, the debt level adjusts each period to reflect either insufficient or surplus receipts. \(X_t\) continues to increase for high defaults, when due interest exceeds the available interest cashflow. Conversely for moderate default levels, interest diversions decrease \(X_t\), what rating agencies call "deleveraging".

2.4.2 Results

Two separate components have been added to the basic model: an extra decision period and the performance-based interest diversion feature. I analyze the effects sequentially, focusing on a manager who does not own debt \((v^X = 0)\). Prior results are supplemented by the following:

**Proposition 6** For good to moderate default realizations, the manager makes the same risk-shifting decisions as in the two-period model. However effort and efficiency are decreased by the ability to switch later.

Neither the \(t = 1.5\) interest distribution nor the manager’s option to switch following it changes the manager’s optimal \(t = 1\) risk-shifting policy, for good performance. Even though the intermediate interest distribution reduces initial leverage \((X_0\) must be reduced by approximately the amount of interest released early, or \((c - Y)F_0/2\), the manager compares a lower debt level owed to a lower \(t = 2\) cashflow. Ex-post, per \(d_1\), leverage is roughly similar. He will continue to remain in good collateral where he would have before, and continue to switch where he would have before. (Note that the volatility benefit of risk-shifting declines as time progresses. As default uncertainty resolves, the price differential between the collateral types, and hence equity’s upside, becomes smaller \((\beta_{1.5} < \beta_1)\). The manager stands only to lose by waiting). Ex-post effort levels and valuations are roughly the same for \(d_1\) realizations where the manager continues to switch.

However, wherever he chooses not to switch, he now has the option to switch later, which he will exercise if high defaults subsequently realize. This option lessens his incentive for early effort. Since the time period is cut in half, and he only anticipates switching for particular \(d_{1.5}\) realizations, the ex-post loss from the delayed risk-shifting option is small. For instance less than 5% of the cost earlier switching. But given the higher probability of reaching these less severe \(d_1\) realizations, the delayed risk-shifting option adds approximately 12% - 25% to
ex-ante risk-shifting costs.

In essence, a continuing subdivision of periods gives the manager more and more time periods to act, and leads to widespread, postponed risk-shifting behavior.

**Proposition 7** *Equity’s intermediate interest option increases the frequency of risk-shifting when performance is poor.*

For high \(d_1\) realizations, the \(t = 1.5\) interest option may remain in-the-money even when the manager’s final \(t = 2\) principal option is out-of-the-money. In this situation the manager will always risk-shift in an attempt to boost his residual \(t = 1.5\) coupon cashflow. He only ceases risk-shifting when both options are completely out-of-the-money.

This behavior is basically short-termism on the part of the manager. Terminal default risk factors heavily into the market price ratio \(\beta_1\). However the manager whose date 2 principal option is already out-of-the-money is unconcerned with principal repayment. To him the high-quality collateral is priced too richly. A manager with no long-term option value trades long-term portfolio value for a higher stream of current cashflow.

Gains to equity are larger, the larger the amount of excess interest in the deal. If there is no interest arbitrage, that is if \(c = Y\), the manager’s interest option travels almost one-for-one with his principal option, eliminating the extra risk-shifting. As \(c - Y\) increases, the base of cashflow which the manager can profitably divert increases. Debtholders absorb the entire transactions cost loss on the remaining principal in addition to whatever volatility gains the equity tranche makes. The ex-ante loss is small due to the low probability of attaining the higher default realizations.

In terms of practical implications, debtholders should be wary of any securities with a tilt towards interest over principal, for instance high-coupon, premium bonds, high-spread assets, interest-only securities, or unfunded credit-default swaps. A manager may invest in such securities in order to maximize short-term interest flows which leak out to equityholders.

The manager’s behavior for a typical parameter set is illustrated in Figure 2-8 (again effort is restricted to zero to focus on the risk-shifting incentive). The topmost figure shows ex-post risk-shifting costs to the CDO in the two-period model. The second figure shows the new risk-shifting for \(d_1 = 7\) and higher in the three-period model. For \(d_1 \leq 6\), \(t = 1\) behavior is
unchanged, illustrating the result in Prop. 6. The cost from delayed risk-shifting can also be discerned at $d_1 = 3$.

**Proposition 8** Interest-diverting coverage tests serve as a deterrent to risk-shifting trades by lessening leverage as defaults realize. The contingent payments to debtholders allow initial leverage to be increased. When initial leverage is increased, the primary effect of coverage tests is to prevent interest-driven risk-shifting.

Coverage tests are an effective way to lessen the manager’s risk-shifting incentives. When defaults are high and the manager is most motivated to risk-shift, such tests pay down debt and increase equity’s share of the deal. This makes risk-shifting less profitable and increases the manager’s share of the transactions costs loss from risk-shifting. In addition, the manager’s equity interest options can be valuable enough that he would prefer staying in less risky collateral to avoid possibly tripping the tests. When initial debt levels are kept constant, coverage tests always lead to less risk-shifting.

However, the diverted interest payments alone are a large consideration to debtholders and can significantly increase the security of debt. This increases the amount of debt a CDO can issue, increasing equity return. Basically, diversion features allow equityholders to operate at a higher level of leverage in good performance regimes, when the deal can support it, but decrease leverage back to the original level in bad performance regimes.

When $X_0$ is increased to maintain default risk, time 1 risk-shifting behavior will improve less than before but in general will not be worse than without an OC test. The intuition is that the amount of risk, loosely speaking, is the key to the manager’s decision to risk-shift. $X_0$ is increased to maintain risk, so risk-shifting will not increase past where it would be without a coverage test. Further, coverage test failures can remove interest options. This counteracts the manager’s incentive to risk-shift in cases where he is only motivated by the intermediate interest option cashflows.

Manager behavior for a range of OC trigger levels is illustrated in Figure 2-8. The second plot shows ex-post risk-shifting costs to the CDO in the three-period model without an OC test. The next three plots add $\overline{OC}$ test levels of 100%, 102%, and 105%. Risk-shifting declines steadily as $\overline{OC}$ increases. Even a moderate test level can eliminate much of the interest-driven
risk-shifting; at $\overline{OC} = 105\%$, which is already failing given the initial debt level, risk-shifting returns to roughly its level in the two-period model (top figure). In both the two-period model and the three-period model with $\overline{OC} = 105\%$, risk-shifting is driven entirely by the manager’s $t = 2$ principal option, occurring at $d_1 = 4...6$.

Note that the benefit of coverage tests to debtholders comes more through diverted interest payments than reduced risk-shifting. For instance, in the parameter set presented, there is less than $1/5000$ of a % chance that $d_1 > 6$, where risk-shifting can be curtailed. The diverted interest payments, paid as soon as the tests break, occur with much greater frequency. The $102\%$ test level supports a $1.75\%$ increase in the debt ratio and begins to fail at $d_1 \geq 2$ ($10.37\%$ likelihood); the $105\%$ test supports a $4.25\%$ increase in the debt ratio and is already failing (i.e. $100\%$ likelihood). Though these test levels can cut ex-post risk-shifting losses to debt by up to $50\%$, the diverted interest is over $4 - 6$ times the prevented risk-shifting costs. Both factors make the gains from decreased risk-shifting less significant, in ex-ante terms, than the diverted interest.

I find no support for the hypothesis that coverage tests induce risk-shifting in order to prevent interest diversions. Specifically, I find no increased tendency to make risk-shifting trades due to a desire to circumvent the test and preserve interest flows to equity. Rather, my model predicts that coverage tests’ ability to prevent risk-shifting (assuming leverage is increased to its maximum) is precisely when interest flows alone would have prompted risk-shifting. In situations where a coverage test is failing, the manager often wants to risk-shift anyway, for principal considerations. The only situation in which a failing coverage test might affect his behavior is if a test is set unreasonably high, bearing little relation to debt repayment. However, it is unlikely that with such a high hurdle switching collateral types could restore his interest flow, so he has little to gain from switching. The general effect is to a decline in risk-shifting.

**Proposition 9** Intermediate interest payments induce higher managerial effort when default performance is poor. Coverage tests increase effort at moderate default levels through reductions in leverage. However they counteract effort increases for high $d_1$ as they remove the manager’s equity interest option.
The manager's incentive for effort in the three-period model is increased when he holds a valuable equity interest option. Even when defaults have wiped out the equity principal option, he remains interested in the deal’s progress, as evidenced by his new risk-shifting in this realm, and will provide positive as opposed to zero effort. The majority of the benefits of such effort accrue to debt since the equity principal option is out-of-the-money. Whether these benefits outweigh the costs of the new risk-shifting depends on the effort specification. For example, in the Medium 2.5% parameterization, the manager risk-shifts at $d_1 = 7...19$ where he was completely out-of-the-money before. However for each realization the benefits of now-positive effort outweigh the costs of risk-shifting by about 5% - 10%. Conversely with the Weak parameterization, the costs of new risk-shifting outweigh the benefits of increased effort by about 150%. As before, the ex-ante significance is low because of the low probability of attaining the higher default realizations.

An unambiguous benefit however of coverage tests is to boost effort early on. For moderate $d_1$ levels, debt paydowns restore equity’s interest in the structure and prompt higher effort levels than would otherwise result.

Figures 2-9 and 2-10 illustrate the effect of coverage tests on the manager’s effort and risk-shifting decisions, showing ex-post value changes to the CDO. In the Medium 2.5% parameterization (Figures 2-9), effort is sufficiently beneficial to overcome the costs of risk-shifting. Coverage tests, beginning with $\overline{OC} = 100\%$ and proceeding to $\overline{OC} = 105\%$, increasingly eliminate effort gains in the high $d_1$ region but increase effort gains for the low $d_1$ region. By $\overline{OC} = 105\%$, manager behavior is roughly as it was in the two-period model. The most efficient test level is a quantitative trade-off of more and less probable effort gains (in this case near $\overline{OC} = 100\%$).

In the Weak 2.5% parameterization, Figure 2-10, the risk-shifting costs in the high $d_1$ region outweigh the effort gains. Here coverage tests have an unambiguously positive effect since they both eliminate risk-shifting losses for high $d_1$ realizations and increase effort gains for low $d_1$ realizations. The most efficient test level is a quantitative trade-off of more probable effort gains and less probable risk-shifting gains (again near $\overline{OC} = 100\%$).

**Proposition 10** As before, the effort benefits of the PEC generally outweigh the risk-shifting costs for the CDO as a whole and for each tranche. CDO returns are roughly the same in the
two- and three-period model. However, equity returns increase significantly as coverage tests support an increase in initial leverage.

The analysis of returns for the 2-period model holds true here. That is, for most effort parameterizations the benefits of the PEC outweigh the costs. As before the PEC becomes more costly as concentration, transactions costs, or bad-type default risk increases.

Table 7 presents return numbers for various parameter sets and \( \bar{OC} \) trigger levels, assuming that debt is priced off a zero-effort, no risk-shifting assumption. Frequently both debt and equity make positive returns through this pricing assumption. Uniformly, equity rate-of-return increases in \( \bar{OC} \) even if the higher hurdle produces less effort or a lower CDO return. Leverage, at the margin, is more beneficial to equityholders than effort. Debtholders also generally benefit from higher \( \bar{OC} \) levels.

2.5 CDO Structural Modifications

Allegations of risk-shifting in CDOs have led to a search for effective structural alterations and indenture restrictions. Investment banks, rating agencies, and managers have worked to design new features that are administratively reasonable and make CDO structures more tamper-proof. Proposed modifications include various manager compensation schemes, alternate interest diversion features, restrictions on low-priced or low-rated assets, etc. Successful solutions will be able to promote value-adding effort while curtailing risk-shifting trades. Managers will require the most oversight when performance is bad.

I present six representative proposals and evaluate their efficacy in the context of my model. Tables 8A-8F present leverage and return results for all extensions, for various parameter sets. I assume here that debt is sized and priced taking risk-shifting into account. Thus the measured value of each proposal gives equity credit for less risk-shifting in terms of greater leverage and debt price.\(^{24}\) Success is judged by equity and CDO rates-of-return. While equity returns increase with leverage, CDO returns are unchanged in leverage and reflect only the net effect of effort and risk-shifting. Section 2.5.7 compares results.

\(^{24}\)I continue to assume rating agencies give no credit to effort, since they profess blindness with regards to talent evaluation. However, for simplicity I assume debtholders price in the benefits of both effort and risk-shifting, making a zero expected return. This focuses the discussion on CDO and equity returns.
2.5.1 Extension 1: OC Whole Interest Diversions

One simple alteration which can potentially add value to CDO structures is more punitive coverage tests. The more punitive the outcome to failing, the more effectively coverage tests deleverage a transaction in stress. Suppose that upon a test failure, all available residual interest is diverted, rather than the amount necessary to cure a test. This has limited applicability to the small region where the test is failing but not badly enough to require a complete diversion.

This measure limits risk-shifting slightly through increased deleveraging in times of stress and by occasionally removing the manager’s $t = 1.5$ interest option. It can support increased debt levels, generally for moderate test levels. Moderate test levels are mild enough to not always require full interest diversion, but tight enough to still be reasonably likely to fail. The larger the available excess interest, the larger the effect.

The effect on effort is small and generally positive. In this as in future extensions, lower leverage usually induces greater effort. Here, by more aggressive deleveraging, the whole-interest diversion feature prompts higher effort at low default levels. This is offset by two potential decreases in effort: First, at moderate default levels, the higher diversions can remove the manager’s interest option completely, rather than partially, lowering effort. Second, there may be a small decrease in effort for good performance if the more punitive diversions support a higher initial debt level $X_0$. Occasionally the latter effect dominates, making the net effort effect negative.

Equityholders benefit most from this modification if it increases initial leverage. For instance, for most parameterizations equity gains the most when $\overline{OC} = 102\%$. The CDO usually gains the most when $\overline{OC} = 105\%$ as this instance generates the greatest increase in effort and decrease in risk-shifting. Though equityholders gain less from this measure at the 105% test level, they still prefer the higher test level absolutely, due to the greater initial leverage.

2.5.2 Extension 2: Trade-based Management Fees

Non-contingent management fees are too expensive to use as an effort incentive since they consist largely of a debt incentive.\textsuperscript{25} However it is possible that contingent fees could specifically reward

\textsuperscript{25}Most managers are paid senior and subordinate fees at a constant rate $\tilde{s}$ on collateral face value. Subordinate fees are paid between the debt and equity distribution and as such are similar to equity ownership in terms of effort.
the manager for not risk-shifting. I take the approach of rewarding the manager for a declining trading volume. The deal contracts to pay a senior, contingent time 2 management fee at rate $\gamma$ based on the cumulative proportion of traded collateral.

There is always some level $\gamma$ which can completely curtail risk-shifting. However as a practical matter it is useless to pay $\gamma$ when performance is good, as the manager is not inclined to risk-shift then, and it may disincentivize effort. To save costs the manager will only be paid if an OC failure has occurred in the deal, and at the lowest $\gamma$ which completely curtails risk-shifting.

There is little effect on effort, because the fee is only relevant for high default levels when OC tests are failing. In these cases, effort provision is generally small, so the trades eliminated are the down-in-quality, risk-shifting trades. (Assuming the fee / trade schedule is not set too restrictively). This measure almost always results in CDO gains from reduced risk-shifting, when including the fee payments made to the manager.

However, the benefit to equityholders depends very much on the prevalence and cost of risk-shifting. When the default risk added by risk-shifting is largest, the fee payments can support an increase in initial leverage and improve equity returns. For instance in the Single-$B$ cases (Tables 8E-8F), by eliminating risk-shifting at $d_1 = 8...15$ and $d_1 = 6...13$, respectively, these fees support a 2.55% (OC = 102%) and 2.00% (OC = 105%) increase in the debt ratio. In contrast in the Medium 2.5% parameterization (Table 8A), although performance fees eliminate risk-shifting, the initial debt level $X_0$ must actually decline. The amount of fees paid out is more detrimental to debtholders than the previous risk-shifting costs. Thus in Tables 8A-8D, the 2.5% and 5% parameterizations, equity returns decline in two-thirds of the cases, whereas in Tables 8E-8F, the Single-$B$ parameterization, equity returns increase for all OC levels.

Clearly, for many parameterizations, contingent fees may be too costly a way of eliminating trading in bad performance regimes. Equityholders may be able to achieve the same effects without incurring additional cost. See Sections 2.5.4 for a non-contingent trading limit and 2.5.6 for a contingent trading prohibition.

and risk-shifting incentive. Senior fees can be exactly replicated in the two-stage model by adding $\nu = \hat{\gamma}/(1 + c)$ to both $\nu^A$ and $\nu^Q$. Insofar as proportionate ownership is overly costly with regards to effort stimulation, the same is true of management fees. One reason to pay fees is in case of costly manager bankruptcy risk. If not being compensated in poor performance times could lead to bankruptcy, and it is costly to the deal to replace managers, it may be beneficial to pay subsistence level management fees.
2.5.3 Extension 3: OC Haircuts

One criticism of coverage tests is that they rely on par values and do not reflect more gradual declines in collateral credit quality. Theoretically managers can rapidly trade a few deteriorating bonds, delaying default and keeping portfolio interest flowing to equityholders. That is, currently only realized collateral losses are factored in, not unrealized losses.

Many market participants have both suggested and begun using strengthened OC tests which weight assets based on low ratings or market values. Unrealized losses would then influence cash distributions and provide more protection to debtholders. These "haircuts" take various forms, for instance carrying C-rated assets at a 50% weight, single-B assets at 80%, any assets with a dollar price < $70 at their current estimated market value, etc. I use a simple market value weighting of 100% for assets identified as good, and $1/\beta_{1.5}$ for assets identified as bad. Asset haircuts have an effect only at $t = 1.5$ since the manager is not able to trade prior to $t = 1$.

This measure tightens coverage tests equally whether the manager provides effort or risk-shifts. Surprisingly, it has very little effect on risk-shifting. When tests are already failing, the haircuts are superfluous. When tests are in no danger of failing, the manager is not likely to risk-shift anyway. Haircuts occasionally lessen risk-shifting for moderately poor default outcomes. There they can lower leverage or put a manager’s $t = 1.5$ interest payment in jeopardy, discouraging trades. Saved costs to debt are usually less than 5% of prior risk-shifting costs, although for select $d_1$ realizations they can be cut by up to 30%.

The most beneficial effect is to increase interest diversions, which increase the amount of debt that can be issued. In this respect are they similar to whole-interest diversions. Especially in scenarios with high levels of effort, test failures are both more unavoidable and engender larger interest diversion payments to debtholders. For instance haircuts increase the supportable debt ratio by 0.60% ($\overline{OC} = 100\%$), 1.05% ($\overline{OC} = 102\%$), and 0.80% ($\overline{OC} = 105\%$) in the Single-B scenarios (Tables 8E-8F). These changes are larger than occur with whole-interest diversions (Extension 1). However haircuts don’t increase leverage in the 2.5% portfolio concentration scenarios (Tables 8A-8B), contrasting unfavorably. There whole-interest diversion tests support a 1.00% ($\overline{OC} = 100\%$) and 1.15% ($\overline{OC} = 102\%$) increase in the debt ratio.

Haircuts have an ambiguous but potentially negative effect on effort. They penalize effort in
that the more effort the manager provides, the more likely coverage tests are to fail. However, increasing interest diversions lowers leverage which can yield higher effort. The net effect is that haircuts are most valuable to equityholders when gains from effort are highest (Single-B scenarios). Gains stem more from higher diversions and increased leverage than from higher effort or reduced risk-shifting. CDO returns typically travel inversely with equity returns, due to negative effort effects. 26

2.5.4 Extension 4: Portfolio Turnover Limits

Currently CDOs have discretionary trading limits of typically 15% - 25% annually. However there are no limits on credit-improved or credit-impaired sales and it is the manager who makes this designation. As such, trading limits do not appear to limit trading in practice at all. Trading limits would be more helpful if all trades were classified as discretionary. Here I impose various hard trading caps ($T = 25\%, 50\%$) which are applied to all trades. Depending on the trade cap and effort parameterization, there are occasions where such limits will prevent effort-based trades or allow risk-shifting trades.

Frequently the manager will choose to risk-shift where he would have before, but to the lesser extent allowed by the trade limit. However, the trade limit lessens the amount the manager can gain from risk-shifting (lower upside). It can thereby prevent risk-shifting in situations when the manager cannot no longer boost volatility enough to gain. Risk-shifting costs to debt are now inversely related to the trade cap. They are usually cut by up to either 75\% or 50\%, and in some cases by nearly 100\%. The decrease in risk-shifting supports a small increase in initial leverage, but not to the extent increased interest diversions do.

This scenario is naturally very sensitive to the trade cap level. As the trade limit goes to 0\%, the CDO becomes essentially a static deal; as it approaches 100\%, there is no effect on the deal. Set too low, the trade cap blocks optimal effort, and returns will be lower for all participants (See Table 8E). It is ideal to set $T$ slightly above what would be the trading

26 There are various complications to market-value adjustments. Since the adjustment is meant to reflect credit risk alone, fixed asset prices must be adjusted by an agreed-upon method to eliminate the effect of interest rate moves. (Corporate bonds are predominantly fixed while loans are generally floaters). In addition, the reliability of prices can be an issue. Illiquidity, especially for corporate loans or high yield debt, can lead to wide bid-ask spreads while prices obtained from monthly dealer quotes can be inaccurate if the assets are rarely traded. Lastly, requiring market value adjustments can impose sizable administrative costs on deals.
level $\xi(\epsilon_{NS})$ resulting from the no-switch effort choice. Effort levels are then maintained and risk-shifting is greatly reduced in scope, boosting both equity and CDO returns simultaneously. Even if leverage does not increase, debtholders pay more for their shares since risk-shifting costs decline.

In general, trading limits do a better job of increasing CDO returns but a worse job than OC-based reforms of boosting equity returns. This is particularly true when coverage test levels are low (None, 100%) or high (105%) since OC diversions and haircuts have little effect with such test levels. It is also true when the risk-shifting costs to debtholders are largest (5% Concentration, Single-B scenarios). There curtailing the larger risk-shifting costs is more beneficial. For instance, in the Medium 5% scenario with $OC = 100\%$ (Table 8C), of all considered proposals $R^C$ is at its highest with the 50% trading cap. However $R^Q$ actually declines since there is no increase in initial leverage. The same occurs for the weaker effort case (Table 8D) at the 25% trading limit.

In practice certain types of CDOs are indeed static or "lightly managed". These are frequently CMBS (commercial real estate mortgages) or other ABS asset types (residential real estate mortgages, securitized auto and credit card loans, etc.). Corporate loan or debt CDOs tend to have the most lenient trading restrictions. This may attest to the type of value-added activity investors perceive. Securitized products may be viewed as more diversified and stable, thus benefiting less from active management. The lower trade limit may allow enough room for any effort-driven trades the manager wants to make.\footnote{Hard trading caps can be approximated by various ratings-based limits, which are currently included in most CDOs. Weighted-average rating limits, known as WARF tests, limit wholesale increases in risk. The manager will either limit his low-rated purchases or be restricted to smaller changes in $\Lambda_B - \Lambda_C$. Various rating baskets are also limited, such as below-investment-grade, single-B, and triple-C assets. This allows unlimited trading at the initial rating level, but prevents bifurcation of the portfolio (which increases volatility). These tests, however, do not prevent the manager from buying riskier collateral that the agencies have not yet downgraded, and there is frequently a lag between market and agency reassessments. The efficacy of such measures clearly depend on rating agency accuracy and timeliness, and as such may be less effective than hard trading caps.}

In most cases hard trading caps are more efficient than paying the manager contingent fees based upon trading levels (Extension 2) because of the extra cost. However in some cases the reverse is true, especially if the caps are overly restrictive. The manager can choose to ignore the fee if effort returns are high enough, boosting overall return.
2.5.5 Extension 5: Collateral Purchases from Excess Interest

Another trading restriction currently in use is to require all low-rated or low-priced assets to be purchased only from available $t = 1$ or $t = 1.5$ excess interest. The manager can choose to divert current equity proceeds and use it for purchase of new collateral. Proceeds are pooled and the new collateral supports debt just as original collateral does. Not surprisingly, this measure completely eliminates risk-shifting trades. There is no value-added in buying the bad type collateral, since they are priced fairly. A value-maximizing manager will not trade certain equity distributions for uncertain, subordinate future distributions.\(^\text{28}\)

A manager will only choose to invest equity funds if he can add value through effort. For him to do so, the value-added would have to accrue mainly to equityholders, i.e. only when default performance is good. Purchases are limited to the amount of available interest, indicating this measure will allow the most effort in deals with high interest arbitrage ($c \gg Y$) or low coverage test levels. This restriction is most advantageous, and supports the greatest increases in initial leverage, when effort levels are not too large to begin with and the costs from risk-shifting are large.

Indeed this restriction, of all considered, usually allows for the greatest increases in leverage (See Tables 8C-8D, $OC = \text{None}$ and 100% and Tables 8E-8F, all $OC$ levels) since risk-shifting is completely eliminated. (Recall rating agencies give no credit to managerial effort, so increases or decreases in effort do not factor into initial leverage). However because the measure does not distinguish among good and bad performance regimes, i.e. is overly restrictive, value is destroyed in all effort parameterizations considered. The manager provides much less effort in good regimes and either chooses not to, or is unable to, provide any effort or in bad regimes. CDO and equity returns decline precipitously, more so as effort is more beneficial and coverage levels higher.

In general this restriction should only be considered when effort benefits are low and risk-shifting costs high, or excess interest high. Else the loss of effort in good default outcomes outweighs the prevention of risk-shifting. See Section 2.5.6 for a comparison with a contingent

\(^{28}\)This measure provides an interesting way for a manager to bolster his reputation. When investing equity interest would be basically a transfer to debtholders, the manager may choose to do so anyways out of a concern to deliver a promised return.
trading prohibition.

2.5.6 Extension 6: Contingent Trading Prohibition

A more targeted trading measure than in Extensions 2, 4, and 5 would be simply to eliminate all purchases (or low-rated purchases) when coverage tests are failing. With no or low coverage test levels, this measure has little effect. With higher test levels the constraint becomes tighter, eliminating the most risk-shifting and supporting the greatest increases in leverage.

This measure also restricts effort, overly so when tests are extremely tight (for instance, in all cases when \( OC = 105\% \)) but less so when test levels are moderate. For instance, in the 2.5% and 5% parameterizations (Table 8A-8D, \( OC = 102\% \)), a contingent trading prohibition yields the second highest equity return of all extensions considered, although a reduced CDO return. At an OC test level of 105%, both CDO and equity returns decline drastically from reduced effort provision.

On the whole such a restriction is beneficial in eliminating risk-shifting but may be too onerous in not allowing effort. It will work best if it has its own supplemental trigger level not coinciding with \( OC \).

2.5.7 Discussion

On the whole, the most effective and widely applicable measures are Extensions 1 and 4, which succeed in boosting both CDO and equity returns. Whole-interest diversions are very effective when the coverage levels are moderate (otherwise they add little in severity to the current test level). Trading caps are extremely effective across all coverage levels but only when the trade cap is not set too restrictively. Contingent trading restrictions (Extension 6) are effective at boosting equity returns by increasing initial leverage. However they can be costly in terms of restricting effort, particularly when coverage test levels are high, and CDO returns frequently decline.

When there are large gains to both effort and risk-shifting (Single-B and sometimes 5% scenarios) Extension 2 and Extension 3 are sometimes preferable to stricter trading caps. Both allow the manager discretion in deciding whether potential effort gains are worth foregoing payments - the contingent trading fee and more certain interest receipts, respectively. Extension
5, allowing trades out of excess interest only, is too restrictive for most environments.

Clearly some of these ideas work better in conjunction. Whole-interest diversions, with OC haircuts and a contingent trade cap (a combination of Extensions 4 and 6) may be the most effective combination. They both combat risk-shifting as well as increase profitability.

Lastly, note that many of the above proposals struggle to distinguish between effort-driven and risk-shifting trades. However they all allow (or can be rewritten to allow) unlimited "credit-risk" sales, in which a manager sells a deteriorating bond before the market prices in its greater default risk. Proceeds from such sales can be unambiguously reinvested in good type collateral. This type of effort-driven trade is not modeled in my paper but is clearly an avenue by which managers add value in the real world. The greater the proportion of such trades in the universe of effort-driven trades, the less costly it becomes to strictly restrict purchases of lower-rated collateral.

2.6 Conclusion

In this paper I propose a simple model of CDO management which relies on familiar principal-agent incentive problems. I find that ex-post, debt investors are warranted in complaints about managerial misbehavior. In periods of poor performance, equity-holding managers slack off and risk-shift to the detriment of debt investors. However, ex-ante debtholders are often advantaged by the manager’s incentives. For even conservative assumptions on the capabilities of the manager, the benefits of higher effort outweigh the costs of risk-shifting. In fact debtholders may have a larger ex-ante excess return when the manager’s incentives are aligned with equity rather than debt. Due to the low cost of equity, it is the most efficient incentive to give the manager. It yields a higher return to equity investors even when debt investors price in anticipated risk-shifting costs.

There is always the possibility of parameter misjudgment. If deal or default parameters are misspecified, the costs of risk-shifting and effort-shirking can grow. However an analysis using recent default parameters indicates that in all but a few scenarios, financial stress, rather than risk-shifting-induced stress, would have accounted for the bulk of debt and CDO losses.

The numerical results for risk-shifting in this financial context are consistent with those
found in more traditional corporate contexts, in that ex-post risk-shifting costs are modest. However, I find substantially larger increases in debt yields or equity opportunity costs and significant decreases in initial leverage. Perhaps in contrast to corporate firms, in CDOs even a few points of leverage is of paramount importance. CDOs’ raison d’etre is value creation through risk-tranching and less tranching implies smaller equity returns. The proliferation of proposed remedies attests to the desire to control risk-shifting. My numerical analysis provides a flexible framework in which to evaluate these proposals’ ability to add value.

There are several features potentially of relevance to managerial behavior and contract optimality which I have not explored. The most obvious is debtholder risk aversion. Explicit risk aversion costs would increase the costs of equity contracts. When risk-shifting diminishes debt value the most, it always increases volatility the most. A compensatory lower initial debt level or price would require more upfront funds from equity investors.

Risk-aversion on the part of the manager could deter non-value adding trades, making the need for structural improvements less pressing. Risk aversion can exist through a concave objective function brought about by reputational concerns (for both a firm’s CDOs and its traditional asset management business) or company and personal investments. If so, larger institutional managers, or those CDOs whose managers invested personally, should have delivered better debt and collateral performance and less turnover. To the extent that the recent stress period has winnowed out weaker performers, it may have left only CDO shops with significant franchise value and already curtailed the risk-shifting problem.

A last consideration is contingent contracts. One can imagine managerial contracts which fluctuate with collateral performance. However, there are both verifiability and coordination issues. As a practical matter, credit deterioration is not a binary event, but rather a gradual process which may be difficult to measure. Even portfolio market value would require strict rules and oversight over the obtainment of bids and pricing assumptions, particularly in less liquid or assumption-dependent sectors. Credit or trade classifications would inevitably be to the detriment of certain parties and likely engender protracted, expensive legal negotiations.

This model makes specific predictions about trading activity, trading returns, and collateral performance. As performance worsens managers are predicted to add volatility, either through default risk or sectoral concentration, and interest-heavy assets which convert deal principal
into interest cashflow. In a related paper I test empirically for the strength of these managerial risk-shifting incentives.
2.7 Appendix 1: Calibration Detail

2.7.1 Description

In all cases, the initial collateral level is scaled to $F_0 = 10$ and deal parameters scaled to a six-year time frame. Default parameters are from Moody’s Research Services. Default intensities $\Lambda_k$ represent six-year cumulative default rates. Along with $\phi$, they are based off the six-year collateral loss rate for their rating level, assumed recovery rate, and choice of concentration level. The "good" type collateral is based off Baa-rated collateral. The debt default limit of 0.25% is taken from Moody’s six-year cumulative default rate for Aaa-rated collateral.\textsuperscript{29}

I take two approaches to the default calibration. In the first, or "average" approach, I use average historical default rates from the 1970-2003 period, as well as the average historical corporate recovery rate of 40%. This is appropriate for new deals since rating agencies evaluate based on historical outcomes.

In the second, or "amplified" approach, I magnify certain factors to mimic what took place during recent years. This gives an indication of the observed costs of risk-shifting. I use higher default rates from the 1998-2003 period, which I further magnify to account for the disparity between dollar-volume weighted default rates and issuer-weighted default rates. Dollar-volume weighted default rates more accurately describe actual CDO collateral losses and were several times higher.\textsuperscript{30} I also use lower, observed corporate recovery rates for the 1998-2003 period.\textsuperscript{31}

In addition I increase loss severities to reflect hedging losses that can result from early defaults and declining interest rates. Corporate bonds are fixed-rate assets, and CDOs generally hedge their floating-rate liabilities by entering into a fixed-floating swap. As defaults mount, the deal has less fixed-rated collateral generating cashflow to cover the fixed payment, and the swap contract becomes more onerous as LIBOR declines (over 5% in this period). I assume associated hedge costs increased collateral losses by 33%.\textsuperscript{32}

\textsuperscript{29}I use corporate default statistics given their longer history. Moody’s stated policy on loan and structured finance assets is to make their ratings equal to corporate ratings, in terms of default risk and recovery rate. To the extent performance bears this out, the results suggested by my numerical analysis are applicable to CDOs of all collateral types.

\textsuperscript{30}On average dollar-weighted defaults match issuer-weighted default rates. However the late 1990’s saw many high-volume defaults which skewed the ratio. Dollar-volume weighted statistics are only available going back to 1994. Source: Moody’s Investors Service (2004a).

\textsuperscript{31}Source: Moody’s Investors Service (2004a).

\textsuperscript{32}See, for instance, Fitch Ratings (2002a).
Compensation parameters $\nu^X$ and $\nu^Q$ are varied in the text and figures. Ownership shares are informed by current CDO market structure. Managers commonly own up to 33% of the equity tranche, and I explore debt ownership shares equivalent in terms of total deal ownership. Transactions costs are scaled to a cost of 0.50%, which translates to $50,000 on a $10 million dollar trade. This figure is roughly consistent with observed bid-ask spreads in various asset markets.

Interest parameters $c$, $Y$, and $OC$ are also varied in the text and figures. The spread between collateral asset coupons $c$ and liability coupon $Y$ is reasonably between 1% and 5% for most collateral types. The debt tranche $X_0$ is sized per rating agency assumptions off either a static portfolio assumption ($\epsilon = 0$, no trading) or one acknowledging risk-shifting ($\epsilon = 0$, risk-shifting).
### 2.7.2 Default Parameters

Parameterizations listed below are the basis for many examples and figures referenced in the text. The interest parameters shown are $c = 0.05$, $Y = 0.02$, and no coverage test. Supported debt levels shown for a static portfolio assumption $(X_{0,NT})$ and one that accounts for risk-shifting $(X_0)$. Amplified scenarios apply higher default rates according to the discussion in Section 2.7.1. Deviations in the text are noted.

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<td>0.5437</td>
<td>0.5437</td>
<td>0.2719</td>
<td>0.5437</td>
<td>0.5437</td>
</tr>
<tr>
<td>$\Lambda_B$</td>
<td>2.8194</td>
<td>2.8194</td>
<td>1.4097</td>
<td>8.1504</td>
<td>16.5053</td>
</tr>
<tr>
<td>$\beta_1$ (2-pd)</td>
<td>1.0349</td>
<td>1.0349</td>
<td>1.0349</td>
<td>1.1214</td>
<td>1.2718</td>
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<tr>
<td>$\beta_1$ (3-pd)</td>
<td>1.0344</td>
<td>1.0344</td>
<td>1.0344</td>
<td>1.1198</td>
<td>1.2679</td>
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<tr>
<td>$\beta_{1.5}$ (3-pd)</td>
<td>1.0173</td>
<td>1.0173</td>
<td>1.0173</td>
<td>1.0590</td>
<td>1.1277</td>
</tr>
</tbody>
</table>

#### B. Amplified Default Rates 1998-2003

<table>
<thead>
<tr>
<th>Variable</th>
<th>2.5%</th>
<th>$\Psi = 1$</th>
<th>5%</th>
<th>Single-B</th>
<th>Triple-C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda_G$</td>
<td>1.8092</td>
<td>1.8092</td>
<td>0.9046</td>
<td>1.8092</td>
<td>1.8092</td>
</tr>
<tr>
<td>$\Lambda_B$</td>
<td>2.2731</td>
<td>2.2731</td>
<td>1.1366</td>
<td>10.5846</td>
<td>34.91676</td>
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<tr>
<td>$\beta_1$ (2-pd)</td>
<td>1.0108</td>
<td>1.0108</td>
<td>1.0108</td>
<td>1.2256</td>
<td>2.1543</td>
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<tr>
<td>$\beta_1$ (3-pd)</td>
<td>1.0107</td>
<td>1.0107</td>
<td>1.0107</td>
<td>1.2224</td>
<td>2.1301</td>
</tr>
<tr>
<td>$\beta_{1.5}$ (3-pd)</td>
<td>1.0054</td>
<td>1.0054</td>
<td>1.0054</td>
<td>1.1071</td>
<td>1.4678</td>
</tr>
</tbody>
</table>
### 2.7.3 Effort Parameters and Returns

Effort strength is the primary unknown in the model. I vary effort parameters to generate a range of first-best CDO returns and calculate efficiency results for each case. As presented in Proposition 5, to establish that subordinated manager contracts are preferable to senior contracts, the assumptions on effort effectiveness are extremely conservative.

The effort return function is $\xi(e) = 1 - e^{-\zeta_1 e}$, ranging between 0 and 1. The cost function is $\Xi(e) = \zeta_2 e^{\zeta_3}$. Values for $\zeta_1$, $\zeta_2$, and $\zeta_3$ imply six-year cumulative first-best CDO excess returns $\mathcal{R}_{FB}$ and combine with ownership shares and default parameters in Section 2.7.2 to generate manager behavior and tranche valuations.

<table>
<thead>
<tr>
<th>Return</th>
<th>2.5%</th>
<th>5%</th>
<th>Single-B</th>
<th>Triple-C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{R}_{FB}$ (2-pd)</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\mathcal{R}_{FB}$ (3-pd)</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

**A. Zero Effort:** $(e = 0)$: $\zeta_1 = 0.0$, $\zeta_2 = \zeta_3 = \infty$

| $\mathcal{R}_{FB}$ (2-pd) | 0.6707 | 0.8213 | 0.6706 | 3.6705 | 10.1332 |
| $\mathcal{R}_{FB}$ (3-pd) | 0.6586 | 0.8087 | 0.6585 | 3.6092 | 9.9553 |

**B. Weak Effort Strength:** $\zeta_1 = 0.5$, $\zeta_2 = 0.0750$, $\zeta_3 = 4$

| $\mathcal{R}_{FB}$ (2-pd) | 1.4396 | 1.7395 | 1.4394 | 6.9843 | 17.8175 |
| $\mathcal{R}_{FB}$ (3-pd) | 1.4153 | 1.7145 | 1.4152 | 6.8764 | 17.5291 |

**C. Medium Effort Strength:** $\zeta_1 = 0.5$, $\zeta_2 = 0.0025$, $\zeta_3 = 4$

| $\mathcal{R}_{FB}$ (2-pd) | 2.4232 | 2.8738 | 2.4229 | 10.1249 | 23.8672 |
| $\mathcal{R}_{FB}$ (3-pd) | 2.3863 | 2.8365 | 2.3860 | 9.9822 | 23.5112 |
2.8 Appendix 2: Expanded (Three-Period) Model Detail

The two additions to the basic model are an extra decision period and a performance-based interest diversion feature.

The time period from \( t = 1 \) to \( t = 2 \) is subdivided into two half periods. Interest payments are now made at both \( t = 1 \) and \( t = 1.5 \). The manager still provides effort at \( t = 1 \) but now has the option to risk-shift at \( t = 1.5 \) if he has not done so at \( t = 1 \). He cannot switch back if he switched earlier.

In addition, certain performance criteria ("coverage" tests) regulate the flow of excess interest each period. As in the two-period model, excess interest cash is paid out each period to equityholders. However, if performance criteria fail, excess interest is used to redeem debt early, decreasing CDO leverage. The calculation of these tests is detailed below.\(^{33}\)

2.8.1 Time \( t = 0 \)

The deal is structured and priced as before. \( V_t \) represents the cumulative value of anticipated cash distributions at time \( t \). \( B_t^0 \) and \( R^t \) remain the initial dollar price and rate of appreciation, or expected return of a tranche. Collateral market prices now factor in half coupon payments at \( t = 1.5 \) and \( t = 2 \). This implies an initial market price for a type \( \kappa \) bond of

\[
b_0^\kappa = \sum_{d=0}^{\infty} \Pi_\kappa^d \phi^d \left( c + b_1^t \right) = ce^{-\Lambda\kappa(1-\phi)} + \frac{c}{2} e^{-\frac{3\Lambda\kappa}{2}(1-\phi)} + \left( 1 + \frac{c}{2} \right) e^{-2\Lambda\kappa(1-\phi)}
\]  

(2.30)

using the \( t = 1 \) collateral price \( b_1^\kappa = \frac{c}{2} e^{-\frac{\Lambda\kappa}{2}(1-\phi)} + \left( 1 + \frac{c}{2} \right) e^{-\Lambda\kappa(1-\phi)}. \)

2.8.2 Time \( t = 1 \)

Interest Payment

After the \( d_1 \) default realization, full payment of \( c \) and \( Y \) occurs as before. Equityholders continue to collect residual interest, however now certain performance tests regulate the distribution of excess interest. A variety of "coverage" tests quantify the ratio of excess par or

\(^{33}\)The subdivision of the final period and restriction of effort to time 1 keep the effect of effort constant across the two- and three-period models. Additional effort at \( t = 1.5 \), or extra periods over which \( t = 1 \) effort operates, would distort numerical comparisons.
collateral interest to the amount of liability debt or liability interest due. Most deals have several tests with various provisos. I focus on the most simple "over-collateralization" or OC test. This test mandates that whenever the collateral-to-debt ratio falls below a trigger level of $OC$, residual interest cashflow is directed towards debt paydowns, to the extent necessary to "cure" the test.

The OC test is failing at time 1 if $F_1 < OC$. The amount of principal reduction required to cure the test, $A_1$, can be solved for as

$$A_1 = \begin{cases} 0 & \text{if } F_1 \geq X_0OC \\ X_0 - F_1/OC & \text{if } F_1 < X_0OC \end{cases}$$

The $t = 1$ intermediate distribution is now

$$N_1^X = cF_1 - \max (0, cF_1 - X_0Y - A_1)$$

$$N_1^Q = \max (0, cF_1 - X_0Y - A_1)$$

$$N_1^M = \nu^X N_1^X + \nu^Q N_1^Q$$

With the possibility of interest diversion the debt level changes to reflect either insufficient or surplus receipts according to

$$X_1 = X_0(1 + Y) - N_1^X$$

$X_1$ increases for high defaults, when due interest exceeds the available interest cashflow. Conversely interest diversions decrease $X_1$. The higher $OC$, the more likely interest is diverted to debtholders. When $OC$ is as low as $Y/c$, there is effectively no coverage test and interest parameters alone are used in determining intermediate distributions.

**Effort and Risk-Shifting Decision**

At $t = 1$ the manager has the exact same effort and risk-shifting decisions as in the two-period model. But if he does not switch at $t = 1$ he has the option to do so at $t = 1.5$. He cannot switch back.\textsuperscript{34}

\textsuperscript{34}Ross (1997) finds reversals are never optimal.
Collateral market prices reflect the new half-coupon payments. Using the \( t = 1.5 \) collateral price from (2.44), the \( t = 1 \) collateral price and market value ratio are

\[
\begin{align*}
b_t' &= \frac{\xi}{2} e^{-\Lambda_t (1-\phi)} + \left(1 + \frac{\xi}{2}\right) e^{-\Lambda_\alpha (1-\phi)} \\
\beta_1 &= \frac{b_t^G}{b_t^F} = \frac{\xi}{2} e^{-\Lambda_\phi (1-\phi)} + \left(1 + \frac{\xi}{2}\right) e^{-\Lambda_B (1-\phi)}
\end{align*}
\] (2.34)

If the manager decides not to switch after providing effort, portfolio face value and default intensity remain at \( F_1'(\varepsilon) \) and \( \Lambda_G \) as before. If he decides to switch, portfolio face value becomes \( F_1',SW_1 = \beta_1 \Psi F_1 \) as before however the resultant portfolio default intensity must be solved for numerically. Let \( F_1',SW_1(\Psi=1) \) be the hypothetical par value which would result if the manager switched without incurring transactions costs, that is

\[
F_1',SW_1(\Psi=1) = \beta_1 F_1 (1 - \xi(\varepsilon) + \xi(\varepsilon)\Psi)
\]

The portfolio default rate, \( \Lambda_{1,SW_1} \), is such that

\[
\left( \frac{c}{2} \exp^{-\Lambda_{1,SW_1} (1-\phi)} + \left(1 + \frac{c}{2}\right) e^{-\Lambda_{1,SW_1} (1-\phi)} \right) F_1',SW_1(\Psi=1) = b_1^G F_1',NS_1
\] (2.35)

\( \Lambda_{1,SW_1} \) is again derived from a no-arbitrage equation, ensuring that the remaining collateral is switched on a value-neutral basis. Note that the parenthesis term in (2.35) is the market price of a hypothetical asset of type \( \kappa = SW_1 \). For any effort level \( \varepsilon \), \( b_1^{SW_1}(\Lambda_{1,SW_1}) \) equates the market value of the non-switched and switched portfolios, \textit{when remaining transactions costs are zero}. For positive transactions costs, \( \Lambda_{1,SW_1} \) is too high, and the manager’s behavior imposes a loss on the portfolio.\textsuperscript{35}

\textsuperscript{35} As before the \( \Lambda_{1,SW_1} \) approximation is chosen to keep the default distribution univariate. This approximation posits a switch to a bond of type \( \kappa = SW_1 \) rather than having a bi-modal portfolio composed of both high and low-default-intensity bonds.
The manager’s NPV and maximal switching policy is

\[
V_1^{M^*} = \max \left[ \max \left( E_1 N_{1,5,SW_1}^M + E_1 V_{1,5,SW_1}^M - \Xi(\epsilon) \right), \max \left( E_1 N_{1,5,NS_1}^M + E_1 V_{1,5,NS_1}^M - \Xi(\epsilon) \right) \right]
\]

\[
\epsilon^* = \arg \max [V_1^M]
\]

\[
\kappa_1^* = \begin{cases} 
G & \text{if } \epsilon^* = \epsilon_{NS}^* \\
SW_1 & \text{if } \epsilon^* = \epsilon_{SW}^*
\end{cases}
\]

(2.36)

His effort level and switching decision are used in the calculation of date 1 valuations

\[
V_1^X(d_1) = E_1 N_{1,5}^X(\epsilon^*(d_1), \kappa_1^*(d_1), d_1) + E_1 V_{1,5}^X(\epsilon^*(d_1), \kappa_1^*(d_1), d_1)
\]

\[
V_1^Q(d_1) = E_1 N_{1,5}^Q(\epsilon^*(d_1), \kappa_1^*(d_1), d_1) + E_1 V_{1,5}^Q(\epsilon^*(d_1), \kappa_1^*(d_1), d_1)
\]

\[
V_1^M(d_1) = \nu^X V_1^X(d_1) + \nu^Q V_1^Q(d_1) - \Xi(\epsilon^*(d_1))
\]

(2.37)

2.8.3 Time \( t = 1.5 \)

At \( t = 1.5 \) there is a new default realization, pay period, and decision period.

**Interest Payment**

Current face value is adjusted to reflect new defaults \( d_{1.5} \)

\[
F_{1.5} = \phi^d_{d_{1.5}} F_{1,\kappa_1}^d
\]

(2.38)

As before if the OC test is passing \( \left( \frac{F_{1.5}}{X_1} \geq \overline{OC} \right) \), equity receives all residual interest. Otherwise, excess interest pays down debt principal. The amount of principal reduction necessary to "cure" the test is

\[
A_{1.5} = \begin{cases} 
0 & \text{if } F_{1.5} \geq X_1 \overline{OC} \\
X_1 - F_{1.5}/\overline{OC} & \text{if } F_{1.5} < X_1 \overline{OC}
\end{cases}
\]

(2.39)

The \( t = 1.5 \) intermediate distribution is then

\[
N_{1,5}^X = \frac{\nu}{2} F_{1.5} - \max \left( 0, \frac{\nu}{2} F_{1.5} - X_1 \frac{Y}{2} - A_{1.5} \right)
\]

\[
N_{1,5}^Q = \max \left( 0, \frac{\nu}{2} F_{1.5} - X_1 \frac{Y}{2} - A_{1.5} \right)
\]

\[
N_{1,5}^M = \nu^X N_{1,5}^X + \nu^Q N_{1,5}^Q
\]

(2.40)
and the debt level adjusts to
\[ X_{1.5} = X_1 \left( 1 + \frac{Y}{2} \right) - N_{1.5}^X \quad (2.41) \]

**Risk-Shifting Decision**

The manager can choose to risk-shift if he has not already. If \( \kappa_1^* = SW_1 \), he has no decision to make; let
\[
F_{1.5,SW_1} = \phi^{d_{1.5}} F_{1,SW_1} \\
A_{1.5,SW_1} = A_{1,SW_1} (\epsilon^*_{SW})
\]
with \( \epsilon^*_{SW} \) taken from the manager’s optimal effort choice in (2.36) and \( A_{1,SW_1} \) taken from the no-arbitrage condition in (2.35).

Otherwise if \( \kappa_1^* = G \), the manager may elect to switch the remaining good collateral, \((1 - \xi(\epsilon^{*\_NS}))\) \(d_{1.5}F_1\), into bad collateral. If he remains invested in good bonds the collateral par and default rate remain at
\[
F_{1.5,NS_{1.5}} = \phi^{d_{1.5}} F_{1,NS_{1.5}} \\
A_{1.5,NS_{1.5}} = A_{G}
\]
If he switches he does so at the same transactions cost loss of \( \Psi - 1 \) but at lower par gain. The \( t = 1.5 \) collateral price and market value ratio are
\[
b_{1.5} = (1 + \frac{\xi}{2}) e^{-\frac{\Delta}{2}(1-\phi)} \\
\beta_{1.5} = \frac{b_{1.5}^G}{b_{1.5}^B} = e^{(1-\phi)(A_B - A_G)/2}
\]

Portfolio par value and default intensity then increase to
\[
F_{1.5,SW_{1.5}} = \Psi F_1 (\beta_{1.5} + \xi(\epsilon^{*\_NS}) (\beta_1 - \beta_{1.5})) \\
A_{1.5,SW_{1.5}} = 2 \ln \frac{\beta_{1.5} + \xi(\epsilon^{*\_NS}) (\beta_1 \Psi - \beta_{1.5})}{1 + \xi(\epsilon^{*\_NS}) (\beta_1 \Psi - 1)}/(1 - \phi) + A_G
\]
As usual, \( A_{1.5,SW_{1.5}} \) is derived to ensure that remaining collateral is switched on a value-neutral basis, when remaining transactions costs are zero.
The manager’s NPV and maximal switching policy are

\[ V_{1.5}^M = \left\{ \begin{array}{ll}
EN_{2,SW}^M & \text{if } \kappa_1^* = SW_1 \\
\max \left[ EN_{2,SW,1.5}^M, EN_{2,NS,1.5}^M \right] & \text{if } \kappa_1^* = G \\
SW_1 & \text{if } \kappa_1^* = SW \\
G & \text{elseif } EN_{2,NS,1.5}^M \geq EN_{2,SW,1.5}^M \\
SW_{1.5} & \text{elseif } EN_{2,NS,1.5}^M < EN_{2,SW,1.5}^M
\end{array} \right. \] (2.46)

His effort level and switching decision are used in the calculation of time 1.5 valuations

\[ \begin{align*}
V_{1.5}^X(d_{1.5}) &= E_{1.5}N_{2}^X(\kappa_{1.5}^*(d_{1.5}), d_{1.5}) \\
V_{1.5}^Q(d_{1.5}) &= E_{1.5}N_{2}^Q(\kappa_{1.5}^*(d_{1.5}), d_{1.5}) \\
V_{1.5}^M(d_{1.5}) &= \nu^X V_{1.5}^X(d_{1.5}) + \nu^Q V_{1.5}^Q(d_{1.5})
\end{align*} \] (2.47)

2.8.4 Time \( t = 2 \)

At date 2, non-defaulted bonds mature, making a principal and half-coupon payment. The final principal amount is

\[ F_2 = \phi^d_2 F_{1.5,\kappa_{1.5}} \] (2.48)

Date 2 distributions are not influenced by any coverage tests since it is the final period. They are

\[ \begin{align*}
N_2^X(d_2) &= (1 + \frac{\delta}{2}) F_2(d_2) - \max \left( 0, (1 + \frac{\delta}{2}) F_2(d_2) - X_{1.5} \left( 1 + \frac{Y}{2} \right) \right) \\
N_2^Q(d_2) &= \max \left( 0, (1 + \frac{\delta}{2}) F_2(d_2) - X_{1.5} \left( 1 + \frac{Y}{2} \right) \right) \\
N_2^M(d_2) &= \nu^X V_2^X(d_2) + \nu^Q V_2^Q(d_2)
\end{align*} \] (2.49)

2.8.5 Solution

The model is solved as before (backwards induction). The first difference is that participants must take into account the possibility of interest diversion at \( t = 1 \) and \( t = 1.5 \). In the two-period model one solves for \( \overline{d_1} \) and \( \overline{d_{2,\kappa}} \), points at which the equity tranche finishes out-of-the-money. Now one solves for \( \overline{d_1} \) and \( \overline{d_{1.5,\kappa}} \) on the basis of OC test failures, not interest
defaults. This yields equity breakpoints of

\[
\begin{align*}
\bar{d}_1 &= \ln \frac{X_0 \Omega \Omega C}{\bar{F}_0} / \ln \phi \\
\bar{d}_{1.5, \kappa_1} &= \ln \frac{X_{1.5} \Omega \Omega C}{\bar{F}_{1.5, \kappa_1}} / \ln \phi \\
\bar{d}_{2, \kappa_{1.5}} &= \ln \frac{X_{1.5} (1+Y/2)}{\bar{F}_{1.5, \kappa_{1.5}}} / \ln \phi
\end{align*}
\] (2.50)

As before, changing $X_t$ levels must be accounted for in all NPV calculations.

Second, the manager now anticipates his possibility of switching later when evaluating his switching decision at $t = 1$. He looks ahead to each $d = 1.5$ realization to ascertain whether he would switch or remain and his resultant NPV. Value expectations and effort first-order conditions are similar once the optimal $t = 1.5$ switching policy has been calculated.
### 2.9 Tables and Figures

Table 2.1: U.S. CDO Tranche Downgrade Rates, by Vintage, Cumulative To 2003. Shown in Percent*

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Investment Grade Bonds</td>
<td>—</td>
<td>—</td>
<td>10</td>
<td>55</td>
<td>32</td>
<td>52</td>
<td>41</td>
</tr>
<tr>
<td>High Yield Bonds</td>
<td>83</td>
<td>95</td>
<td>86</td>
<td>79</td>
<td>60</td>
<td>16</td>
<td>63</td>
</tr>
<tr>
<td>High Yield Loans</td>
<td>80</td>
<td>56</td>
<td>49</td>
<td>4</td>
<td>9</td>
<td>4</td>
<td>15</td>
</tr>
<tr>
<td>Emerging Market Debt</td>
<td>50</td>
<td>33</td>
<td>40</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>15</td>
</tr>
<tr>
<td>Synthetic (Credit Default Swap)</td>
<td>—</td>
<td>100</td>
<td>72</td>
<td>33</td>
<td>57</td>
<td>80</td>
<td>63</td>
</tr>
</tbody>
</table>

Table 2.2: Average 1-Year Downgrade Rates For CDO Collateral, by Type and Rating Level, 1996-2002. Shown in Percent*

<table>
<thead>
<tr>
<th>Underlying Collateral</th>
<th>Aaa</th>
<th>Aa2</th>
<th>Baa2</th>
<th>Baa3</th>
<th>Ba2</th>
<th>Ba3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corporate Bonds</td>
<td>7</td>
<td>16.3</td>
<td>20.5</td>
<td>21.8</td>
<td>36.6</td>
<td>19.8</td>
</tr>
<tr>
<td>High Yield Loans</td>
<td>0.4</td>
<td>6</td>
<td>3.6</td>
<td>10.7</td>
<td>10.5</td>
<td>5.8</td>
</tr>
<tr>
<td>Emerging Market Debt</td>
<td>0</td>
<td>9.3</td>
<td>3.7</td>
<td>17.2</td>
<td>9.1</td>
<td>14.3</td>
</tr>
<tr>
<td>Synthetic (Credit Default Swap)</td>
<td>8.8</td>
<td>6.7</td>
<td>26.7</td>
<td>37.5</td>
<td>31.3</td>
<td>50</td>
</tr>
<tr>
<td>All</td>
<td>5.2</td>
<td>13.4</td>
<td>14.3</td>
<td>17.5</td>
<td>20.4</td>
<td>16.8</td>
</tr>
<tr>
<td>Corporate Bond Market</td>
<td>6</td>
<td>13.5</td>
<td>17.1</td>
<td>14.3</td>
<td>26.2</td>
<td>24.1</td>
</tr>
</tbody>
</table>

*Source: Moody’s Investors Service (2003a)

Table 2.3: Distribution of Structured Finance and Corporate Ratings, 1985-2002. Shown in Percent / Number*

<table>
<thead>
<tr>
<th>Rating Level / Year</th>
<th>STRUCTURED FINANCE</th>
<th>CORPORATE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aaa</td>
<td>91.7</td>
<td>36.4</td>
</tr>
<tr>
<td>Aa</td>
<td>0</td>
<td>58.2</td>
</tr>
<tr>
<td>A</td>
<td>8.3</td>
<td>1.8</td>
</tr>
<tr>
<td>Baa</td>
<td>0</td>
<td>2.9</td>
</tr>
<tr>
<td>Ba</td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Caa-C</td>
<td>0</td>
<td>0.2</td>
</tr>
<tr>
<td>Investment Grade</td>
<td>100</td>
<td>99.4</td>
</tr>
<tr>
<td>High Yield</td>
<td>0</td>
<td>0.6</td>
</tr>
<tr>
<td>Ratings Outstanding</td>
<td>12</td>
<td>649</td>
</tr>
</tbody>
</table>

*Source: Moody’s Investors Service (2003g)
Table 2.4: Various Costs of Risk-shifting to the Debt Tranche \((X)\) and CDO \((C)\), by Parameter Set in the Two-period Model with Zero Effort. The debt tranche is sized and priced off a static portfolio assumption \((NT)\) using average default parameters; costs are measured for both A) average and B) amplified default parameters. The inability to prevent managerial risk-shifting imposes (maximum) ex-post and ex-ante costs to each tranche \(i\), where \(\Delta V^i\) is the ex-post cost \(V^i - V_{1,NT}^i\) and \(\Delta V^0\) the ex-ante cost \(V^0 - V_{0,NT}^0\). Also shown is the increased default risk and ratings degradation to the debt tranche, the resultant decrease in supported debt ratio \((\Delta X_0 = X_0 - X_{0,NT})\), and the opportunity cost to equityholders of having to supply additional funds, measured from equity return in the relevant parameterization with Medium effort strength. The manager owns equity \((\nu^Q > 0)\) and no debt \((\nu^X = 0)\). All numbers shown in percent.

### A. AVERAGE DEFAULT RATES

<table>
<thead>
<tr>
<th>Risk-Shifting Cost</th>
<th>Parameter Set / Alternate Pricing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.5%</td>
</tr>
<tr>
<td></td>
<td>2.5%</td>
</tr>
<tr>
<td>Max (\Delta V^X_{1,NT}/V^X_{1,NT})</td>
<td>-0.9121</td>
</tr>
<tr>
<td>Max (\Delta V^C_{1,NT}/V^C_{1,NT})</td>
<td>-0.5000</td>
</tr>
<tr>
<td>(\Delta V^X_0/V^X_{0,NT})</td>
<td>-0.0017</td>
</tr>
<tr>
<td>(\Delta V^C_0/V^C_{0,NT})</td>
<td>-0.0011</td>
</tr>
<tr>
<td>(X_{0,NT}) Def. Prob.</td>
<td>0.09</td>
</tr>
<tr>
<td>Implied NT Rating</td>
<td>Aaa</td>
</tr>
<tr>
<td>(X_0) Def. Prob.</td>
<td>0.19</td>
</tr>
<tr>
<td>Implied Rating</td>
<td>Aaa</td>
</tr>
<tr>
<td>(\Delta X_0/F_0)</td>
<td>0.00</td>
</tr>
<tr>
<td>Opportunity Cost</td>
<td>0.00002</td>
</tr>
</tbody>
</table>

### B. AMPLIFIED DEFAULT RATES

<table>
<thead>
<tr>
<th>Risk-Shifting Cost</th>
<th>Parameter Set / Alternate Pricing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.5% Amp</td>
</tr>
<tr>
<td></td>
<td>2.5%</td>
</tr>
<tr>
<td>Max (\Delta V^X_{1,NT}/V^X_{1,NT})</td>
<td>-0.5262</td>
</tr>
<tr>
<td>Max (\Delta V^C_{1,NT}/V^C_{1,NT})</td>
<td>-0.5000</td>
</tr>
<tr>
<td>(\Delta V^X_0/V^X_{0,NT})</td>
<td>-0.0821</td>
</tr>
<tr>
<td>(\Delta V^C_0/V^C_{0,NT})</td>
<td>-0.0748</td>
</tr>
<tr>
<td>(X_{0,NT}) Def. Prob.</td>
<td>48.87</td>
</tr>
<tr>
<td>Implied NT Rating</td>
<td>B3</td>
</tr>
<tr>
<td>(X_0) Def. Prob.</td>
<td>49.85</td>
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<tr>
<td>Implied Rating</td>
<td>B3</td>
</tr>
</tbody>
</table>
Table 2.5: Returns to Tranches by Manager Compensation and Parameter Set in the Two-period Model. Showing first-best, flat, debt, and equity ownership across various parameter sets, all modeled for no interest \((c = Y = 0)\). Returns \(R^i\) based on a static portfolio pricing and sizing assumption (zero effort, no trading) for the debt tranche and shown in percent, cumulative over a six-year period. CDO return reduced by effort costs.

A. Medium Effort Strength, 2.5% Deal Concentration

<table>
<thead>
<tr>
<th>Return / Compensation</th>
<th>(\nu^i = 100%)</th>
<th>(\nu^i = 2.5%)</th>
<th>(\nu^X = 2.7%)</th>
<th>(\nu^Q = 33.8%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R^X)</td>
<td>0.0013</td>
<td>0.0007</td>
<td>0.0003</td>
<td>0.0011</td>
</tr>
<tr>
<td>(R^Q)</td>
<td>30.0728</td>
<td>13.8769</td>
<td>0.9212</td>
<td>24.8991</td>
</tr>
<tr>
<td>(R^C)</td>
<td>1.4955</td>
<td>0.8113</td>
<td>0.0544</td>
<td>1.3769</td>
</tr>
</tbody>
</table>

B. Weak Effort Strength, 2.5% Deal Concentration

<table>
<thead>
<tr>
<th>Return / Compensation</th>
<th>(\nu^i = 100%)</th>
<th>(\nu^i = 2.5%)</th>
<th>(\nu^X = 2.7%)</th>
<th>(\nu^Q = 33.8%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R^X)</td>
<td>0.0008</td>
<td>0.0003</td>
<td>0.0001</td>
<td>-0.0005</td>
</tr>
<tr>
<td>(R^Q)</td>
<td>14.9218</td>
<td>5.3011</td>
<td>0.3016</td>
<td>11.2477</td>
</tr>
<tr>
<td>(R^C)</td>
<td>0.6939</td>
<td>0.3097</td>
<td>0.0178</td>
<td>0.611</td>
</tr>
</tbody>
</table>

C. Strong Effort Strength, 2.5% Deal Concentration

<table>
<thead>
<tr>
<th>Return / Compensation</th>
<th>(\nu^i = 100%)</th>
<th>(\nu^i = 2.5%)</th>
<th>(\nu^X = 2.7%)</th>
<th>(\nu^Q = 33.8%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R^X)</td>
<td>0.0014</td>
<td>0.0014</td>
<td>0.0008</td>
<td>0.0014</td>
</tr>
<tr>
<td>(R^Q)</td>
<td>42.8861</td>
<td>33.3018</td>
<td>3.6796</td>
<td>43.1934</td>
</tr>
<tr>
<td>(R^C)</td>
<td>2.4552</td>
<td>1.9501</td>
<td>0.2169</td>
<td>2.4675</td>
</tr>
</tbody>
</table>

D. Various Medium Parameter Sets: \(\nu^X = 0\%, \nu^Q = 33.8\%\)

<table>
<thead>
<tr>
<th>Return / Parameter Set</th>
<th>(\Psi = 1)</th>
<th>5%</th>
<th>Single-B</th>
<th>Triple-C</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R^X)</td>
<td>-0.0012</td>
<td>0.0087</td>
<td>0.0015</td>
<td>0.0016</td>
</tr>
<tr>
<td>(R^Q)</td>
<td>30.1367</td>
<td>24.4050</td>
<td>122.3795</td>
<td>316.9965</td>
</tr>
<tr>
<td>(R^C)</td>
<td>1.6663</td>
<td>1.3643</td>
<td>6.8284</td>
<td>17.6440</td>
</tr>
</tbody>
</table>

E. Various Weak Parameter Sets: \(\nu^X = 0\%, \nu^Q = 33.8\%\)

<table>
<thead>
<tr>
<th>Return / Parameter Set</th>
<th>(\Psi = 1)</th>
<th>5%</th>
<th>Single-B</th>
<th>Triple-C</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R^X)</td>
<td>-0.0132</td>
<td>-0.0115</td>
<td>0.0013</td>
<td>0.0016</td>
</tr>
<tr>
<td>(R^Q)</td>
<td>14.0665</td>
<td>11.1022</td>
<td>62.2946</td>
<td>174.8593</td>
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<tr>
<td>(R^C)</td>
<td>0.7480</td>
<td>0.5967</td>
<td>3.4053</td>
<td>9.5125</td>
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</tbody>
</table>
Table 2.6: Tranche Returns by Parameter Set in the Two-period Model with Amplified Default Rates. The debt tranche is sized off a static portfolio assumption (zero effort, no trading) and priced off a positive effort, no-switch assumption, using average default parameters. Returns to tranche \( i \) are shown for optimal manager behavior (\( R^i \)) as well as when restricted to no-switching (\( R^i_{NS} \)) for amplified default rates. Shown in percent, cumulative over a six-year period. CDO return is reduced by effort costs. The manager owns equity (\( \nu^Q = 33.8\% \)) and no debt (\( \nu^X = 0 \)).

<table>
<thead>
<tr>
<th>Parameter Set / Alternate Pricing</th>
<th>2.5% Amp</th>
<th>( \Psi = 1 ) Amp</th>
<th>5% Amp</th>
<th>Single-B Amp</th>
<th>Triple-C Amp</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A. ( \epsilon = 0 ) Scenarios</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( R^X_{NS} )</td>
<td>-2.1117</td>
<td>-2.1117</td>
<td>-2.0672</td>
<td>-2.8716</td>
<td>-2.1117</td>
</tr>
<tr>
<td>( R^X )</td>
<td>-2.1921</td>
<td>-2.2254</td>
<td>-2.1591</td>
<td>-2.8716</td>
<td>-3.8338</td>
</tr>
<tr>
<td>( R^Q_{NS} )</td>
<td>-53.2321</td>
<td>-53.2321</td>
<td>-46.0892</td>
<td>-44.8785</td>
<td>-53.2321</td>
</tr>
<tr>
<td>( R^Q )</td>
<td>-53.1890</td>
<td>-51.9835</td>
<td>-45.9580</td>
<td>-44.8785</td>
<td>-39.6702</td>
</tr>
<tr>
<td>( R^C_{NS} )</td>
<td>-6.3802</td>
<td>-6.3802</td>
<td>-6.3799</td>
<td>-6.3798</td>
<td>-6.3802</td>
</tr>
<tr>
<td>( R^C )</td>
<td>-6.4503</td>
<td>-6.4500</td>
<td>-6.4500</td>
<td>-6.3798</td>
<td>-6.8262</td>
</tr>
<tr>
<td>B. Weak Effort Scenarios</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( R^X_{NS} )</td>
<td>-2.0895</td>
<td>-2.0499</td>
<td>-2.0354</td>
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<td>-0.4443</td>
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<tr>
<td>( R^X )</td>
<td>-2.1516</td>
<td>-2.1506</td>
<td>-2.1192</td>
<td>-9.0730</td>
<td>-0.7948</td>
</tr>
<tr>
<td>( R^Q_{NS} )</td>
<td>-52.8310</td>
<td>-52.3870</td>
<td>-45.8366</td>
<td>-64.9294</td>
<td>10.4151</td>
</tr>
<tr>
<td>( R^Q )</td>
<td>-52.7945</td>
<td>-51.4334</td>
<td>-45.7040</td>
<td>-64.9294</td>
<td>11.7964</td>
</tr>
<tr>
<td>( R^C_{NS} )</td>
<td>-6.3293</td>
<td>-6.2649</td>
<td>-6.3338</td>
<td>-13.7372</td>
<td>0.0262</td>
</tr>
<tr>
<td>( R^C )</td>
<td>-6.3955</td>
<td>-6.2756</td>
<td>-6.3964</td>
<td>-13.7372</td>
<td>-0.1294</td>
</tr>
<tr>
<td>C. Medium Effort Scenarios</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( R^X_{NS} )</td>
<td>-2.0565</td>
<td>-1.9893</td>
<td>-2.0196</td>
<td>-2.8246</td>
<td>-0.0218</td>
</tr>
<tr>
<td>( R^X )</td>
<td>-2.1229</td>
<td>-2.0688</td>
<td>-2.0882</td>
<td>-2.8246</td>
<td>-0.0528</td>
</tr>
<tr>
<td>( R^Q_{NS} )</td>
<td>-52.2004</td>
<td>-50.9924</td>
<td>-45.2740</td>
<td>-43.9223</td>
<td>82.3890</td>
</tr>
<tr>
<td>( R^Q )</td>
<td>-52.1741</td>
<td>-50.3934</td>
<td>-45.1383</td>
<td>-43.9223</td>
<td>82.3076</td>
</tr>
<tr>
<td>( R^C_{NS} )</td>
<td>-6.2488</td>
<td>-6.0928</td>
<td>-6.2614</td>
<td>-6.2612</td>
<td>6.2580</td>
</tr>
<tr>
<td>( R^C )</td>
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<td>-6.1125</td>
<td>-6.3100</td>
<td>-6.2612</td>
<td>6.2290</td>
</tr>
</tbody>
</table>

138
Table 2.7: Returns to Tranches for Varying Coverage Test Hurdles, by Parameter Set in the Three-period Model. The initial debt level $X_{0,NT}$ and returns $R^d$ are based on a static portfolio pricing and sizing assumption (zero effort, no trading). Returns shown in percent, cumulative over a six-year period. CDO return reduced by effort costs. The manager owns equity ($v^Q = 33.8\%$) and no debt ($v^X = 0$).

<table>
<thead>
<tr>
<th>OC</th>
<th>Tranche</th>
<th>Medium 2.5%</th>
<th>Weak 2.5%</th>
<th>Medium 5%</th>
<th>Weak 5%</th>
<th>Medium Single-B</th>
<th>Weak Single-B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$R^X$</td>
<td>0.0011</td>
<td>-0.0009</td>
<td>-0.0002</td>
<td>-0.0048</td>
<td>0.0015</td>
<td>0.0013</td>
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<tr>
<td></td>
<td>$R^Q$</td>
<td>14.3351</td>
<td>6.495</td>
<td>12.4508</td>
<td>5.6578</td>
<td>70.4262</td>
<td>35.9682</td>
</tr>
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<td>$R^C$</td>
<td>1.3037</td>
<td>0.5798</td>
<td>1.3009</td>
<td>0.5776</td>
<td>6.4627</td>
<td>3.2333</td>
</tr>
<tr>
<td>100%</td>
<td>$X_{0,NT}$</td>
<td>9.4050</td>
<td>9.4050</td>
<td>9.2550</td>
<td>9.2550</td>
<td>9.4050</td>
<td>9.4050</td>
</tr>
<tr>
<td></td>
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<td>0.0002</td>
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<td>-0.0033</td>
<td>0.0008</td>
<td>0.0007</td>
</tr>
<tr>
<td></td>
<td>$R^Q$</td>
<td>14.3424</td>
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<td>12.4675</td>
<td>5.6538</td>
<td>70.4382</td>
<td>35.9776</td>
</tr>
<tr>
<td></td>
<td>$R^C$</td>
<td>1.3038</td>
<td>0.5811</td>
<td>1.3015</td>
<td>0.5781</td>
<td>6.4627</td>
<td>3.2334</td>
</tr>
<tr>
<td></td>
<td>$R^X$</td>
<td>0.0039</td>
<td>0.0015</td>
<td>0.0019</td>
<td>0.0008</td>
<td>0.0049</td>
<td>0.0047</td>
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<tr>
<td></td>
<td>$R^Q$</td>
<td>19.5399</td>
<td>8.8444</td>
<td>12.462</td>
<td>5.6451</td>
<td>96.1417</td>
<td>49.0791</td>
</tr>
<tr>
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<td>$R^C$</td>
<td>1.3037</td>
<td>0.5803</td>
<td>1.3034</td>
<td>0.5808</td>
<td>6.4627</td>
<td>3.2333</td>
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<tr>
<td></td>
<td>$R^X$</td>
<td>0.0058</td>
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<td>-0.0003</td>
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<td>10.9882</td>
<td>14.1582</td>
<td>6.4153</td>
<td>119.5102</td>
<td>60.9706</td>
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<td>0.5795</td>
<td>1.3032</td>
<td>0.5796</td>
<td>6.4627</td>
<td>3.2322</td>
</tr>
</tbody>
</table>
Table 2.8: Return Comparison for Various Structural Modifications, by Parameter Set in the Three-period Model (See Section 2.5 for details). Initial debt level $X_0$ is sized off a zero-effort, positive risk-shifting assumption, and priced off a positive effort, positive risk-shifting assumption. Returns $R^i$ shown in percent, cumulative over a six-year period. The debt pricing assumption implies $R^X = 0$. CDO return reduced by effort costs and includes any fees paid to the management. The manager owns equity ($\nu^Q = 33.8\%$) and no debt ($\nu^X = 0$).

### A. Medium 2.5% Extension Results

<table>
<thead>
<tr>
<th>OC</th>
<th>$R^i$</th>
<th>BASE</th>
<th>EXT1</th>
<th>EXT2_35bp</th>
<th>EXT3</th>
<th>EXT4_25%</th>
<th>EXT4_50%</th>
<th>EXT5</th>
<th>EXT6</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$R^C$</td>
<td>1.3037</td>
<td>1.3037</td>
<td>1.3037</td>
<td>1.3037</td>
<td>0.6957</td>
<td>1.3037</td>
<td>0.0977</td>
<td>1.3037</td>
</tr>
<tr>
<td></td>
<td>$R^C$</td>
<td>1.3038</td>
<td>1.3038</td>
<td>1.3038</td>
<td>1.3038</td>
<td>0.6957</td>
<td>1.3038</td>
<td>0.0979</td>
<td>1.3038</td>
</tr>
<tr>
<td></td>
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<td>17.3682</td>
<td>8.9698</td>
<td>17.7967</td>
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<td>19.2672</td>
</tr>
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<td>1.3038</td>
<td>1.3038</td>
<td>1.3039</td>
<td>1.3038</td>
<td>0.6957</td>
<td>1.3038</td>
<td>0.0882</td>
<td>1.2814</td>
</tr>
<tr>
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<td>1.3037</td>
<td>1.3033</td>
<td>1.3038</td>
<td>0.6957</td>
<td>1.3032</td>
<td>0.0004</td>
<td>0.7643</td>
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### B. Weak 2.5% Extension Results

<table>
<thead>
<tr>
<th>OC</th>
<th>$R^i$</th>
<th>BASE</th>
<th>EXT1</th>
<th>EXT2_10bp</th>
<th>EXT3</th>
<th>EXT4_25%</th>
<th>EXT4_50%</th>
<th>EXT5</th>
<th>EXT6</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>0.5798</td>
<td>0.5798</td>
<td>0.5798</td>
<td>0.5798</td>
<td>0.5810</td>
<td>0.5808</td>
<td>0.0977</td>
<td>0.5798</td>
</tr>
<tr>
<td></td>
<td>$R^C$</td>
<td>0.5811</td>
<td>0.5807</td>
<td>0.5811</td>
<td>0.5811</td>
<td>0.5811</td>
<td>0.5811</td>
<td>0.0979</td>
<td>0.5811</td>
</tr>
<tr>
<td></td>
<td>$R^C$</td>
<td>0.5809</td>
<td>0.5809</td>
<td>0.5808</td>
<td>0.5808</td>
<td>0.5810</td>
<td>0.5810</td>
<td>0.0882</td>
<td>0.5709</td>
</tr>
<tr>
<td></td>
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<td>11.0124</td>
<td>11.0176</td>
<td>7.5340</td>
<td>11.0158</td>
<td>11.1277</td>
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Figure 2-2: Typical CDO Legal Structure. Issuance-related fees are paid upfront when the deal closes. Periodic payments include (in order of priority) hedge fees, trustee administration fees, deal expenses, manager fees, debt interest, and equity residual. The manager participates in tranche distributions to the extent of his ownership shares. Source: McDermott (2000).
Figure 2-3: Growth in the Structured Finance (SF) and Corporate (CORP) Bond Market 1985-2003. Source: Moody’s Investors Service (2003d), Moody’s Investors Service (2003h).
Figure 2-4: Effort Provision at Time 1 by Manager Compensation and Collateral Performance in Two-period Model. Showing first-best, flat, debt, and equity ownership for parameter set Medium 2.5% (no interest).
Figure 2-5: Ex-post Effort Gains to Tranches by Manager Compensation in Two-period Model (NS). The following illustrates gains from $t = 1$ effort to the CDO as a whole and to the debt and equity tranches, assuming switching is restricted, conditional on $d_1$. CDO gain is equal to debt and equity gain minus effort costs $\Xi(\epsilon)$. Showing flat, debt, and equity ownership for the parameter set Medium 2.5% (no interest). Units are dollar gains on a CDO of size 10.
Figure 2-6: Ex-post Risk-Shifting Gains to Tranches in Two-period Model with Zero Effort. Expected gains from the manager's $t = 1$ risk-shifting decision are shown conditional on $d_1$ and assuming equity ownership ($\nu^X = 0$, $\nu^Q > 0$). Not shown are CDO gains, equal to the sum of debt ($\Delta V_1^X$) and equity gains ($\Delta V_1^Q$). Parameter sets presented are (1) 2.5% deal concentration 2) 5% deal concentration 3) $\Psi = 1$ (no transactions costs) and 4) Single-B bad type collateral. Units are dollar gains on a CDO of size 10.
Figure 2-7: Ex-post Effort and Risk-Shifting Gains to Tranches in Two-period Model. Expected gains from the manager’s $t = 1$ effort and risk-shifting decisions are shown conditional on $d_1$ and assuming equity ownership ($\nu^X = 0$, $\nu^Q = 33.8\%$). Effort gains measured from a static baseline and risk-shifting gains measured from the no-switch effort solution. Total CDO gain (solid) is equal to the sum of debt and equity gains (dashed, dotted) minus effort costs $\Xi(\epsilon)$. Parameter set is Medium 2.5\% (no interest). Units are dollar gains on a CDO of size 10.
Figure 2-9: Ex-post Effort and Risk-Shifting Gains to CDO in Two- and Three-period Models over Range of Coverage Test Levels. Effort gains (solid lines) are shown from a static portfolio baseline. Risk-shifting gains (dotted lines) are shown from the no-switch effort solution. Plots 1 and 2 present the two- and three-period models without coverage tests. Plots 3 - 5 show the three-period model with coverage test levels of $OC = 100\%, 102\%, \text{ and } 105\%$. Units are dollar gains on a CDO of size 10; Parameter set is Medium effort, 2.5% portfolio concentration. The manager owns equity ($\nu^Q = 33.8\%$) and no debt ($\nu^X = 0$).
Figure 2-10: Ex-post Effort and Risk-Shifting Gains to CDO in Two- and Three-period Models over Range of Coverage Test Levels. Effort gains (solid lines) are shown from a static portfolio baseline. Risk-shifting gains (dotted lines) are shown from the no-switch effort solution. Plots 1 and 2 present the two- and three-period models without coverage tests. Plots 3 - 5 show the three-period model with coverage test levels of $OC = 100\%, 102\%, \text{ and } 105\%$. Units are dollar gains on a CDO of size 10; Parameter set is Weak effort, 2.5% portfolio concentration. The manager owns equity ($\nu^Q = 33.8\%$) and no debt ($\nu^X = 0$).
Figure 2-10: Ex-post Effort and Risk-Shifting Gains to CDO in Two- and Three-period Models over Range of Coverage Test Levels. Effort gains (solid lines) are shown from a static portfolio baseline. Risk-shifting gains (dotted lines) are shown from the no-switch effort solution. Plots 1 and 2 present the two- and three-period models without coverage tests. Plots 3 - 5 show the three-period model with coverage test levels of $\bar{OC} = 100\%, 102\%, \text{ and } 105\%$. Units are dollar gains on a CDO of size 10; Parameter set is Weak effort, 2.5% portfolio concentration. The manager owns equity ($\nu^Q = 33.8\%$) and no debt ($\nu^X = 0$).
Chapter 3

Bad Luck or Bad Incentives? An Empirical Investigation into CDO Manager Trading Behavior

3.1 Introduction

The recent spell of "irrational exuberance" saw questionable and occasionally criminal decision-making on the part of many professional investors. Investor dissatisfaction has extended to the relatively young CDO market, a rapidly growing offshoot of the structured finance market. These new and highly leveraged investment vehicles were designed to take advantage of ratings and funding arbitrages in corporate credit markets, but experienced disastrous performance from 1998 to 2003. Many large and well known institutional investors took heavy losses. Recently vociferous allegations of manager misbehavior have been accompanied by lawsuits. Barclays and Bank of America announced settlements in the spring of 2005, both accused of insufficiently revealing risk factors.¹

Though no one has yet prosecuted a CDO management team, several market participants

¹Insurance companies were among the largest early investors in the CDO market, both as investors and guarantors. For instance, Financial Security Assurance (FSA) took large charges on its $78 billion CDO exposure in August 2002. On MBIA’s $66 billion CDO exposure in early 2003, outside investors calculated the mark-to-market loss as between $5.3 billion and $7.7 billion. Other large insurers with significant exposure included Conseco and AIG. Source: Weil and Sender (2003), Moody’s Investors Service (2003c).
have accused them of having had conflicts of interest which led to deleterious trading decisions. Moody’s Investors Services, for instance, identifies managers’ "aggressive... practices" which "deviate from the spirit of the indenture" as a significant factor in collateral losses. Others have claimed specifically that managers’ subordinated compensation structure led them to reinvest into more volatile securities. This strategy, "risk-shifting", is aimed to benefit subordinate investors and equityholders to the detriment of senior investors.

It is not surprising that fund managers may have been motivated by their compensation structure in a situation of delegated management. As Chevalier and Ellison (1999) note, the Investment Co. Act of 1940 was amended to outlaw option-like incentive schemes for retail mutual funds precisely to avoid undesirable manager behavior. However, in private and institutional vehicles such as CDOs, managers are given all manner of incentive fees. The situation is compounded by the highly complex structure of CDOs, their relative novelty, and the extreme credit stress which occurred from 1998 to 2003. If, as Moody’s states, rating agencies did not properly acknowledge, understand, and model manager risk, investors may be justified in their complaints.

Prior theory on principal agent problems in the investment management arena includes Modigliani and Pogue (1975), Holmstrom (1999), Starks (1987), Heinkel and Stoughton (1994), Narayanaswamy, Schirm, and Shukla (2001), Admati and Pfleiderer (1997), and Papastaikoudi (2004b). The risk-shifting problem in particular is discussed theoretically in Jensen and Meckling (1976), Myers (1977), and Hart and Moore (1998) and computationally in Leland (1998), Parrino and Weisbach (1999), and Ross (1997). These risk-shifting studies focus mainly on traditional firm settings and conclude that managerial risk-shifting is a minor problem relative to the benefits of leverage. I argue that the CDO context differs, due to the much higher leverage typically observed in CDOs, and that in fact CDOs are an ideal situation in which to study risk-shifting. There are clear measures of volatility (asset prices or ratings) as well as trading and compensation data which are fairly easy to interpret. For a theoretical exposition

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2Source: Moody’s Investors Service (2003i). In a 2002 study of high yield CDOs, they conclude that CDO "downgrades are not solely explained by high-yield stress" and that "managers have introduced risk... beyond that justified". The aggressive practices include the "purchase of Caa / deeply discounted securities".

3See, e.g., Thompson, Reeves, Weaver, and Folkerts-Landau (2002) and UBS Warburg (2003).

4Source: Moody’s Investors Service (2003i). Moody’s notes that managers’ behavior was "problematic because not contemplated in initial ratings" and state that their rating approach will move to a "more realistic modeling of manager trading strategies and reinvestment."
and calibration of the risk-shifting problem in CDOs, see the second chapter of this thesis, as well as Duffie and Garleanu (2001) and Gibson (2004) who quantify by simulation the effect of volatility on CDO tranches.

In this paper I look for empirical evidence of risk-shifting in CDOs. I attempt to discern whether CDO debt investors were simply the victims of bad timing, whether managers succumbed to general overconfidence, or whether managers were in fact motivated by their subordinate incentive claims to make value-shifting decisions. I use a unique and newly available panel data set to study the actions of CDO managers across a wide range of CDO types. Data includes performance statistics, compensation parameters, and trading decisions during the time period 2001-2004. In particular I am looking for any evidence that managers sought to add risk to the portfolio or to manipulate certain deal features which would have protected senior investors. If managers were motivated by subordinate interests to shift value from senior to junior investors, then such activities should have increased in the size and probability of receiving incentive fees or equity distributions.

In the regression analysis I find a statistically significant link between performance problems, manager compensation, and risk-shifting behavior in CDO deals. Managers’ tendencies to make discount purchases and premium sales are highly and significantly correlated with collateralization levels and fee structure. Both discount purchases and premium sales are a way to add volatility to a portfolio by selling higher-quality, lower-risk assets and reinvesting in lower-quality, higher-risk assets. In addition, they subvert specific deal features known as coverage tests which are meant to protect senior debtholders. When coverage tests are failing, interest which would go to subordinate and equity investors is instead paid to senior debtholders as early principal paydowns. But because coverage tests are based on par, not market value, making par gains improves this metric of deal performance, however cosmetically. The benefit of this is that it keeps interest flowing to equityholders.

The relationship is highly non-linear in performance. If performance is so bad as to eliminate any prospect of the manager receiving junior and incentive fees, manager activity declines. When deal performance is marginal the manager is most active. When performance is good and manager cashflows are fairly secure, there is less activity. I also find a significant tendency to group premium and distressed sales together, a strategy which minimizes par erosion but
also increases collateral volatility by removing higher quality assets unnecessarily.

Manager reputation is considered, using assets-under-management as a proxy for reputa-
tional concerns. The influence of reputation on manager decisions has been developed in
Hirshleifer and Thakor (1992), Diamond (1989), and Huddart (1999). I find that in deals with
bad performance, manager size decreases par-building trades. This suggests that managers
with a large client base were less willing to jeopardize goodwill by angering clients.

For empirical studies of investment manager incentives, see work by Chevalier and Ellison
an overview of the treatment of standard errors in various financial panel data applications.

The setup of this paper is as follows: Section 2 describes the data set. Section 3 examines
the transactions data in more detail, while Section 4 presents the regression analysis. Section
5 concludes.

3.2 Data Description

CDOs are a new type of structured finance asset. Managers choose an initial portfolio of
collateral assets, say BBB-rated corporate debt assets. These are placed in a special purpose
vehicle which then issues debt to CDO investors at varying ratings levels. The difference
between the average collateral and liability spread is excess interest which accumulates to the
benefit of equityholders. For an overview of the CDO market, see Goodman and Fabozzi
(2002).

Deals considered for my data set are those rated by Moody’s Investor Services and listed
on their "CDO Deal Score" report from March 2004. These 347 deals are issued from 1996
to 2003 and are grouped according to the collateral backing them. They fall into four broad
categories, Emerging Markets, Loan, Investment-grade and High-yield Corporate.5 I draw
descriptive, collateral, and capital structure data from Moody’s "CDO Enhanced Monitoring

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5There is little or no issue of survivorship bias. Deals either mature naturally, are called early through an
optional or mandatory liquidation process (usually when collateral balance declines to 10% of initial balance),
or in some few cases can be forcefully liquidated by the controlling class upon bad performance. In general, the
deals in this sample are too young to have matured naturally or through the auction call process. Further there
do not seem to be any instances of forced early liquidation outside the sample, as only a minority of deals have
such a provision and it is generally difficult to enact.
Service" (EMS) reports, available by subscription. This is a new and heretofore unaccessed data set, as Moody’s first began to compile, back-fill, and market their CDO data to investors in 2003.

Descriptive information, manager compensation structure, and transactions information is culled from Intex Solutions, Inc., a modeling service for structured finance assets. This data source, available by subscription, is new as well. As the CDO market grew, it became advantageous for liquidity purposes to have a standardized modeling tool available to investors. Deal structurers began to have Intex model every deal starting around 2003. The financial data is unique in that there is specific historical asset information on holdings and trades. Whenever possible, data is cross-checked between Moody’s and Intex. Intex models 325 of the 347 deals.

Lastly, manager size information is taken from a public Pensions & Investments, Inc. (P&I) report for the year 2003. They list the top 500 investment firms by U.S. institutional assets-under-management (AUM). 55 of the 129 total deal managers are included, covering 58% of deals in the sample.6

3.2.1 Descriptive Information

Descriptive deal characteristics are taken from both Moody’s and Intex. They include the deal closing date, end reinvestment date, lead investment banker, CDO collateral manager (original and replacement if applicable), and pay frequency. There is a wide range of deal vintages and types which ensures a mix of performance outcomes. Deal issuance stretches from 1996 to 2003, with 41% of the deals timed to avoid the worst collateral performance (i.e. issued between 1996-1998 and 2002-2003). Nearly half of the deals (46%) are backed by primarily loan or emerging markets assets which had much better performance than corporate debt over the time period studied.

The reinvestment period, normally between 4 - 7 years, is significant because after this period ends the manager is usually restricted in his purchases. I generally confine regressions on manager behavior to within the reinvestment period. The pay frequency is used to determine whether a deal has primarily floating-rate or fixed-rate collateral, which is used when collateral

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6I am deeply indebted to State Street Research & Management, Ltd. for access to these data sets. The identities of all trustees, bankers, managers, investors, deals, and assets are codified so as to remain confidential.
interest rates are included as independent variables in regressions.

Fixed effects for deal, deal type, lead banker, and collateral manager will be considered.

3.2.2 Moody’s Data

EMS Data - Capital Structure

Capital structure information is provided on a snapshot basis, both original and current. Original tranche sizes and collateral levels are used to compute the initial equity percentage of the deal. Table 3.1 summarizes capital structure across deal types. Riskier collateral necessitates a larger equity cushion, evident in a 4.22% average Investment-grade equity share and a 15.20% average Emerging Markets equity share. Equity share is an important measure of how likely a manager is to receive subordinate and incentive fees and thus is an indicator of his incentive to risk-shift.

MDS Data

"Moody’s Deal Score" or MDS is a metric quantifying the ratings changes to each deal’s liability tranches, measured cumulatively to February 2005. Scores are used to summarize a deal’s overall ratings performance and to sort deals for sub-sampling in regressions. Table 3.1 provides a breakdown by deal type. Only 3 deals of the 347 - 2 emerging markets and 1 loan deal - have seen net upgrades while 149 have seen net downgrades. By far the worst performers are the Investment-grade and High-yield categories, with an average of 2.0 and 4.6 notches downgraded, respectively, across the entire capital structure (3 notches = 1 letter grade, e.g. Baa1 to Ba1). As with most of the performance data, there is a great deal of dispersion in the data, enabling efficient regression analysis.

EMS Data - Collateral

Monthly collateral data is provided for varying timeframes. The beginning point is either the deal origin or some later point at which Moody’s first began storing data on the deal; ending point is approximately July 2004. Between 2 - 33 data points are available per deal, on average
Table 3.1: Capital Structure and Liability Performance by Deal Type. Original deal size is the sum of all liability and equity tranches’ face value. Average notches upgraded is the deal-wide average of rating changes to constituent tranches. It is implied by the final Moody’s Deal Score for each deal, measured from an assumed A1 average initial deal rating. Upgrade / downgrade percentage shows deals with positive or negative net changes.

<table>
<thead>
<tr>
<th>Deal Type</th>
<th>Obs.</th>
<th>Variable</th>
<th>Mean</th>
<th>St. Dev.</th>
<th>5%</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALL</td>
<td>347</td>
<td>Orig. Deal Size ($MM)</td>
<td>408.1</td>
<td>193.3</td>
<td>195.7</td>
<td>297.7</td>
<td>372.5</td>
<td>500.0</td>
<td>800.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Orig. Equity %</td>
<td>9.08</td>
<td>3.70</td>
<td>3.77</td>
<td>7.06</td>
<td>8.43</td>
<td>10.76</td>
<td>16.39</td>
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<tr>
<td></td>
<td></td>
<td>Notches Upgraded</td>
<td>-2.40</td>
<td>3.66</td>
<td>-10.04</td>
<td>-4.98</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
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<tr>
<td></td>
<td></td>
<td>Upgrade / Downgrade %</td>
<td>0.86</td>
<td>42.94</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EM</td>
<td>19</td>
<td>Orig. Deal Size ($MM)</td>
<td>243.5</td>
<td>122.8</td>
<td>60.0</td>
<td>132.8</td>
<td>256.8</td>
<td>304.2</td>
<td>587.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Orig. Equity %</td>
<td>15.20</td>
<td>6.34</td>
<td>0.00</td>
<td>12.68</td>
<td>16.70</td>
<td>18.83</td>
<td>27.34</td>
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<td></td>
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<td>Notches Upgraded</td>
<td>-0.65</td>
<td>1.91</td>
<td>-6.49</td>
<td>-1.38</td>
<td>0.00</td>
<td>0.00</td>
<td>2.75</td>
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<td></td>
<td></td>
<td>Upgrade / Downgrade %</td>
<td>10.00</td>
<td>30.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HY</td>
<td>156</td>
<td>Orig. Deal Size ($MM)</td>
<td>347.6</td>
<td>146.8</td>
<td>181.1</td>
<td>254.0</td>
<td>306.4</td>
<td>400.0</td>
<td>656.5</td>
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<tr>
<td></td>
<td></td>
<td>Orig. Equity %</td>
<td>10.21</td>
<td>3.25</td>
<td>5.41</td>
<td>8.27</td>
<td>10.04</td>
<td>12.29</td>
<td>15.85</td>
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<tr>
<td></td>
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<td>Notches Upgraded</td>
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<td>4.22</td>
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<td>-8.51</td>
<td>-5.00</td>
<td>0.00</td>
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<tr>
<td></td>
<td></td>
<td>Upgrade / Downgrade %</td>
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<td></td>
<td></td>
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<td>32</td>
<td>Orig. Deal Size ($MM)</td>
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<td>172.4</td>
<td>255.0</td>
<td>329.8</td>
<td>448.4</td>
<td>503.0</td>
<td>987.8</td>
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<tr>
<td></td>
<td></td>
<td>Orig. Equity %</td>
<td>4.22</td>
<td>1.07</td>
<td>2.23</td>
<td>3.65</td>
<td>4.14</td>
<td>4.97</td>
<td>6.00</td>
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<tr>
<td></td>
<td></td>
<td>Notches Upgraded</td>
<td>-2.00</td>
<td>2.82</td>
<td>-7.84</td>
<td>-3.59</td>
<td>0.00</td>
<td>0.00</td>
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<td></td>
<td>Upgrade / Downgrade %</td>
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<td>Orig. Deal Size ($MM)</td>
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<td>Orig. Equity %</td>
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<td>1.99</td>
<td>4.86</td>
<td>7.29</td>
<td>8.00</td>
<td>8.90</td>
<td>11.43</td>
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<tr>
<td></td>
<td></td>
<td>Notches Upgraded</td>
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<td>0.87</td>
<td>-1.99</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
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<tr>
<td></td>
<td></td>
<td>Upgrade / Downgrade %</td>
<td>0.71</td>
<td>14.89</td>
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</table>
Table 3.2: Moody’s Deal Collateral Data, by Deal Type and Deal Average over Time. Various statistics include share of defaulted securities, weighted-average rating factor and coupon levels, and over-collateralization metrics. WARF and OC levels also shown in relation to their respective deal limits. Closing date standard deviation in years.

<table>
<thead>
<tr>
<th>Obs.</th>
<th>Variable</th>
<th>Mean</th>
<th>Dev.</th>
<th>5%</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
<th>95%</th>
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<tr>
<td>All Deals</td>
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<td>May-00</td>
<td>1.72</td>
<td>Jul-97</td>
<td>Mar-99</td>
<td>Jun-00</td>
<td>Aug-01</td>
<td>Apr-03</td>
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<tr>
<td>347</td>
<td>Defaulted Par (%)</td>
<td>6.19</td>
<td>6.26</td>
<td>0.00</td>
<td>1.22</td>
<td>4.34</td>
<td>9.28</td>
<td>19.91</td>
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<tr>
<td>347</td>
<td>WA Rating Factor</td>
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<td>1096</td>
<td>919</td>
<td>2121</td>
<td>2761</td>
<td>3524</td>
<td>4742</td>
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<td>347</td>
<td>Ratio to Deal Limit</td>
<td>1.21</td>
<td>0.31</td>
<td>0.84</td>
<td>0.96</td>
<td>1.16</td>
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<td>1.82</td>
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<td>325</td>
<td>WA Fixed Coupon (%)</td>
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<td>9.33</td>
<td>9.77</td>
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<td>284</td>
<td>WA Floating Spread (%)</td>
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<td>1.12</td>
<td>0.99</td>
<td>2.96</td>
<td>3.24</td>
<td>3.50</td>
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<td>Seniormost OC Ratio (%)</td>
<td>1.28</td>
<td>0.92</td>
<td>0.96</td>
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<td>1.23</td>
<td>1.29</td>
<td>1.49</td>
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<tr>
<td>347</td>
<td>Ratio to Deal Limit</td>
<td>1.06</td>
<td>0.52</td>
<td>0.83</td>
<td>0.97</td>
<td>1.04</td>
<td>1.10</td>
<td>1.20</td>
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<td>347</td>
<td>Juniormost OC Ratio (%)</td>
<td>1.00</td>
<td>0.12</td>
<td>0.76</td>
<td>0.95</td>
<td>1.02</td>
<td>1.06</td>
<td>1.17</td>
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<td>347</td>
<td>Ratio to Deal Limit</td>
<td>0.96</td>
<td>0.11</td>
<td>0.73</td>
<td>0.90</td>
<td>1.00</td>
<td>1.03</td>
<td>1.07</td>
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<td>347</td>
<td>OC Par Cushion (%)</td>
<td>-2.43</td>
<td>8.36</td>
<td>-16.12</td>
<td>-1.53</td>
<td>0.01</td>
<td>0.16</td>
<td>0.85</td>
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<th>Emerging Markets Deals</th>
<th>Closing Date</th>
<th>Sep-98</th>
<th>1.71</th>
<th>May-96</th>
<th>Aug-97</th>
<th>Mar-98</th>
<th>Oct-99</th>
<th>Oct-02</th>
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<tr>
<td>19</td>
<td>Defaulted Par (%)</td>
<td>7.64</td>
<td>6.77</td>
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<td>1.72</td>
<td>6.14</td>
<td>12.94</td>
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<td>WA Rating Factor</td>
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<td>897</td>
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<td>2558</td>
<td>3374</td>
<td>4621</td>
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<td>0.37</td>
<td>0.68</td>
<td>0.93</td>
<td>1.21</td>
<td>1.48</td>
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<td>19</td>
<td>OC Par Cushion (%)</td>
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<td>7.65</td>
<td>-8.34</td>
<td>-4.56</td>
<td>0.24</td>
<td>1.76</td>
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<th>High-Yield Deals</th>
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<th>1.42</th>
<th>Mar-97</th>
<th>Dec-98</th>
<th>Oct-99</th>
<th>Oct-00</th>
<th>Dec-01</th>
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<tbody>
<tr>
<td>156</td>
<td>Defaulted Par (%)</td>
<td>9.11</td>
<td>6.79</td>
<td>0.72</td>
<td>4.25</td>
<td>7.86</td>
<td>12.55</td>
<td>24.60</td>
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<td>156</td>
<td>WA Rating Factor</td>
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<td>886</td>
<td>2200</td>
<td>2986</td>
<td>3426</td>
<td>4018</td>
<td>5286</td>
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<tr>
<td>156</td>
<td>Ratio to Deal Limit</td>
<td>1.32</td>
<td>0.29</td>
<td>0.97</td>
<td>1.12</td>
<td>1.24</td>
<td>1.47</td>
<td>1.86</td>
</tr>
<tr>
<td>156</td>
<td>OC Par Cushion (%)</td>
<td>-5.49</td>
<td>11.31</td>
<td>-29.02</td>
<td>-4.39</td>
<td>-1.21</td>
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<th>1.14</th>
<th>Jul-98</th>
<th>Jul-00</th>
<th>Jan-01</th>
<th>Jul-01</th>
<th>Jul-02</th>
<th>Aug-02</th>
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<tbody>
<tr>
<td>32</td>
<td>Defaulted Par (%)</td>
<td>1.47</td>
<td>0.96</td>
<td>0.10</td>
<td>0.60</td>
<td>1.37</td>
<td>2.27</td>
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<tr>
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<td>WA Rating Factor</td>
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<td>578</td>
<td>776</td>
<td>911</td>
<td>1003</td>
<td>1223</td>
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<td>Ratio to Deal Limit</td>
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<td>0.30</td>
<td>1.16</td>
<td>1.29</td>
<td>1.45</td>
<td>1.61</td>
<td>2.07</td>
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<td>32</td>
<td>OC Par Cushion (%)</td>
<td>0.08</td>
<td>0.16</td>
<td>-0.11</td>
<td>0.01</td>
<td>0.05</td>
<td>0.18</td>
<td>0.42</td>
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<th>Dec-97</th>
<th>Oct-99</th>
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<tbody>
<tr>
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<td>0.26</td>
<td>2.43</td>
<td>5.73</td>
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</tr>
<tr>
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<td>140</td>
<td>Ratio to Deal Limit</td>
<td>1.03</td>
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<td>0.95</td>
<td>1.06</td>
<td>1.51</td>
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<tr>
<td>140</td>
<td>OC Par Cushion (%)</td>
<td>-0.07</td>
<td>2.12</td>
<td>-1.77</td>
<td>0.01</td>
<td>0.09</td>
<td>0.18</td>
<td>0.81</td>
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</table>
Collateral data is described below and summarized in Table 3.2.

**Par Information:** Par information includes the face value of total collateral, assets in default, and cash. Cash is designated as either principal collections (from collateral sales, early principal payment, or default recoveries) or interest collections. Principal cash is intended to be reinvested if the deal is within its reinvestment period and passing all coverage tests. Interest cash is to be distributed on the next paydate, as either fees or liability interest. All trading statistics and other cash measures are converted into percentages of total portfolio par, defined as the sum of collateral face value and principal cash.

Principal cash is used as an independent variable in regressions on the manager’s trade activity. Defaulted assets are used as control variables in regressions on the amount of credit-risk sales. Over the time period studied corporate defaults hit CDOs hard, with an average default exposure to each portfolio of 6.2%.8

**Weighted Average Coupon / Spread Information:** The weighted average coupon (WAC) and spread (WAS) rates are calculated for fixed and floating-rate collateral, respectively. Collateral interest levels are used as control variables in regressions on transactions volume.

**Diversity Score Information:** Moody’s Diversity Score (DIV) is a metric meant to represent the amount of diversification in a deal. The larger the score, the more the implied diversity in the deal. Diversity score is tested as an independent variable in regressions on transactions volume.

7Excluding observations where portfolio par, defaulted par, rating factor, or any coverage test report is missing.

8The five-year cohort from 1998 to 2002 had the worst issuer-weighted default performance on record. In addition dollar-volume weighted default rates, more accurately describing actual CDO collateral losses, were several times higher due to the large number of high-volume defaults. Familiar multi-billion dollar defaults include, in chronological order: Russia ($42.7), Daewoo ($5.6), Ecuador ($6.6), Peru ($4.9), Southern California Edison ($3.6), Pacific Gas & Electric ($5.0), Finova Capital ($6.3), Asia Pulp & Paper ($5.2), PSINet ($2.9), Comdisco ($2.8), MYCAL ($3.1), Exodus Communications ($2.8), Federal-Mogul ($2.8), XO Communications ($4.9), Argentina ($82.3), Enron ($9.9), Global Crossing ($3.8), Kmart ($2.5), McleodUSA ($2.9), Nextel ($2.3), United Pan-Europe Communications ($5.1), Metromedia Fiber ($2.6), Call-Net ($2.2), NTL Communications ($8.5), Williams Communications ($3.0), Adelphia ($6.9), Intermedia Communications ($3.1), WorldCom/MCI ($25.8), Conseco ($5.1), Marconi ($3.3), AT&T Canada ($3.0), NRG Energy ($4.0), Telewest ($5.2), Genuity ($2.0), TXU ($2.3), Qwest ($12.9), United Air Lines ($3.6), Healthsouth ($3.4), and Mirant ($4.6). Source: e.g. Moody’s Investors Service (2004a).
**Weighted Average Rating Information:** Moody’s assigns numeric "rating factors" to its ratings meant to correspond to ten-year cumulative default rates. The par-weighted average of these rating factors is reported by the trustee each month as the deal WARF. The average level is 2836, corresponding to a B2 portfolio. A more informative statistic is the ratio to the maximum test limit. The average is 1.21, indicating that the average deal is 21% past its rating limit. When the test is failing, the manager is typically not allowed to make any trades which worsen the test.

As with most performance metrics during this time period, WARF levels are highly skewed. At the fifth percentile, there is a 16% cushion to the deal limit while the 95th percentile is over by 82%. The wide dispersion of ratings performance persists even when parsing the sample by deal type.

Ratings levels are a significant control variable in regressions on transactions volumes. Additionally I divide the sample by WARF failure since the manager faces reinvest constraints if the test is failing.

**Coverage Test Information:** Over-collateralization (OC) and interest coverage (IC) ratios measure the amount of principal par or interest in the deal, relative to the liability par or liability interest due at a particular rating level. That is, five OC and five IC ratios are listed, "Senior 1", "Senior 2", "Mezzanine 1", "Mezzanine 2", and "Subordinate", corresponding to liability ratings of Aaa, Aa, A, Baa, and Ba or below, respectively. For example, the "Mezzanine 2" OC ratio is defined as the ratio of total portfolio par to the amount of outstanding debt rated Baa or higher at deal issuance.

OC and IC levels are key variables in regressions, for two reasons. First, if reported monthly levels are below their trigger level, the manager is typically not allowed to make trades which worsen the test. I control for test failures in regressions as well as parse the sample by OC and IC failures. However, as explored in Section 3.3, this may not be a tightly binding constraint, since managers are usually allowed to worsen coverage levels when selling distressed assets.

Second, OC / IC levels indicate whether subordinate investors are receiving cashflow or not. If tests are failing, on paydates interest and principal cash will be redirected to senior debtholders (away from subordinate investors and reinvestment collateral) in order to delever
the deal. Thus near a coverage test failure, managers may have more incentive to trade into weaker collateral and boost par levels. Alternatively, when deals are performing so poorly that equity investors may never receive any more cashflow, managers may have little incentive to risk-shift.

I construct a variable called the "OC Par Cushion", defined as the amount of collateral par losses the deal would have to sustain in order to make any of the five OC tests fail. The cushion is negative if tests are already failing. The aim is to convert different levels of coverage ratios into a single deal performance metric that is comparable across deals with different capital structures. This number expresses, in percentage format, how close a deal is to OC failure and resultant interest diversions. It is an estimate since it is based on original rather than current tranche sizes.

The median deal is just passing all coverage tests. For instance, looking at the most junior OC test in Table 3.2, the ratio of level to test limit is exactly 1 at the median. But again the sample is highly skewed - the average OC par cushion is -2.43% per deal, ranging from -37.18% to 6.57% at the P(1) and P(99) levels, respectively. I.e., though half the deals are passing all OC tests, the average deal would have to increase par by 2.43% to restore test passage.

3.2.3 Intex Data

Fee Schedules

CDO managers are paid with some combination of senior, subordinate, and incentive fees. In addition, it is common practice (almost universal during the time period studied) to require an equity purchase in order to incentivize effort. Though compensation information is rarely specified, it is possible to glean fee information from Intex. Fee data, interacted with measures of collateral performance or capital structure, are then used in regressions to capture the size of a manager’s cashflow incentive and the probability of achieving such distributions. His subordinated interests are a primary measure of his incentive to risk-shift. While I have no information on managerial equity ownership, I consider various other measures as a proxy in

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9I focus on OC tests rather than IC tests since OC tests are almost universally tighter. In only 34 instances out of 9571 observations is an IC test failing without an OC test failing, compared to 2924 instances of the reverse.
regressions, including lead banker dummies and the ratio of senior to subordinated fees.

To estimate manager fee rates I run cashflow analyses on manager fees and divide the output cashflow stream by the collateral par value. (Intex models a variety of CDO deal features, including hedge schedules, trustee fees, and manager fees, in addition to tranche cashflows). I follow the following procedure. First, fees modeled by Intex are listed and classified as senior, subordinate, or incentive. These fee types generally have the following characteristics:

1. Senior fees: paid after deal fees and hedge costs but before liability interest, at a small rate of collateral par

2. Subordinate fees: paid after liability interest but before equity payments, at a small rate of collateral par

3. Incentive fees: paid after subordinate fees and before equity payments, at either a small rate of collateral par or a sizable rate of remaining cashflow. Only paid after equityholders have reached a specified internal-rate-of-return, known as the hurdle or target return.

I then run each fee class from deal issuance and take the second positive cashflow number. This is divided by the original collateral par and payperiod length to estimate the annual fee rate. When incentive fees are defined off remaining cashflow, rather than collateral par, this converts them into a comparable measure.

Fee characteristics implied by the procedure above are shown in Table 3.3. While senior fees, averaging just over 20 basis points, are smaller than fee levels in the retail fund management business, CDO fees compare favorably to typical institutional fee rates. In addition CDOs offer to managers the lucrative upside potential of subordinate and incentive fees. Almost 75% of the sample pays subordinate fees, averaging a 32.77 basis point rate. Approximately 60% of the sample pays at least one incentive fee, averaging 37.67 basis points for the first incentive fee rate. This data indicates the extreme convexity of CDO manager compensation.

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10 The second number is used to avoid the first, often uneven payperiod.
11 I exercise judgment in assigning fees to categories, however names and other data provide a guide. Fee classes are often labeled descriptively. In addition, incentive fees are usually accompanied by modeling classes for the specified hurdle return - these will be labeled as "TARGET - IRR15%" for instance. The presence of such classes allows one to match the right number of incentive fees. Classifications and rates are checked against any modeling notes made by programmers.
Table 3.3: Fee Characteristics. Includes the number of deals paying, the annualized percentage rates paid, and the incremental equity internal rate-of-return hurdles for the payment of incentive fees.

<table>
<thead>
<tr>
<th>Fee Type</th>
<th>No. Deals Paying</th>
<th>Fee % of Deal Par</th>
<th>Incremental Equity Return Hurdle (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Senior</td>
<td>310</td>
<td>0.2044</td>
<td>na</td>
</tr>
<tr>
<td>Subordinate</td>
<td>275</td>
<td>0.3277</td>
<td>na</td>
</tr>
<tr>
<td>Incentive (1)</td>
<td>193</td>
<td>0.3767</td>
<td>13.05</td>
</tr>
<tr>
<td>Incentive (2)</td>
<td>56</td>
<td>0.2934</td>
<td>6.93</td>
</tr>
<tr>
<td>Incentive (3)</td>
<td>7</td>
<td>0.3255</td>
<td>10.00</td>
</tr>
</tbody>
</table>

Compensation is potentially several times more convex than corporate executive compensation, given CDOs’ extreme leverage levels. The more convex manager compensation, the greater the incentive to risk-shift or otherwise manipulate deals to subordinate investors’ advantage.

Transactions Data

Intex has recently begun accumulating transactions data from trustee transactions files. This data details the date of the trade, whether it was a sale or purchase, the name and market ID of the security, the principal amount, price, and any given reason for the transaction. Data is cumulative through approximately July 2004 with varying starting points. For most deals, data is available for an 10 - 18 month period, with a minimum of 1 month and a maximum of 32 months.\(^\text{12}\)

Fewer deals have this information - 286 rather than 325 - and in total there are 66,211 transaction line items. I eliminate 3,218 transactions which involve money-market, commercial paper, or Treasury placeholder securities. I eliminate 26 transactions with either garbled or clearly incorrect dates, leaving 62,967, or 220 transactions per deal on average.

The object was to compile this data into monthly line items capturing the manager’s discretionary trading decisions. The trade rationales help to spotlight which transactions the manager most directly controlled. They are grouped into six broad categories:

1. Redeemed assets: During this time period the main source of mandatory early redemp-

\(^{12}\) Maximum of 32 months overlapping with collateral data. Several deals go back further (up to 60 months) however without matching EMS collateral data.
Table 3.4: Manager-Specified Trade Rationales by Type. Trade rationales are grouped into six categories of trades, after removing transactions involving placeholder securities, and listed by sale and purchase volume.

<table>
<thead>
<tr>
<th>Type</th>
<th>Given Rationale</th>
<th>Sales No.</th>
<th>Purchases No.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>25,283</td>
<td>37,684</td>
</tr>
<tr>
<td></td>
<td></td>
<td>40.15%</td>
<td>59.85%</td>
</tr>
<tr>
<td>Redemptions</td>
<td>Early Redemption / Call</td>
<td>3006</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>Refinance</td>
<td>14</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>Interest-Only Paydown</td>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>Exchanges</td>
<td>Exchange Offer</td>
<td>951</td>
<td>763</td>
</tr>
<tr>
<td></td>
<td>Tenders</td>
<td>970</td>
<td>48</td>
</tr>
<tr>
<td></td>
<td>Bankruptcy Exchange</td>
<td>120</td>
<td>31</td>
</tr>
<tr>
<td></td>
<td>Restructuring</td>
<td>321</td>
<td>241</td>
</tr>
<tr>
<td></td>
<td>Other Corporate Action</td>
<td>311</td>
<td>176</td>
</tr>
<tr>
<td>Defaulted</td>
<td>Defaulted / Deferred Interest</td>
<td>1389</td>
<td>0</td>
</tr>
<tr>
<td>Equity</td>
<td>Equity Disposition / Equity Exchange</td>
<td>1033</td>
<td>75</td>
</tr>
<tr>
<td></td>
<td>Equity Security</td>
<td>809</td>
<td>944</td>
</tr>
<tr>
<td></td>
<td>Warrants Security</td>
<td>256</td>
<td>294</td>
</tr>
<tr>
<td>Discretionary</td>
<td>Credit Risk</td>
<td>4693</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Credit Improved</td>
<td>7394</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Elective / Substitution / Replacement</td>
<td>1572</td>
<td>19</td>
</tr>
<tr>
<td>Unknown</td>
<td>Other</td>
<td>174</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>12.##*</td>
<td>1394</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Missing</td>
<td>868</td>
<td>35070</td>
</tr>
</tbody>
</table>

*such trades are labeled according to an unknown indenture classification.
tions was the issuer calling debt in order to refinance it. Loan assets (floaters) are always subject to refinance risk while corporate assets (fixed rate) would depend on asset covenants.

2. Exchanged assets: Both cosmetic and distressed exchanges occur widely in the sample. An example of a cosmetic exchange would be a corporate entity changing its name or buying a firm and needing to reissue any outstanding debt. Alternatively, a firm in distress might restructure its debt.

3. Defaulted assets: Defaulted assets must generally be removed from the portfolio within a year of the date of default. Within that timeframe they are sold at the discretion of the manager.

4. Equity transactions: All trades that are specifically denoted as equity transactions or whose security name indicates stocks, shares, or warrants are summed separately. Not all deals allow equity securities and their prices and par amounts would distort calculations.

5. Discretionary trades: For purchases, discretionary transactions consist of all reinvestment decisions unrelated to redemptions, exchanges, or equity assets. For sales, they include both credit-improved and credit-risk sales as well as those listed as elective, substitution, replacement, or optional decisions. (This latter category is collectively labeled as "Elective" decisions). I view sales of credit-improved and credit-risk securities as being discretionary since there is generally no obligation for a manager to make such trades and no deal rules which clearly identify such securities.

6. Unknown: Trades whose motives are unclear are those whose rationales are left blank, are labeled as "Other", or reference a section of the deal’s indenture. For instance, 228 sales were explained as "12.1a" referring to some deal-specific definition. Approximately 10% of the sales and 93% of the purchases had unclear trade rationales. The purchases appear to be reinvestment decisions which managers habitually did not designate. I include all unknown transactions in the "Elective" discretionary category when compiling trade statistics.

The approach in regressions is to test only on voluntary manager actions. I hold equity
transactions apart, treat redemptions and exchanges as non-discretionary, test defaulted sales as possibly discretionary, and include all other transactions as wholly discretionary. One can argue that distressed exchanges and defaulted sales are the result of choosing not to act on credit-risk situations earlier, and thus are in a sense discretionary. I test on both assumptions. Table 3.4 presents a breakdown of trade rationales.

Using the trade rationales, I sort and sum all transactions over periods corresponding to a deal’s monthly collateral data. I compile the total redemption, exchange, and equity volume bought and sold, as well as weighted-average prices when applicable. I compile the total discretionary trade volume bought and sold, the volume sold above $100 and bought below $100, and the sold volume breakdown between credit-improved, credit-risk, elective, and defaulted rationales. The weighted-average sale and purchase prices for each of the preceding categories is computed as well.

Some 4534 transactions had either zero or missing prices. I view transactions with missing prices as legitimate trades for which the trustee did not supply price data. However I treat zero prices as an indication of some sort of exchange and include in exchange calculations. When prices are missing, the weighted-average transaction price is calculated off the priced volume and serves as a proxy for the total volume.

I assume Intex’ transactions data is comprehensive over the timeframe provided, that is, that there is zero trading if none is listed.

### 3.3 Trade Data Analysis

First I look at the trade data in more detail, for trends and to judge whether it is reliable. I also investigate whether it is in accord with common deal restrictions, with an eye towards how this affects regression analysis.

Table 3.5 gives an overview of Intex’s monthly trading and price data. There is quite a bit of turnover, with an average monthly volume of approximately 1.43% discretionary sales and 3.24% discretionary reinvestment (as a percentage of the prior month’s portfolio par). Credit-improved sales are the largest component of discretionary sales, at 0.65% monthly volume.

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13 Missing prices most often accompanied loan securities.
Table 3.5: Monthly Trade Volume and Price Data for Sales and Purchases across Trade Types. Trade volumes are measured by face value, and both trade volumes and monthly par gains are expressed as percentages of the prior month’s portfolio par. Weighted-average prices (WAP) are shown in dollars per 100 units for the months in which applicable transactions occurred. Totals include all transactions, both sales (S) and purchases (B), both discretionary and non-discretionary.

<table>
<thead>
<tr>
<th>Obs.</th>
<th>Variable</th>
<th>Mean</th>
<th>St.</th>
<th>5%</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Dev.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3351</td>
<td>Sales Vol. Disc. (%)</td>
<td>1.43</td>
<td>2.40</td>
<td>0.00</td>
<td>0.00</td>
<td>0.65</td>
<td>2.05</td>
<td>5.31</td>
</tr>
<tr>
<td>3351</td>
<td>Sales Vol. Disc. &gt; $100 (%)</td>
<td>0.70</td>
<td>1.34</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.93</td>
<td>2.34</td>
</tr>
<tr>
<td>3355</td>
<td>Sales Vol. Credit Risk (%)</td>
<td>0.46</td>
<td>1.05</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.49</td>
<td>2.34</td>
</tr>
<tr>
<td>3355</td>
<td>Sales Vol. Credit Impv. (%)</td>
<td>0.65</td>
<td>1.44</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.74</td>
<td>3.17</td>
</tr>
<tr>
<td>3352</td>
<td>Sales Vol. Elective (%)</td>
<td>0.32</td>
<td>1.11</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.19</td>
<td>1.70</td>
</tr>
<tr>
<td>3353</td>
<td>Sales Vol. Defaulted (%)</td>
<td>0.21</td>
<td>0.69</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>1.43</td>
</tr>
<tr>
<td>3356</td>
<td>Sales Vol. Redemptions (%)</td>
<td>0.49</td>
<td>1.10</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.44</td>
<td>2.67</td>
</tr>
<tr>
<td>3356</td>
<td>Sales Vol. Exchanges (%)</td>
<td>0.53</td>
<td>1.07</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.70</td>
<td>2.55</td>
</tr>
<tr>
<td>3351</td>
<td>Sales Vol. Equity (%)</td>
<td>0.02</td>
<td>0.20</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.02</td>
</tr>
<tr>
<td>3356</td>
<td>Bought Vol. Disc. (%)</td>
<td>3.24</td>
<td>4.43</td>
<td>0.00</td>
<td>0.00</td>
<td>1.81</td>
<td>5.30</td>
<td>10.63</td>
</tr>
<tr>
<td>3356</td>
<td>Bought Vol. &lt; $100 (%)</td>
<td>1.09</td>
<td>2.53</td>
<td>0.00</td>
<td>0.00</td>
<td>0.33</td>
<td>1.54</td>
<td>4.16</td>
</tr>
<tr>
<td>3348</td>
<td>Total Vol. (%)</td>
<td>6.20</td>
<td>6.11</td>
<td>0.00</td>
<td>1.83</td>
<td>5.06</td>
<td>8.75</td>
<td>16.28</td>
</tr>
<tr>
<td>2115</td>
<td>WAPS Disc. ($)</td>
<td>92.37</td>
<td>17.84</td>
<td>51.13</td>
<td>89.87</td>
<td>98.90</td>
<td>101.02</td>
<td>108.62</td>
</tr>
<tr>
<td>1280</td>
<td>WAPS Credit Risk ($)</td>
<td>80.53</td>
<td>22.62</td>
<td>32.00</td>
<td>70.85</td>
<td>87.22</td>
<td>97.24</td>
<td>104.91</td>
</tr>
<tr>
<td>1276</td>
<td>WAPS Credit Improved ($)</td>
<td>102.63</td>
<td>7.33</td>
<td>94.03</td>
<td>100.28</td>
<td>101.63</td>
<td>105.61</td>
<td>113.26</td>
</tr>
<tr>
<td>833</td>
<td>WAPS Elective ($)</td>
<td>94.76</td>
<td>15.46</td>
<td>65.00</td>
<td>96.17</td>
<td>99.62</td>
<td>100.45</td>
<td>104.99</td>
</tr>
<tr>
<td>561</td>
<td>WAPS Defaulted ($)</td>
<td>47.55</td>
<td>29.37</td>
<td>3.00</td>
<td>23.72</td>
<td>44.90</td>
<td>72.00</td>
<td>97.13</td>
</tr>
<tr>
<td>1132</td>
<td>WAPS Redemptions ($)</td>
<td>102.88</td>
<td>10.78</td>
<td>100.23</td>
<td>102.86</td>
<td>104.09</td>
<td>104.98</td>
<td>110.00</td>
</tr>
<tr>
<td>536</td>
<td>WAPS Equity ($)</td>
<td>3600.6</td>
<td>14879.1</td>
<td>11.2</td>
<td>261.2</td>
<td>1108.9</td>
<td>2753.6</td>
<td>9735.1</td>
</tr>
<tr>
<td>2023</td>
<td>WAPB Disc. ($)</td>
<td>98.34</td>
<td>6.05</td>
<td>89.33</td>
<td>97.90</td>
<td>99.62</td>
<td>100.04</td>
<td>104.05</td>
</tr>
<tr>
<td>3351</td>
<td>Par Gain Disc. Sales (%)</td>
<td>-0.08</td>
<td>0.32</td>
<td>-0.55</td>
<td>-0.04</td>
<td>0.00</td>
<td>0.00</td>
<td>0.12</td>
</tr>
<tr>
<td>3353</td>
<td>Par Gain Defaulted Sales (%)</td>
<td>-0.12</td>
<td>0.48</td>
<td>-0.78</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>3356</td>
<td>Par Gain Redemptions (%)</td>
<td>0.01</td>
<td>0.12</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.01</td>
<td>0.10</td>
</tr>
<tr>
<td>3356</td>
<td>Par Gain Exchanges (%)</td>
<td>-0.27</td>
<td>1.08</td>
<td>-1.98</td>
<td>-0.23</td>
<td>0.00</td>
<td>0.00</td>
<td>0.43</td>
</tr>
<tr>
<td>3356</td>
<td>Par Gain Equity Vol. (%)</td>
<td>0.04</td>
<td>0.82</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.10</td>
</tr>
<tr>
<td>3356</td>
<td>Par Gain Disc. Purchases (%)</td>
<td>0.04</td>
<td>0.27</td>
<td>-0.06</td>
<td>0.00</td>
<td>0.00</td>
<td>0.03</td>
<td>0.27</td>
</tr>
<tr>
<td>3348</td>
<td>Total Par Gain (%)</td>
<td>-0.38</td>
<td>1.52</td>
<td>-2.48</td>
<td>-0.60</td>
<td>0.00</td>
<td>0.04</td>
<td>0.60</td>
</tr>
</tbody>
</table>
followed by credit-risk sales at 0.46% monthly volume. Sales of defaulted securities add 0.21% to sold volume, but are sold at the largest loss, with an average sales price of $47.55. In general, the manager reinvests at higher prices than he sells, or $98.34 versus $92.37, but still at less than par. (All prices are expressed in dollars per 100 units).

The bottommost lines estimate par loss due to each transaction type. The largest average monthly losses stem from defaulted sales and exchanges, -0.12% and -0.27%, respectively, although the exchange number is an estimate dependent upon the equivalence of the security types being exchanged. The manager makes small par gains on redemptions and purchases. Monthly defaulted and total par losses, the only two measures directly comparable with Moody’s collateral data, are significantly correlated with Moody’s numbers. Despite the reduction in data points, I use Intex transactions information in all regressions because it is less noisy and more detailed.

It is important to see whether the data meshes with common CDO restrictions. One must account for trading restrictions in the regression analysis or risk interpreting mandatory actions as voluntary. Rules on the types of allowable trades (defaulted, credit-risk, credit-improved, and elective security sales) specify when they may be made (mandatory / optional, limited / unlimited, before / after the reinvestment period, etc.) and how the proceeds should be reinvested (timing, price, quality test considerations, etc.). I have little information on specific trading restrictions, but am operating under certain assumptions.

First, there are annual limits on elective trading which I do not view as binding. This is because typically credit-risk and credit-improved sales are allowed in unlimited quantities and a manager can classify a sale candidate as he wishes. Indeed, analysis of the monthly trade data, presented in Table 3.6, suggests very little elective trading, both absolutely and relatively. This table differs from Table 3.5 in that the trade volumes are annual rather than monthly and reported on a per deal basis rather than per observation. The median deal has just 1.15% annualized elective trading (as a percentage of 12 month’s prior portfolio par) versus 7.58% of credit-risk or credit-improved sales trading. Indeed, just 10 deals (3.5%) have more than 15% annualized elective trading and just 1 (0.35%) has more than 25%, the two most frequent elective trading limits.

Second, volume and price characteristics before and after the given reinvestment period
Table 3.6: Annualized Elective Sales Trading, by Deals’ Average Volume. Volumes are measured in face value, as percentages of 12 month’s prior portfolio par.

<table>
<thead>
<tr>
<th>Deals</th>
<th>Sales Type</th>
<th>Mean</th>
<th>St. Dev.</th>
<th>5%</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>285</td>
<td>Elective (%)</td>
<td>3.20</td>
<td>4.72</td>
<td>0.00</td>
<td>0.00</td>
<td>1.15</td>
<td>4.65</td>
<td>12.30</td>
</tr>
<tr>
<td>285</td>
<td>Credit Risk, Improved (%)</td>
<td>11.82</td>
<td>13.89</td>
<td>0.00</td>
<td>2.74</td>
<td>7.58</td>
<td>15.72</td>
<td>38.90</td>
</tr>
<tr>
<td>285</td>
<td>Elective Share (%)</td>
<td>21.29</td>
<td>34.07</td>
<td>0.00</td>
<td>0.00</td>
<td>8.05</td>
<td>29.74</td>
<td>86.48</td>
</tr>
</tbody>
</table>

Table 3.7: Average Monthly Trading Frequencies and Volumes Before and After Reinvest Period, by Trade Type. Sale (S) and purchase (B) volumes are measured by face value, and expressed as percentages of the prior month’s portfolio par. Frequency indicates the percent of months in which a transaction of the applicable type took place, relative to the total available observations, and prices are conditional on a transaction taking place. Prices are shown in dollars per 100 units.

<table>
<thead>
<tr>
<th>Trade Type</th>
<th>During Reinvest Period</th>
<th>After Reinvest Period</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Obs.</td>
<td>Freq. %</td>
</tr>
<tr>
<td>(S) Credit Impv.</td>
<td>2656</td>
<td>44.50</td>
</tr>
<tr>
<td>(S) Credit Risk</td>
<td>2656</td>
<td>41.08</td>
</tr>
<tr>
<td>(S) Elective</td>
<td>2653</td>
<td>32.98</td>
</tr>
<tr>
<td>(S) Defaulted</td>
<td>2654</td>
<td>16.69</td>
</tr>
<tr>
<td>(S) Redemptions</td>
<td>2657</td>
<td>34.29</td>
</tr>
<tr>
<td>(S) Exchanges</td>
<td>2657</td>
<td>40.23</td>
</tr>
<tr>
<td>(S) Equity</td>
<td>2652</td>
<td>8.45</td>
</tr>
<tr>
<td>(B) Discretionary</td>
<td>2657</td>
<td>75.72</td>
</tr>
</tbody>
</table>

171
suggest that managers have some latitude but are subject to binding constraints.\textsuperscript{14} Table 3.7 presents various trade statistics compiled during and after the reinvest period. There is an observable fall-off in discretionary activity. For instance the average volume and frequency of credit-improved and elective sales fall by between 55% and 77%. Distressed transactions do not decline as much: credit-risk volume falls minutely while defaulted volume actually increases. Redemptions and exchanged volume approximately double, likely due to aging. Average monthly reinvest volume falls by nearly 5 times but still occurs over 28% of the time. The main difference in prices is a general decline, with the exception that redeemed assets’ prices are static (indicating assets are called at par or at prices set by covenants). In regressions I condition on the reinvest period so as to hone in on unconstrained managerial behavior.

Third, and perhaps most importantly, deal guidelines and test violations may play a key role in the manager’s decisions and specifically his ability to risk-shift. For instance, almost universally, if a deal’s WARF and OC tests are in compliance, the manager may reinvest freely; when out of compliance, he must maintain or improve the test metrics. Thus if an OC test is failing, the manager must generally reinvest at a price less than that of the removed security, or remove a high-priced asset from the portfolio simultaneously. Likewise if the WARF test is failing, the manager must reinvest into a security rated at least equally to the one removed from the portfolio.\textsuperscript{15} A breakdown of sales and purchase prices by trade type and test failure is presented in Table 3.8.

There is mixed evidence that a failing WARF test (second section) prompts investment into higher quality assets. I do not have data on ratings of sold and bought items but use prices as a proxy for quality. Though the average reinvest price falls, the ratio of reinvest to sale price increases. This indicates that on average managers are buying slightly more expensive assets than they sell, which may be linked to higher ratings. However, the median ratio is less than 1 (and declines from the no-failure scenario), so that in over 50% of observations many managers are apparently building par even while maintaining ratings. As participants have alleged, this may be an indication that managers have adversely selected assets with low prices but still

\textsuperscript{14}Many deals permit reinvestment after the reinvest period has technically ended so as not to discourage value-adding sales. The manager receives fees on portfolio assets and equityholders receive excess interest, so a profit-maximizing manager might be disinclined to remove securities from the portfolio.

\textsuperscript{15}Less common (or more varied) restrictions, for instance, deals allowing differing exposures to cheaper sectors, would be accounted for in performance regressions by deal type, deal ID, or deal manager dummies.
Table 3.8: Monthly Weighted Average Discretionary Trade Prices for Coverage and Ratings Failures. Showing breakdown between Credit-improved, Credit-risk, and Elective Sales. Price ratio of discretionary purchases to total discretionary sales (WAP_B / WAP_S) shown for monthly observations in which both occur. Prices are shown in dollars per 100 units.

<table>
<thead>
<tr>
<th>Trade Type</th>
<th>No Failure</th>
<th></th>
<th></th>
<th>WARF Fail Only</th>
<th></th>
<th></th>
<th>OC / IC Failure</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(S) Credit Impv.</td>
<td>503</td>
<td>101.29</td>
<td>100.88</td>
<td>361</td>
<td>104.23</td>
<td>103.22</td>
<td>405</td>
<td>102.86</td>
</tr>
<tr>
<td>(S) Credit Risk</td>
<td>310</td>
<td>85.48</td>
<td>91.51</td>
<td>333</td>
<td>83.81</td>
<td>88.20</td>
<td>630</td>
<td>76.45</td>
</tr>
<tr>
<td>(S) Elective</td>
<td>388</td>
<td>97.21</td>
<td>99.75</td>
<td>238</td>
<td>95.24</td>
<td>99.59</td>
<td>199</td>
<td>89.61</td>
</tr>
<tr>
<td>(S) Total Disc.</td>
<td>706</td>
<td>97.58</td>
<td>99.91</td>
<td>521</td>
<td>96.25</td>
<td>99.23</td>
<td>876</td>
<td>85.87</td>
</tr>
<tr>
<td>(B) Total Disc.</td>
<td>731</td>
<td>99.54</td>
<td>99.82</td>
<td>530</td>
<td>98.69</td>
<td>99.50</td>
<td>749</td>
<td>96.98</td>
</tr>
<tr>
<td>WAP_B / WAP_S</td>
<td>665</td>
<td>1.0381</td>
<td>0.9985</td>
<td>471</td>
<td>1.0493</td>
<td>0.9954</td>
<td>587</td>
<td>1.2988</td>
</tr>
</tbody>
</table>

equivalent ratings, and also suggests ratings may not be the best proxy for quality. I use ratings metrics as control variables in regressions but in general it does not appear that ratings failures would have significantly impeded a manager's ability to risk-shift. Particularly in a rapidly worsening credit environment, rating agencies may have been slow to assess downgrades, and managers been thus fairly free to acquire riskier assets. For the OC failure cases it is crucial to establish whether reinvestment into low-quality assets is forced or optional on the part of the manager. That is, to confirm that the manager has the discretion to sell a defaulted asset and then reinvest at par. Despite the general requirement to maintain failing OC levels, most deals allow for a passing coverage test to fail, or a failing coverage test to worsen, after the sale and reinvestment of credit-risk or defaulted securities. This meshes with most deals' guidelines "recommending"\textsuperscript{16} that credit-risk and defaulted sales proceeds be reinvested at any price less than par ($100).

This latitude is indeed evidenced in the data in several ways. First, though there is a tendency to reinvest at lower prices following credit-risk or defaulted sales, it is not a dramatic one. The average reinvest price falls nearly a dollar ($0.88) comparing elective and credit-risk reinvest prices. However, considering the much lower sales price ($95.34 vs. $82.77) this is a small decline. It does not appear that managers are churning, or going from one distressed asset to an equally distressed one, in hopes of delaying defaults. Further, average reinvest

\textsuperscript{16}That is, to the best belief and ability of the manager, in contrast to hard-and-fast restrictions.
prices following distressed sales are significantly larger than the sold prices, at all percentile ranges.

Additionally, in at least half the cases with OC failures, managers are on net losing par through trades. This point is made in Table 3.8 (third section). The average reinvest price with an OC failure is about 2.5 points lower than when passing, at $97 versus $99.5. This could signify that managers must or wish to reinvest at lower prices in order to maintain OC levels. However, the average ratio of reinvest to sale price increases dramatically, over 25%. This indicates the manager, while still purchasing assets below par, is on net losing par. The median ratio is near 1, so that in half the cases, the manager is reinvesting at a lower price, and in half, actually higher.

I conclude that the manager has the discretion to reinvest at a higher price, and in regressions I condition on OC failures but interpret results based on the conjecture that par-building trades are not required. Because it appears managers are not churning, to find risk-shifting activity one must look in the composition and timing of trades.

3.4 Regression Analysis

Because of their subordinated and incentive fees and equity ownership, managers of CDOs have extremely convex payoffs as a function of deal performance. Essentially they own a time-series of call options (with various strike prices) on collateral interest and principal payments. As such, they are in the familiar position of favoring any value process which boosts the value of equity at the expense of debt. In the regression analysis I look for evidence that managers of badly performing deals were motivated by their subordinate interests and sought to shift value away from more senior investors.

One way they could do this is by adding volatility to the deal, e.g. through exchanging higher-quality for lower-quality assets. A second way is to manipulate par-based coverage tests by boosting par levels. Building par - buying low and selling high - is an inclusive metric for risk-shifting since it is an outcome of both methods. Primarily focusing on par-building trades, I explore the influence of deal performance and compensation structure on managers’ trading decisions. I analyze whether regression results support the hypothesis of risk-shifting
and attempt to disentangle the risk-shifting explanation of managerial behavior from others (default regime, overconfidence, etc.).

Because of the optionality in managers’ contracts, one would expect a greater incentive to increase risk around strike prices. In the CDO setting there are two types of strike prices, pertaining to the sequence of interest options and the final principal option. OC test levels can be considered as interest strike prices, in that they control the flow of subordinated interest and fees on every paydate. (The manager’s stream of fees and equity payments are very large incentives, since CDOs are typically issued with a large amount of excess interest flowing through the capital structure). The final principal strike price is the total amount of liability payments due prior to fee and equity payments.

Though it is possible interest and principal option payments could diverge, in practice they are linked. Deals had to structure OC test levels in a certain manner, due to a common CDO rating methodology. That was to set OC test levels such that given expected collateral defaults, final collateral value would be sufficient to repay liabilities. Thus if collateral levels are too low to pass OC tests, signifying the principal options might fail, interest options would also fail and interest be diverted to senior noteholders. In regressions I focus mainly on OC test passage as a metric for how close a manager is to his strike prices.

For the manager to be less active in risk-shifting away from strike prices, any of several assumptions suffice. One is simply that manager activity is costly, since the potential gains from risk-shifting are smaller when the deals’ payouts are more certain. These costs could be the time spent analyzing trade ideas, cost in implementing trades, or opportunity costs. Managers may focus more on other portfolios from which they expect higher returns to effort (e.g. a later-issue, better-performing CDO).

Alternately, one might postulate a ceiling on variance, given market illiquidity and CDO deal restrictions. That is, a manager of a badly-performing deal cannot identify one risky security and reinvest wholly into it. There may not be enough exposure available to impact a large portfolio and furthermore CDOs generally restrict issuer exposures to no more than 1-2%. Similarly, the manager is restricted in his choice of securities (no esoteric credit or rate options, no defaulted securities) since CDOs also restrict collateral types. The prospect of having to make numerous transactions to appreciably impact equity prospects may deter risk-shifting
activity when deal performance is bad.

Lastly, when deal performance is good, managers may be deterred from risk-shifting by either reputation or franchise value considerations. The future stream of payments may be valuable enough, relative to his other prospects, that the manager does not wish to increase risk all at once and chance a blow-up in the next period. Indeed CDO fee payments are generally larger than what managers make in other portfolio management lines. Likewise for a small improvement in equity prospects, the manager may not wish to anger potential future clients and dry up his future deal stream. In regression analysis I assume a risk-shifting manager will be most active when deal performance is marginal (measured by OC ratios) and less so when performance is extreme in either direction. Evidence that this is so would refute the overconfidence argument by suggesting that managers were rationally considering a cost-benefit analysis rather than indiscriminately believing they could pick "winners".

To analyze the relationship between trading decisions, manager incentives, and deal performance, I run OLS panel regressions with fixed effects for time and specific manager or deal ID. I consider many possible dependent variables, all some form of trading metric. In general volume and par trading gains work better than price statistics, simply because there is more data: if no trades were made, the volume and gains are zero whereas the price is undefined. For regressors I consider mechanical factors such as deal age, other fixed effects such as deal type, lead investment banker, and manager size, other simultaneous trading decisions, manager incentive structure, and various measures of collateral performance including default levels, rating levels, and par and interest coverage levels, particularly with respect to their test levels. The basic equation can be expressed as

\[ \text{Trade}_{it} = \beta_0 + \beta_{1t} + \beta_{12t} + \beta_3 \text{DealStats}_{it} + \beta_4 \text{Trade}_{it} + \beta_5 \text{Incentive}_{it} + \beta_6 \text{Performance}_{it} + \epsilon_{it} \]

in which \( \beta_{1t} \) and \( \beta_{12t} \) are fixed effects on time and either deal or manager ID.

When performing panel data regressions, it is important to consider the treatment of standard errors. Correlation between observations can lead to large errors in calculating signifi-

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In all regressions I performed coefficients, standard errors, and adjusted $R^2$ were markedly different when including $\beta_{1t}$ and $\beta_{12}$ dummy variables, signifying the presence of strong fixed effects. All reported regressions use fixed effects to control for any time-invariant time-series and cross-sectional residual correlation, and are further corrected for heteroskedasticity. I also consider deal- or manager-clustered standard errors to ensure unbiased estimates when fixed effects may vary over time.\footnote{See Petersen (2005) for a discussion of optimal error treatment in panel data regressions. Using time dummies and clustering by firms (cross-sectional) is appropriate when the time series is short and the number of clusters are large. In simulations performed by Petersen, the error in standard error estimation fell to 1% with 100 clusters. In the CDO data set there are enough managers and deals to cluster by either.}

It is problematic to include constant independent variables (say fee rates) in a time-series regression and not have the effect be lost in the observation’s general fixed effect coefficient. This issue is addressed by using alternate fixed effects. For instance, with deal structure variables, manager dummies are used, while with manager variables, deal dummies are used. In general, despite the amount of dummy variables employed, and the conditioning of the sample on reinvest period, coverage test failure, manager switches, etc., I am able to get several consistently significant relationships. $R^2$ levels are high, considering that levels of 20% are not unusual in financial regressions. Coefficients should be interpreted with care. In the fixed effects model, one effectively looks at each observation’s deviation from its mean, rather than its absolute level.

### 3.4.1 Grouping Decisions

The first aspect of the manager’s trading behavior I look at is any tendencies to group trades together so as to avoid par impairment. One sign of potential risk-shifting would be a manager lowering his reinvest price to accompany credit risk sales or raising the amount of credit-improved sales. Evidence suggests that many managers did this - for instance though the weighted-average sale price of credit-risk securities falls dramatically when the OC test is failing, the credit-improved sales price actually rises. (See Table 3.8, third section).

Consider the manager’s decision about when to make credit-improved and credit-risk sales, presented in Table 3.9. The manager is not obligated to make credit-improved sales in conjunction with credit-risk sales, even when failing coverage tests, because as discussed in Section
3.3 he is allowed to worsen par coverage in distressed sales. Note first that the later vintage a deal was (Closing Date), the more likely it was to have credit-improved sales, controlling for the stress located in earlier deals. Also, credit-improved sales are positively related to deal age, since assets have more time to deviate from new-issue quality. When the manager is unrestrained by ratings test failures, as in column (1) of Table 3.9, average monthly credit-improved sales volume increases significantly, nearly 60%, in credit-risk sales volume. Whenever there is a WARF failure, as in columns (2) and (4), the coefficient falls by nearly two-thirds, suggesting the manager is not able to make as many credit-improved sales as he would like. Selling high-dollar assets would tend to worsen WARF. This could indicate that there are simply fewer high-quality assets in the portfolio to sell, given that ratings levels are low. However this interpretation is belied by the coefficient when OC tests are failing, presented in column (3), which is actually at its maximum. One would expect an equally, if not more, distressed portfolio when OC tests are failing. But when the OC test is failing, selling high-dollar assets tends to build par, improving coverage tests. That managers make credit-improved sales when they have the discretion to do so suggests a desire to preserve par whenever permitted, not the lack of assets to sell.

The same trend occurs with elective and defaulted sales volume although with less significance. (There are fewer defaulted or elective sales relative to credit-improved and credit-risk sales, increasing standard errors). The overall trend suggests that managers chose to worsen the average quality of the portfolio in favor of maintaining par, rather than letting par decline and keeping high-quality, low-risk assets in the portfolio. It also suggests that ratings constraints did bind to some extent. Note that redeemed assets and exchanged assets played little role. That is, it appears managers specifically paired discretionary trades to avoid taking losses, whereas with non-discretionary trades they were less able to manage timing.

I also look at another method of building par, which is to reinvest at a low dollar price. Buying lower-quality assets adds risk to the portfolio. Par trading gains from purchases are analyzed in equations (5) and (6) of Table 3.9. When OC tests are passing, par gains from purchases and sales are inversely related, suggesting that managers attempt to match sales par losses with reinvest par gains. On the other hand when coverage tests are failing, the relationship weakens (nearly 90%). Either more distressed sales are taking place without
Table 3.9: Relationship between Contemporaneous Sales and Purchase Decisions for Coverage and Ratings Failures. Monthly trade volumes are measured by face value, and both monthly trade volumes and par gains are expressed as percentages of the prior month’s portfolio par. T-stats, adjusted for heteroskedasticity, are in brackets. Credit-improved sales volume is regressed on simultaneous sales activity and presented for four scenarios: (1) both WARF and OC tests passing (2) WARF failing (3) OC failing and (4) both WARF and OC tests failing. Purchase par gains are regressed on simultaneous sales par gains and presented for two scenarios: (5) both OC and IC tests passing and (6) either OC or IC test failing. Month, deal type, and either deal (1)-(4) or manager (5)-(6) fixed effects are included in regressions but not reported. For (1)-(4), trades are restricted to the reinvest period.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Credit Improved Sales</th>
<th>Purchase Par Gains</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Closing Date</td>
<td>0.19</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>[1.35]</td>
<td>[2.70]**</td>
</tr>
<tr>
<td>Deal Age</td>
<td>0.19</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>[1.37]</td>
<td>[1.65]</td>
</tr>
<tr>
<td>Credit Risk Sales Vol.</td>
<td>0.59</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td>[2.02]*</td>
<td>[3.10]**</td>
</tr>
<tr>
<td>Elective Sales Vol.</td>
<td>0.22</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>[3.04]**</td>
<td>[1.09]</td>
</tr>
<tr>
<td>Defaulted Sales Vol.</td>
<td>0.70</td>
<td>0.33</td>
</tr>
<tr>
<td></td>
<td>[1.87]</td>
<td>[2.11]*</td>
</tr>
<tr>
<td>Redemption Sales Vol.</td>
<td>0.13</td>
<td>-0.08</td>
</tr>
<tr>
<td></td>
<td>[0.79]</td>
<td>[0.78]</td>
</tr>
<tr>
<td>Exchanged Sales Vol.</td>
<td>0.09</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>[1.03]</td>
<td>[1.03]</td>
</tr>
<tr>
<td>Equity Sales Vol.</td>
<td>0.39</td>
<td>1.17</td>
</tr>
<tr>
<td></td>
<td>[0.60]</td>
<td>[1.39]</td>
</tr>
<tr>
<td>Par Gain Disc. + Def. Sales</td>
<td>-0.24</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[4.63]**</td>
<td></td>
</tr>
<tr>
<td>Par Gain Non-Disc. Sales</td>
<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>[0.83]</td>
<td>[0.91]</td>
</tr>
<tr>
<td>Constant</td>
<td>-99.96</td>
<td>-12.26</td>
</tr>
<tr>
<td></td>
<td>[1.35]</td>
<td>[2.14]*</td>
</tr>
<tr>
<td>Observations</td>
<td>812</td>
<td>624</td>
</tr>
<tr>
<td>Deals</td>
<td>96</td>
<td>85</td>
</tr>
<tr>
<td>Adjusted R²</td>
<td>0.5123</td>
<td>0.5101</td>
</tr>
</tbody>
</table>

179
compensatory par reinvest gains or the manager is now seeking par gains whenever he reinvests without regard to sales activity. As before, the discretionary reinvest par gains show little relationship with non-discretionary reinvest par gains.

The tendency to group trades to avoid par losses is a central argument that managers were at the very least gambling on riskier assets in lieu of keeping solid assets in the portfolio. Whether this managerial behavior actually qualifies as risk-shifting, rather than what I will call "tactical" behavior, is complex. One might suppose that managers had a basic preference for stability and good appearances, and managed portfolios accordingly. For instance, managers may not like investors to see a stream of constant losses, and view realized sales gains as more reassuring to investors than hypothetical mark-to-market gains. As well, managers may have extreme sensitivity to test failures. Collateral losses that cause marginal tests to fail may be more noticeable and cause greater headaches through increased client communication, rating agency inquiries, etc.¹⁹ These tactical preferences would lead to increased activity when incurring credit-risk and defaulted asset losses and near test failures.

Risk-shifting is an incentive-based explanation, whereas tactical management is more a behavioral tendency. One should note that a "managing to tests" mentality may not be motivated by equityholders' cashflow interests but certainly benefits them. That is, it is difficult to pinpoint the motives for par-building behavior because it is at the confluence of manager preferences for greater volatility and "tactical" test improvement. A pattern of such trading is suggestive but not conclusive. The smoking gun for the risk-shifting interpretation will be in the role of compensation structure on manager behavior, analyzed in the next section.

3.4.2 Discretionary Trading and Incentive Structure

My second line of argument for risk-shifting is the influence of performance and incentive measures on managers' trading decisions. Were managers simply prone to overconfidence, or deals simply victims of a bad default regime, trading decisions should not be tightly linked to manager's compensation. In addition to their economic incentives, managers have reputations to consider. A desire to keep or attract new investors to future deals or other asset management

¹⁹See, for instance, findings in Bertrand and Mullainathan (2003) that suggest a preference for less activity when managers are insulated from takeover pressures.
accounts may also have influenced manager choices. Again if manager’s reputational concerns have a demonstrable effect of trade decisions, it indicates that bad investment choices were not simply the result of overconfidence, bad luck, or tactical style.

**Par-building Sales**

I look first at the discretionary volume sold above $100, that is at a par gain. Such sales can be made because of credit improvement in the portfolio, higher interest rates in the portfolio, or a desire to build par. Table 3.10 presents regression results from par-building sales volume on various deal and performance parameters.

First note that generally the deal type dummies are strongly and significantly negative, compared to the baseline Emerging Market deal type, which had very good credit performance over the timeframe. Also, while not significant here, in a trend that is significant elsewhere, the later the closing date, the better the overall performance, and the higher the amount of high-dollar or credit-improved sales. Cash generally has a negative effect on any sort of sales volume, perhaps because managers would rather reinvest cash prior to generating more. Also, rating factor is a negative influence on high-dollar sales, simply due to poor credit quality.

The most significant performance-related variable is the level of over-collateralization. This metric is significant in various forms, including the most senior OC level, the OC par cushion, and the latter squared. The senior OC ratio is somewhat orthogonal to the latter two variables, since generally junior tests have much less cushion in them. What one observes is that a larger senior level generally has a negative effect on par-building sales. Even though deals with lower coverage levels would tend to have worse collateral, nonetheless they sell more volume at high prices.

Care must be taken with OC analysis because there might be endogeneity or serial correlation. Roughly three things could conceivably raise OC levels relative to deal means: A) good default performance B) mandatory principal paydowns stemming from earlier OC failures (bad performance) or C) earlier par-building sales. It is extremely unlikely that bad performance and subsequent mandatory paydowns are responsible for the implied marginal decline in par-building sales. First, mandatory paydowns occur sporadically, only on paydates. Secondly in regressions (3), (5), and (6), in which all OC tests are passing, the effect is also negative
and strong. If OC tests are passing there are no mandatory paydowns inside the reinvestment period.

It is also unlikely that the decline in par-building sales is due to having already previously removed premium assets from the portfolio. This is because credit-improved sales (the only group of sales with average price > $100) are so linked with credit-risk sales. Concomitant credit-risk and credit-improved sales would generally accompany lower OC levels, and in future periods reduced credit-improved sales volume would be linked with low rather than high OC levels. In addition credit-improved sales are positively and significantly linked with three lags of prior credit-improved sales. That is, if the OC test is high due to prior par-building or credit-improved sales, it is more likely that credit-improved sales will continue to be high, raising OC levels. One can conclude that the most obvious reason, good default performance, is most responsible for high OC levels. This makes the negative marginal impact (i.e. deals with worse OC levels making more high-dollar sales) even more striking.

The effect of the minimum OC par cushion is complex due to the squared term. In the most general regression, (1), both coefficients are significant and positive although the squared term is small. This implies that the larger the minimum cushion is, including as the cushion travels from negative to positive, the manager makes more par-building sales. This could reflect that in better performing deals, there is more good collateral to sell. But also, for badly performing deals, it may be that an improvement in performance can increase risk-shifting behavior. The manager is more likely to be rewarded for aggressive, par-building behavior when coverage levels are still within range of passing.

The effects can be analyzed better by breaking down the sample into deals whose OC tests are passing and failing. When passing, the OC par cushion is above zero by definition. The coefficient in column (3) on the first term becomes strongly and significantly negative. The combined numeric effect of the two coefficients depends on the cushion. When the cushion is small, the marginal impact is generally determined by the negative term, so that activity declines in a growing cushion. However when the deal is performing very well and the cushion is sufficiently large, the marginal impact can become positive due to the positive coefficient on the squared term. I speculate that in the tight range, the majority of deals’ managers react in a sit-tight manner. If deal performance is tolerable, they are less active, but as
tests become endangered (cushion shrinks) they take steps to boost par, hence the negative correlation. On the other hand deals with extremely large cushions are generally the deals with superb performance and hence have more credit-improved securities to sell, producing a positive combined effect. This interpretation is borne out in columns (5) and (6) in which I break down by the magnitude of the par cushion.

An analogous effect can be discerned when OC tests are failing, as in column (4). There, both coefficients are positive. Because the par cushion is negative, an increase decreases the squared term, and makes the marginal impact of the squared term negative. However the coefficient is small and the combined marginal impact of a growing cushion is generally positive. As the cushion gets nearer to passing the pace of activity quickens. This suggests that managers may give up on worse deals and are more motivated to build par when they have
a chance to restore test passage. This analysis is borne out in a detailed look, columns (7) and (8) in which I break down by the level of par cushion.

Figure 3-1 illustrates the marginal impact of growing OC par cushions, using the coefficients from columns (3) and (4). Note that the graphed observations are in absolute terms (not deviations from means) and the fitted line has been shifted to reflect this. The marginal impact is largest (and positive) at the extremes of the sample. The estimate for extremely poor performance (cushions < −20%) probably reflects a very low collateral denominator, accentuating percentage trading statistics. The upswing in par-building sales for good performance (cushions > 2.5%) probably reflects better collateral from which to choose.20 In the inner region, par-building sales activity spikes dramatically around OC test passage points, the strike levels for subordinate interest distributions. Again, it is difficult to rule out a tactical explanation from this alone, but it seems unlikely that there would be such heavy activity in the moderately bad performance range (cushions < 0 but > −10%) were managers simply worried about the bad appearance of failing tests. The trend is highly suggestive of risk-shifting, in that deals with marginal but worsening performance begin to par-build more; while in badly performing deals, the more hope a manager has of eventually receiving funds, the more volatility he seeks to add.

The significant relationship between manager behavior and compensation is an important argument for the risk-shifting explanation. I use the ratio of senior to junior fees as an indicator of the level of the manager’s subordinated interest (and perhaps also as a proxy for his equity share). It is significantly negative, so that as a manager’s interests become less subordinated the amount of par-building sales decline. I also look at an interaction of the manager’s junior fees and equity share. As the equity share increases, the debt share decreases, and a manager is more likely to actually receive his subordinated fees on paydates. This factor, while just below significance in column (1), is negative and significant in other regressions (column (9)). Again, the less subordinated the manager’s interests are, the fewer sales he makes above par. This is a key sign of risk-shifting since it suggests that it is the manager’s subordinated cashflows - junior and incentive fees, equity shares - which are influencing trading.

20 Although ratings levels are used in regressions to control for collateral quality, issuers improving in credit quality tend to wait longer for upgrades, especially during volatile time periods.
Table 3.10: Relationship between Par-building Sales Volume and Deal Structure and Lagged Performance for Various Coverage Scenarios. Trades are restricted to the reinvest period. Monthly trade volumes are measured by face value, as percentages of the prior month’s portfolio par. T-stats, adjusted for heteroskedasticity, are in brackets. The regression is presented for ten scenarios: (1) no manager switch (2) worst quartile of deals, by final MDS score (3) OC tests passing (4) OC tests failing (5) OC tests passing but par cushion tight (≤ 1%) (6) OC tests passing and par cushion loose (> 1%) (7) OC tests passing but par cushion tight (≥ -1%) (8) OC tests failing and par cushion loose (< -1%) (9) IC tests passing and (10) IC tests failing. For (1)-(8), month, banker, and manager fixed effects are included in regressions but not reported. For (9)-(10), month and manager fixed effects are included in regressions but not reported.

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</table>
Another fee variable I consider in Table 3.10 is the level of incentive fee hurdles. The larger this variable, the higher the return equityholders must receive before the manager receives his bonus incentive fees. As expected, the coefficient is strongly positive. The higher the hurdle rate, it seems the more the manager is inclined to boost par and take chances. In a subsection of badly performing deals, column (2), the coefficient turns negative. I take this as more evidence for the theory that managers give up on badly performing deals. As it becomes less likely that managers receive any bonus, discretionary trading falls.

The effect of the IC test is an interesting proxy for whether a manager’s interest are completely out-of-the-money or not. It is rare to see an IC test failing without its complementary OC test failing, but not vice-versa. When an IC test is failing, indicating that there is not even enough collateral interest to cover liability interest, usually the deal is hopelessly underwater and subordinate investors are holding worthless claims. Across the board, either with an IC failing dummy or by subdividing the sample in columns (9) and (10), par-building sales generally lessen in IC failures. The coefficient in (1) on the IC fail dummy is negative and significant. In column (9), in which passing IC tests are sampled, the constant amount of high-dollar sales is significantly larger than in column (10), where IC tests fail. As well the coefficients on OC cushions are larger and more positive. This suggests the manager is more active when interest is still flowing through the deal. If excess interest is available, the manager may still receive subordinated fees or equity distributions in the future.

One observes that for worse performing deals, the marginal impact of a larger equity share is positive and larger than that for well-performing deals, which is ambiguously signed. This may be due to a larger equity share keeping a manager’s subordinated interests more in-the-money (10). When a deal is performing well, a larger equity share will absorb more bad performance and can lessen risk-shifting as in columns (5) and (9). On the other hand, when coverage tests are close but failing (6) a larger equity share is very conducive to increasing risk-shifting volume. When the cushion is largely negative (7) equity share is smaller and insignificant. This is another example of the non-linear compensation relationship. In deals with small equity shares, bad performance will quickly wipe out subordinate investors, turn off excess interest distributions permanently through severe coverage test failures, and in general reduce managers’ upside completely. When upside is completely gone, there is little point in
risk-shifting activity, especially if trading imposes some cost on managers or there is a limit to the amount of volatility managers can add.

**Par-building Purchases**

The results on par-building sales hold for sales par gains and prices as well, but with sales statistics it is somewhat laborious to disentangle structural and incentive motivations from general credit stress influences. Therefore I also look at par-building purchases which are not as linked with credit stress in the portfolio. That is, a manager does not have to reinvest in dubious assets at below-par prices just because the default regime is poor. One comment rating agency analysts made is that they had expected managers to take refuge in high-quality assets and let the deals delever naturally, to the benefit of senior debt investors. Any relationship between incentives and purchase decisions would strongly suggest that managers were exploiting structures. Table 3.11 presents a regression of par purchase gains on various deal parameters and coverage test scenarios. Both the closing date and age of a deal are strongly and significantly negative, which is explained by the general credit environment. During the wave of corporate defaults, credit spreads widened dramatically which would have resulted in cheaper prices and earlier deals having more opportunities for par gains. Further, in 2003 and 2004 credit spreads tightened dramatically, which would correspond with younger deals (those from 2002-2003) and older deals (1996 deals in 2002 versus 1996 deals in 1996) having less ability to buy cheaply. Also, prior principal cash increases the amount of possible volume one could buy and is a positive factor in all cases, significant in columns (2) and (4).

As before, the manager’s fees and their subordination level play a significant role. The more subordinated the compensation that the manager receives, the more he is inclined to par-building purchases. The senior fee level decreases par purchase gains. In contrast, the junior fee level, subordinate to debtholders’ interest payments, has a positive effect on purchase gains. In fact, the junior fee coefficient becomes even larger for subsets of worse performing

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21 Source: Moody’s Investors Service (2002b). In a 2002 case study of a typical 1998 high-yield bond CDO, Moody’s finds that had the CDO held market collateral, none of the 9 senior debt downgrades and only 14 of the 18 junior debt downgrades would have occurred. Further, had the manager pursued a more conservative trading strategy after the onset of stress, CDO investors could have avoided an additional 8 junior debt downgrades (realizing just 6 of 18).
Table 3.11: Relationship between Purchase Par Gains and Deal Structure and Lagged Performance for Various Coverage Scenarios. Trades are restricted to the reinvest period. Monthly par gains are measured as percentages of the prior month’s portfolio par. T-stats, adjusted for heteroskedasticity and intra-deal error correlation, are in brackets. The regression is presented for four scenarios: (1) All (2) worst quartile of deals, by final MDS score (3) IC tests passing and (4) IC tests failing. Month and deal fixed effects are included in regressions but not reported.

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Dependent Variable: $T+1$ Par Gains Purchases
Consistent with the previous analysis of sold volume, the ratio of senior to junior fee level has a negative and significant impact. I take this as a proxy for the manager's level of subordinated interest and possibly his equity share. Also as before, I run an interaction of the original equity share and junior fee level. The larger the equity share, the less subordinated the manager's junior fee is, and consistent with this logic there is again a significantly negative coefficient on par-building trades.

The other interesting effect which is captured in this regression is the effect of manager size. While not uniformly significant, the coefficient is negative and strongly significant in the subsets of worse performing deals (columns (2) and (4)). This suggests that when deals go bad, larger managers pursue less aggressive or risk-shifting behavior. It is also an argument for the risk-shifting explanation of manager behavior. If managers are "managing to tests" in a preference for stability, one would expect larger managers, i.e. those with more assets at risk and perhaps a more sedate corporate culture, to have an even more pronounced preference. That larger managers eschew the riskier trades suggests it is in fact an appetite for risk driving such trades; that the par-building activity peaks around test passage points points to risk-shifting motivations.

3.4.3 Non-discretionary Volume

A last argument lies in a brief look at non-discretionary sold volume (redemptions, exchanges, and defaults). Collateral characteristics, with the exceptions of ratings levels and deal age indicators, were insignificant factors in discretionary volume regressions. In contrast, non-
Table 3.12: Relationship between Non-Discretionary Sales Volume and Deal Structure and Lagged Performance. Transactions are restricted to the reinvest period. Monthly trade volumes are measured by face value, as percentages of the prior month’s portfolio par. T-stats are in brackets, adjusted for both heteroskedasticity and intra-manager error correlation in (1) and (2) and for heteroskedasticity alone in (3) and (4). The regression is presented for two definitions of non-discretionary sales: (1) and (3) redemptions and exchanged volume and (2) and (4) redemptions, exchanged, and defaulted volume. Month and manager fixed effects are included in regressions but not reported.

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<td>Diversity Score</td>
<td>-0.0206</td>
<td>-0.0216</td>
<td>-0.0206</td>
<td>-0.0216</td>
</tr>
<tr>
<td></td>
<td>[2.46]*</td>
<td>[2.21]*</td>
<td>[3.25]**</td>
<td>[3.01]**</td>
</tr>
<tr>
<td>Incentive Fee x Eq. %</td>
<td>0.0015</td>
<td>0.0016</td>
<td>0.0015</td>
<td>0.0016</td>
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<tr>
<td></td>
<td>[0.40]</td>
<td>[0.26]</td>
<td>[0.35]</td>
<td>[0.27]</td>
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<tr>
<td>Incentive Hurdles</td>
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<td>0.4419</td>
<td>0.5021</td>
<td>0.4418</td>
</tr>
<tr>
<td></td>
<td>[3.04]**</td>
<td>[1.40]</td>
<td>[2.82]**</td>
<td>[1.46]</td>
</tr>
<tr>
<td></td>
<td>[3.89]**</td>
<td>[3.03]**</td>
<td>[4.16]**</td>
<td>[3.47]**</td>
</tr>
<tr>
<td>Observations</td>
<td>1949</td>
<td>1948</td>
<td>1949</td>
<td>1948</td>
</tr>
<tr>
<td>Managers</td>
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<td>89</td>
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<td>89</td>
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<tr>
<td>Adjusted R²</td>
<td>0.2992</td>
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discretionary sold volume is more dependent on deal performance and less dependent on deal structure and fee characteristics. Table 3.12 illustrates this. The best fit is heavily dependent on collateral characteristics. Again, closing date and deal age are highly significant. In this case, both coefficients are positive. Later vintage deals may have had more loan exposure and therefore been more prone to calls. Likewise, older deals would experience more calls as bonds age. Along the same lines, coupon rates were positive and highly significant. The higher the portfolio coupon rate, the more likely assets were to be redeemed during the general decline in interest rates and credit spreads.

Not surprisingly, the level of defaults was positive and highly significant. Interestingly, diversity score had a beneficial effect on exchanges and defaults, suggesting that less concentrated portfolios (higher Diversity scores) tended to have lower turnover. Rating factor was largely insignificant, in various forms. This makes sense since a declining rating is not likely to be forcefully redeemed or exchanged until an asset is actually in some sort of default. Prior cash went from being generally significant for voluntary trading to insignificant.

Coverage test parameters and most fee parameters were for the most part either inconsistently signed or insignificant. Some appeared as if they went in directions previously analyzed, suggesting maybe such parameters led to riskier behavior and ultimately to more non-discretionary turnover. One factor that remained significant was the incentive fee hurdle rate. As before it had a positive effect, increasing turnover.

3.5 Conclusion

I feel this is a unique opportunity to study managerial risk-shifting in a setting in which there are clear measures of volatility and in which both manager actions and compensation structure are known. Prior studies of managerial behavior in more traditional firm settings have concluded that manager risk-shifting is extremely rare or that the costs are small. In the financial arena, in particular the CDO market, the situation is different because CDOs are so much more highly leveraged. This makes both the costs and frequency of risk-shifting behavior likely to be larger. In addition instead of undefined investments we have a clear idea of the manager’s choice set and can measure volatility using asset prices or ratings.
My goal was to discern any evidence that managers with more subordinated compensation added more volatility to the portfolios they managed. Volatility can be added by purchasing discount, lower-quality securities or selling premium, higher-quality securities. I find a significant tendency to group credit-improved with credit-risk trades, a strategy for avoiding par erosion in badly performing deals but one which increases default volatility. I also find that there is a strong and significant relationship between manager trading, compensation, and deal performance. As deal performance worsens, managers become more likely to make such par-building trades, a trend which intensifies if their subordinated fee levels or hurdle rates for incentive fees are larger. This is what one would expect from managers' owning calls on total portfolio value.

As one would expect given the optionality in the managers' subordinated contracts, the relationship is highly non-linear. As deal performance worsens to the point that subordinated investors are hopelessly underwater, manager activity declines. For badly performing deals, but ones in which managers may still get paid, activity is highest. For marginal or well-performing deals, managers are less inclined to par-building. It appears that managers actively seek to add volatility the nearer they get to strike values for their various options, as proxied by over-collateralization levels.

I find a negative correlation between manager size and risk-shifting activity, particularly in deals with worst performance. This suggests that managers with greater reputational concerns were less likely to add volatility in the hopes of achieving subordinated fees. Results on compensation structure and manager reputation suggest that it is not regime, overconfidence, or manager style leading to aggressive trading strategies, but rather rational risk-shifting motivations.

These findings are consistent with theoretical predictions of the principal-agent problem in a setting of delegated management. They are also consistent with allegations from numerous market participants to the effect that manager behavior worsened CDO performance by unnecessarily adding risk to portfolios. Rating agencies in particular have been frank in admissions that they did not properly account for reinvestment risk. One important implication of this paper is that the attention being given to revision of CDO structures in order to prevent future manager misbehavior is well-spent. It also implies that for investors manager reputation is an
important asset to consider.

Areas of future research include a more detailed investigation of trading activity, including ratings and return performance. In this manner one could judge the long-term performance of apparently risk-shifting trades. Also, it would be useful to focus on the behavior patterns of managers who continued to issue CDOs.
Bibliography


Landier, A., and D. Thesmar (2003):.


