Energy Absorption of Reticulated Foams Filled with Shear-Thickening Silica Suspensions

By

Giorgia Bettin

BA, Mechanical Engineering (2002)

University of California, Berkeley

Submitted to the Department of Mechanical Engineering in Partial Fulfillment of the Requirements for the Degree of Master of Science in Mechanical Engineering at the Massachusetts Institute of Technology

June 2005

© 2005 Massachusetts Institute of Technology
All rights reserved

Signature of Author...............................

Department of Mechanical Engineering
May 9th, 2005

Certified by.................................

Gareth H. McKinley
Professor of Mechanical Engineering

Accepted by.................................

Anand
Chairman, Department Committee on Graduate Students

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

NOV 7 2005

BARKER

LIBRARIES
Energy Absorption of Reticulated Foams Filled with Shear-Thickening Silica Suspensions

by

Giorgia Bettin

Submitted to the Department of Mechanical Engineering
On May 9th, 2005 in partial fulfillment of the requirements for the Degree of Master of Science in Mechanical Engineering

ABSTRACT

The need for smarter and adaptive, energy absorption materials especially for human protection applications has fueled the interest in new and alternative energy absorbing composites. In this thesis a ‘novel’ energy absorbing fluid-composite that utilized a shear thickening fluid is developed. Shear-thickening fluids are a class of field responsive fluids that have the ability to transition from low viscosity to high viscosity under an imposed deformation field.

Two different types of silica particles are used to create shear thickening fluids. The first are polydisperse and non spherical, with a median diameter of $1.7 \pm 1.4 \, \mu m$, while the second are monodisperse spherical particles of $0.3 \pm 0.03 \, \mu m$ diameter. The particles are dispersed in ethylene glycol at volume fractions of up to $\phi=47\%$ for the polydisperse sample and up to $\phi=60\%$ for the monodisperse spheres. The behavior of the silica suspensions is studied under steady shear, small and large amplitude oscillatory shear flow and also in transient extensional flow. The viscosity of the polydisperse suspension is found to be much greater than the monodisperse one due to the difference in particles shape. Oscillatory experiments indicate that both the onset and magnitude of the shear thickening depends on the frequency and strain applied and show that rapid time-varying deformations result in maximum energy dissipation. Two different regimes are found through extensional flow measurement: at low extension rates the suspensions respond as a viscous rate-thinning fluid, whereas beyond a critical extension rate, the suspension strain-hardens and ultimately fractures in a solid-like fashion.

Polyurethane open cell or ‘reticulated’ foam with relative density of 0.03 and average cell size of $360 \, \mu m$ is chosen to envelop the concentrated silica suspensions. The behavior of this nonlinear fluid-solid composite is studied over a range of filling fractions under quasi-static deformation rates (strain rates between $10^{-2} - 1\, s^{-1}$), under dynamic impact loading (with energy densities of $e = 10^5 - 10^6 \, J/m^3$) and under high strain-rate deformations (strain rates up to $800 \, s^{-1}$). Results show that, if the foam is filled with a
shear thickening suspension, the composite stiffens even at strain rates of $10^{-2} \text{ s}^{-1}$ as the impregnated fluid shear-thickens due to the high local strain rates that develop on cellular length scales. High impact load experiments show two different mechanisms for energy absorption: at lower impact energies viscous dissipation is dominant; whereas, after a critical impact energy is reached, the fluid undergoes a transition from liquid-like to solid-like. High-speed digital video-imaging shows that cracks form and propagate through the sample and the impact energy is absorbed by viscoplastic deformation. The addition of these shear-thickening fluids in polyurethane foam is shown to increase the composite energy absorption capability by 35-fold.

Thesis Supervisor: Gareth H. McKinley
Title: Professor of Mechanical Engineering
Acknowledgments

I would like to thank my advisor, Gareth McKinley for his support and constant encouragement. A special thanks also goes to my group mates and at the Non-Newtonian Fluids lab for the endless discussions and support throughout the past few years. Finally, I would like to extend my thanks to the Institute for soldier Nanotechnologies (ISN) for providing financial support.
Table of Content

ABSTRACT 2
ACKNOWLEDGMENTS 4

CHAPTER 1 7
1. INTRODUCTION 7
   1.1. MOTIVATION 7
   1.2. ENERGIES AND RATES - METHODS OF DEPLOYMENT 8
   1.3. NEEDS OF SOLDIER- ISN 9

CHAPTER 2 10
2. SHEAR THICKENING FLUIDS: INTRODUCTION & REVIEW 10
   2.1. INTRODUCTION 10
   2.2. MECHANISM OF SHEAR THICKENING 11
      2.2.1. Order-Disorder Theory 11
      2.2.2. Hydrocluster Theory 12
      2.2.3. Scaling Models 13
   2.3. DESCRIPTION OF THE PHENOMENA 15
      2.3.1. Volume Fraction Dependence 16
      2.3.2. Particle Size Dependence 19
      2.3.3. Particle Size Distribution Dependence 22
      2.3.4. Particle Shape Dependence 23
      2.3.5. Particle-Particle Interaction Dependence 24
      2.3.6. Viscosity of Carrier Fluid Dependence 26
   2.4. RHEOLOGY OF SHEAR THICKENING FLUIDS 26
      2.4.1. Steady shear rheology 26
      2.4.2. Dynamic Shear rheology 26
      2.4.3. Extensional rheology 29
   2.5. APPLICATIONS 30
      2.5.1. Hip Padding 30
      2.5.2. Ballistic Protection 32

CHAPTER 3 33
3. CELLULAR SOLIDS: REVIEW 33
   3.1. STRUCTURE AND PROPERTIES OF CELLULAR SOLIDS 33
      3.1.1. Cell Structure 33
      3.1.2. Properties of Cellular Solids 35
   3.2. MECHANICS OF FOAM 36
      3.2.1. Mechanics of Foam Compression 36
      3.2.2. Fluid-Filled Open Cell Foam 39
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.2.3.</td>
<td>Poroelastic Model</td>
<td>41</td>
</tr>
<tr>
<td>3.2.4.</td>
<td>Energy Absorption of Foams</td>
<td>43</td>
</tr>
<tr>
<td>3.2.5.</td>
<td>Optimization Criteria</td>
<td>45</td>
</tr>
<tr>
<td>3.3.</td>
<td>Testing Apparatus for Foams Material Properties</td>
<td>47</td>
</tr>
<tr>
<td>3.3.1.</td>
<td>Texture Analyzer</td>
<td>47</td>
</tr>
<tr>
<td>3.3.2.</td>
<td>Dynatup Drop Tower</td>
<td>48</td>
</tr>
<tr>
<td>3.3.3.</td>
<td>Split Hopkinson Pressure Bars</td>
<td>49</td>
</tr>
</tbody>
</table>

**CHAPTER 4**

4. Shear-Thickening Fluids: Rheology and Characterization  

4.1. Materials and Characterization  
4.1.1. Particles Characterization  
4.1.2. Preparation Techniques  
4.2. Shear Rheology  
4.2.1. Steady Shear Rheology  
4.2.2. Dynamic Shear Rheology  
4.3. Extensional Flow Response  
4.4. Slip

**CHAPTER 5**

5. Energy Absorption System: Characterization  

5.1. Foam Specification and Sample Preparation  
5.2. Characterization of Empty Foam  
5.3. Low Strain Rates: Characterization of Glycerol Filled Foam  
5.3.1. Rate dependence  
5.3.2. Volume fraction dependence  
5.4. Low Strain Rates: Characterization of STF Filled Foam  
5.4.1. Rate dependence  
5.4.2. Volume fraction dependence  
5.5. High Impact Energy Testing: Dynatup  
5.6. High Rates Testing: Split Hopkinson Pressure Bars

**CHAPTER 6**

6. Conclusion
Chapter 1

1. Introduction

1.1. Motivation

Shear thickening fluids (STF) have sparked much interest and are currently widely studied because of their unique ability to transition from a low viscosity to a high viscosity fluid, almost in a solid-like state. Their particular behavior makes these fluids ideal for energy absorption applications; while they are in low viscosity state under normal conditions, and therefore allow for greater user mobility and flexibility, they undergo a transition when impacted (under large stresses) which absorbs some of the energy of the impact and help shunt the remaining energy. Additionally, the interesting feature about these fluids, is that the onset of this transition can be picked and tuned a priori for the specific application by choosing the appropriate shear thickening fluid components, for example particles types and size or solvent.

One of the major concerns when trying to create an effective energy-absorbing composite with these fluids is the interaction between the fluid and the geometry which holds it in place, the envelopment geometry. The rheological response of shear thickening fluids is greatly affected by the local geometry and therefore the local deformation rates at which it is strained. For this reason, open cell elastomeric foam is the ideal packaging for these energy absorption systems; reticulated foams, in fact, can be manufactured with different size pores, which are responsible for the amount of local strain the fluid experiences.

Additionally, foam is flexible and can conform to various surfaces, as noted earlier that
helps with flexibility, but at the same time keeps the fluid contained while allowing fluid flow in between pores.

In this work we studied the rheological behavior of shear thickening fluids and how these fluids interact with a complex envelopment such as reticulated open cell foam.

**1.2. Energies and Rates - Methods of Deployment**

The behavior of foam samples filled with shear thickening fluid depends on stress and rate of loading; that is the composite is compliant at low stresses or slow deformation rates, but stiffens at higher rates. As mentioned earlier the critical stress necessary to ‘activate’ the fluid can be varied and one can adapt the fluid-foam system to react and therefore protect against a number of injuries that occur at different timescales. Table 1.1. illustrates the most common types of injuries and their timescales.

<table>
<thead>
<tr>
<th></th>
<th>Immobilization and support</th>
<th>Blows and Falls</th>
<th>Bullets and Blasts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency Range</td>
<td>0 -10 Hz</td>
<td>10 -1000 Hz</td>
<td>1-100 kHz</td>
</tr>
<tr>
<td>Pressure Sore</td>
<td>Fracture</td>
<td></td>
<td>Laceration</td>
</tr>
<tr>
<td>Sprains, Strains</td>
<td>Vibration syndrome</td>
<td></td>
<td>Internal organ damage</td>
</tr>
</tbody>
</table>

Table 1.1 Characteristic timescales of commonly encountered injuries.

Shear thickening fluids are also of particular interest because their ‘method of deployment’. In fact shear thickening fluids do not required any external activation, as from the user, but they will activate (transition from low to high viscosity) only when subjected to a critical amount of stress. This is not the case, for example, of electrorheological and magnetorheological fluids, which have also been studied for energy absorption applications, but instead require an imposed electric or magnetic field.

It is the focus of this investigation to measure the amount of energy that STF based systems are able to absorb, their limitations, and their optimum operational timescales.
The response times is also a key element for these systems performance and have been studied.

1.3. Needs of Soldier - ISN
The Institute for Soldier Nanotechnologies has the goal to improve soldier survivability in the battlefield by designing a suit which integrates ballistic, chemical and biological protection. In particular, the scope of this project is to develop a switchable ‘smart fluid’ base system for energy shunting and load transfer. For this purpose we focus our effort to study and develop a shear thickening fluid/reticulated foam system.

When designing an efficient system for soldier protection applications one needs to address a number of issues that are not necessarily related to the level of protection, as in this case was weight, power requirement, integration of the system with current suit design, and of course comfort and flexibility of the final product.

Using STF-foam systems offers a number of advantages from the soldier needs prospective as it doesn’t require ‘user activation,’ as explained earlier, or any power for operation, it is also compliant under normal operation and therefore allows for greater mobility and flexibility as compared to standard body armor. Weight minimization is also a very important aspect of the design in which shear thickening fluids can offer an advantage over other types of energy absorbing material. In fact, if necessary, the density of the material could be decreased by making STF from hollow spheres. Additionally because these systems do not require power for operation they reduce the need for battery and external power supplies, which is instead required for other ‘smart fluid’ based systems as for magnetorheological and electrorheological fluids systems.
Chapter 2

2. Shear Thickening Fluids: Introduction & Review

2.1. Introduction

Shear thickening fluids are part of a class of fluids commonly referred as “field-activated;” the mean of activation being imposed stress. This phenomenon is in fact described as a steplike increase in viscosity as the applied stress reaches a critical value; the fluids being low viscosity at imposed stresses lower then the critical value, and high viscosity when stresses exceed the critical value (see Figure 2.1). Shear thickening fluids are composed of highly concentrated suspensions disperse in a carrier fluid. The thickening behavior of these suspensions is normally seen at high volume fraction so that the particles are dense enough that their separation is less then the particles diameter and therefore multiple-body and lubrication interaction between particles are of importance (Stickel and Powell 2005).

![Figure 2.1 Viscosity of 400 nm Latex particles suspension in diethylene glycol at volume fraction of 50.4 % vs. shear rate as measured in a modified Weissenberg rheogoniometer and Sandwich Rheometer. (Laun, Bung et al. 1991) For this case the critical shear rate \( \dot{\gamma}_{cr} = 10 \text{ s}^{-1} \).]
The shear thickening behavior of these fluids have been studied for well over a half century. Some comprehensive reviews of the field can be found in Barnes (Barnes 1989), Hoffman (Hoffman 1982; Hoffman 1998) and Laun (Laun, Bung et al. 1991) and more recently Stickel and Powell (Stickel and Powell 2005).

2.2. Mechanism of Shear Thickening

Only relatively recently, with the improvement of stress controlled rheometer and with the help of different techniques, neutron scattering (Laun, Bung et al. 1992; Bender and Wagner 1996; Maranzano and Wagner 2001; Maranzano and Wagner 2002), rheo-optical devices (Bossis and Brady 1989; D’Haene, Mewis et al. 1993), it was possible to rigorously study the mechanism of this phenomena and its stress-induced structure. Additionally, computer simulations have recently contribute to the understanding of the mechanism of shear thickening. Stokesian Dynamics techniques (Bossis and Brady 1989; Boesma, Laven et al. 1995; Catherall, Melrose et al. 2000; Foss and Brady 2000) are commonly used to simulate the behavior of every particles in the suspension, but other methods, like dissipative particles dynamics (Boek, Coveney et al. 1997), and Lagrange multiplier fictitious domain method (Singh, Hesla et al. 2003) are also used. Two different mechanism have emerged from literature to explain these fluids behavior: the Order-Disorder theory and the ‘Hydrocluster’ theory.

2.2.1. Order-Disorder Theory

The order-disorder mechanism was first suggested by Hoffman (Hoffman 1972) which observed that monodisperse suspension under shear generate diffraction patterns. An example of these patterns can be seen in figure 2.2. Hoffman (Hoffman 1972) developed...
a model to characterize this diffraction phenomena and showed that figure 2.2 (A) corresponds to spheres layered in hexagonal packing while figure 2.2 (B) suggests randomly positioned spheres.

According to this theory, as the suspension is sheared, particles align in hexagonally packed layers parallel to the plane of shear. After a critical stress is reached, flow instabilities start to grow and they induce particles out of their ordered layers; these particles then collide and jam into each other and produce the rise in viscosity.

Figure 2.2. Diffraction patterns from monodisperse plastisols sheared in simple shearing flow. (A) Sample shear below critical shear rate. (B) Sample shear above critical shear rate (Hoffman 1972).

2.2.2. Hydrocluster Theory

Another possible explanation for the shear thickening behavior of concentrated suspension is found in the ‘Hydrocluster’ theory. This theory is based on a model that accounts for the force balance between the hydrodynamic forces imposed by a shearing flow and forces arising by interparticles interactions.

According to this theory at low stresses (below the critical value) interparticles interactions, either electrostatic or Brownian, allow the particles to easily flow past each other; this result in a ‘low viscosity’ state. As the stress is increased, lubrication forces between the particles grow until, when the lubrication and interparticles forces are of the
same order of magnitude, the particles in suspension ‘jam’ into a stress-bearing cluster, commonly referred as a ‘hydrocluster’; this is what creates the increase in the suspension viscosity. This theory was first suggested by Brady (Bossis and Brady 1989) as a result from Stokesian Dynamics simulations and consequently supported by rheo-optical experiment by Wagner (Bender and Wagner 1995) and D’Haene (D’Haene, Mewis et al. 1993).

2.2.3. Scaling Models

Scaling models are extremely useful for understanding and to be able to predict the shear thickening phenomena. Bender and Wagner (Bender and Wagner 1996) proposed a scaling based on the ratio between Brownian and hydrodynamic forces. From Bachelor (Batchelor 1976) we obtain an expression for the Brownian force as:

\[ F_{\text{Brown}} = -k_B T \frac{\partial \ln g(r)}{\partial r} \]  

(2.1)

where \( k_B \) is Boltzmann’s constant, \( T \) absolute temperature, and \( g(r) \) is the equilibrium pair distribution function. We can also express the hydrodynamic force as:

\[ F_{\text{hydro}} = 3 \pi \mu a^3 \dot{\gamma} / 2h \]  

(2.2)

where \( \mu \) is the solvent viscosity, \( a \) is the particles size, \( \dot{\gamma} \) is the shear rate, and \( h \) is the characteristic distance between particles. We therefore define a dimensionless critical stress as the ratio between hydrodynamic and Brownian forces as follows:

\[ \tau^{\text{crit}}_{\text{Br}} = \frac{F_{\text{hydro}}}{F_{\text{Brown}}} = \frac{3 \pi \mu a^3 \dot{\gamma} / 2h}{-k_B T \frac{\partial \ln g(r)}{\partial r}} \bigg|_{r=2a+h} \]  

(2.3)
The hydrodynamic force can be also adjusted to account for the effect of the ‘surrounding particles’. This mean field correction allows us to arrive to a similar expression for the scaling model as presented by Bender and Wagner (Bender and Wagner 1996).

\[
\frac{\tau_{\text{crit}} a^3}{k_B T} = -h \eta \left( \frac{\partial \ln g(r)}{\partial r} \right)
\]

where \( \tau_{\text{crit}} \) is the dimensional critical stress for the onset of shear thickening. Bender and Wagner used this scaling to collapse all their data for bimodal dispersions as illustrated in figure 2.3.

Figure 2.3. (a) Viscosity vs. stress plotted for two monodisperse suspensions and the bimodal mixtures at \( \phi = 0.64 \). (b) Reduced viscosity, \( \eta/\mu \), vs. reduced stress \( \sigma_{\text{crit}} a^3/k_B T \). The labels indicate the mixing of small (160 nm) to large (330 nm) particles by volume percent.

Similar scaling can be obtained for electrostatically stabilized particles where the electrostatic forces are dominant over Browning forces. By using Derjaguin
approximation (in the limit of very dense suspensions) of the Poisson-Bolzmann equation (Hiemenz and Rajagopalan 1997), we can obtain a relation for the electrostatic forces:

\[ F_{\text{electro}}(h \to 0) = 2\pi\varepsilon_0\varepsilon_r\psi_S^2\kappa a/2 \]  

(2.5)

where \( \varepsilon_0 \) is the permittivity in vacuum, \( \varepsilon_r \) is the relative permittivity, \( \psi_S \) is the potential at the surface, and \( 1/\kappa \) is the Debye length. The scaling between the two forces then becomes:

\[ \tau_{\text{elec}}^{\text{cr}} = P_{\text{elec}}^{\text{cr}} = \frac{F_{\text{hydro}}}{F_{\text{elec}}(h \to 0)} = \frac{3\pi\mu a^3 \dot{\gamma}/2h}{2\pi\varepsilon_0\varepsilon_r\psi_S^2\kappa a/2} \]  

(2.6)

The important relation to note is that the dimensionless stress or Peclet number have different dependence on particles size based on the type on interparticles interaction; in fact, for Brownian particles \( \tau_{\text{crit}} \sim 1/a^3 \), but for electrostatically stabilized particles as \( \tau_{\text{crit}} \sim 1/a^2 \). The interesting point emerging from this analysis is that, according to this scaling, one can then control the onset of the thickening effect, \( \tau_{\text{crit}} \), by varying the particles size, the smaller the particles, the greater \( \tau_{\text{crit}} \).

2.3. Description of the Phenomena

The complexity in modeling the rheological behavior of shear-thickening fluids arises form the great number of variables that affect the magnitude of the shear-thickening effect, the onset of it (the critical stress), the yield-stress etc. Many studies have successfully correlated the effect of some of the parameters on the fluid rheology and the reviews from Laun (Laun, Bung et al. 1991), Barnes (Barnes 1989) and Powell (Stickel and Powell 2005), summarized some of these findings. The rheology of a shear thickening fluids is a function of the particles size, shape and particle size distribution,
the particles density, charge, roughness, and in some cases the chemical interaction
between the particle surface and the carrier fluid. Additionally, it is also a function of the
number of particles present in the fluid (the volume fraction), the fluid viscosity and
density, and Brownian energy. In the following sections we will try, when possible, to
describe each variable dependence separately.

2.3.1. Volume Fraction Dependence

The rheology of shear thickening fluids depends significantly on the volume fraction ($\phi$) of the suspension. In order to observe shear-thickening effects the volume fraction needs to be high enough that the particles ‘feel’ each other’s presence, that is, the particles force field and behavior is affected by the presence of surrounding particles. These interparticles forces depend on the particles charge as well as the polarity of the fluid. It is therefore hard to define a critical volume fraction at which shear thickening is observed to occur, but in within the shear rates that are achievable with standard rheometry we should expect the appearance of shear thickening on most suspension with a volume fraction greater then 40%. As the volume fraction increases we note three different trends in the rheology of the suspension (Figure 2.4)

1. An increase of the overall viscosity as volume fraction is increased.

2. A shift of the critical stress or $\dot{\gamma}_{crit}$ at which shear thickening occurs; as $\phi$
   increases the critical stress decreases.

3. An increase of the magnitude of the viscosity jump as the volume fraction
   increases.
The increase in viscosity with volume fraction is no surprise. In fact in dilute suspension this dependence has been known and studied extensively. As first introduced by Einstein and then extended by Batchelor (Batchelor 1976) the viscosity relation for hard spheres suspensions follows the relation:

$$\eta = \eta_0 \left(1 + 2.5\phi + 6.2\phi^2 + O(\phi^3) + \ldots\right)$$

(2.7)

where the first order term in $\phi$ represent the contribution from viscous dissipation of single spheres in fluids, the second order term is the contribution from two-particles hydrodynamic interactions, the third order term from three-body hydrodynamic interaction and so on.

At high volume fractions the relation because more complex and even small changes in volume fraction result in large variation of viscosities. Additionally, at very high $\phi$ the
viscosity became extremely sensitive to particles roughness inhomogeneity, shape etc. (Larson 1999).

It has also been showed that the onset of shear-thickening decreases with $\phi$. This means that if the number of particles in suspension decreases, and therefore they are farther apart from each other, ones need to impose a greater stress to achieve the thickening of the suspension. This change in critical stress is not linear $\phi$; by comparing and analyzing the rheological data from different authors Barnes (Barnes 1989) has showed that the rate of change in the critical shear stress, $\tau_{\text{crit}}$, is very high at lower $\phi$ (below $\approx 45\%$) plateau around $50\%$, and again changes rapidly at $\phi$ greater then $50\%$. This is illustrated in figure 2.5.

![Figure 2.5 Schematic of the dependence of critical shear rate for the onset of shear thickening ($\dot{\gamma}_c$) as a function of the phase volume of the disperse phase ($\phi$)]
2.3.2. Particle Size Dependence

Many studies have focused on the particles size dependence of the suspension rheology and on the prediction of the critical stress. Barnes (Barnes 1989) and also Hoffman (Hoffman 1998) reported tables with a summary of some of these studies of the type and size of particles together with the value of critical stress. According to the scaling model presented in section 2.2.3, we would expect that for monodisperse spherical particles, the critical shear stress be proportional to $1/a^3$ for electrostatically stabilized particles and $1/a^2$ for Brownian suspension.

These scalings have been confirmed through experimental results from Wagner, at least for electrically stabilized particles, as illustrated in figure 2.6. Barnes (Barnes 1989) has also shown this relation by comparing the work of many authors in his review (see figure 2.7) and also found a relation of $1/a^2$. His data being more scattered because a collection of work of many authors with different 'controlled conditions', as particles charge and chemistry, solvent viscosity etc., which can affect the critical stress.
# Compilation of Particle Size vs. Critical Shear Rate Data

<table>
<thead>
<tr>
<th>Author(s)</th>
<th>System</th>
<th>Particle size (μm)</th>
<th>Approximate critical shear rate (cP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Albert (27)</td>
<td>Clay in water</td>
<td>0.5</td>
<td>~15</td>
</tr>
<tr>
<td>Aline &amp; Lepoutre (25)</td>
<td>Calcium carbonate in water</td>
<td>0.7</td>
<td>~300-1000</td>
</tr>
<tr>
<td>Beasley (23)</td>
<td>Clay in water</td>
<td>0.25</td>
<td>120</td>
</tr>
<tr>
<td>Berentz et al. (92)</td>
<td>PVC in water</td>
<td>1.0</td>
<td>10</td>
</tr>
<tr>
<td>Bullivant &amp; Jones (84)</td>
<td>Clay in water</td>
<td>0.25</td>
<td>125 → 200</td>
</tr>
<tr>
<td>Butters &amp; Roberts (8)</td>
<td>PVC in water</td>
<td>0.8</td>
<td>300</td>
</tr>
<tr>
<td>Cawthra et al. (31)</td>
<td>PVC</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>Chiu et al. (86)</td>
<td>Blood cells in plasma</td>
<td>0.1</td>
<td>1</td>
</tr>
<tr>
<td>Chai (87)</td>
<td>Polystyrene spheres in silicone oil</td>
<td>0.422</td>
<td>≤10 (44cP, 10)</td>
</tr>
<tr>
<td>Clarke (35)</td>
<td>Quartz in water</td>
<td>102-211</td>
<td>~10</td>
</tr>
<tr>
<td>Eastwood &amp; Barnes (53)</td>
<td>Starch particles in glycerol-water</td>
<td>10</td>
<td>1-20</td>
</tr>
<tr>
<td>Forquet (87)</td>
<td>PVC</td>
<td>0.3-4</td>
<td>10</td>
</tr>
<tr>
<td>Freundlich &amp; Röder (39)</td>
<td>Quartz powder in water</td>
<td>0.1-5</td>
<td>10</td>
</tr>
<tr>
<td>Gillespie (88)</td>
<td>PVC in DOP</td>
<td>1</td>
<td>50</td>
</tr>
<tr>
<td>Gunnerson &amp; Gallagher (32)</td>
<td>PVC in DOP</td>
<td>-1</td>
<td>1-20</td>
</tr>
<tr>
<td>Hoffman (54)</td>
<td>PVC in DOP</td>
<td>1.25</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>PSAN in ethylene glycol</td>
<td>0.445</td>
<td>200</td>
</tr>
<tr>
<td></td>
<td>PVC in dioctyl phthalate</td>
<td>1.25</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>PVC in 18-14000 DP</td>
<td>0.24</td>
<td>0.02-0.27</td>
</tr>
<tr>
<td></td>
<td>Starch/glycerol</td>
<td>15-60</td>
<td>52% 0.02; 56% 0.02</td>
</tr>
<tr>
<td></td>
<td>PVC</td>
<td>0.506</td>
<td>4 in glye, 20 in DOC</td>
</tr>
<tr>
<td>Kaltwasser et al. (18)</td>
<td>Glass beads in glycerol</td>
<td>45-65</td>
<td>~0.05</td>
</tr>
<tr>
<td></td>
<td>80-100</td>
<td>0.0025</td>
<td></td>
</tr>
<tr>
<td></td>
<td>PVC in DOP</td>
<td>0.8</td>
<td>30-50</td>
</tr>
<tr>
<td>Laser (10)</td>
<td>Typical plastisols</td>
<td>1</td>
<td>15</td>
</tr>
<tr>
<td>Moore &amp; Davies (89)</td>
<td>Alumina casting slip</td>
<td>few μm</td>
<td>~10</td>
</tr>
<tr>
<td>Morgan (90)</td>
<td>Red iron oxide pigment in water</td>
<td>0.2 ± 0.1</td>
<td>~300 by extrap.</td>
</tr>
<tr>
<td>Patzold (61)</td>
<td>Glass beads</td>
<td>50</td>
<td>~2 × 10⁹</td>
</tr>
<tr>
<td>Price &amp; Hagemeyer (26)</td>
<td>Carbons</td>
<td>1.1-1.2</td>
<td>~0.2</td>
</tr>
<tr>
<td>Schmidt-Schönbein et al. (55)</td>
<td>Blood cells in plasma</td>
<td>8</td>
<td>0.1</td>
</tr>
<tr>
<td>Severe &amp; Austin (51)</td>
<td>Vinyl chloride resin</td>
<td>~1</td>
<td>3-10</td>
</tr>
<tr>
<td>Shikatama et al. (91)</td>
<td>PVC in DOP</td>
<td>0.7</td>
<td>20</td>
</tr>
<tr>
<td>Strivens (58)</td>
<td>PMMA</td>
<td>1</td>
<td>~600</td>
</tr>
<tr>
<td>Taylor et al. (43)</td>
<td>Polystyrene latex in water</td>
<td>9.2</td>
<td>~10³</td>
</tr>
<tr>
<td>Todd (11)</td>
<td>Vinyl paste</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>Underdal et al. (28)</td>
<td>Resins in DOP</td>
<td>Mixtures</td>
<td>~</td>
</tr>
<tr>
<td>Van Wazer (45)</td>
<td>Shale &amp; limestone in water</td>
<td>~1</td>
<td>100</td>
</tr>
<tr>
<td>Verwey &amp; de Boer (46)</td>
<td>Iron particles in CCl₄</td>
<td>3</td>
<td>~1</td>
</tr>
<tr>
<td>Wagstaff &amp; Chaffey (29)</td>
<td>M H homopolymer or copolymer with ethyl acetate</td>
<td>0.027</td>
<td>2 × 10⁴</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0018</td>
<td>1.5 × 10⁴</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.111</td>
<td>10⁴</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.256</td>
<td>10⁴</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.016</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0715</td>
<td>200</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.118</td>
<td>10²</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.286</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.082</td>
<td>2 × 10³</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.251</td>
<td>800</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.07</td>
<td>8 × 10⁶</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0418</td>
<td>15 × 10⁵</td>
</tr>
<tr>
<td>Willey &amp; Macosko (56, 60)</td>
<td>PVC in DOP</td>
<td>0.619</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.2</td>
<td>9</td>
</tr>
<tr>
<td>Williams et al. (92)</td>
<td>PS in DOP, BBP, &amp; DOA</td>
<td>2.88</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.76</td>
<td>90</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.56</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.87</td>
<td>&lt;5</td>
</tr>
<tr>
<td>Williamsen &amp; Heckert (22)</td>
<td>Cornstarch in various solvents</td>
<td>10.3</td>
<td>~0.5</td>
</tr>
</tbody>
</table>

Table 2.1 Comparison of particle size vs. critical shear rate for a number of studies (Barnes 1989)
Figure 2.6. Critical shear stress plotted against particles radius for monodisperse silica particles. The line is a power law regression with slope of $-2.2\pm0.16$. (Maranzano and Wagner 2001)

Figure 2.7. Critical shear rate as a function of average particle size. (Barnes 1989)

This finding is of particular interest from a design and application point of view. In fact, this allows one to ‘tune’ the shear-thickening fluid, as to pick the appropriate critical stress, for the intended application by choosing the right particles size.
2.3.3. Particle Size Distribution Dependence

The particle size distribution of the particles in suspension affect the maximum value of volume fraction, $\phi_m$, possible; that is, if the particles are of different size they can pack more efficiently because smaller particles can fit in within voids between greater particles; which allows for greater volume fractions. If comparing two different suspensions with the same volume fraction, but one with monodisperse and the other polydisperse particles, the monodisperse suspension will show a stronger shear thickening effect at a lower critical stress. This again relates to the fact that polydisperse suspensions pack more efficiently and particles (at constant $\phi$) are therefore farther apart from each other. Consequently, the broader the particle size distribution, the higher the critical shear stress and less severe the shear thickening (Barnes 1989).

Particular attention has also been given to bimodal suspensions and their rheology. Some early work on the dependence of size ratio carried out Wagstaff & Chaffey (Wagstaff and Chaffey 1977) have shown that a size ratio of 4 to 1 or greater greatly reduced the shear thickening of the suspension, the ratio 4 to 1 being optimal for maximum packing.

Additionally, Bender and Wagner (Bender and Wagner 1996) showed that they were able to control, and increase the critical stress by increasing the ratio of small to large particles. The particles used in this study were silica spheres, which were electrostatically stabilized. Similar results, but for sterically stabilized particles were presented by D’Haene (D'Haene and Mewis 1994).
2.3.4. Particle Shape Dependence

The shape of particles in a suspension has a great impact on its rheology. In fact, non-spherical particles tend to align with the flow field and show much greater elastic effect than spherical particles suspensions (Larson 1999).

At relatively low $\phi$, suspensions of anisotropic particles exhibits shear thinning behavior at much lower volume fractions than for spherical particles suspensions (usually at $\phi > 30\%$).

![Figure 2.8. The effect of particles shape on shear thickening at volume fraction of 20% (Barnes 1989)](image)

For high volume fraction suspensions there is no complete, quantitative studies on the effect of particle shape on suspension rheology, but some qualitative studies have been conducted. As Barnes summarized in his review article, the rheology of suspensions of anisotropic particles shows a smaller critical stresses for the onset shear thickening and the thickening is also observable at lower volume fractions than for spherical particles.
suspensions. Figure 2.8 illustrates an example of the rheological behavior of suspensions made of different shapes particles at a constant volume fraction of 20%.

Wagner (Egres and Wagner 2005) also presented some work on acicular precipitated calcium carbonated and showed that increase in particles aspect ratio enhances the shear thickening effect that can be observed at much lower volume fraction than for spherical particles.

2.3.5. Particle-Particle Interaction Dependence

Interparticles interactions are extremely important in determining the shear thickening response of a suspension. Flocculated suspensions will not show any shear thickening (Barnes 1989), but instead they will exhibit shear thinning, as shown schematically in figure 2.9.

![Figure 2.9 Schematic representation of the effect of chemically induced flocculation on shear thickening. (Barnes 1989)](image)

To observe shear thickening, the particles in the suspension need to be neutral (or what is commonly referred as Brownian or hard spheres) or repel each other (electrostatically
charged particles). Hard sphere suspensions tend to show less severe shear thickening if compared to electrostatically charged particles. Figure 2.10 illustrates the different rheological responses of silica particles with and without a surface charge; the higher the surface charge the more severe the shear-thickening effect is.

![Figure 2.10. Comparison of viscosity data between 600 nm silica particles dispersion with 0.066 M HNO₃ and 0 M HNO₃ plotted against applied shear rate. (Maranzano and Wagner 2001)](image)

This effect is due principally to the fact the surface charge or grafted layer of polymers (for sterically stabilized particles) on the particles surface, does not allow particles to get too close to each other. This can be viewed as having particles with effective size \( a + \Delta \), where \( a \) is the particles radius and \( \Delta \) is the thickness of the layer. This varies the effective volume fraction which can then be expressed as:

\[
\phi = \phi_0 \left(1 + \frac{\Delta}{a}\right)^3
\]  

(2.8)

where \( \phi_0 \) is the volume fraction of the hard spheres. In principle by increasing the particles charge or polymer layer thickness the effective volume fraction increases and shear thickening effect increases at lower nominal volume fractions.
2.3.6. Viscosity of Carrier Fluid Dependence

The viscosity of a shear-thickening suspension is directly related to the viscosity of the carrier fluid. "All things being equal, doubling the continuous phase viscosity doubles the suspension viscosity" (Barnes 1989). This is also supported by the scaling model presented earlier, where, in both cases, electrostatically stabilized or Brownian particles, the Peclet number depends linearly on the fluid viscosity.

Some difficulty arises from getting exact experimental measurements of the effect of solvent viscosity due to the fact that changing solvents also affect interparticles interactions and it may be difficult to separate the two effects. Heating has been used to test the carrier fluid viscosity dependence on the shear-thickening effect, but particular attention needs to be given to the effect of particles swelling and the effect of the particles Browning energy increase (Barnes 1989).

2.4. Rheology of Shear Thickening Fluids

2.4.1. Steady shear rheology

As described earlier in this chapter the steady state rheology of a shear thickening suspension is characterized by a shear thinning region at low stresses followed by shear thickening (see figure 2.1); the details of the rheology influenced by the suspension volume fraction and particles characteristics. These results are representative of the equilibrium structure that forms under shear stress, but do not illustrate any of the suspension time dependence response.

2.4.2. Dynamic Shear rheology

Only relatively recently (Laun, Bung et al. 1991; Raghavan and Khan 1997; Lee, Wetzel et al. 2003) authors have began to study the dynamic response of these shear-thickening
fluids. Under dynamic loading these concentrated suspensions strain-thicken, that is, their complex viscosity increases at a critical combination of $\omega \gamma_0$. This viscoelastic response can be used to estimate the suspension relaxation time and also the minimum strain requirements to initiate shear thickening. Some controversy in the applicability of the modified Cox-Merz rule or Delaware-Rutgers rule for dense suspension is still ongoing. This rule relates the steady state viscosity to the dynamic fluid response as follows:

$$\left| \eta^* (\omega \gamma_0) \right| = \eta ( \dot{\gamma} )_{\gamma = \omega \gamma_0}$$

(2.9)

where $\eta^*$ is the complex viscosity, $\omega$ is the oscillation frequency, $\gamma_0$ is the oscillation maximum strain, $\eta$ the steady state viscosity, and $\dot{\gamma}$ the steady state shear rate.

Ragavan and Khan (Raghavan and Khan 1997) have shown that for their fumed silica suspension, the modified Cox-Merz rule applies (see figure 2.11), while Wagner (Lee, Wetzel et al. 2003) has argued that the dynamic and steady response should superimpose (follow the Delaware-Rutgers) only in the shear thinning regions and should not apply in the highly non-linear shear thickening regime; this agrees with their experimental data shown in figure 2.12.

It has also been suggested (Laun, Bung et al. 1991; Raghavan and Khan 1997) that there is a minimum strain of order 1 below which shear thickening does not occur (independently of the frequency of oscillation). This is consistent with the idea that one needs to move the particle at least past each other (a distance on the order of particle size) in order to see shear thickening. Some authors (Lee, Wetzel et al. 2003) don't agree and have suggested that the trend in experimental data can be explained by slip.
Figure 2.11. Comparison of steady and dynamic data for a 10% fumed silica suspension. Here, $\eta^\ast$ is plotted against the maximum dynamic shear rate $\omega\gamma_0$, where $\gamma_0$ is the appropriate strain amplitude in the frequency sweep. (Raghavan and Khan 1997)

Figure 2.12. Application of the Cox-Merz and Delaware-Rutgers rules for 60 vol. % aqueous silica dispersion (Lee, Wetzel et al. 2003)
2.4.3. Extensional rheology

The extensional behavior of concentrated suspensions is an aspect of their rheology that has not been explored. The transient extensional flow rheology is a very active field in polymer solution, but very limited experimental work has been done on concentrated suspensions. Nonetheless numerical simulations have been carried out by Brady (Sami and Brady 1996) on dense suspensions and have shown that the extensional viscosity of these fluids relate to the shear viscosity as follows:

\[ \eta_e(Pe_e) = 4\eta_s\left(\sqrt{2Pe_s}\right) \] (2.10)

This is illustrated in figure 2.13.

Figure 2.13. Shear and Extensional viscosity plotted against Peclet number. Open symbols refer to shear viscosities while full symbols to extensional viscosities. Additionally plotted are also the hydrodynamic and Brownian contributions of these viscosities as a function of Pe. (Sami and Brady 1996)
2.5. Applications

The shear thickening fluids rheological response to stress and the associated energy absorption capability make them very good candidate as the bases for composites with human protection applications. These fluids are particularly interesting because they don’t require an external activation mechanism, as magneto-rheological or electro-rheological fluids do, but they ‘activate’ under stress. A number of authors have tried to exploit these fluids mechanical and rheological properties in ingenious ways; a few examples are hip protection pads and ballistic protection garment.

2.5.1. Hip Padding

One of the first applications of these shear-thickening fluids was a hip-protection garment patented by McMahon (Hayes, Robinovitch et al. 1997). This specific design consisted of a series of pouches filled with shear-thickening fluids, that made up the garment (see figure 2.14). The final product was supposed to be flexible and comfortable to wear under normal circumstances, but hard and energy absorbent in the case of a fall (or high imposed stress).
Figure 2.14 (a) Schematic of pouches filled with shear thickening fluid. (b) Garment made of shear-thickening filled pouches.

Figure 2.15 Plots of Load vs. Time for three different impacts. The broken line represent the total load, the solid line the load experience by the femur. Impact 131 refers to impact to unpadded surrogate pelvis, 132 impact after affixing a pelvis device according to U.S. patent 4,573,216, and 133 impact to surrogate pelvis protected by with shear thickening fluid composed garment (Hayes, Robinovitch et al. 1997).

Experimental results of impact tests have shown that the garment is able to reduce the femoral load to about 50% of its total load. A sample plots of the impacts load for these experiments as shown in figure 2.15.
2.5.2. Ballistic Protection

One of the most recent applications for these fluids was presented by Wetzel and Wagner (Lee, Wetzel et al. 2003) which tried to incorporate a shear thickening fluid in a multiplayer composite for use as a body armor for ballistic protection. Their composite was made of a multiple layers of Kevlar sheets impregnated in shear thickening fluid. Ballistic testing on these multiplayer structures showed that the ballistic penetration resistance was enhanced by the presence of STF impregnated Kevlar, as compared to Kevlar or shear thickening fluid alone.

Figure 2.15 Clay witnessed placed behind shooting target. (a) was protected by a pouch of shear thickening fluids and four layers of Kevlar, (b) was protected by 4 layers of Kevlar all impregnated with shear thickening fluid (Lee, Wetzel et al. 2003).
Chapter 3

3. Cellular Solids: Review

Cellular solids are assembly of cells with solid edges and faces packed together so that they fill space (Gibson and Ashby 1988). When the structure of these cellular materials is made of parallel, prismatic cells, which form a two-dimensional structure, they are called honeycombs. Example of honeycomb structures can be found in nature in bee’s honeycombs and cork. When the cells are polyhedra and therefore form a three-dimensional structure, they are called foams; examples of foams include sponges, cancellous bones, corals, breads etc.


3.1.1. Cell Structure

The shape and structure of a cell is of particular importance because it directly affects the foam material properties. Honeycomb two-dimensional structures are made of parallel, prismatic cells; most cells are hexagonal, but square, triangular or random cells can also be made, see figure 3.1.

Foams on the other hand can be classified as open-cell or closed-cell. In the case of open-cell foam the material is concentrated into struts that form the cell edges. For the case of closed-cell, a thin film of the material is also present in each cell wall. The difference in structure between the two types of foams is also illustrated in figure 3.2.
Figure 3.1. Two-dimensional cellular material: (a) aluminum honeycomb, (b) paper-phenolic resin honeycomb, (c) ceramic honeycomb with square cells, (d) ceramic honeycomb with triangular cells (Gibson and Ashby 1988).

Figure 3.2. Three-dimensional cellular material: (a) open-cell polyurethane, (b) closed-cell polyurethane (Gibson and Ashby 1988).
3.1.2. Properties of Cellular Solids

“The mechanical response of...foam is controlled by the way in which cell elements behave cooperatively under localized strain...” (Hilyard and Cunningham 1994). Foams material properties depend on two factors: the geometry of their structures and the properties of the material they are composed of.

The most important structural characteristic of foams is its relative density, which is defined as \( \frac{\rho^*/\rho_s} {\rho_s} \), where \( \rho^* \) is the density of the cellular material, and \( \rho_s \) is the density of the solid from which the foam is made. The cell size and shape can be related to the relative density, but the relation varies depending on the type of foam, open or closed cell. We can define the cell edge \( l \) and the cell wall thickness \( t \) as seen in figure 3.3, then the relation between relative density and cell size can be express as

\[
\frac{\rho^*}{\rho_s} = C \left( \frac{t}{l} \right)^n
\]

where \( n \) is 2 for open-cell foam and 1 for honeycombs and closed-cell foams. The constant \( C \) depends on the shape of the cell and is of order 1.

![Figure 3.3 The packing of polyhedra to fill space: kaidecahedra (Gibson and Ashby 1988).](image)
Another characteristic of foam is its porosity, the fraction of open space, which is simply defined as $1-(\rho^*/\rho_s)$.

The cell shape is also important because it is related to the isotropy or anisotropy of the foam. Random shaped cells will result in foams that have isotropic material properties as, for example, polymeric foams. Elongated cells instead result in highly anisotropic material properties as in wood or cancellous bone.

Additionally, as one would expect, the foam material properties also depend on the material properties of the solid from which the foam is made, such as its density $\rho_s$, Young’s modulus $E_s$, and plastic yield strength, $\sigma_{ys}$.

3.2. Mechanics of foam

We focus our attention on three-dimensional foam because it is more suited for our intended energy absorption applications. Most man-made foams are used for energy absorption for impacts or for the construction of lightweight materials, additionally they are widely used as thermal insulators, in the construction of floating devices and as filter. Because of the widespread use, the mechanics of foam has been well studied and documented.

3.2.1. Mechanics of Foam Compression

The material response of a typical elastomeric foam is characterized by an initial linear regime followed by a long plateau which ends in a densification region. This is illustrated in figure 3.4.
The initial linear regime is associated with the bending of the cell wall and characterized by a modulus $E^*$. By modeling the open cell wall as a cubic array, a simple scaling to relate the modules $E^*$, $G^*$ and $\nu^*$ to the relative density of the foam can be derived:

$$\frac{E^*}{E_s} = C_1 \left( \frac{\rho^*}{\rho_s} \right)^2 \quad (3.2)$$

$$\frac{G^*}{E_s} = C_2 \left( \frac{\rho^*}{\rho_s} \right)^2 \quad (3.3)$$

$$\nu^* = C_3 \quad (3.4)$$

Details of this model and scaling arguments can be found in Ashby and Gibson (Gibson and Ashby 1988). By comparing the scaling with experimental data it is found that for open cell foam $C_1 \approx 1$, $C_2 \approx 3/8$, $C_1 \approx 1/3$. A detailed discussion on micromechanical model of open cell foam can be found in Hilyard and Cunningham (Hilyard and Cunningham 1994).

![ELASTOMERIC FOAM COMPRESSION](image)

**Figure 3.4** Schematic compressive stress-strain curves for foams, showing the three regimes of linear elasticity, collapse and densification for an elastomeric foam (Gibson and Ashby 1988).
Following the elastic region the stress-strain curve flattens into a plateau at a stress $\sigma_{el}^*$ called elastic collapse stress which, as the word suggests, is associated with the collapse of the cells structure. This value of stress is particularly important because it is in the region where most of the energy absorption occurs and, if following similar scaling arguments as for the modulus, we can derive a relation between $\sigma_{el}^*$ and the relative density as follows:

$$\frac{\sigma_{el}^*}{E_s} = C_4 \left( \frac{\rho^*}{\rho_s} \right)^2 \left[ 1 + \left( \frac{\rho^*}{\rho_s} \right)^{\frac{1}{2}} \right]^2$$

(3.5)

Again, if comparing the prediction with actual data for open cell foam we found that $C_4 \approx 0.03$.

As the foam is collapsed the cell walls are brought closer and closer together until the cells are crushed together. At this strain, usually denominated $\varepsilon_D$ densification strain, the stress rapidly increases approaching $E_s$, the modulus of the solid material. The densification strain is not exactly the porosity of the foam as one may expect, but from experimental results was found it to be

$$\varepsilon_D = 1 - 1.4 \left( \frac{\rho^*}{\rho_s} \right)$$

(3.6)

A schematic of the foam microstructure at the various states of compression is illustrated in figure 3.5.
3.2.2. Fluid-Filled Open Cell Foam

When foams are filled with a viscous fluid their material properties are, as expected, affected by the presence of the fluid. As the foam is compressed the fluid is squeezed out, and its stress contribution proportional to the viscosity of the fluid. Additionally, the presence of the fluid adds a greater strain rate dependence on the fluid-solid material properties. A relationship between the fluid stress contribution and the material properties can be derived (Gibson and Ashby 1988) by treating the foam as a porous medium.

According to Darcy's law:

$$ u = \frac{K \, dp}{\mu \, dx} $$  \hspace{1cm} (3.7)
where \( u \) is the velocity of the fluid, \( \mu \) is the fluid viscosity and \( dp/dx \) the pressure gradient. From the geometry in figure 3.6 (unconstrained compression), we can estimate the fluid flux through the vertical faces of the foam as

\[
q = \frac{VL}{2H} = \frac{\dot{e}L}{2}
\]

(3.8)

where \( \dot{e} \) is the strain rate. By substituting 3.8 into Darcy’s law, and making some estimates about \( K \) and \( dp/dx \) we arrive to the final relation:

\[
\sigma_f^* = \frac{C\mu\dot{e}(L)}{1-\varepsilon}\left(\frac{L}{l}\right)^2
\]

(3.9)

where \( C \) is a geometric constant of order 1, \( \varepsilon \) is the strain, \( l \) is the cell-edge length. The total stress plateau for fluid filled foam can then be found by adding the two stress contributions as

\[
\sigma_{tot} = E_C 4 \left(\frac{\rho^*}{\rho_s}\right)^2 \left(1 + \left(\frac{\rho^*}{\rho_s}\right)^{\frac{3}{2}}\right) + C\mu\dot{e}\left(\frac{L}{l}\right)^2
\]

(3.10)

This scaling and modeling presented here consider the effect of the presence of a Newtonian fluid; the analysis becomes more complex if the fluid behaves as non-Newtonian. Some previous work has been carried on the effect of non-Newtonian fluid flowing through porous material and has been summarized by Pearson (Pearson and Tardy 2002) in a review article. These models are very complex and necessitate numerical simulation to account for the material microstructure and the non-Newtonian properties of the fluid.
3.2.3. Poroelastic Model

A different way of looking at the problem of fluid flow through a deforming solid matrix comes from the application of poroelastic models. Poroelastic models were first introduced by Biot (Biot 1956) and is now commonly used to study wave propagation in fluid saturated soils (Kim, Parizek et al. 1997; Parra 2000; Williams, Jackson et al. 2002) as well as, more recently, in studying the mechanical response of biological tissue (Kim, Bonassar et al. 1995; Li, Buschmann et al. 2000; Korhonen, Laasanen et al. 2003). These three dimensional models often required complex computer simulations that account for pore microstructure as well as solid deformations, and in the case of biological tissue, the matrix surface charges. For our case a simple one-dimensional model can be derived by accounting for the force and momentum in a solid matrix filled with a viscous fluid as shown in figure 3.7. In this model the sides of the matrix are constrained to allow for a

![Figure 3.6 The compression of a fluid-filled, open-cell foam. The pressure gradient, of order σL, expels the fluid through the two vertical faces (Gibson and Ashby 1988).](image-url)
purely one dimensional model. By assuming that the solid matrix behaves as a hookean solid we can first write the constitutive relation for the matrix:

\[ \sigma_{11} = H e_{11} - p \]  

(3.11)

where \( \sigma_{11} \) is the stress in the vertical direction, \( H \) is the compression bulk modulus, \( e_{11} \) is the vertical strain, and \( p \) is pressure. Additionally we can apply Darcy’s law, which in 1D is written as

\[ v_f = \frac{K}{\mu} \frac{\partial p}{\partial x} \]  

(3.12)

where \( v_f \) is the fluid velocity, \( K \) represent the permeability of the solid matrix, \( \mu \) is the fluid viscosity and \( \frac{\partial p}{\partial x} \) the pressure gradient. Mass conservation is written as:

\[ v_f = -\frac{\partial u}{\partial x} \]  

(3.13)
where \( u \) is the solid matrix displacement. The negative sign in the expression accounts for the fluid to flow always in the opposite direction than the solid matrix displacement in this geometry. Finally the momentum balance is written as:

\[
\frac{\partial \sigma}{\partial x} = \rho \frac{\partial^2 u}{\partial x^2}
\]  

(3.14)

where \( \rho \) is the density of the solid matrix. These equations can be combined to obtain the well-known Telegraph equation.

\[
\rho \frac{\partial^2 u}{\partial t^2} + \mu \frac{\partial u}{\partial t} = H \frac{\partial^2 u}{\partial x^2}
\]

(3.15)

3.2.4. Energy Absorption of Foams

The popularity of elastomeric reticulated foams comes from their ability to absorb energy from an impact at a relatively constant stress. Their mechanical response to impact, added with the lightweight properties, makes them perfect for this type of application. Figure 3.7 illustrates a typical stress-strain curve of a elastomeric foam; the energy absorption or the work done (per unit volume) by the foam is represented from the area under this curve. Most of the energy is absorbed along the flat plateau, and therefore at constant load and can be calculated as

\[
W = \int \sigma d\varepsilon
\]

(3.16)
Figure 3.7 Stress-strain curves for an elastic solid and a foam made from the same solid, showing the energy per unit volume absorbed at a peak stress $\sigma_p$ (Gibson and Ashby 1988).

and because the stress is almost constant throughout the compression we can estimate it to be

$$W = \sigma^* \varepsilon_f$$  \hspace{1cm} (3.17)

If the foam is also filled with a Newtonian fluid then the total work needs to also account for the fluid component and can be estimated with the following expression:

$$W_{\text{fluid}} = \int_0^{\varepsilon_f} \sigma_j^* d\varepsilon = C \mu \dot{\varepsilon} \left( \frac{L}{l} \right)^2 \left( \frac{1}{1.4 \rho^*/\rho_s} \right)$$  \hspace{1cm} (3.18)
3.2.5. Optimization Criteria

On choosing a particular foam for an application requires not only knowledge of the foam material properties but also the amount of energy that needs to be absorbed, the rate of impact and the maximum allowable stress. Peak stress is in fact the parameter that is of most concern, for example: even if a particular foam has the capacity of absorbing a great amount of energy during impact, if its plateau stress is greater than the one allowable by the system, the stress will grow and exceed the system maximum stress before the foam has a chance to reach the plateau and absorb the impact energy.

For these reasons it is useful to construct optimization curve. This is commonly done by integration of the stress-strain curve to find the work done by the foam and replotting the data as normalized work vs. normalized stress for different strain rates (fig 3.8(b)). The optimum energy absorbed is the region of the curve inflection, which corresponds to the beginning of the densification region. An optimization line is drawn to connect the inflection points of different strain rates characterizations (broken line in figure 3.8 (b)). A number of these lines can be also derived for different density foams and a complete energy absorption diagram can be constructed as in figure 3.8 (c).

When foams are filled with a fluid, optimization becomes more complex. As seen earlier the fluid contribution to stress depends on strain and strain rate and, especially if the fluid viscosity is relatively high, there is no clear transition between the plateau and densification region.
Nevertheless we can still represent the stress-strain response in a similar fashion. For this case there is no clear optimization line but what we find is an area of optimization.

There are a number of other ways to characterize energy absorption. Some of the most popular methods is to use Cushion or Jensein factors; more detail can be found in Gibson and Ashby (Gibson and Ashby 1988).
3.3. Testing Apparatus For Foams Material Properties

As explained earlier the material properties of reticulated foams and especially fluid-filled foams, are functions of strain rates. To be able to characterize these foams for large variations in strains rates (many orders of magnitude) different apparatus and techniques are necessary. Table 3.1 illustrates some of the most common testing apparatuses and the typical strain rates ranges that they are able to achieve. In the present work three apparatuses were used: texture analyzer and drop tower impact tester, and split Hopkinson pressure bars.

<table>
<thead>
<tr>
<th>Texture Analyzer</th>
<th>Servohydraulic frames</th>
<th>Drop Test</th>
<th>Hopkinson Pressure Bar</th>
<th>Taylor Impact Bar</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\dot{\varepsilon} = 10^{-4} - 10^{0} s^{-1}$</td>
<td>$\dot{\varepsilon} = 10^{-1} - 10^{1} s^{-1}$</td>
<td>$\dot{\varepsilon} = 10^{0} - 10^{3} s^{-1}$</td>
<td>$\dot{\varepsilon} = 10^{2} - 10^{4} s^{-1}$</td>
<td>$\dot{\varepsilon} = 10^{4} - 10^{6} s^{-1}$</td>
</tr>
</tbody>
</table>

Table 3.1. Strain rates ranges achievable by different testing techniques and apparatuses.

3.3.1. Texture Analyzer

The Texture Analyzer (TA.XT Plus) by Stable Microsystem is a mechanical test widely used for ‘low strain rates’ ($10^{-6}-10^{0}$ 1/s) characterization of solids or semi-solid materials. The apparatus comes with interchangeable fixtures (see figure 3.9) for a variety of applications, as for example, food industry, pharmaceutics, electronics, adhesive etc. The apparatus comes with a load cell that can measure forces up to 50 kg with resolution of 0.1 g. The moving top plate can achieve velocity from 0.01-40 mm/s, with linear or exponential velocity profiles, and the vertical displacement is measured with a resolution of $10^{-3}$ mm.
3.3.2. Dynatup Drop Tower

Drop Tower Impact testing were carried out with a Dynatup 9850 from Instron (see figure 3.10). Drop Towers are commonly used to determine impact resistance of plastic, metal composites, rubber, etc. This specific model includes spring assist velocity accelerator that can achieve impact velocity of 20 m/s and impact energies of 1600J. The load cell is able to record forces up to 45 KN, at a speed of 5MHz. The tup and bottom fixture can be modified to either produce a 'penetration impact,' as for example to test sheets of composites or a simple compression, with a wide and flat tup.
3.3.3. Split Hopkinson Pressure Bars

The split Hopkinson pressure bars apparatus is commonly used for material characterization in uniaxial compression at very high strain rates ($10^2-10^4$ s$^{-1}$). This
method was first derived by Hopkinson in 1914 who calculated that by using long elastic bars to compress small specimens, as illustrated in figure 3.11, the stress and strain in the specimens could be derived directly from the displacement and wavelength of the pressure wave in the bars. More specifically the strain rate of the specimen is defined as:

$$\frac{d\varepsilon}{dt} = \frac{V_1 - V_2}{L}$$

(3.19)

where $V_1$ is the velocity at incident bar interface, $V_2$ is the velocity at the transmission bar interface and $L$ is the length of the specimen. $V_1$ and $V_2$ can be defined as a function of the bars properties and strains as follows:

$$V_1 = C_0 (\varepsilon_i - \varepsilon_r)$$

(3.20)

$$V_2 = C_0 \varepsilon_T$$

(3.21)

where $C_0$ is the wave speed inside the bars and is defined as $\sqrt{\frac{E}{\rho}}$ and $\varepsilon_i$, $\varepsilon_r$, $\varepsilon_T$ are the incident, reflected and transmitted strain respectively. By substituting 3.20 and 3.21 into 3.19 we obtain the following expression:

$$\frac{d\varepsilon(t)}{dt} = \frac{C_0}{L} \left[ \varepsilon_i(t) - \varepsilon_r(t) - \varepsilon_T(t) \right]$$

(3.22)

Figure 3.12 Schematic of specimen sandwiched between two pressure bars.
Similarly we can define the stress at the specimen/bars interface as:

\[ \sigma(t) = \frac{F_1 + F_2}{2A} \]  

(3.23)

where \( F_1 \) and \( F_2 \) can be written as \( F_1 = E[\varepsilon_i(t) + \varepsilon_r(t)]A_0 \) and \( F_2 = EA_0\varepsilon_T(t) \). E is the bar modulus, \( A \) is the bar’s diameter and \( A_0 \) is the specimen diameter. Combining these equation we derive at the following expression:

\[ \sigma(t) = \frac{E}{2A} \left[ \varepsilon_i(t) + \varepsilon_r(t) + \varepsilon_T(t) \right] \]  

(3.24)

After impact stress waves propagate through the sample and only after some time, the time for the sample to reach equilibrium, the sample will be deforming uniformly. If deforming uniformly the following relation holds:

\[ \varepsilon_i(t) + \varepsilon_r(t) = \varepsilon_T(t) \]  

(3.25)

When this relation is substituted in equation 3.22 and 3.24 we obtain:

\[ \frac{d\varepsilon(t)}{dt} = \frac{-2C_0}{L} \varepsilon_r(t) \]  

(3.26)

\[ \sigma(t) = \frac{E}{A} \varepsilon_T(t) \]  

(3.27)

These two expressions are the fundamental basis used for the calculation of stress and strain rates in the specimen. Strains are obtained by integration 3.26 in time. There are a number of assumptions and conditions that need to be met in order to be able to use these expressions. As mentioned earlier the specimen has to be deforming uniformly and additionally the wave propagation in the bars needs to be one-dimensional. The latter condition can usually be achieved by using very long bars, at least 10 times their diameter, and a ratio of radius to wavelength \( R/\lambda \ll 1 \).
This method was originally used for the high rates characterization of metals and
ceramics but recently scientists have tried to modify this technique to measure the
characteristic of much softer material like foams (Zhao 1997; Chen, Lu et al. 2002; Chen,
Lu et al. 2002), a biological tissue. The major problem with the characterization of these
softer materials is that their impedance is much smaller than the classical steel or
aluminum bar impedance. This impedance mismatch doesn’t allow the incident wave to
transmit to the sample and it results in a transmission signal that is too small to measure.
For this reason hollow aluminum bars (Chen, Zhang et al. 1999) with higher sensitivity
strain gauges (Chen, Lu et al. 2000) or polymeric bars (Gary, Klepaczko et al. 1992;
Wang, Labibes et al. 1992) have recently been used.
Chapter 4

4. Shear-thickening fluids: Rheology and Characterization

4.1. MATERIALS AND CHARACTERIZATION

4.1.1. Particles Characterization

The shear thickening fluids used in this study were composed of silica particles dispersed in ethylene glycol. Two types of silica particles were used: one monodisperse and the other polydisperse. The monodisperse silica particles were supplied by Fuso chemical in powder form with a nominal diameter of 300 nm. A micrograph of the particles, taken by SEM, is shown in figure 4.1a, and the corresponding particle size distribution (PSD) obtained by dynamic light scattering in figure 4.1b. The density of these particles was determined to be 2.1 gr/cc, and their zeta potential in water, measured with a Brookhaven Zeta Pals was found to be -77.6 mV.

![Micrograph of monodisperse silica particles with diameter of 300 nm.](a)

![Particle size distribution of the same silica particles.](b)

Figure 4.1. (a) Micrograph of monodisperse silica particles with diameter of 300 nm. (b) Particle size distribution of the same silica particles.

The second type of silica particles, supplied in powder form by US Silica, is polydisperse and faceted in shapes with nominal size of 1 μm. The density was measured to be 2.6
gr/cc and the zeta potential -79.8mV. A micrograph and PSD of these particles are shown in figure 4.2.

Figure 4.2 (a) Micrograph of polydisperse silica particles with nominal diameter of 1 μm. (b) Particle size distribution of these silica particles.

4.1.2. Preparation Techniques

The two different types of particles were dispersed in ethylene glycol. At first the particles were suspended at relatively low volume fraction (not greater then 20 %) through vigorous mixing (with Vortex Mixer) and sonication to ensure complete dispersion. The low volume fraction suspension was then centrifuged and the supernatant removed. The crystallized suspension is then diluted to the desired volume fraction, by again, vigorous mixing and sonication to ensure homogeneity. The monodisperse silica suspension final volume fraction was 60 % and up to 47 % for the polydisperse one.
4.2. SHEAR RHEOLOGY

4.2.1. Steady Shear Rheology

Steady shear experiments were carried out with AR2000 stress controlled rheometer. The polydisperse suspension was found to shear thicken at a volume fraction ($\phi$) above 40%, and extreme shear thickening was observed at $\phi$ of 47%. This volume fraction is relatively low if compared to monodisperse spherical particles suspensions. In fact, the monodisperse suspension starts to show shear thickening at $\phi$ of 54% and extreme shear thickening does not occur until $\phi$ up to 58-61% (see figure 4.3). This difference in volume fraction comes from the different particles shape of the two suspensions, as discussed in section 2.3.4.

![Graph showing steady shear response of both silica suspensions.](image)

Figure 4.3 Steady shear response of both silica suspensions. The rheology of different volume fraction suspensions is shown to indicate the critical volume fraction for the appearance of shear thickening effect. (in blue) Viscosity of the solvent ethylene glycol.
In fact, the more faceted the particles are, the more likely it is for them to jam into each other when they get close, and therefore the easier for the suspension to shear thicken, even at relatively low $\phi$.

To note also the great difference between the two suspension overall viscosity: according to data shown in figure 4.3 even at relatively low $\phi$ at which shear thickening appears (40% for polydisperse suspension and 54% for the monodisperse one) the viscosity of the polydisperse suspension is two orders of magnitude greater than the monodisperse. This phenomenon is again related to the shape of the particles in suspension. Intuitively, it takes a greater stress to move ‘pointy’ faceted particles past each other than to do the same for spherical particles.

We also note the different value of stress at the critical onset of shear thickening. This is consistent with the fact, derived from scaling arguments, that suspensions of smaller particles will show greater critical stress. Particularly for electrostatically stabilized particles $\tau_{\text{crit}} - 1/a^2$, this obviously not rigorously followed by the two suspensions because of different particles characteristics.

The rheological data of both suspensions (at high $\phi$) has been fitted to a mathematical model. In order to capture the yield stress seen in both suspensions the Cross model was utilized with a modification of $n$ to account for the shear thickening of the suspension.

The equations used follows

$$\sigma = K\dot{\gamma}^n + \frac{\dot{\gamma} (\eta_0 - \eta_\infty)}{1 + \left(\frac{\dot{\gamma}}{\dot{\gamma}_c}\right)^{1/2}}$$

(4.1)
where $\sigma$ represent the shear stress and $\dot{\gamma}$ the shear rate. $K$, $n$, $\eta_0 - \eta_\infty$ and $\dot{\gamma}_c$ are found by fitting the model to the data and these values for both dispersion are reported in table 4.1.

<table>
<thead>
<tr>
<th>Dispersions</th>
<th>$K$ (Pa.s$^{-n}$)</th>
<th>$n$</th>
<th>$\eta_0 - \eta_\infty$ (Pa.s)</th>
<th>$\dot{\gamma}_c$ (1/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monodisperse</td>
<td>$5.0 \cdot 10^{-4}$</td>
<td>3.7</td>
<td>$4.5 \cdot 10^3$</td>
<td>$7.5 \cdot 10^{-4}$</td>
</tr>
<tr>
<td>Polydisperse</td>
<td>43</td>
<td>3.2</td>
<td>$3.5 \cdot 10^5$</td>
<td>$1.0 \cdot 10^{-4}$</td>
</tr>
</tbody>
</table>

Table 4.1 Fitted values of the modified Cross model for the monodisperse and polydisperse suspensions.

As seen in figure 4.4 this model captures very well the behavior of the polydisperse suspension, even though it has trouble following the slower viscosity drop of the monodisperse suspension in the shear thinning region.

The shear thickening fluid dependence on temperature was also studied. According to the scaling arguments derived earlier (section 2.2.3) the shear thickening effect should
depend only weakly on temperature, but because the onset of thickening directly depends on the solvent viscosity we expect the critical stress to shift with temperature. This is precisely what we see in figure 4.5, which illustrates the fluid viscosity dependence on temperatures for the monodisperse silica suspension.

The temperature dependence of the suspension can be collapsed into a mastercurve by applying proper scalings. The shear stress is scaled with thermal energy and the horizontal axis becomes a function of the peclet number, defined as $Pe = \frac{\tau a^3}{k_B T}$. The magnitude of the viscosity also needs to be scaled, and a reduced viscosity can be defined as $\eta_r = \frac{\eta}{\eta_T}$, where $\eta_T$ is the temperature shift factor see figure 4.6. We also observe that the shift in the viscosity is coming directly and only form the solvent viscosity dependence. In fact as seen in the inset in figure 4.6, the shift factor dependence
on temperature for the fluid coincide with the dependence of the suspension solvent, ethylene glycol.

Figure 4.6. Rheology of the monodisperse silica suspension at various temperature plotted as reduced viscosity versus Pe. (top-right) shift factor as a function of $1/T$ of the shear thickening fluids and the solvent ethylene glycol.

### 4.2.2. Dynamic Shear Rheology

The monodisperse silica suspension has been extensively studied under dynamic shear flow; these experiments have shown that the dynamic complex viscosity is related to the steady shear viscosity. In particular when the magnitude of the complex viscosity, $|\eta^*|$, is plotted as a function of the product $\omega \gamma_0$ the curve for the steady shear viscosity and complex shear viscosity superimpose. This is illustrated in figure 4.7. The product $\omega \gamma_0$ represents the ‘dynamic shear rate’, and this correspondence is referred as the Delaware-Rutgers rule which can be expresses as:

$$\eta(\dot{\gamma}) = |\eta^*(\gamma_0 \omega)|$$  \hspace{1cm} (4.2)
This finding is particularly important because it infers that there are two ways to achieve thickening, the first obtainable by shearing at a steady rate \( \dot{\gamma} > \dot{\gamma}_{\text{crit}} \), the second by imposing oscillatory shearing at \( \dot{\gamma} = \gamma_0 \omega > \dot{\gamma}_{\text{crit}} \). There are ‘infinite’ combinations of \( \gamma_0 \omega \) that can induce the fluid to thicken and in order to study this in a more systematic way we represent the dynamic viscosity as both a function of strain and frequency using a Pipkin diagram as seen in figure 4.8. The value of complex viscosity is plotted as function of both parameters and the figure has been color coded; vertical lines in the plot corresponds to strain sweeps, while horizontal ones to frequency sweeps. This representation of the data also suggests that there is a limit on how small of a strain is necessary for the
suspension to shear thicken. This has been seen in various work on shear thickening studies by Wagner (Lee and Wagner 2003) and Laun (Laun, Bung et al. 1991) but sometimes attributed to wall slip effects. Dynamic shear flow experiments were not carried out with the polydisperse suspension because the complete elimination of slip was not successful.

4.3. **Extensional Flow Response**

The extensional viscosity of shear thickening fluids was measured with a filament stretching rheometer (FiSER), as shown in figure 4.9. This instrument is composed of two circular plates, the bottom stationary, and the top able to move upward. The fluid sample is loaded in between the two plates and stretched. The bottom plate is attached to a load cell that records the force as a function of time. The instrument also measures the
value of the midplane diameter by using a laser micrometer mounted on a moving stage. As the top plate moves, the micrometer stage travels at half the speed; this ensures that the micrometer always reads the midplane diameter. From the diameter reading we can calculate the rate of strain as follows:

\[
\dot{\varepsilon}_{mid} = -\frac{2}{D_{mid}(t)} \frac{dD_{mid}(t)}{dt} \tag{4.3}
\]

it is our goal to be able to measure extensional viscosity at constant strain rates, which means that the top plate needs to be moving up exponentially.

![Diagram of Filament Stretching Extensional Rheometer (FiSER)](image)

Figure 4.9. Filament stretching extensional rheometer (FiSER) Shelley Anna (PhD thesis, 2001)

The force measurement readings at the bottom plate needs to be adjusted to account for gravity and surface tension effects. An expression for the force balance (Szabo and McKinley 2003), can be used to estimate the contribution from each components to the total force experienced by the fluid. We can write:
\[ \tau_{zz} = \frac{\rho(g + \ddot{L}/2)V_0}{2\pi R_s^2} + \frac{F_{p,\text{bottom}} - \rho(g + \ddot{L}/2)V_0}{\pi R_s^2} + \frac{\sigma}{R_s} \left(1 + R_s R_s'' \right) \] (4.4)

where inertia has been neglected. By ignoring the surface tension term that contains \( R_s'' \) and by rearranging (4.4) we obtained an expression for the viscous force:

\[ F_{\text{viscous}} = F_{p,\text{bottom}} - \frac{\rho(g + \ddot{L}/2)V_0}{2} + \pi\sigma R_s \] (4.5)

By using the direct measures of diameter as function of time together with the viscous force measurements, stress-strain diagrams can then be constructed. Experiments carried out for the monodisperse silica suspension show two different flow regimes: at low extension rates the suspensions respond as a viscous rate-thinning fluid, whereas beyond a critical extension rate, the suspension strain-hardens and ultimately fractures in a solid-like fashion. This behavior is clearly observable from a series of images taken during the experiments; figure 4.10.

The stress-strain behavior of the monodisperse silica suspension is shown in figure 4.11 where each curve corresponds to a different nominal strain rate. The oscillation of the stress values seen at high extension rate can be related to the FiSER vibration during high rates experiments. The characteristic stress for each extension rate was obtained by averaging the stress measurements (data before strain of 0.1 were not included into the averages calculation.)

The characteristic stress obtained from figure 4.11 are then used to find nominal extensional viscosities as:

\[ \eta_E = \frac{\tau_{\text{avg.}}}{\dot{\varepsilon}_{\text{nom.}}} \] (4.6)
which are plotted in figure 4.12. We note that viscosity measured in extension is comparable qualitatively to the shear viscosity, but it seems to have a greater and earlier (at smaller strain rates) thickening response in extension. This is not consistent with the finding from Brady (Sami and Brady 1996), but one possible explanation for the difference may lay in the use of nominal strain rates rather then true strain rates, due to experimental limitation. In fact we expect the true extensional strain rates to be greater then the nominal, which would effectively shift the viscosity-strain rate curve to the right, and also (because viscosity is calculated by dividing by strain rates) down closer to the shear viscosity curve.

![Figure 4.10 Video images from extensional flow experiments for the monodisperse silica suspension at $\phi$ of 61%, (Top) Strain rate 0.5 s$^{-1}$, (Bottom) Strain rate 6 s$^{-1}$.](image)
Figure 4.11. Stress-strain curves from extensional flow experiments of 60% volume fraction silica suspension. Extension rates ranges from 0.5-6 s⁻¹.

Figure 4.12. Comparison between extensional and shear rheology for the monodisperse silica suspension at \( \phi \) of 60%, shear rates have been divided by square root of 3 to account for the second invariant.
4.4. SLIP

The effect of wall slip, commonly encountered in suspensions, has been considered in this study: the presence of this phenomenon was detected in both the monodisperse and polydisperse suspensions. In order to minimize or possibly eliminates its effect, different roughen surfaces have been used to carry out rheological experiments.

The magnitude of slip occurring at the fluid-solid interface was quantified by applying the classic analysis by Yoshimura & Prud'homme (Yoshimura and Prudhomme 1988), and therefore the best slip-reducing surface found.

The polydisperse suspension showed a significant amount of slip with all the surfaces studied. The one most efficient to reduce wall slip was obtained by gluing particles (same as used in the shear thickening fluid) to the surface. As expected, the slip velocity was found to vary linearly in shear stress until a critical value of stress was

![Graph](image-url)

Figure 4.13. Wall slip velocity dependence on shear stress. The slip velocity increases until it reaches twice the moving wall velocity. (a) Velocity profile at $r = R$ for stresses smaller then critical one. (b) Velocity profile at $r = R$ for stresses greater then critical one.
excided. After this critical stress the slip velocities showed no increase with stress: this behavior is shown in figure 4.13. By looking at the speed of the moving wall as a function of stress we can sketch the velocity profile of the fluid at the outer radius. Below the critical stress the velocity profile is similar to the one sketched in 4.13a, where the shear rate is reduced by the presence of wall slip. As the shear stress increases so does the slip velocity, which consequently reduces the ‘real’ shear rate. At the critical stress the slip velocity approaches twice the wall velocity and the velocity gradient in the fluid approached zero (see figure 4.13b). After this point the fluid moves in a ‘solid body rotation’. Physically what is observed is the thickening of the fluid due to the increasing stress, until a change in structure occurs, at the critical stress, and the fluid ‘jams’ into a solid state. The presence of a small lower viscosity layer closest to the walls allows the top and bottom plate to slide and slip even if the fluid in between is in a solid state.

A different behavior was observed for the monodisperse suspension; a set of plates polished to a roughness of the order of microns was found to completely eliminate
the presence of slip. A micrograph of the ‘roughen’ surface is showed in figure 4.14.
5. Energy Absorption System: Characterization

5.1. Foam specification and sample preparation

The polyurethane foam used for the foam/shear thickening fluid systems was supplied by Crest Foam Industries with the following product specifications: average cell size of 360 μm, relative density of 0.03 and $E_s=70$ MPa. Prior to testing the samples were cut to sample size with a puncher to assure accurate dimensions and sample reproducibility. Two different sample sizes were used: for Texture Analyzer and Dynatup experiments the diameter measured 2.8 cm with a height of 2 cm, while for the split Hopkinson pressure bars a much smaller sample with diameter of 1.3 cm and height of 0.5 cm was used. The foam samples were impregnated with either glycerol or shear thickening fluid (monodisperse silica suspension at volume fraction $\phi$ of 61%) at different volume fraction of fluid into foam, $\psi$, by suction forces. The volume fraction of the sample, $\psi$, was found by direct weighting prior and after impregnation. Figure 5.1 shows micrographs of polyurethane foam and foam impregnated with a shear thickening fluid. As fluid is introduced into the foam it tends to collect around the cell walls, or trusses of the foam structure. As the amount of fluid is increased (as the volume fraction $\psi$ increases) the fluid starts to collect in between the pores, until the fluid fills the whole space. In order for the fluid to remain in place without draining, a small value of yield stress in the fluid is necessary.
5.2. Characterization of Empty Foam

The mechanical properties of air filled polyurethane foam were characterized with a texture analyzer at constant strain rates that varied between $10^{-2}$-1 s$^{-1}$. As expected the foam showed an initial linear regime up to a strain of 10% followed by a long plateau characterized by a stress that varied between 3 to 6 kPa, depending on the strain rate. In fact, the stress-strain response of this polyurethane foam is weakly strain rate dependent as can be clearly seen in figure 5.2. After the long plateau region the foam densifies at a strain of about 80%.

The energy absorption characteristics of these foams were calculated by integration of the stress-strain curve. Optimization curves were therefore constructed according to the methods described in section 3.2.5 and shown in figure 5.3. Foams with different cells size were also tested and, as expected, this feature didn’t affect the mechanical response of air filled polyurethane foam.
Figure 5.2 Stress-strain response of polyurethane foam with relative density of 0.03 and $E_s = 70$MPa at strain rates that varied between $10^{-2} - 1 \text{ s}^{-1}$.

Figure 5.3 Optimization chart for polyurethane open cell foam with density 0.03 and $E_s = 70$MPa.
5.3. Low Strain rates: Characterization of Glycerol Filled Foam

5.3.1. Rate dependence

As explained earlier in chapter 3, the presence of a viscous fluid inside foams greatly affects their material response; in particular the stress response of foam acquires a greater strain rate sensitivity. This is illustrated in figure 5.4 where the stress-strain response of glycerol filled foam is shown at different strain rates. Additionally, we note that the “stress plateau” not only depends on strain rates but also depends on strain and increases as $1/(1 - \varepsilon)$ (as derived earlier in section 3.2.2). This can be physically explained by the nature of the fluid stress contribution. In fact, as the foam is compressed the fluid is pushed out of the pores, with the resultant stress being proportional to $-\eta \dot{\varepsilon}_0 - \eta \frac{V}{l}$, where $V$ is the velocity of foam compression and $l$, the cells size. As the strain $\varepsilon$ increases the size of the pore and therefore the area through which the fluid flow,

Figure 5.4 The stress-strain response of polyurethane foam filled with glycerol at $\phi=100\%$ for 3 different strain rates. (In symbols) Predictions of the scaling model as described in section 3.2.2.
decreases as $\sim (1 - c)^{1/2}$, which results in a much greater stress contribution. Also plotted in figure 5.4 is the prediction from the scaling model presented in section 3.2.2, the model works well for ‘low strain rate’, but fails to capture the behavior of the foam at strain rates greater than 11/s. The strain dependence of the plateau stress is not very evident at low strain rates because in that case $V$ is small and the stress contribution from the fluid is small compared to the contribution form the solid matrix. This is illustrated in figure 5.5 where the response of glycerol filled foam is compared to the empty foam response for each strain rate. This effect, on the other hand, became very important above strain rates of about 11/s for this specific sample. Looking carefully at figure 5.5 one notice that, before the fluid contribution to stress became significant, the stress response

![Stress-strain graph showing the comparison between air filled foam and foam filled with glycerol at three different strain rates.](image)

**Figure 5.5** Comparison between air filled foam and foam filled with glycerol at three different strain rates. (Solid lines) stress-strain response of polyurethane foam filled with glycerol at $\phi=100\%$. (Dashed lines) stress strain response of air filled polyurethane foam.
of the empty foam is greater than the response of glycerol-filled one. This is not surprising if one realizes that the cell wall of the glycerol-filled foam are now “wet” with fluid which reduces the stiffness of the solid walls. As for the air-filled foam the energy absorption characteristics were calculated by integration of the stress-strain curve and optimization curves were constructed according to the methods described in section 3.2.2 and shown in figure 5.6.

![Stress vs. Work/ES](image)

Figure 5.6 Optimization chart for polyurethane foam filled with glycerol at ϕ=100%

### 5.3.2. Volume fraction dependence

The stress response of fluid-filled foams has been studied as a function of volume fraction ϕ. At relatively low strain rates, when the fluid contribution to stress is small, there isn’t any experimentally detectable difference between foams filled with glycerol at different volume fractions. Figure 5.7(a) illustrates this phenomenon; no observable difference in
between sample of different volume fraction is observed but all the fluid-filled samples stress response was lower than the stress of the air-filled sample.

![Figure 5.7 Stress-strain response of polyurethane foam filled with glycerol.](image)

(a) Strain rate of $10^{-2}$ s$^{-1}$, volume fraction vary between 100 to 0 %. (b) Strain rate of $10^{-1}$ s$^{-1}$, volume fraction vary between 100 to 0 %. (c) Strain rate of 1 s$^{-1}$, volume fraction vary between 100 to 0 %. (d) close up of stress-strain curved at strain rate of $10^{-1}$ s$^{-1}$, volume fraction vary between 100 to 0 %.

At strain rates of $10^{-1}$ 1/s one starts to observe a small volume fraction dependence, figure 5.7(b). In this case the empty foam stress response is higher than the ‘wet’ foam at low strains, but as the strain increases, so does the stress contribution from the fluid, and the glycerol-filled foam stress response overtakes the stress of the air-filled foam. The point at which the empty foam and glycerol filled foam stress response cross is a function
of the volume fraction. This is illustrated in figure 5.7(d). As the strain rate is further increased this phenomenon becomes even more evident as illustrated in figure 5.7(c).

5.4. Low Strain rates: Characterization of STF Filled Foam

5.4.1. Rate dependence

Even at relatively low strain rates ($\dot{\varepsilon} = 10^{-2} \text{s}^{-1}$) the STF filled foams behaves very differently then the glycerol filled foam. In fact, as explained earlier in chapter 2 the viscosity of shear thickening fluid depends on the ‘local’ strain rate and therefore is very sensitive to the local geometry. We can estimate the local strain rate as

$$\dot{\varepsilon}_{\text{local}} = C \frac{\dot{\varepsilon}}{d_i \left(1 - \varepsilon\right)^{\frac{3}{2}}} > 250 \text{s}^{-1} \gg \dot{\gamma}_{\text{Critical}}$$

(5.1)

where $C$ is a constant order 1 that depends on the foam cell geometry, $d_i$ is the average cell size (360 $\mu$m for the foams used in these experiments), $\dot{\varepsilon}$ is the foam strain rate and $\varepsilon$ is the ‘macroscopic’ strain. According to equation 5.1 a strain rates of $10^{-2}$ $\text{s}^{-1}$ will still impose a local strain rate of approximately 250 $\text{s}^{-1}$ which is much greater than the fluid critical shear rate. The foams filled with shear thickening fluids were therefore always strained (even at low nominal rates) beyond the fluid critical deformation rate.

This is consistent with the data shown in figure 5.8, where the stress response of STF filled foams ($\psi = 100\%$) is compared to with glycerol filled foam (also filled to $\psi = 100\%$). At this volume fraction the shear thickening fluid foam is always stiffer than the glycerol filled one.
Figure 5.8 (solid lines) Stress-strain response of polyurethane foam filled with monodisperse silica suspension at $\phi = 61\%$ at strain rates that varied between $10^{-2}$ to $1 \text{ s}^{-1}$. (dashed lines) Stress-strain response of polyurethane foam filled with glycerol at strain rates that varied between $10^{-2}$ to $1 \text{ s}^{-1}$.

### 5.4.2. Volume fraction dependence

The stress-strain response of polyurethane foam/shear-thickening fluid composites depends on the volume fraction of impregnation, $\psi$; this dependence has been studied and characterized. As expected, the higher the volume fraction of fluid inside the foam, the greater the stress response of the composite as illustrated in figure 5.9. The shape of these stress-strain curves remains self-similar, as it doesn’t change with volume fraction as it can be observed in the case of glycerol. In fact, the response of these samples superimposes until a strain of about 5-10 % is reached and the stress grows proportionally to the volume fraction of STF in the sample. This is not consistent with the idea introduced earlier for the case of glycerol filled foam that the fluid contribution to stress will dramatically grow at a strain of $(100 - \psi)$. 
Figure 5.9 (dashed line) Stress-strain response of polyurethane foam filled with glycerol at different volume fraction $\psi$. (solid lines) Stress-strain response of polyurethane foam filled with monodisperse silica suspension ($\phi=61\%$) at different volume fraction $\psi$. Strain rate $1 \text{s}^{-1}$.

The stress response of the composite will vary if the fluid shear-thickening characteristics change. Illustrated in figure 5.10 is the stress-strain response of the composite filled to a volume fraction $\psi = 65\%$ with monodisperse silica suspension ($\phi=58\%$). In this case the stress response of STF filled foam is much lower than the previous examples and it is ‘comparable’ in magnitude and shape to the response of glycerol filled foam. This suggests that the shear thickening fluid inside the foam cell is not experiencing the high strain rates as captured by equation 5.1 because of the air content in the foam. In fact, at $\dot{\varepsilon}=1 \text{s}^{-1}$ the stress response of STF foam starts to increase dramatically around a strain of $40-45\%$. 

78
The different volume fraction dependence of the two composites, one made with STF with $\phi=61\%$ and the other $\phi=58\%$ lays on the extreme shear-thickening behavior of the most concentrated suspension. In this case the fluid will thicken even if not forced to flow through the pores, but by simply following the deformation of the cell walls at small strain.

![Stress-strain response of polyurethane foam filled with glycerol (dashed line) and shear-thickening fluid (solid lines) at a volume fraction $\nu = 65\%$.](image)

Figure 5.10 Stress-strain response of polyurethane foam filled with glycerol (dashed line) and shear-thickening fluid (solid lines) at a volume fraction $\nu = 65\%$.

5.5. **High impact energy testing: Dynatup**

Because of apparatus limitations constant strain rates testing was not possible with the texture analyzer for strain rates greater than 1 s$^{-1}$. A different technique was then utilized which included the use of a drop tower impact tester; details of the apparatus are described in section 3.3.2. The Dynatup was modified to achieve unconstrained
compression of foam composites: the tup was replaced by a 3 cm diameter aluminum rod and a flat aluminum plate was placed under the sample (see figure 5.11)

Figure 5.11 (left) original set up of Dynatup for sample penetration. (right) modified Dynatup set up for unconstrained compression.

5.4.3.1. Glycerol filled samples

For comparison samples filled with glycerol were tested first. Even at relatively low $e$ (energy per unit volume), the glycerol filled foam “bottom out” and expelled most of the fluid. At the lowest impact energies obtainable with this apparatus ($e = 1.3 \times 10^5$ J/m$^3$ for this specific samples) the glycerol sample was able to absorb up to 60% of the incoming energy, but as the impact energy increased its performance dropped down to 25%; this is illustrated in figure 5.12.
Figure 5.12 Percentage of energy absorbed for glycerol filled foam at \( \psi = 100\% \) vs. \( e \), impact energy per unit volume. The sample absorbed a percentage of the incoming energy before bottoming out.

From each impact the load vs. time data was collected and a representative pool is shown in figure 5.13(a). This data can be used to calculate the ‘nominal’ stress-strain curve (nominal because not a material function since the deformation rates were not constant during experiments) and can be compared to the stress-strain curves at low strain rates as shown in figure 5.13(b). High-speed video images of the impacts were also collected and a series is shown in figure 5.14.

Figure 5.13 (a) Load versus time data for \( e \) (impact energies /volume) that vary between \( 1.3 \times 10^5 \) J/m\(^3\) for glycerol sample filled to 100 %. (b) Calculated nominal stress versus strain for glycerol sample filled to 100 %. Stress-strain curve from low rate Texture Analyzer, \( \dot{\varepsilon} = 1 \) s\(^{-1}\), experiment also plotted for comparison.
Figure 5.14 High-speed video images of impact for a glycerol filled foam at $\psi =100\%$, $e=4.1\times10^5$ J/m$^3$, 55% energy absorbed.

5.4.3.2. Shear thickening fluid filled samples

Drop tower experiments were also carried out with foams sample filled with shear thickening fluid. Results have shown that STF filled foams are able to absorb most of the impact energy and the tup stops and rebound before the sample is fully strained. A typical response at relatively low $e$ (impact energy per unit volume) is shown in figure 5.15 with the corresponding video images. To note is that the peak in the stress does not correspond to the max strain in the foam and therefore the point at which the impact velocity is zero. This is a consequence of the highly non-linear response of this foam/fluid system. As the impact energies are increased the foam/fluid system response varies, the peak stress increases and the duration of the impact decreases; this is illustrated in figure 5.16. In these experiments the material was able to absorb about 90 % of the incoming energy, while the remaining 10% was returned in the rebound. As we increased the impact energies even further ($e > 6\times10^5$ J/m$^3$) we note a transition in the material response of the system. As the impact energy is further increased the peak stress reaches a maximum and then plateau (see figure 5.17), which suggests that a
Figure 5.15 (top) High-speed video images of impact for foam filled with 61% volume fraction silica suspension in ethylene glycol at $e=1.3 \times 10^5$ J/m$^3$, 89% energy absorbed. (bottom) Load vs time data for the same experiment. 7.4 kg is the weight of the crosshead, 1 m/s is the impact velocity which correspond to $e=1.3 \times 10^5$ J/m$^3$.

different mechanism for energy absorption comes into play. A clue on the mechanism for energy absorption comes from the high-speed video images of the sample during impact. As can be clearly seen in figure 5.18 after an initial deformation the fluid in the sample thickens and undergoes a transition form liquid to solid. If the impact energy is great enough the sample fractures and deforms in a viscoplastic fashion.
Figure 5.16 Impact data for foams completely filled with 61% volume fraction silica suspension in ethylene glycol, \( e \) varied between \( 1.3 \times 10^5 \) J/m\(^3\) to \( 4.1 \times 10^5 \) J/m\(^3\), 89% energy absorbed.

During the fracture and the propagation of the cracks the magnitude of the stress remain constant. This is further illustrated in figure 5.19 where the stress vs. strain has been plotted for varying magnitude of \( e \). For each experiment the stress reached a maximum value of 2.8 MPa after which it plateaus. The duration of this plateau (the amount of strain) dependents on the impact energy. These deformations are plastic in nature and very little recoil is observed.
Figure 5.17 Impact data for foams filled with 61% volume fraction silica suspension in ethylene glycol, $e$ varied between $3.2 \times 10^5 - 2.3 \times 10^6$ J/m$^3$. The data from the impact of glycerol filled foam is also shown for comparison.

Figure 5.18 High-speed video images of impact for foam filled with 61% volume fraction silica suspension in ethylene glycol at, $e = 1.6 \times 10^6$ J/m$^3$.

The amount of energy absorbed in these experiments varied with the impact energy $e$, and the percentage of energy absorbed (energy absorbed over impact energy) fluctuated between 80-90% (see figure 5.20) where the remaining energy was returned in rebound.
Figure 5.19 Stress-strain curves for foams filled with 61% volume fraction silica suspension in ethylene glycol, $e$ varied between $3.2 \times 10^5 - 1.6 \times 10^6$ J/m$^3$. The magnitude of the stress reaches a maximum of 2.8 MPa and then plateaus. Little recoil is observed.

Figure 5.20 Percentage of energy absorbed vs. impact energy for both foams filled with glycerol or shear thickening fluid.
5.6. **High rates testing: Split Hopkinson Pressure bars**

High strain rates experiments on foams filled with shear thickening fluid were carried out by using a split Hopkinson Pressure bar apparatus. Details on the apparatus and data reduction are presented in section 3.3.3. Aluminum pressure bars with a diameter of 19.05 cm and modulus of 72GPa were used.

The stress-strain response of these STF/foam composites at a strain rate of 800 s\(^{-1}\) is shown in figure 5.21. Only data greater than a strain of 4\% in shown because only then the sample reached equilibrium and was therefore in a condition of uniform deformation.

Higher strain rates experiments were also carried out but in this case the sample never reached a state of equilibrium.

The response of these composites at high strain rates is consistent with their behavior as seen from Dynatup drop tower experiments. In fact the stress in the material is shown to

![Stress-strain response](image-url)

*Figure 5.21 Stress-strain response of polyurethane foam filled with monodisperse silica suspension with \(\phi=61\%\) under constant strain rate compression. Strain rate of 800 s\(^{-1}\).*
grow until a plateau is reached (at around 15% strain). The value of stress at this plateau is also consistent with the one found by drop tower experiments, and it is around 2 to 3 MPa. The data comparison between experiments carried out with Dynatup and the high strain rates experiments of the split Hopkinson bars are shown in figure 5.22. By integration of the stress strain curve for the SHB experiment we can estimate the energy per unit volume that the composite absorbs during compression to be $1.0 \times 10^5$ J/m$^3$, which is in the same order of magnitude that the energy recorded during Dynatup experiments.

Unfortunately because of the limitation of this technique in measuring material properties of soft material (as foams) it was not possible to test the specimen at strains higher then 18% and therefore to sample the behavior along the stress plateau region.

![Stress-strain curves](image)

Figure 5.22 (solid lines) Stress-strain curves for foams completely filled with 61% volume fraction silica suspension in ethylene glycol, $e$ varied between $3.2 \times 10^5$ – $1.6 \times 10^6$ J/m$^3$. (in symbols) Stress-strain curve of same sample at strain rate of 800 s$^{-1}$. 
Chapter 6

6. Conclusion

This study was concerned with two major goals: understand and being able to predict the rheological response of shear thickening fluids, and characterize the energy absorption behavior of the fluid-foam system at different impact rates.

The rheological response of highly concentrated silica suspensions in both shear and extensional deformations was studied. Results show that in shearing deformations, extreme shear thickening occurs at higher volume fraction for the monodisperse spherical particles, \( \phi \geq 54\% \) than in the polydisperse suspensions where it occurs at \( \phi = 45\% \).

Additionally, the viscosity of the polydisperse suspension is found to be much greater than the monodisperse one; this phenomena is related to the particle shape, and shows that shear thickening fluid particles don’t necessarily have to be spherical or of regular shape (which usually translate to expensive), but can be made with cheaper particles, as in the case of polydisperse silica, obtained by grounding a solid substrates.

Experiments over a wide range of imposed oscillatory deformations were carried out and indicated that both the onset and magnitude of the shear thickening depend on the frequency and strain applied. These results are represented on a Pipkin diagram (which is shown in figure 4.6) and show that rapid time-varying deformations result in maximum energy dissipation. This diagram also suggests that there is a limit on how small of a strain is necessary for the suspension to shear thicken.

In shear flow experiments particular attention is given to the problem of slip and a number of different plates were used to try to eliminate this problem. It is found that an
aluminum plate polished to a 1μm roughness (i.e. on the same length scale as the particle size) minimizes the effect of wall slip, and therefore is used for all steady and oscillatory shear flow experiments.

The shear thickening fluid was also tested under extensional flow and results showed that at low extension rates the suspensions responds as a viscous rate-thinning fluid, whereas beyond a critical extension rate, the suspension strain-hardens and ultimately fractures in a solid-like fashion. By using a ‘Filament Stretching Extensional Rheometer’ the nominal extensional viscosity of the suspension was measured and compared with the corresponding shear viscosity (see figure 4.10), which was found to follow the same qualitative trend.

Open cell or ‘reticulated’ foam was chosen to envelop the fluid because by varying pore size one can control the local rates at which the fluid is strained and therefore control its material response. Also, the foam is flexible and can conform to various surfaces, and keeps the fluid contained but allows for fluid flow in between pores (which dissipated energy).

Constant strain rates compression test (\(\dot{\varepsilon} = 10^{-2} - 1 \text{ s}^{-1}\)) showed that even at low strain rates the STF filled foam at \(\psi\) of 100 % stiffened under compression. This is due to the effect of local straining of the shear thickening fluid, which occurs, at the length scale of the cell size. A more compliant foam (at low strain rates) was obtained by reducing the fluid volume fraction (\(\phi\)) and therefore the magnitude of the shear thickening effect.

The volume fraction of fluid inside the foam (\(\psi\)) was found to influence the system response and could be used as a parameter to vary the system material response.
Higher rates experiments (speed varying from 1-4 m/s and $e$ between $1.3 \times 10^5$ to $2.3 \times 10^6$ J/m³) were carried out with a drop tower system and have shown two different mechanisms for energy absorption. At lower impact speed the energy is absorbed by fluid viscous dissipation (same mechanism seen by low constant rates experiments), while after a critical impact energy is reached the fluid undergoes a transition from fluid to solid, cracks form and propagate and most of the energy is absorbed by “fracture” and plastic deformation (see figure 5.18).

Even higher strain rates (800 s⁻¹) were achieved by using a split Hopkinson bars apparatus. The material response of the composite in this high rate regime was found to be consistent with the data collected by the drop tower experiments. Unfortunately, in this regime we were not able to reach strains greater than 18%.
Bibliography


Hayes, W. C., S. N. Robinovitch, et al. (1997). Bone fracture prevention garment and method. USA, Beth Israel Hospital, Boston; President and Fellos of Harvard College.


