Synthetic Aperture Radar Interferometry with 3 Satellites

by

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ABSTRACT

Our study investigates interferometric SAR (InSAR) post-processing height retrieval techniques. We explore the possible improvements by adding a third satellite to the two already in orbit, and examine some potential uses of this setup. As such, we investigate three methods for height retrieval and compare their results with the original 2-satellite method. The first approach is data averaging; a simple method that extends from the results obtained using the 2-satellite method. The 3 sets of data obtained per sampling look are grouped into pairs, and the 2 statistical best pairs are selected to be averaged, producing a better estimate of the digital elevation map (DEM) height. The second approach is the unambiguous range magnification (URM) method, which seeks to ease the reliance on phase unwrapping steps often necessary in retrieving height. It does so by expanding the wrapped phase range without performing any phase unwrapping, through the use of different wrapping speeds of the 3 sets of satellite pairings. The third method is the maximum likelihood estimation technique, an asymptotically efficient method which employs the same phase expansion property as the URM to predict the closest phase estimate which best fits most (if not all) of the data sets provided.

Results show that for a handful of flyover looks, the data averaging method provides for an efficient and non-computationally intensive method for improving retrieved height results. This method can also help eliminate the need of GCPs in height retrieval, though such performance is limited by the presence of noise. The maximum likelihood method is shown to be asymptotically favorable over the data averaging method, if given a large number of flyover looks. The URM method performs worst, because it depends on the shortest baseline, which is most sensitive to noise, for unwrapping.

Results are entirely simulation-based, using the engineering tool Matlab Version 6.1. Single- and multiple- trial simulations are compared for 1-dimensional interferograms only. In most cases, the root-mean-square error will be used as the metric for comparison.

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Chapter 1

Introduction

1.1 Introduction to SAR Interferometry (InSAR)

Synthetic Aperture Radar (SAR) techniques have been in constant development since its advent in the 1950s. These techniques include geolocation, remote sensing, and interferometry, amongst many others, and one can find their use in fields ranging from agriculture to oceanography to even space exploration [1]. Interferometry, which subsequently leads to our area of interest (i.e. height retrieval), refers to the method of obtaining topographic information of a specified region by comparing phase values from 2 or more non-identical SAR images. This resultant phase plot image is commonly referred to as an interferogram. Potential uses of SAR interferometry include predicting seismic activity, tracking the motion of specified targets, and obtaining a three-dimensional digital height map, otherwise known as a Digital Elevation Map (DEM). Examples of an interferogram and its corresponding DEM are provided in Figure 1.

Figure 1: An Example of an Interferogram and corresponding DEM.
(a) Interferogram example. Axes are slant range ‘rho’ and ground range ‘index’.
The different methods used for height retrieval through interferometric SAR (InSAR) and their results will form the basis of our investigation. Unlike other methods of obtaining DEMs, such as tacheometry or photogrammetric methods [2], SAR interferometry provides the advantage that its results are largely independent of weather elements such as clouds, rain or atmospheric attenuation, making it easy to obtain terrain information throughout the year. In addition, unlike conventional passive radiometers which record data by collecting data from external sources like reflected solar energy, the SAR operates actively, meaning that it provides its own signal to obtain the SAR images, hence enhancing its capacity to operate anytime anywhere.

Many systems today use a 2-antenna setup for InSAR height retrieval [10]. These antennas are often mounted on separate satellites (space-borne) or on the same airplane (airborne) [1] – [3]. We will assume for the rest of this study that the SAR system is space-borne and thus satellite-mounted on separate systems taking slightly different flight paths, and collectively use the term “satellites” to refer to the mounted SAR “antennas”. The SAR images collected by the satellites are overlapped to reveal the interferogram through their respective phase data, and this “phase map” is subsequently processed to obtain the DEM of the specified area.

However, as we shall see, this configuration produces fairly inaccurate DEMs limited by not only the presence of noise but also in the correct estimation of its system orbit.
parameters. Furthermore, this method encompasses the need for ground control points (GCP) to resolve an accurate height estimation of the terrain-in-scope. GCPs are points on the actual terrain that have predetermined height, i.e. ground truth, measured either empirically, or by other methods of height estimation.

In addition, the phase data obtained from the SAR images are not given as absolute phase estimates, but rather are the absolute phase estimates modulo $2\pi$, i.e. the phase is wrapped, which cause fringe lines to surface on the interferograms obtained. This results in ambiguity when trying to obtain the DEM, since the fringe lines obstruct the ease of directly calculating the height from the phase. In addition, noisy data may cause unnecessary phase jumps (i.e. from 0 to $2\pi$) in the phase unwrapping procedure. Such phase ambiguity can cause slight errors in height retrieval, but may propagate to cause large global errors if not resolved early. In fact, even though phase unwrapping algorithms have been developed extensively over the past years [7]—[9], these techniques have largely proven elusive or complex for obtaining general solutions in obtaining the original phase for rough terrain, and hence phase unwrapping remains as a challenging problem in InSAR, prompting the development of the multi-baseline approach [21] – [26].

1.2 Problem of Interest

Our problem(s) of interest can be summarized as follows:
- To demonstrate the improvements that are possible with the 3 proposed methods of 3-satellite height retrieval.
- For the data averaging method, demonstrate that despite its simplicity it can still solve for previous problems encountered for the 2-satellite case, and in particular solve for baseline uncertainty in orbit parameter estimation.
- For the data averaging method, demonstrate that optimization in satellite positioning can be made possible to further improve retrieval results.
- For the unambiguous range magnification method, demonstrate the reduction in reliance on phase unwrapping techniques previously seen in the 2-satellite case.
- For the maximum likelihood estimation method, demonstrate asymptotically that improvements in height retrieval over the 2-satellite approach, data averaging as well as the URM methods are possible, and to a possibly large extent depending on the number of looks recorded.
- Between the 3 proposed methods, compare and contrast each of their results, limitations and conditions of use.

Note that although it is mentioned that 3 methods of height retrieval will be introduced, in truth all 4 methods (including the 2-satellite approach) introduced in this study will use the same height conversion formula, based entirely on geometric modeling of the 2-satellite system. The averaging method will use the height retrieval results directly from a pair of the 2-satellite configuration, whilst the unambiguous range magnification and maximum likelihood methods try to achieve the phase values for the 2-satellite height
conversion first, without yet using the height retrieval results for 2 satellites. We shall elaborate more of their differences in the upcoming chapters.
Chapter 2

Fundamentals of SAR Interferometry

2.1 SAR System Configuration

Analysis of SAR interferometry to obtain DEMs entails a need to first understand the SAR and its environment. To do so, it is necessary to clarify what it means by the Synthetic Aperture Radar (SAR), and how it differs from other conventional radar array configurations. The synthetic aperture radar obtains its name from its behavior as an antenna array using only a single radar system, as compared to classic antenna array configurations that require more than 1 physical array component present [1].

Consider Figure 2 of the SAR system.

Figure 2: The SAR system
Figure 2 shows a SAR system in orbit over a terrain of interest. The SAR system can be attached onto either a satellite or an airplane, and the satellite/airplane carries with it a single active antenna that both transmits and receives signals used to measure the terrain information. The frequency of the signals transmitted is within the microwave region (\( \lambda \approx 10^{-2} \) m), and the signals can be understood as a train of constant amplitude wave pulses emitting from the SAR antenna. The system is taken to be orbiting the Earth’s surface, and is not in geosynchronous orbit, with the system moving much faster with respect to the Earth’s spin motion. When this system passes over the specified area, it emits its signal(s) over the terrain. The microwave signals impinge on the ground below, and a portion of the signal energy gets reflected back to the relatively slow moving SAR receiver. We assume here that scattering effects allow a significant portion of the reflected data to return to the SAR system. The SAR system is moving at a speed of \( v_{\text{SAR}} \) and the time it takes for the signal to transmit and reflect back is given by \( t_{\text{send}} \). During this time, the SAR system would already have moved from its original position of transmission by a distance:

\[
d_{\text{SAR}} = v_{\text{SAR}} \cdot t_{\text{send}}
\]

This distance would correspond to the unit antenna-antenna distance of a linear equally spaced antenna array; provided that the SAR sends another signal the same instant it receives the reflected one. This way, when repeated over a few \( d_{\text{SAR}} \) lengths, the behavior of the SAR would mimic that of an antenna array. Since the SAR system emits pulses from an aperture that determines the terrain resolution, and the other antenna components in this “array” are virtual or synthetic, hence the term synthetic array (or aperture) radar is coined. Note here however that for the remainder of this thesis the effect of the time differences of arrival are ignored, since we are concerned with the post-processing aspect of the SAR only. Also, we have ignored the Earth’s curvature as well as its spin, hence adopting a flat-Earth assumption.

We now define the 2-dimensional coordinate system commonly used for sideward-looking (or along-track) SAR systems in interferometry. The first dimension is the slant range direction \( \rho \) in which the SAR transmits/receives its signals while the second dimension is the orthogonal flight path in which the system moves. The slant angle of beam release/capture is commonly referred to as the look angle \( \theta \). This coordinate system has already been demonstrated in Figure 2 above.

### 2.2 SAR Interferometry

Figure 2 depicts only a single SAR system. However, in interferometric SAR (InSAR), at least 2 systems will be required, each providing a different view of the same terrain. Yet, the physical existence of both systems within the same local space at the same time can be compromised, since height retrieval is a post-processing step done long after data collection. Conversely, this means that as long as 2 non-identical SAR images are taken from different viewpoints on the same area, a reasonable interferogram (and hence DEM) can be obtained. These 2 SAR images can therefore be taken by 2 SAR systems in the
same orbit (known as single-pass) or by a single SAR system flying over the same region in consecutive orbit passes (known as repeat-pass). Figure 3 demonstrates a comparison between both methods of SAR image retrieval. The exact shape of the orbits of the SAR systems is chosen depending on the type of data required, and is not of real interest in our investigation. We shall assume nonetheless a circular orbit.

![Diagram of SAR systems](image)

**Figure 3:** (a) Single-pass versus (b) Repeat-pass SAR interferometry.

The 2 variations described for image capture have their own advantages and disadvantages. In the case of repeat-pass image retrieval, the obvious advantage is in its lower cost, since the same single SAR satellite is used twice without the need of another coexistent system. However, as we see in Figure 3(b), the flight path of the repeat-pass SAR system is not the same path as it took on its previous orbit. This creates difficulty in estimating its system parameters, particularly its baseline. The accuracy of the baseline contributes significantly to the accuracy of the height retrieved, and hence the repeat-pass method is limited by its baseline accuracy. However, the single-pass 2-SAR system is not affected by its baseline accuracy, since the distance between the 2 systems on the same orbit can be found almost instantaneously. Likewise, cost becomes the major consideration when using single-pass InSAR techniques, since more SAR systems are required. We will adopt the single-pass retrieval process for the rest of this study, and briefly explore the possibility of solving for baseline uncertainty effects of both the single- and repeat-pass retrieval processes using the data averaging method in Chapter 4.3b.

Be it the single-pass or repeat-pass methods, it is nonetheless essential that the 2 looks of the system be different in order to produce any conceivable results through InSAR. The importance of this can be seen in Figure 4.
Figure 4 above depicts a flat view (1-dimensional) of 2 orbiting satellites in situ on the same plane at a certain point in time. This configuration is feasible in both single and repeat pass modes, and will be used repeatedly through the remainder of the thesis. This configuration of 2 satellites forms a single baseline, defined as the straight line distance between the 2 satellites. From Figure 4, we see that by arranging the 2 systems such that their positions do not overlap in the range direction, this allows 2 different look angles at a specific point on the terrain of interest. This allows a stereoscopic view of the terrain, much like how the eyes of a person allow him to judge distances from objects with a three-dimensional viewpoint. This stereoscopy forms the basis of the InSAR method, and works only for non-zero values of $B$.

The equivalent signal values returning to each of the 2 SAR systems are recorded in terms of their magnitude and phase. Therefore, for each pixel in each SAR image (corresponding to a point on the ground), we can write the signal value at that pixel as $Ae^{i\phi}$, where $A$ is the relative signal strength and $\phi$ is the signal phase. Since there are 2 SAR systems, we obtain 2 signal values for each pixel on the ground: $A_1e^{i\phi_1}$ and $A_2e^{i\phi_2}$. With these 2 values, we can obtain a coherence value between the 2 using the coherence estimator:

$$ Ae^{i\phi} = (A_1e^{i\phi_1}) \cdot (A_2e^{i\phi_2})^* $$

(2)

where $A = (A_1A_2)$ and $\phi = \phi_1 - \phi_2$. 

22
With this coherence value, we obtain a relationship between the 2 differing phase values for each look on the same SAR image pixel, and we use this phase relationship to obtain the corresponding height for the DEM. While the phase values are crucial to the height retrieval process, the magnitude of the coherence estimator cannot be neglected as well. This value indicates the degree of reliability between the 2 pixel images, and a high coherence ($A \sim 0.8$ to 1) indicates that using the phase value given by that estimator will produce a good height estimate. Conversely, a low coherence ($A \sim 0$ to 0.2) will not retrieve a good height estimate. Furthermore, the coherence value is sensitive to both noise and height of terrain, which will significantly affect the values obtained from SAR signal retrieval. For simplicity, the coherence effects will be ignored for most of Chapters 3 and 4 (i.e. we assume coherence estimator $A = 1$), while in Chapters 5 the definite variability of these values will be necessary in providing good results for the maximum likelihood estimation method.

In general, to avoid low coherence, one simple method would be to adopt the critical baseline condition, of which baselines exceeding this value will cause the signal values to become completely decorrelated [44], i.e. $A = 0$.

The critical spatial baseline length [44] is given by the following:

$$B_{\text{critical}} = \frac{A \rho_0 \sin(\theta_0 - \alpha)}{c} BW \quad (3)$$

where $\rho_0$ is the starting range distance (i.e. distance between SAR$_1$ and the closest pixel to the nadir), $\theta_0$ is the angle for the starting range, $\alpha$ is the slope angle and $BW$ is the system bandwidth. In all instances in this paper, we shall set our simulation parameters such that they exceed this demand.

In an ideal situation (i.e. noiseless and full coherence), an exact DEM can be obtained through geometric calculations [10]—[12]. However, in real-time scenarios, the height retrieval is often not so easily obtained. Effects of noise, orbital parameter estimation errors, as well as the curvature of the Earth all contribute to complicating the process of height retrieval. We will deal with most of these problems in this study, with the exception of the Earth's curvature, though this can be corrected for as well [12]. Nonetheless, the height retrieval process from SAR image data can be simplified into several consecutive steps, as shown in the flowchart in Figure 5 [1].
The processes of SAR data processing (not to be confused with the post-processing steps in interferogram evaluation), co-registration, coherence determination and phase unwrapping are ignored for the remainder of this study, since they are not within the scope of our objectives. Our scope, on the other hand, will involve modifying the existing methods of noise filtering and interferogram evaluation such that improvements are evident without a need for introducing a new phase unwrapping procedure. Furthermore, the effects of low coherence due to difficult terrain, like layover and shadowing, are also out of scope, and hence we shall use simple terrain simulations that conveniently ignore the need to account for these problems. We briefly mention the method used to account for foreshortening in the next section.

Figure 5: Height Retrieval Flow-Chart (from [1])
2.3 Height Retrieval through Geometric Modeling

In this section, we shall explore the method of converting the phase values measured from the SAR systems into their corresponding height, i.e. we interpret the interferogram such as in Figure 1(a) to produce its corresponding DEM as in Figure 1(b). This method is analytical, and is based on geometric modeling of the SAR satellite models. For simplicity, the 2-satellite configuration is used.

In truth, as mentioned in Chapter 1, the three 3-satellite height retrieval methods introduced in this thesis will also use this geometric model of height retrieval for the 2-satellite configuration. The 3-satellite model of the data averaging method will be using this method two more times to account for the 2 additional interferograms resultant from the additional satellite. For both the URM and the maximum likelihood models, the 2-satellite height conversion model will be used for the two furthest SAR satellite positions after a new phase estimate has been obtained.

We are now ready to investigate how geometric methods can be used to obtain height estimates of the 2-satellite InSAR method. To do so, we first work through the reverse problem, i.e. given the actual height and system parameters we obtain the phase values for the interferogram. As mentioned in the previous section, these phase values are physically derived from the reflected signals measured by the two SAR systems (see Equation 2). Using this, we then move to the forward problem, and directly solve for the height using simple geometric calculations and the phase values, assuming ideality in the InSAR environment, i.e. no noise and no errors in system parameters. Note that the system parameters refer to parameters that affect the orbit of the SAR system, such as baseline $B$, look angle $\theta$, baseline slant angle $\alpha$, height of the system $H$ and slant-range projection $X$. The next section will explore the importance of accuracy of baseline uncertainty.

2.3.1 Obtaining Phase Relationship from Terrain Features

We start with a 1-dimensional (i.e. flat) view of the terrain, and seek to obtain the phase difference between the 2 receiving SARs. The 2-dimensional interferogram can be obtained by taking snapshots of the terrain (at every azimuthal pixel width) and joining the 1-dimensional SAR images together. Note that all the simulations performed in this thesis will be based on the 1-dimensional view only, since extension to the 2-dimensional view is not difficult and is not the focus point here. Figure 6 on the following page shows the 1-dimensional single baseline configuration previously seen. We assume that the orbital parameters are known and are accurate, i.e. we have the following parameters: height $H$ of the system SAR$_1$, distance $X$ from the base of SAR$_1$ to the region of signal incidence, pixel ground range distance $\Delta x$ within the region of signal incidence, the look angle $\theta$, the baseline $B$, the baseline slant angle $\alpha$, the wavelength of each pulse $\lambda$ and the height of the terrain $\Delta z$ at the point of incidence $P$. 

25
We first obtain the range distances $\rho_1$ and $\rho_2$ from a particular point $P$ to each of the SAR satellites.

\[
\rho_1 = \sqrt{(H - \Delta z)^2 + (X + \Delta x)^2}
\]

\[
\rho_2 = \sqrt{(H - \Delta z + B \sin \alpha)^2 + (X + \Delta x - B \cos \alpha)^2}
\]

The range distances are used to obtain the interferometric path length $\delta$, which refers to the distance traveled by each signal back and forth for each SAR antenna.

\[
\delta = 2(\rho_1 - \rho_2).
\]

The interferometric phase difference is then obtained through the relationship:

\[
\phi = k\delta = \frac{2\pi}{\lambda} \delta = 4\pi \left(\frac{\rho_1 - \rho_2}{\lambda}\right).
\]

The phase difference in this case would correspond to the phase difference of the complex vector obtained in Equation (2). Hence, each pixel on the pixilated terrain map would have a phase difference value given by this relationship.
2.3.2 The Wrapped Phase Problem

However, the absolute phase difference calculated in Equation (7) is continuous, while the phase difference measured is discontinuous, since the phase follows a periodic cycle. This means that the phase obtained through the real SAR systems is the absolute phase wrapped modulo $2\pi$. This hinders the real phase from being used to directly retrieve the height, and the wrapped phase has to be unwrapped before the height can be retrieved. The $2\pi$ jumps appear on black and white interferograms as white lines called fringe lines (see Figure 1(a)). Note here that the true value of the quotient when wrapping is performed (i.e. the quotient obtained when dividing the phase value by $2\pi$; remainder obtained is the wrapped phase value) is forever lost, and the only way to retain this value with respect to the 2-satellite setup is through Ground Control Points (GCPs), which contain the true height values on the ground obtained through other methods of measurement.

The most obvious way to unwrap the phase is to just add $2\pi$ whenever a $2\pi$ phase jump occurs. However, this technique is severely limited by the presence of background noise, which will inevitably occur in real scenarios, since backscattering signals received by the SARs may also come from thermal radiation, or from any other possible emitting source outside the SAR system. The presence of noise disrupts the phase unwrapping procedure of adding $2\pi$ since noise near the $2\pi$ regions may cause even more phase jumps to occur. This would mean that adding $2\pi$ around the additional fringe lines might cause unwanted height jumps in the height retrieval process, leading to inaccurate height estimates.

The wrapped phase noise problem and the loss of the quotient value have formed the bane of SAR interferometry researchers over the history of the SAR and no complete solution has been truly derived. Yet, there are several ways proposed to get around them. For instance, many phase unwrapping algorithms are available to estimate the height in various types of terrain; however this is beyond our scope here. For the remainder of our thesis, the unwrapping algorithm adopted will be that provided by Matlab Version 6.1’s unwrap algorithm, which provides a simple tool for unwrapping—whenever an instance of a preset tolerance level is exceeded between 2 adjacent points, the unwrap algorithm will add a value of $2\pi$ to the second point to account for the phase jump. The default tolerance level is set at $\pi$. It is important to know that this simple algorithm cannot account for many instances of noise perturbation, but works well for relatively low noise cases, which in our case still makes good for comparison.

2.3.3 Obtaining Terrain Height from Geometric Calculations

With the phase values unwrapped directly from the interferogram, we can directly obtain the terrain height $\Delta z$ (assuming we have the values $H$, $X$, $a$, $B$) using geometric calculations. In this case, we find the values of $\Delta x$ and $\Delta z$. Using the phase relationship, we can get back the path difference.

Rearranging Equation (7),

$$\delta = \frac{\lambda}{4\pi} \varphi$$

(8)
Using cosine rule, we obtain the relationship between the look angle $\theta$ and the path length $\delta$, which is given by:

$$\theta = \alpha + \arcsin \left( \frac{\delta + B}{2\rho_i} - \frac{\delta^2}{2B\rho_i} \right)$$  \hspace{1cm} (9)

Finally, the height $\Delta z$ at the point $P$ is given by:

$$\Delta z = H - \rho_i \cos \theta$$  \hspace{1cm} (10)

Therefore, the height variation for the entire 1-dimensional case can be found assuming equidistant values of $\Delta x$. Yet, these height variations may not exactly reflect the ground-height variations. The derived height values have to be modified a little, because misalignment of the height coordinates in the horizontal direction can occur due to foreshortening.

Foreshortening refers to the effect of having irregularly spaced $\Delta x$ components (i.e. uneven pixel widths) when retrieving the height. This effect occurs when the pulses that are emitted from the SAR impinge on undulating terrain and their reflected signals do not return the SAR systems at the same time. The SAR system receives data according to increasing values of the range distance (in time), since the pulses that reach nearer range distances are collected by the SAR system first. This, when tabulated and worked backwards according to increasing range order, creates the illusion that taller objects seem nearer than they really are if their range distances are shorter, and shorter objects seem further than they really are from the SAR in the ground range direction if their range distances are further. Hence, for a retrieved $\Delta z$ value, we correct for foreshortening by aligning this height value with the corrected $\Delta x$ value, $\Delta x'$, where $\Delta x'$ is given by:

$$\Delta x' = \Delta x + (\Delta z)\tan \left( \frac{\pi}{2} - \theta \right)$$  \hspace{1cm} (11)

Using Equation (11), taller objects that seemed nearer in range direction now are placed further away in ground range direction, and vice versa. Thus, position $P = (\Delta x', \Delta z)$ instead of $(\Delta x, \Delta z)$, which will give the correct height to ground range position. It is wise to note that if noise is present, the foreshortened ground range distance given by (11) will also be affected by noise, since it is dependent on the retrieved $\Delta z$ value. Therefore, noise will significantly affect the arrangement and acquisition of terrain features.

Finally, the three-dimensional DEM can be reconstructed from the two-dimensional interferogram using this height retrieval method by combining several such two-dimensional interferograms spanning a fixed ground range distance.
2.4 The Test Model

2.4.1 Simulation Parameters and Ideal Test Terrain

We are almost ready to perform our simulations. However, we need to first specify the test terrain to be used for comparison purposes. Naturally, a perfect match in height for all the pixels used in the test plot would be the ideal solution. For simplicity, we shall use a simple sinusoid (see Figure 7), coupled with the system parameters specified in Table 1 for the 2-satellite system. Modifications will be made for the 3-satellite scenario, and will be presented later on in the relevant chapters.

The system parameters are given by:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Parameter Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline Length [m]</td>
<td>200</td>
</tr>
<tr>
<td>Pixel Range Resolution [m]</td>
<td>1</td>
</tr>
<tr>
<td>Number of pixels</td>
<td>128</td>
</tr>
<tr>
<td>Baseline elevation angle, ( \alpha ) [°]</td>
<td>35</td>
</tr>
<tr>
<td>Height of SAR\textsubscript{i} from nadir, ( H ) [m]</td>
<td>5E5 (i.e. space-borne)</td>
</tr>
<tr>
<td>Distance from nadir, ( X ) [m]</td>
<td>3E5</td>
</tr>
<tr>
<td>Wavelength, ( \lambda ) [m]</td>
<td>0.03</td>
</tr>
<tr>
<td>Bandwidth, ( BW ) [MHz]</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 1: Simulation Parameters for 2-satellite Configuration

The sinusoid is given by:

\[
\Delta h = 10 \sin \left( \frac{\Delta x + X}{20} \right)
\]  

(12)

where \( \Delta h \) is the terrain height and \( \Delta x \) is the pixel separation on the ground. As before, \( X \) is the distance from the base of SAR\textsubscript{1} \( O \) to the region of signal incidence.
2.4.2 Noise Model

It is difficult in general to exactly characterize each of the noise contributions to the SAR signals measured in practical situations, but alas noise characterization is amongst the least of our concerns here. Thus, it should not be an overwhelming concern if we were to adopt a simple noise model to be added to the ideal model. This noise model should be deemed qualified if all 3 methods demonstrated should adopt the same model, and that the model should not show any bias towards a particular method. With this, we introduce the simple noise model to be used in all 3 instances:

To the calculated range distances $\rho$, we add values generated from the following noise model:

$$\Delta \rho_{\text{noise}} = \frac{\lambda}{4 \cdot 180} n$$

(13)

where the value $n$ refers to the variance of a specified noise level (in degrees) to be added. This noise model would hence be a zero mean white Gaussian noise model of variance specified by Equation (13). This value would be added directly to the range values obtained through Equations (4) and (5), performed on the ideal terrain sinusoid given by Equation (12). This value will be obtained with the help of Matlab Version 6.1’s \texttt{randn} function, which generates a zero mean random variable with unit variance. To obtain the variance values specified, we multiply that square root of that value (i.e. the standard deviation) to the \texttt{randn} function. The absence of a bias (i.e. zero mean) demonstrates no systematic errors in our noise model.
Note here that the exact characterization of noise onto the interferogram phase values through this setup would not be exactly clear, since firstly the interferogram value involves the complex multiplication of 2 SAR signals with Gaussian distribution (of which the exact resultant random variable’s distribution becomes complex and hard to solve [5] – [6]), and secondly the wrapping of absolute phase distorts the random variable’s expected behavior. Once again, the exact characterization for this would not be necessary, albeit having it might have simplified the model even further. Furthermore, it would be wise to note that this adds a significant degree of correlation ambiguity between all 3 interferogram values, which may or may not be the case for the real scenario. Nonetheless, the model proposed by Equation (13) will suffice. However, as we shall see, the maximum likelihood approach will adopt a different noise model from this, in an effort to be consistent with the original model for maximum likelihood estimation with 3 satellites.

2.5 Metric Used

For each method, we will first demonstrate their applicability for a single random simulation, and show their corresponding terrain plots with respect to the noiseless one.

In order to account for the presence of random error generated from Matlab’s randn function, we perform Monte Carlo simulations for each method introduced, and find the average RMS height error for repeated trials of about 100 simulations. The mean RMS error would therefore be our metric used to measure improvement, with a lower mean RMS value indicating better performance. Units will be presented according to the International System of Units (SI) format, i.e. in meters for the mean RMS height error.

The mean RMS height error of a single pixel is given by Equation (14) below.

\[
\text{r.m.s height} = \sqrt{\frac{1}{N} \sum_{k=1}^{N} \left[h_{\text{noise}} - h_{\text{orig}}\right]^2}
\]  

(14)

where \(N\) is the number of trials for the same pixel, \(h_{\text{noise}}\) is the noise-affected retrieved height using the proposed methods, and \(h_{\text{orig}}\) is the noise-free height.

2.6 Plot Alignment

Because of the loss of the quotient value when wrapping (even in ideal scenarios), this would cause a constant value misalignment in the unwrapped phase value, after applying Matlab’s unwrap function as mentioned before. To account for this, it would be necessary for some ground truth indicator (a.k.a. a GCP) to provide for this constant value, of which height comparison would be impossible without. Since our test model is
itself simulated, we have to provide for our own ground truth. This is done quite simply by assuming that the ground truth for the first pixel in the interferogram is provided for. This would provide for an approximation to account for the misalignment to the unwrapped phase value.

Yet, this approximate ground truth value would only provide accuracy for the first pixel point. To further fine-tune the best results obtainable in our simulations, as well as to remove any systematic errors and biases that might occur, we perform plot alignment between the simulated results of each method (with noise) with the noise-free original terrain plot. This alignment is performed through overlapping the "centers" of both the vertical and horizontal axes (i.e. mean of the heights and medians of the range), followed by interpolation of the noisy height results onto the pixel intervals of the noise-free plot. Finally, the RMS height errors would then be calculated with respect to the noise-free plot. In truth, this might not be the best result achievable through plot alignment for each method, but nonetheless it will be used for the following reasons: simplicity and fairness of comparison. Another point to note would be that for an actual scenario it would be unreasonable to apply this alignment method, and using ground truth through GCPs might be a more viable method.

To perform the plot alignments, we need to find the central point of all the curves, and align them such that this central point overlaps for all the curves. This central point is given by 2 values, one for the horizontal, and the other the vertical. The vertical value is given by the arithmetic mean of the heights given by all the pixels. Hence, the vertical center is:

$$h_{\text{vertical}} = \frac{1}{N} \sum_{k=1}^{N} h(x_k)$$

where $N$ is the number of pixels, and $h(x_k)$ is the height which is dependent on the horizontal position. Since the distance from the first to the last pixel may be different for the curves in the presence of noise (due to foreshortening), hence we set the horizontal center to be the midpoint of each of the curves. Aligning these curves would hence involve shifting all the values to match the curve centers. In addition, linear interpolation of the heights has been applied such that the horizontal spacing is aligned for all plots, and is evenly distributed according to the original curve (after foreshortening). This allows for direct comparison of the heights. At the ends, whenever interpolation is not possible, we hold the values to be constant and equal to the last interpolated value.
Chapter 3

2-Satellite InSAR Height Retrieval

3.1 2-Satellite SAR System Setup

With the test model provided in Chapter 2.4, we can now begin simulating the results for the 2-satellite configuration. We shall use the same simulation parameters as stated in Table 1.

3.2 Simulations and Results for 2-satellite Height Retrieval

We start by presenting the results of a single simulation trial, with noise level \( n \) set at 5°. Figure 8(a) shows the height retrieval results (see Chapter 2.3) before plot alignment as described in Chapter 2.6, while 8(b) shows the aligned plots. To demonstrate the absence of a non-zero bias in our plots, we display a histogram of the error between the aligned retrieved height profile and the original terrain plot, as in Figure 8(c). This error is simply the height difference between the 2 pixels. Furthermore, we can calculate the average RMS error for that single trial, by assuming that equal noise weighting has been placed on each pixel. With that, we can simply use Equation (14) to find this value, with \( N \) replaced to \( M \), the number of pixels per interferogram. We then increment the noise levels from 5° to 25° at regular intervals of 5°; Table 2 shows this. Data presented on Table 2 include the maximum error amongst all pixels on the interferogram and the RMS error for \( N = \) pixel count = 128.
Figure 8: 2-satellite height retrieval for $B = 200m$.
(a): Unaligned Height Plot for $B = 200m$, (b): Aligned Height Plot.
Figure 8: 2-satellite height retrieval for $B = 200$ m.
(c) Histogram for height difference between original and retrieved height profiles.

![Histogram for Error Count of B13 with noise 5°](image)

Table 2: Error distribution for 2-satellite height retrieval. As noise level increases, RMS and maximum error also increases.

<table>
<thead>
<tr>
<th>Noise Level [°]</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Error [m]</td>
<td>0.7426</td>
<td>2.9163</td>
<td>5.5038</td>
<td>4.0792</td>
<td>6.3644</td>
</tr>
<tr>
<td>RMS Error [m]</td>
<td>0.4200</td>
<td>0.8441</td>
<td>1.3107</td>
<td>1.3875</td>
<td>1.7902</td>
</tr>
</tbody>
</table>

From Table 2, it is clear that both RMS and maximum errors increase for increasing noise levels. However, the increments presented here are irregular and hence unreliable, since they represent the results of only 1 trial. To reduce the effect of random error, we perform 100 simulations and find the mean and standard deviations RMS values for these simulations. These results are tabulated into Table 3.
<table>
<thead>
<tr>
<th>Noise Level [°]</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline [m]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>1.6161</td>
<td>3.2020</td>
<td>4.8371</td>
<td>6.2352</td>
<td>7.5570</td>
</tr>
<tr>
<td>100</td>
<td>0.8285</td>
<td>1.6377</td>
<td>2.5386</td>
<td>3.3040</td>
<td>4.0975</td>
</tr>
<tr>
<td>150</td>
<td>0.5686</td>
<td>1.1116</td>
<td>1.6577</td>
<td>2.2317</td>
<td>2.7835</td>
</tr>
<tr>
<td>200</td>
<td>0.4286</td>
<td>0.8461</td>
<td>1.2583</td>
<td>1.6515</td>
<td>2.0672</td>
</tr>
<tr>
<td>250</td>
<td>0.3507</td>
<td>0.6752</td>
<td>1.0252</td>
<td>1.3475</td>
<td>1.6626</td>
</tr>
<tr>
<td>300</td>
<td>0.2927</td>
<td>0.5748</td>
<td>0.8293</td>
<td>1.1240</td>
<td>1.3788</td>
</tr>
<tr>
<td>Baseline [m]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>0.2123</td>
<td>0.3916</td>
<td>0.5553</td>
<td>0.7491</td>
<td>0.8417</td>
</tr>
<tr>
<td>100</td>
<td>0.0967</td>
<td>0.1954</td>
<td>0.2759</td>
<td>0.3819</td>
<td>0.5040</td>
</tr>
<tr>
<td>150</td>
<td>0.0823</td>
<td>0.1400</td>
<td>0.2216</td>
<td>0.2336</td>
<td>0.3375</td>
</tr>
<tr>
<td>200</td>
<td>0.0548</td>
<td>0.1188</td>
<td>0.1623</td>
<td>0.2101</td>
<td>0.2465</td>
</tr>
<tr>
<td>250</td>
<td>0.0487</td>
<td>0.0787</td>
<td>0.1215</td>
<td>0.1631</td>
<td>0.2185</td>
</tr>
<tr>
<td>300</td>
<td>0.0385</td>
<td>0.0789</td>
<td>0.0962</td>
<td>0.1436</td>
<td>0.1501</td>
</tr>
</tbody>
</table>

Table 3: Simulation Results for mean and standard deviation of RMS height error using 2-Satellite Height Retrieval, with varying Baseline Lengths and Noise Levels.

From this Table, we observe that the increments in mean RMS error follow a linear path, as shown in Figure 9. Furthermore, if we vary the baseline length value, and set it at different lengths while keeping the noise value constant, we observe that the mean RMS errors are directly proportional too. This is also shown in Table 3, where we have simulated the mean RMS errors of 2 other lengths as well.

With this, we propose the relationship between the mean RMS error with the baseline length and noise level.

$$[\text{Mean RMS Error}]_{2\text{-sat}} \propto \frac{n}{B}$$ (16)

We can check the validity of this postulate by plotting the RMS height values of Table 3 against $n$ and $B$ respectively. These are shown in Figures 9 and 10 respectively.
Figure 9: Plotting the linear relationship for mean RMS height error against $n$.

Figure 10: Plotting the linear relationships for $1/(\text{mean RMS height error})$ against $B$. 
Similar conclusions can be obtained for the standard deviation of the RMS height error, i.e.

\[
[\text{Std RMS Error}]_{2-nst} \propto \frac{n}{B} \tag{17}
\]

Note also that these conclusions only apply for the range of values tested, and might not hold for values outside this range, where the coherence values might interfere with the decrease in RMS error.

### 3.3 System Sensitivities: Baseline Uncertainty

From Equations (9)–(11), one can acknowledge that the forward method of height retrieval is heavily sensitive to the accuracy of the parameters used to estimate the retrieved height. To illustrate the sensitivity towards accuracy of the system parameters (which include \(H, X, \alpha, B\) and \(\varphi\)), we shall investigate the effects of one of these parameters, i.e. the importance of accuracy in the baseline estimate. We do so by demonstrating mismatches in the estimation of our original parameters used for height retrieval. That is, we first “collect” the phase data by simulating the collected phase data for a particular setup (given in Table 1). We follow this by trying to deliberately making a close, but wrong estimation to the baseline parameter, *ceteris paribus*. This parameter, when fitted into the height retrieval algorithm, will reveal the effects of baseline uncertainty to a given height retrieval process. We align the centers of the resultant plot with the original test terrain, and again compare the mean RMS height error. We shall ignore the effects of noise generated through the noise model, and hence only 1 simulation is needed per trial (i.e. the resultant error is systematic and deterministic, and not random).

To simulate the interferometric phase data, we assume that the problem was set up with actual baseline length \(B = 200\)m, as of Table 1. We first start with an intelligent “guess” to our actual \(B\) value, set \(B' = 199.9\)m < \(B\), where we have labeled the “guess” \(B'\). The aligned plot for this guess is shown in Figure 11(a), while after alignment the plot is shown as Figure 11(b).
Figure 11(a): Unaligned Plots for 2-satellite Height Retrieval.

'x': \( B' = 199.9 \text{m} \), '—': \( B = 200 \text{m} \).

Figure 11(b): Aligned Plots for 2-satellite Height Retrieval.

'x': \( B' = 199.9 \text{m} \), '—': \( B = 200 \text{m} \).
From 11(a), we see that a small error in parameter estimation results in a large misalignment in the unaligned plots. Even though this misalignment is somewhat insignificant after alignment (as in Figure 11(b)), it is in fact a huge issue concerning the need for GCPs. The fact that the retrieved height profile could be aligned to the original terrain plot has already assumed the existence of an “original terrain”, or at least a ground truth point. If this point(s) is not present, then only Figure 11(a) would hold true, hence amplifying the importance of the accuracy of the system parameters used. Also, note that in the case of the unaligned plot, the retrieved profile is nearer and lower to the nadir of the satellites than the real terrain really is.

Even though alignment between the original terrain and the retrieved heights may seem good with the use of GCPs as in Figure 11(b), this is really not so in reality. Figure 12 shows the plots when a larger guess difference is taken, this time with $B' = 195m$.

Figure 12(a): Unaligned Plots for 2-satellite Height Retrieval.

‘x’: $B' = 195m$, ‘—’: $B = 200m$. 

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Figure 12(b): Aligned Plots for 2-satellite Height Retrieval.
‘x’: $B' = 195\text{m}$, ‘—’: $B = 200\text{m}$.

From Figure 12, we see that before alignment, the problem encountered by the lack of GCPs is amplified multiple-fold. After plot alignment, it also seems that the retrieved height no longer accurately represents the original profile. In fact, it seems that points before (to the left of) the horizontal midpoint (see Chapter 2.6) appear lower than they should be, while points after (to the right of) the same point appear higher. We shall explain this phenomenon, as well as the alignment problem after we explore the effects that occur for $B' > B$. The case of $B' = B$ is ignored, since we obviously know these implications. Figures 13(a) and (b) show the plots for $B' = 200.1\text{m} > B$, while 13(c) shows the aligned plot for $B' = 205\text{m} > B$. 
Figure 13: Plots for 2-satellite Height Retrieval, using $B' = 200.1\text{m} > B$.
(a) Unaligned Plot, (b) Aligned Plot
Figure 13: Plots for 2-satellite Height Retrieval, using $B' = 205 \text{m} > B$.

(c) Aligned Plot

We see that the effects of height retrieval are reversed when choosing $B' > B$. Here, in the absence of GCPs, the unaligned retrieved profile appears higher and further away than the original terrain (Figure 13(a)). After alignment, the effects of choosing a larger baseline causes the points to the right of the midpoint to appear lower, while points to the left appear higher. Furthermore, when we plot the RMS height values of the aligned terrain profiles with respect to changing $B'$ guesses we will obtain the results in Figure 14.
Figure 14: RMS Height Error Variations for $B'$

Figure 14 shows what one can expect: As the $B'$ estimate improves, the RMS error begins to converge, and vice versa when $B'$ deviates.

To explain the phenomenon of height misalignment as seen in Figures 11–13, one needs to examine the equations used for height retrieval, namely Equations (9), (10) and (11) from Chapter 2. They have been repeated and modified below to fit for $B'$:

\[
\theta' = \alpha + \arcsin \left( \frac{\delta}{B'} + \frac{B'}{2\rho_l} - \frac{\delta^2}{2B'\rho_l} \right) \quad (18)
\]

\[
\Delta z' = H - \rho_l \cos(\theta') \quad (19)
\]

\[
\Delta x'' = \Delta x + (\Delta z') \tan \left( \frac{\pi}{2} - \theta' \right) \quad (20)
\]

The parameters, namely $H$, $\alpha$, $\rho_l$, and $\delta$ remain unchanged. To examine the effect of a change of baseline $B'$ on the look angle $\theta'$, it would be imperative to examine Equation (18) in detail. By plugging standard values for $H$, $\alpha$, $\rho_l$, and $\delta$ along with $B'$, we see that of the 3 terms in the arcsine function in (18), the determining factor in this function is the ratio of $\delta/B'$, since this value is largest in terms of magnitude amongst all 3 terms. This ratio is a negative value for our system, since $\rho_2$ is larger than $\rho_l$ for our configuration.
(see Equations (4) to (6)). Therefore, we can make the following approximation to Equation (18):

\[ \theta' = \alpha + \arcsin \left( \frac{\delta}{B'} \right) \]  

(21)

where \( \left( \frac{\delta}{B'} + \frac{B'}{2\rho_1} - \frac{\delta^2}{2B' \rho_1} \right) = \frac{\delta}{B'} < 0. \)

Here, we suppose that \( B' \) is a small decrement. This leads to an increment in \( 1/B' \), which conversely leads to a decrement for \( \delta/B' \) (since \( \delta < 0 \)). Following the arcsine relationship shown in Figure 15, we see that for a small decrement in \( \delta/B' \) in the region where \( \delta/B' < 0 \), we will expect a corresponding decrement in the arcsine function.

Figure 15: Decrement in \( \delta/B' \) leads to a decrement in arcsine value.

This being so, we can conclude from Equation (21) that the value of \( \theta' \) will also decrease. (Note that \( \theta' \) is always greater than zero for the entire SAR configuration to be physical). Figure 16 shows the result of a small decrement in angle for the cosine function (where \( \theta' > 0 \)), which is an increment in the cosine value.
From Equation (19), an increase in the cosine value will cause a corresponding decrease in the overall $\Delta z'$ function. This means that the height retrieved with a small decrement in $B'$ will be less than with the original $B$ for all heights (i.e. $|\Delta z'| < |\Delta z|$ for $\Delta z > 0$ and $|\Delta z'| > |\Delta z|$ for $\Delta z < 0$). Furthermore, from Equation (20), we see that foreshortening with $\theta'$ causes a conflicting effect between the decreasing negative cosine function for $\Delta z'$ and the increasing tangent function (i.e. $\tan(\pi/2-\theta')$ increases). Yet, it can be easily shown that the tangential increase does not outweigh the cosine function for the values given in our setup, hence $\Delta x''$ will be also less than would the actual foreshortened $\Delta x'$.

With the effects of smaller $\Delta x''$ and $\Delta z'$, the effect of the unaligned height profiles retrieved would cause the height profile due to $B'$ to be much nearer and lower to the origin than the original height profile would have, as we saw in Figures 11(a)—13(a). Furthermore, we expect the center of this height profile to have the same characteristics of the overall profile: lower and more forward center. This effect when aligning the plots with their centers causes the plot to the left of the center to be less than the original height profile, while the plot to the right of the center to be greater than the original height profile. Therefore, when aligned, the height retrieved for $B' < B$ appears such that the leftward side appears lower, while the rightward side appears higher, as was seen in Figures 11(b)—13(b), and 13(c) as well.

As described in this section, the effects of parameter uncertainty (and in this case, baseline uncertainty) can have drastic effects on the accuracy of height retrieved. This becomes a bigger problem if no prior ground truth were available, a realistic scenario.
which can be painted for space exploration missions. Also, even if only some ground truth were present, there seems to have been no way to know if the baseline estimation were accurate, since this would only be known if the entire terrain were present, and not just some points as assumed so. Hence, it would be essential for a method which can allow for the elimination of the need for ground truth, as we shall see in the next chapter.
Chapter 4

3-Satellite InSAR Height Retrieval:
Data Averaging Method

4.1 SAR Interferometry using 3 satellites

With a good understanding of the problems of the 2-satellite approach, we can now explore the possibility of improving the results of single baseline interferometry with multiple baselines. For simplicity, we have assumed a 3-satellite (or orbital) configuration. The reason why 3 satellites are adopted is merely to demonstrate that an additional measurement system can greatly reduce the ambiguity problems encountered by having just 2 satellites. Naturally, this could be extended for N-satellite systems, of which one can expect better retrieval results every time a system is added.

As mentioned before, we shall adopt 3 different methods of determining the height needed for constructing a DEM with 3 satellites. Firstly, we look at the data averaging method. With 3 SAR systems we can combine them to form 3 non-identical pairs. This allows us to choose the most suitable height results from the 3 pairs, using the height retrieval methods seen in Chapter 2. (Note: With N-satellites, there will be \( \frac{N!}{(N - 2)!2!} \) interferograms). These results are then averaged to give a better estimate of the height than each pair would give. Intuitively, this method promises better results, since random error fluctuations would be averaged out. Furthermore, prominent error sources from 1 set of interferogram data can be weeded out if the corresponding data from the other 2 sets differ distinctly from those of that set. Hence, we expect better results in terms of smaller root-mean-square error using this method of data averaging. However, there are limitations to the effectiveness of this method, as will be demonstrated. We shall also explore some applications of this seemingly simple method, such as removing the need for GCPs and also satellite position optimization.

The second and third methods were first proposed by F. Lombardini to obtain a maximum likelihood estimator (MLE) for the interferometric phase value to undergo as few phase unwrapping steps as possible [21] – [26]. To do so, it would be necessary to expand the wrapped phase range of within \( 2\pi \) to something possibly larger. Only then will the estimator provide for less unwrapping procedures. This method is made possible with the existence of a third satellite, since the wrapping rates of each satellite would be different, assuming the satellites are asymmetrically distanced from one another. This expansion of wrapped phase is known as unambiguous range magnification (URM), and
it applies simple geometry along with some reasonable assumptions to increase the unambiguous phase range from merely $2\pi$ to a multiple of $2\pi$. In special cases where absolutely no phase unwrapping is necessary; the need for ground control points (GCP), since the absolute phase may be completely retrieved entirely. Using the URM's expanded phase range, the MLE method would then determine the most probable phase value which most accurately approximates the actual noiseless value. Again, conditions apply for the MLE and URM methods to function properly. Due to the close similarities which they share, we shall explore both URM and MLE procedures together in the next chapter.

4.2 3-Satellite SAR System Setup

The 3-satellite (or multiple baseline) configuration used to study the method of data manipulation is simply an extension of the 2-satellite method. As mentioned before, our system will be flying in the single-pass mode of InSAR, where the baseline accuracy ensures that the noise levels will be the main contribution to discrepancies in our data. From Figure 4 seen in Chapter 2, the single baseline is simply the direct distance between the 2 SAR satellites. If an additional SAR satellite is placed somewhere on this baseline (i.e. the 3 satellites are collinear), then the 3 satellites (labeled SAR$_1$, SAR$_2$ and SAR$_3$) can be paired up in $(3!) / 2 = 3$ different. We assume a collinear alignment for simplicity purposes, though this may not necessarily be the case for the data averaging method. The resultant 3 baselines formed will be named $B_{13}$, $B_{12}$ and $B_{23}$, where $B_{13} = B_{12} + B_{23}$, $B_{13}$ is the distance between SAR$_1$ and SAR$_3$, $B_{12}$ for SAR$_1$ and SAR$_2$, and $B_{23}$ for SAR$_2$ and SAR$_3$. We shall use these 3 baselines to investigate the effect of data averaging. As we have seen in Chapter 3, $B_{13}$ is equal to the length of the best 2-antenna InSAR method (since it is longest). Figure 17 below shows the configuration used for the 3-satellite configuration.
The only modifications made to Table 1 are the baseline lengths:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Parameter Value [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_{13}$</td>
<td>200</td>
</tr>
<tr>
<td>$B_{12}$</td>
<td>150</td>
</tr>
<tr>
<td>$B_{23}$</td>
<td>50</td>
</tr>
</tbody>
</table>

Table 4: Additional Simulation Parameters to Table 1 for the 3-satellite Configuration

Note that the baseline parameters adopted satisfy the critical baseline condition given in Equation 3 (~3000m for flat slope). Also, another condition is that the baseline lengths are arranged such that SAR$_2$ is closer to one of the satellites, which in our case is towards SAR$_3$. This is a necessary condition for the data averaging method to work, and we will demonstrate in a later section that in fact there will be only a fixed range within which data averaging can be performed.
4.3 Data Averaging

The method of data averaging is simple and straightforward. Because of the randomness (and hence non-uniqueness) of noise, the 3 noise-affected interferograms obtained from the 3 SAR systems will no doubt produce non-identical noise-affected height profiles. Hence, when averaging the height profiles obtained by unwrapping each of the 2-satellite interferograms, we expect that random error will be reduced by this process; producing results that will be at the least perform better than 1 of the height results produced from each interferogram, provided there exists an asymmetry in the arrangement of the 3 SARs.

With careful considerations of the numerical results produced by each of the satellite pairs, one can apply weighting functions such that the final averaged result using all 3 sets of unwrapped height profiles will produce even better results than each of the 3 individual results produced by the satellite pairs. Hence, the appropriate averaging function has to be chosen:

\[
 h_{BAverage} = \frac{w_{12} \cdot h_{B12} + w_{13} \cdot h_{B13} + w_{23} \cdot h_{B23}}{w_{12} + w_{13} + w_{23}}
\]  
(22)

where \( h_{BAverage} \) is the averaged height profile, \( h_{Bij} \) is the height profile for the corresponding baseline separation \( B \) between SARs \( i \) and \( j \), \( w_{ij} \) is the appropriate weighting function and \( \{i,j\} = \{1,2,3\}, \ i \neq j \). The weighting function \( w_{ij} \) would be dependent on parameters like \( B_{ij}, n \), and other external contributions that may affect the noise levels of the corresponding height profiles retrieved. Further optimization may be possible by choosing the best weighting function for different terrain characteristics.

For our case, we consider a simplification of Equation (22).

\[
 h_{BAverage} = \frac{h_{B12} + h_{B13}}{2}
\]  
(23)

where we have simply assigned the following weighting values to the equation:

\[
w_{12} = 1; w_{13} = 1; w_{23} = 0
\]  
(24)

Here we have assumed that the weighting functions \( w_{12} \) and \( w_{13} \) are the same, i.e. we gave equal importance to height profiles \( h_{B12} \) and \( h_{B13} \). This assignment would not lead to an optimum solution. From our setup in Table 4, we see that \( B_{13} > B_{12} > B_{23} \), and also from Equation (16) this would cause \( \text{RMS}_{13} < \text{RMS}_{12} < \text{RMS}_{23} \). Hence, placing equal weighting to the weighting functions \( w_{12} \) and \( w_{13} \) will not result in the best achievable results. Though this conclusion is logical, our motive at present is not to find the best
averaging method that works, but rather to prove that the general method works. Hence, we will first derive simulation results for the assignments given in Equation (24).

Furthermore, we wish to show that the weighting functions chosen will invariably result in determining if the averaging method given by Equation (22) will work. Hence, we shall compare the results of assigning Equation (22) values with that of Equation (25) shown below:

\[ w_{12} = 1; w_{13} = 1; w_{23} = 1 \]  

(25)

Hence, the averaging model to compare is:

\[ h_{BAverage} = \left( \frac{h_{B12} + h_{B13} + h_{B23}}{3} \right) \]  

(26)

4.4 Simulations and Results for Data Averaging

4.4.1 Single and Repeat Trial Results

Like before, we first perform a single simulation trial for each satellite pair to give the reader a rough sense of what to expect in a real data collection scenario. Furthermore, we perform the averaging methods given by Equations (23) and (26) to show how the results will look. Figures 18(a) and (b) demonstrate these results. Our noise level for simplicity has been set at 5°.
Figure 18: Aligned Height Profiles
(a): For the 3 pairs of 2-satellite configurations, using the baseline parameters in Table 4.
(b): Using the data averaging methods of Equations (23) and (26).
Histograms for the 2 averaging methods are given in Figures 19(a) and (b).

Figure 19: Zero-biased Histogram for Data Averaging Method (a) for Equation (23), (b): for Equation (26).
Figure 18(a) clearly demonstrates the conclusions seen earlier in Chapter 2 for the 2-satellite arrangement. In each curve on the plot, we see that for a given noise level, the baseline to RMS error relationship is an inverse one. Figure 18(b) shows the results we are looking for: that the choice of weighting function clearly affects the effectiveness of the data averaging method. Figures 19(a) and (b) are included to show the non-biasness of the original and averaging methods introduced.

We wish to find the error results of the single simulation performed for the data averaging method. These are reflected in Table 5 below.

<table>
<thead>
<tr>
<th>Baseline Length</th>
<th>$B_{13}$</th>
<th>$B_{12}$</th>
<th>$B_{23}$</th>
<th>$B_{AVG-2}$</th>
<th>$B_{AVG-3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Error [m]</td>
<td>1.6754</td>
<td>1.7345</td>
<td>3.8701</td>
<td>1.2327</td>
<td>1.3153</td>
</tr>
<tr>
<td>RMS Error [m]</td>
<td>0.3821</td>
<td>0.5106</td>
<td>1.5576</td>
<td>0.3323</td>
<td>0.5217</td>
</tr>
</tbody>
</table>

Table 5: Simulation Results for the single simulation trial at $5^\circ$ noise. $B_{AVG-2}$ represents the method using Equation (23), while $B_{AVG-3}$ uses Equation (26).

From Table 5, we see that the averaging method performed by Equation (23), for which we label the '2-component averaging method', performs better than the best results amongst the 2-satellite methods (i.e. than $B_{13}$, the longest baseline length). However, the same cannot be said for Equation (26), which we call the '3-component averaging method'. This method only performs better than the 2-satellite methods in 1 case, which is arrangement for $B_{23}$, the shortest length. To be sure, we repeat the simulations for 100 trials, and gather the RMS results to confirm our conclusions. These results are presented in Table 6.

<table>
<thead>
<tr>
<th>Noise [$^\circ$]</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline Mean RMS [m]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B_{23}$</td>
<td>1.7131</td>
<td>3.3687</td>
<td>4.6879</td>
<td>6.3159</td>
<td>7.5588</td>
</tr>
<tr>
<td>$B_{12}$</td>
<td>0.5542</td>
<td>1.1163</td>
<td>1.7344</td>
<td>2.2060</td>
<td>2.7890</td>
</tr>
<tr>
<td>$B_{13}$</td>
<td>0.4246</td>
<td>0.8418</td>
<td>1.2603</td>
<td>1.6737</td>
<td>2.1011</td>
</tr>
<tr>
<td>$B_{AVG-2}$</td>
<td>0.3834</td>
<td>0.7401</td>
<td>1.1211</td>
<td>1.4426</td>
<td>1.8402</td>
</tr>
<tr>
<td>$B_{AVG-3}$</td>
<td>0.6322</td>
<td>1.2487</td>
<td>1.7875</td>
<td>2.3997</td>
<td>2.9471</td>
</tr>
<tr>
<td>Baseline Std RMS [m]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B_{23}$</td>
<td>0.2489</td>
<td>0.3544</td>
<td>0.5802</td>
<td>0.7608</td>
<td>0.7494</td>
</tr>
<tr>
<td>$B_{12}$</td>
<td>0.0720</td>
<td>0.1355</td>
<td>0.2627</td>
<td>0.2744</td>
<td>0.3627</td>
</tr>
<tr>
<td>$B_{13}$</td>
<td>0.0603</td>
<td>0.1011</td>
<td>0.1709</td>
<td>0.2137</td>
<td>0.2727</td>
</tr>
<tr>
<td>$B_{AVG-2}$</td>
<td>0.0523</td>
<td>0.0993</td>
<td>0.1895</td>
<td>0.1881</td>
<td>0.2413</td>
</tr>
<tr>
<td>$B_{AVG-3}$</td>
<td>0.0981</td>
<td>0.1304</td>
<td>0.2111</td>
<td>0.2772</td>
<td>0.2815</td>
</tr>
</tbody>
</table>

Table 6: Simulation Results for 100 Trials with the 3-satellite configuration

We confirm the conclusions made for the data averaging methods from Table 6—Data averaging is a method of which its reliability depends on the weighting functions applied. For example, in the case of the 3-component averaging method, the wrong choice of weighting functions leads to results which perform poorer than 2 pairs of 2-satellite configurations. This is because the error values of the shortest baseline (i.e. $B_{23}$) are so large in comparison to the other 2 sets that they “pollute” the accuracy of the other 2
baselines. This causes improvements compared to the error values of the shortest baseline only, but the reverse can be said to the other 2 baseline results. On the other hand, the 2-component averaging method fares well in our case, which further backs up our conclusions of requiring a good weighting function estimate. We also conclude that with the right weighting functions, data averaging can become a simple yet reliable method for height retrieval. While the mean RMS values indicate the expected RMS value for each method proposed, the standard deviation indicates the range of deviation by which one can expect the RMS values to lie within. Observing Table 6, we see that again the best results (i.e. smallest deviation) are given for the 2-component averaging method, while the 3-component case is only better than that of $B_{23}$.

Using the results of the 2-component data averaging method, we can calculate an equivalent 2-satellite baseline length which would lead to the same mean RMS values as calculated. We tabulate these theoretical lengths in Table 7 using Equation (16).

<table>
<thead>
<tr>
<th>Noise</th>
<th>5°</th>
<th>10°</th>
<th>15°</th>
<th>20°</th>
<th>25°</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Baseline [m]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B_{Eqvt.2}$</td>
<td>221.4306</td>
<td>227.4787</td>
<td>225.4727</td>
<td>232.6931</td>
<td>229.4539</td>
</tr>
<tr>
<td>$B_{Eqvt.3}$</td>
<td>134.5119</td>
<td>134.8379</td>
<td>139.4926</td>
<td>138.2578</td>
<td>140.3638</td>
</tr>
</tbody>
</table>

Table 7: Theoretical Equivalent Baseline Lengths using Data Averaging Results

We see from Table 7 that the theoretical equivalent baseline length for the 2-component averaging method is larger than the length of $B_{13}$, which is the longest baseline length at 200m. Furthermore, the equivalent baseline length for the 3-component averaging method is such that it is shorter than $B_{12}$ (at 150m), but longer than $B_{23}$ (at 50m). These values are also approximately independent of noise level, and also of the critical baseline length (since these values have no real physical equivalence).

Even though the results obtained using the 3-component averaging method seem to provide worse-off results when compared to the 2-component methods, in truth this may only be a result of using the simulation parameters given in Table 4. This implies that we should not be so ready to discard the 3-component averaging method, because there may be instances by which it may provide better results than the 2-component averaging method. These instances, however, are not of concern in our simulations. For the remainder of the Chapter, we shall focus on the effects of the 2-component averaging method instead, and demonstrate the applications and limitations of using the weight functions given by Equation (23) to give the 2-component averaging method.

### 4.4.2 Threshold Baseline Length

With the 2-component averaging method, we now wish to find the limits within which good results can be obtained. Since from Equation (16) we have seen the inverse relationship of mean RMS error to baseline length, and from the 3-component averaging method we have seen how the shortest baseline length can cause the most significant pollution to the data integrity, hence we seek to find the lower bound below which the 2-component averaging method will fail. In other words, we want to find the shortest $B_{12}$
length for a fixed $B_{13}$ length such that the 2-component averaging method will only provide results as good as $B_{13}$'s results. Table 8 and Figure 20 show the variation of RMS error for the 2-component averaging method with respect to variations in $B_{12}$ length, at $n$ set at $5^\circ$ and for $B_{13}$ fixed at 200m. The RMS values for $B_{13}$ are also added to the Table and Figure as a fair test, and should be largely unvarying in this simulation.

<table>
<thead>
<tr>
<th>$B_{12}$ [m]</th>
<th>RMS Error for $B_{12}$ [m]</th>
<th>RMS Error for $B_{13}$ [m]</th>
<th>RMS Error for $B_{\text{Avg-2}}$ [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.4428</td>
<td>0.8209</td>
<td>0.4945</td>
</tr>
<tr>
<td>110</td>
<td>0.4352</td>
<td>0.7838</td>
<td>0.4811</td>
</tr>
<tr>
<td>120</td>
<td>0.4244</td>
<td>0.7156</td>
<td>0.4478</td>
</tr>
<tr>
<td>130</td>
<td>0.4298</td>
<td>0.6498</td>
<td>0.4198</td>
</tr>
<tr>
<td>140</td>
<td>0.4234</td>
<td>0.6000</td>
<td>0.3971</td>
</tr>
<tr>
<td>150</td>
<td>0.4236</td>
<td>0.5676</td>
<td>0.3821</td>
</tr>
<tr>
<td>160</td>
<td>0.4203</td>
<td>0.5237</td>
<td>0.3665</td>
</tr>
<tr>
<td>170</td>
<td>0.4365</td>
<td>0.5090</td>
<td>0.3708</td>
</tr>
<tr>
<td>180</td>
<td>0.4246</td>
<td>0.4737</td>
<td>0.3537</td>
</tr>
<tr>
<td>190</td>
<td>0.4219</td>
<td>0.4518</td>
<td>0.3428</td>
</tr>
<tr>
<td>200</td>
<td>0.4305</td>
<td>0.4304</td>
<td>0.3352</td>
</tr>
</tbody>
</table>

Table 8: Average RMS error of $B_{\text{Avg-2}}$ against $B_{12}$ for 100 trials, $n = 5^\circ$.

Figure 20: Plot of RMS error for $B_{\text{Avg-2}}$, $B_{12}$ and $B_{13}$ against $B_{12}$ length.

We see from the plots that the RMS error for $B_{\text{Avg-2}}$ increases with decreasing $B_{12}$ lengths. From both the Table and plot in Figure 20, we see that there is a lower threshold value of
$B_{12}$ beneath which causes the RMS error of the 2-component averaging method to be greater than that of $B_{13}$, hence resulting in the single baseline method proving more effective. From the curve, this occurs at about $B_{12}$ length $\approx 127m$. Conversely, as $B_{12}$ approaches $B_{13}$ in length, we expect decreasing RMS error for the 2-component averaging method. The best results are hence at the point where $B_{12}$ equals $B_{13}$ in length. It may seem unphysical for this point to exist in the 1-dimensional case (since SAR$_2$ overlaps with SAR$_3$), but as we shall see in the next section, this in fact results in the optimum positioning of antennas possible, provided that the longest baseline length satisfies the critical baseline condition given in Equation (3). Hence, it is possible to set up a range of values between the lower threshold and the upper critical baseline condition in which the 2-component averaging method will work. Thus,

$$B_{\text{threshold}} \leq B_{\text{Avg-2}} \leq B_{\text{Critical}}$$  \hspace{1cm} (27)

In addition, it would be wise to note that in our simulation, we start with $B_{12}$ at half of $B_{13}$'s length, i.e. at 100m. This would correspond to $B_{23} = B_{12}$, which means SAR$_2$ is exactly at the midpoint between SAR$_1$ and SAR$_3$. If $B_{12}$ were less than this value, this would indicate that $B_{23}$ would be longer than $B_{12}$, and hence the setup would be reversed. We therefore expect a reflection of the results we have seen. In short, $B_{23}$ would replace $B_{12}$ in all instances prior seen, and vice versa.

The reason behind why $B_{12}$ and $B_{13}$ should converge for maximum RMS results can be simply explained through reasoning of Equation (16). Since $B_{12}$ is at its maximum when it is equal to $B_{13}$, hence through Equation (16) the RMS height error for $B_{12}$ is at its minimum. When put into the averaging method of Equation (23), it is therefore straightforward to expect minimum RMS height error for the averaging method too.

Therefore, we can conclude the following about data manipulation of the 3-satellite configuration. Above a certain threshold, the 3-satellite configuration provides for a better estimation of terrain features than that of the 2-satellite one using the data averaging technique. An example of this technique has been shown to be the 2-component averaging method in Equation (23), and the results have shown that as long as $B_{12}$ remains close in length to $B_{13}$, there will be significant improvements in height obtained. Furthermore, we have shown that the choice of weighting functions matter in determining the capabilities of the data averaging method. The choice of setup for SAR satellites also matter, as shown in Table 8 and Figure 20. The data averaging method can also be applied easily to 2-dimensional interferograms, and the results obtained here will still hold, since the 2-dimensional method merely involves repeating the 1-dimensional method many times along the flight direction.
4.5a Applications: Optimizing Results

4.5a.1 Optimization Strategy using Data Averaging

Our current setup assumes a collinear alignment of SAR satellites. From the previous section, we saw that as $B_{13}$ and $B_{12}$ approach the same length, the RMS error of the 2-component averaging method decreases steadily (i.e. it improves.) The best RMS values would hence be at the point at which $B_{13} = B_{12}$ in length, as seen in Figure 20. This would seem non-physical for the 1-dimensional view; since this would mean the second and third satellites (i.e. SAR2 and SAR3 from Figure 17) would be on the same point in space. However, if we consider the three-dimensional viewpoint of the SAR satellites, this configuration is actually physical, and is displayed in Figure 21 shown below.

![Figure 21](https://example.com/figure21.png)

**Figure 21:** Physical configuration of the 3-antennas in cartwheel rotation.
(a) Position which results in the general configuration

(b) Position which results in optimum configuration

In Figure 21, we show the 3-dimensional view of the 3 antennas in motion. Here we have assumed that the 3-antennas are moving in a circular cartwheel motion on the same plane, as in [25], [26]. We first assume that the 3 antennas are in the position depicted in Figure 21(a). In the height-azimuth plane (i.e. the right diagram of Figure 21(a)), we see the
cartwheel configuration of the SAR systems, whereby the rotation of the antennas is slow compared to their motion along the azimuth direction. (Ignore the Earth’s spin). The height projections of these antennas are plotted against the range direction (as we see on the left). This results in the general configuration of the three antennas that we are familiar with (as seen in Figure 17). If the antennas are rotated along such that their position becomes that of Figure 21(b), then the height projections of these antennas result in equal baseline lengths for $B_{12}$ and $B_{13}$. This is exactly the optimum configuration as predicted in the previous section, and is obviously not unphysical. Therefore, from the results obtained in the previous section, this configuration should correspond to producing the lowest RMS error through the 2-component averaging method.

However, note that for a circular cartwheel rotation of fixed radius, the length of $B_{13}$ varies with the position of the satellites, as opposed to the fixed $B_{13}$ length assumed in the previous section. We investigate the effect of the rotation on the baseline length $B_{13}$, and see if there is any effect on the RMS error using the 2-component averaging method. From Figure 22 of the equilateral triangle, we obtain the relation between $B_{13}$, $B_{12}$ and the angle of rotation, $\beta$. We define this angle to be that between the flight path (i.e. azimuthal direction) and the normal through the midpoint of SAR$_1$ and SAR$_3$.

From Figure 22, we tilt the satellites such that the lengths of the baselines $B_{13}$ and $B_{12}$ depend on the rotation angle. We set the radius of the circle, $R$, to be such that the distance between SAR$_1$ and SAR$_3$ equals 200m, as seen in the previous section. For this maximum length to be equal to the 1-dimensional projection length, the rotation angle $\beta$ must be set to 0. Hence, the maximum distance between SAR$_1$ and SAR$_3$ would correspond to $B_{13} = 200m$. In this case, SAR$_2$ must bisect the length, i.e. $B_{12} = 100m$. For the lengths corresponding to Figure 21(b), i.e. $B_{13} = B_{12}$, we must find the projection length such that $\beta$ must be set to $\pi/6$. This causes $B_{13} = B_{12} \approx 173m$. Hence we obtain the relationship between $B_{13}$, $B_{12}$ and $\beta$ as in Equations (28) and (29).

\[
B_{13} = R \cos(\beta) 
\] (28)
\[ B_{12} = R \cos \left( \frac{\pi}{3} - \beta \right) \]  

(29)

where \( R \) is the displacement length between SAR_1 and SAR_3 = 200m.

Hence, we construct Table 9 and Figure 23 to compare the RMS values for 100 trials while varying the values of \( \beta \) (and hence \( B_{13} \) and \( B_{12} \)). Note that \( \beta \) varies between 0 and \( \pi/6 \) only. This is because beyond that, \( B_{13} \) will not be the largest baseline length anymore, and hence the changing of components will be necessary for data manipulation. All other parameters are kept the same as in the previous section.

<table>
<thead>
<tr>
<th>( \beta ) [°]</th>
<th>( B_{13} ) [m]</th>
<th>( B_{12} ) [m]</th>
<th>RMS Error for ( B_{13} ) [m]</th>
<th>RMS Error for ( B_{12} ) [m]</th>
<th>RMS Error for ( B_{AVG} ) [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>200</td>
<td>100</td>
<td>0.4337</td>
<td>0.8261</td>
<td>0.4887</td>
</tr>
<tr>
<td>5</td>
<td>199.2389</td>
<td>114.7152</td>
<td>0.4270</td>
<td>0.7528</td>
<td>0.4655</td>
</tr>
<tr>
<td>10</td>
<td>196.9616</td>
<td>128.5574</td>
<td>0.4317</td>
<td>0.6389</td>
<td>0.4177</td>
</tr>
<tr>
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</tr>
<tr>
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<td>0.3963</td>
</tr>
<tr>
<td>25</td>
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<td>163.8303</td>
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<td>30</td>
<td>173.2051</td>
<td>173.2051</td>
<td>0.4879</td>
<td>0.4838</td>
<td>0.3752</td>
</tr>
</tbody>
</table>

Table 9: Comparing how rotation angle \( \beta \) affects RMS error, \( n = 5° \).

Figure 23: Plot of RMS error against rotation angle.
As expected from Equation (16), we see from Figure 23 that the inverse relationship of the baseline with the RMS error causes increasing RMS error for \( B_{13} \) while decreasing RMS error for \( B_{12} \). This corresponds to the increase in \( \beta \) values, which causes \( B_{13} \) to fall and \( B_{12} \) to increase towards \( B_{11} \). Hence, the RMS errors for both \( B_{13} \) and \( B_{12} \) are expected to converge with increasing \( \beta \) values. However, the increase in RMS error for \( B_{13} \) is not enough to offset the improvement of RMS error through the 2-component averaging method, caused by the decrease in RMS error of \( B_{12} \). This is clear from Figure 23, where the RMS error for the averaging method gradually decreases for increasing \( \beta \). Note that at \( \beta \) below 8°, the threshold condition is not exceeded, and hence there is no improvement of RMS error for the averaging method as compared to the best single baseline RMS error. Since the maximum possible rotation angle \( \beta \) is at 30°, therefore the optimum position for the 3 antenna system using the data manipulation method occurs at this angle, i.e. at \( B_{12} = B_{13} \) as in Figure 21(b).

Comparing the improvement in the RMS errors, we see that the optimum configuration provides an average RMS error of about 0.38m (i.e. \( B_{13} = B_{12} \)), and when compared to the best single baseline RMS error of about 0.43m (i.e. longest \( B_{13} \)), our method provides an improvement of about 12% in accuracy. Furthermore, if a more appropriate weighting function was adopted in Equation (22), more improvement might have been possible.

In short, we derived the optimum alignment of the baselines for \( B_{12} \) and \( B_{13} \), assuming Equation (27) is satisfied.

### 4.5b Applications: Solution for Baseline Uncertainty

We saw in a previous chapter the importance of baseline estimation accuracy in the height retrieval method adopted. Also, we emphasized the problem of requiring GCPs before plot alignment can occur within a satisfactory range of accuracy. In this section, we shall solve the problem of GCP requirement, and simultaneously propose a solution to estimating the baseline parameter using the 3-satellite configuration.

Simply put, the problem seen in the 2 satellite case seems tripled, since with 3 satellites we get 3 sets of baselines and hence 3 interferograms. If we fix the position of 1 satellite, say SAR1, we see that the other 2 satellite positions may vary, and hence there will be 2 other baseline parameters that are made uncertain. The third baseline parameter is merely the difference between the 2 uncertain satellites; hence if we are "guessing" the other 2 values, this would make that 3rd parameter "certain".

#### 4.5b.1 Simulations with some/all ground truth

We shall use the data averaging method as a tool to help solve our problem of predicting correctly the baseline length. First of all, we wish to investigate the effects of the averaging method in predicting baseline uncertainty. Figures 24(a)—(e) below show the corresponding variations of RMS height errors as \( B_{12} \) and \( B_{13} \) lengths change. (\( B_{23} \) length variations follow naturally).
Figure 24: Variation of RMS Height Error of Varying Baseline Lengths.

(a) $B_{13}' = 180\, \text{m} < B_{13}$, (b) $B_{13}' = 190\, \text{m} < B_{13}$
Figure 24: Variation of RMS Height Error of Varying Baseline Lengths.
(c) $B_{13}' = 200\,\text{m} = B_{13}$, (d) $B_{13}' = 210\,\text{m} > B_{13}$
Figure 24: Variation of RMS Height Error of Varying Baseline Lengths.

(e) \( B_{13}' = 220\text{m} > B_{13} \)

From Figure 24(a) – (e), we see that as \( B_{13}' \) and \( B_{12}' \) approach their correct parameters, the RMS error for \( B_{\text{Avg}}' \) will approach the zero value. While there can be many instances where RMS error for \( B_{13}' \) and \( B_{12}' \) can be zero on their own, there is only one instance where RMS error of \( B_{\text{Avg}}' \) is zero, i.e. at \( B_{13}' = B_{13}, B_{12}' = B_{12} \). At this point, both RMS error for \( B_{13}' \) and \( B_{12}' \) are zero simultaneously as well. This results are confirmed in Table 10, where we tabulate the RMS height error for \( B_{\text{Avg}}' \) against varying \( B_{13}' \) and \( B_{12}' \) lengths. Again, we see that the RMS error of \( B_{\text{Avg}}' \) is zero at one and only one point (see underlined bold box).

<table>
<thead>
<tr>
<th>( B_{12}' ) [m]</th>
<th>180</th>
<th>185</th>
<th>190</th>
<th>195</th>
<th>200</th>
<th>205</th>
<th>210</th>
<th>215</th>
<th>220</th>
</tr>
</thead>
<tbody>
<tr>
<td>125</td>
<td>2.3241</td>
<td>2.0248</td>
<td>1.7232</td>
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<tr>
<td>130</td>
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<td>1.4994</td>
<td>1.1983</td>
<td>0.9002</td>
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<td>0.2317</td>
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<td>1.5748</td>
<td>1.2743</td>
<td>0.9734</td>
<td>0.6748</td>
<td>0.3912</td>
<td>0.1603</td>
<td>0.2374</td>
<td>0.4765</td>
</tr>
<tr>
<td>140</td>
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<td>1.0488</td>
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<td>0.4085</td>
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<td>145</td>
<td>1.4216</td>
<td>1.1237</td>
<td>0.8236</td>
<td>0.5231</td>
<td>0.2242</td>
<td>0.0716</td>
<td>0.3496</td>
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<td>1.1993</td>
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<td>0.5996</td>
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<td>0.0000</td>
<td>0.0000</td>
<td>0.5699</td>
<td>0.8386</td>
<td>1.0963</td>
</tr>
<tr>
<td>155</td>
<td>0.9872</td>
<td>0.6863</td>
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<td>0.0850</td>
<td>0.2185</td>
<td>0.5086</td>
<td>0.7882</td>
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<td>1.3132</td>
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<tr>
<td>160</td>
<td>0.7884</td>
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<td>170</td>
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<td>0.2316</td>
<td>0.2850</td>
<td>0.5476</td>
<td>0.8385</td>
<td>1.1277</td>
<td>1.4082</td>
<td>1.6771</td>
<td>1.9345</td>
</tr>
</tbody>
</table>

Table 10: RMS height error of \( B_{\text{Avg}}' \) using varying \( B_{13}' \) and \( B_{12}' \) lengths
This ‘zero’ point of which absolute zero height error is obtained (highlighted boxes) can be used as a vantage point whereby we set this up as a marker to check for accuracy in baseline parameter guess.

4.5b.2 Simulations without ground truth

However, we have in the above case assumed that we already knew the total ground truth of the terrain beforehand. Next, we shall investigate the possibility of checking for relative error instead of absolute error; that is, rather than using the total known terrain as a base for comparison, we shall calculate the relative error between $B_{13}'$ and $B_{12}'$ retrieved heights, and likewise for $B_{13}'$ vs. $B_{23}'$, $B_{23}'$ vs. $B_{12}'$, $B_{13}'$ vs. $B_{\text{Avg}}'$, and so on. In order for absolute accuracy to be achieved, we expect a similar ‘zero’ point to appear exactly at a perfect match. We demonstrate this with the following diagrams:

![Plot of RMS Height Error](image_url)

**Figure 25:** Variation of RMS Height Error of Varying Baseline Lengths.
(a) $B_{13}' = 180 \text{m} < B_{13}$
Figure 25: Variation of RMS Height Error of Varying Baseline Lengths.
(b) $B_{13}' = 190\text{m} < B_{13}$, (c) $B_{13}' = 200\text{m} = B_{13}$

(b) $B_{12}$ 
(c) $B_{23}$
Figure 25: Variation of RMS Height Error of Varying Baseline Lengths.
(d) $B_{13}' = 210m > B_{13}$, (e) $B_{13}' = 220m > B_{13}$
To generate Figures 25(a) – (e), we have set the reference terrain as \( B_{13}' \)'s terrain instead. This means that instead of finding the relative RMS error with respect to the original test terrain as described in Chapter 2.4, which we did for all cases seen previously, we instead find the relative RMS error with respect to \( B_{13}' \)'s plot instead. The choice of \( B_{13}' \)'s plot is because if noise were considered in the plots, \( B_{13} \) would be most stable to noise, as we saw in Equation (16). Hence, \( B_{13}' \) would be a logical choice for setting as a reference.

From the Figures, we see that the relative error again reaches zero at one and only one point (underlined bold box), though this time it comes close at 2 other plots as well (bold boxes). Table 11 below demonstrates the absolute zero point again.

<table>
<thead>
<tr>
<th>( B_{13}' ) [\text{m}]</th>
<th>180</th>
<th>185</th>
<th>190</th>
<th>195</th>
<th>200</th>
<th>205</th>
<th>210</th>
<th>215</th>
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</tr>
<tr>
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<td>0.0001</td>
<td>0.3368</td>
<td>0.6604</td>
<td>0.9718</td>
<td>1.2690</td>
<td>1.5537</td>
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<td>0.2523</td>
</tr>
</tbody>
</table>

Table 11: RMS height error of \( B_{Avg}' \) using varying \( B_{13}' \) and \( B_{12}' \) lengths, but with \( B_{13}' \) as reference

From Table 11, we see that only at \( B_{13}' = B_{13} \), and \( B_{12}' = B_{12} \), only then will the RMS error for the averaging method become zero. This demonstrates that even without GCPs or any ground truth, we are still able to find the point at which the baseline parameters can be deemed accurate. However, in this case, there are 2 other points demarcated in Table 11 which have RMS error for \( B_{Avg}' \) very close to zero. Hence, we conclude that if ground truth were not present the degrees of accuracy in RMS error become crucial for estimating the baseline parameters accurately.

4.5b.3 Simulations with noise and without ground truth

The solutions proposed before have assumed a noiseless configuration. However, this is not realistic in a practical scenario, and hence may not be useful in all situations. Thus, we wish to observe the effectiveness of the data averaging method in the case of noisy data. We set our noise level at \( 5^\circ \) and perform the procedures described in 4.5b.2 for a single simulation, then move on to the average of 100 simulations to remove random error effects.

Presented in Figures 26(a) – (c) are the results for a single simulation with \( 5^\circ \) noise. We see that the linearity of Figures 24 and 25 as seen before no longer exist, making it harder to discern the accuracy of the plots.
Figure 26: Variation of RMS Height Error of Varying Baseline Lengths with noise at 5°.
(a) $B_{13}' = 190\text{m} < B_{13}$,  
(b) $B_{13}' = 200\text{m} = B_{13}$
Figure 26: Variation of RMS Height Error of Varying Baseline Lengths with noise at 5°.

(c) $B_{13}' = 210\text{m} > B_{13}$

Table 12 shows the numerical values for RMS height error for data averaging.

<table>
<thead>
<tr>
<th>$B_{12}'$</th>
<th>$B_{11}'$</th>
<th>$B_{10}'$</th>
<th>$B_{9}'$</th>
<th>$B_{8}'$</th>
<th>$B_{7}'$</th>
<th>$B_{6}'$</th>
<th>$B_{5}'$</th>
<th>$B_{4}'$</th>
<th>$B_{3}'$</th>
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<th>$B_{1}'$</th>
</tr>
</thead>
<tbody>
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<td>m</td>
<td>m</td>
<td>m</td>
<td>m</td>
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<td>0.4149</td>
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<td>0.4037</td>
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Table 12: RMS height error of $B_{Avg}'$ using varying $B_{13}'$ and $B_{12}'$ lengths, but with $B_{13}'$ as reference and noise at 5°.

Table 12 shows that with noise, the method of data averaging to estimate the baseline lengths no longer works as well; highlighted together with the correct $B_{13}'$ and $B_{12}'$ values are the values in which the RMS error appears less than the theoretical minimum expected to occur at $B_{13}' = B_{13}$ and $B_{12}' = B_{12}$. Therefore, we conclude that the single
simulation trial (which represents a single look of data collection) will not be comprehensive enough to eliminate GCPs if both noise and baseline estimation inaccuracies result. If we consider instead the asymptotic average values, i.e. the mean RMS error for 100 trials, we obtain the results as shown in Table 13.

<table>
<thead>
<tr>
<th>$B_{12}'$ [m]</th>
<th>180</th>
<th>185</th>
<th>190</th>
<th>195</th>
<th>200</th>
<th>205</th>
<th>210</th>
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<td>0.5057</td>
<td>0.7466</td>
<td>1.0217</td>
<td>1.2875</td>
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</tr>
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<td>0.4426</td>
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</tbody>
</table>

Table 13: Mean RMS height error of $B_{AVG}$ using varying $B_{13}'$ and $B_{12}'$ lengths, but with $B_{13}'$ as reference and noise at 5°, for 100 trials

From Table 13, one can see that by accounting for the random error fluctuations using the mean of 100 repeated simulations, the result of data averaging method again reduces towards the minimum value (with the exception of one case, at $B_{13}' = 215$ m and $B_{12}' = 170$ m, which we shall consider as an outlying point, i.e. it is through randomness that this value is minimum). Hence, it is reasonable to conclude that in the presence of noise, the data averaging method will still provide a good method of detecting accuracies (or inaccuracies) in the estimated baseline length. Asymptotically speaking, if ground truth were not present, using the relative error comparison, the minimum point would then correspond to the “correct” estimates. If noise were absent, then this minimum point would be considered the “zero” point seen earlier.

Thus, we have provided a tenable application for the data averaging method; by finding the minimum relative RMS error of the data averaged height profile, it is possible to alter the baseline parameters such that correct estimates can be obtained. However, one needs to note the limitations of this method in the presence of noise, and for this only asymptotically can one ascertain the baseline parameters accurately. Note also that the averaging method used here was the simple one adopted in Equation (23). If more suitable assignments to the weighting function were adopted, it may be possible to further narrow down the possible values for estimating the baseline parameters. Lastly, it must be noted that our simulations were performed in the 1-D plane only. If this were in a realistic 2-D scenario, the minimum point may be amplified by the fact that it would be minimum in 2 planes instead of just one, hence the other incorrect points can be filtered out through this extension.
Chapter 5

3-Satellite InSAR Height Retrieval: Maximum Likelihood Estimation Method

5.1 3-Satellite SAR System Setup

Since the 2 methods are similar in setup and principles, we shall present both URM and MLE methods together in this chapter. In this chapter, we will look in detail what both these methods mean and how they work, as well as present results for them. For this, the setup used will be the same as Figure 17, only slightly modified such that a straightforward simplification of the model used can be obtained. We redraw Figure 17 to emphasize certain features, as shown below:

![Diagram](image)

Figure 27: A more accurate representation of Figure 17

Note the higher tilt angle of the baseline as compared to Figure 17 (the tilt is exaggerated to demonstrate the perpendicular cutting through \( \rho_3 \); our baseline tilt angle is still 35°.)
The parameter values are the same as that in Table 4. This setup will be used for the URM method described below, which will in turn develop the principles used by the MLE method described subsequently.

5.2 Unambiguous Range Magnification (URM)

5.2.1 Wrapped Phase Problem Revisited

The main advantage which the URM method (and subsequently the MLE) provides lies in the ability to provide magnification of the unambiguous phase range. As we understood from Chapter 2, the interferometric phase obtained from a single SAR satellite is wrapped modulo 2π, and phase unwrapping techniques are necessary in order to determine the absolute phase, which will finally be used for height retrieval. The problem with phase unwrapping techniques occurs in the presence of high noise and steep terrain, in which these methods may be rendered useless due to the ambiguity that may result when unwrapping with the Matlab `unwrap` function. This simple method can provide a good estimation to the overall height in many cases, but is severely limited by the noise level as well as steepness of terrain. If the presence of noise causes data to fluctuate close to a phase jump point, this phase unwrapping technique will cause 2π phase additions to that region more times than necessary, hence elevating the real height by many times. If this occurs early in the phase unwrapping process (i.e. near to the GCP), the unnecessary elevation of height becomes magnified multiple-fold, resulting in huge errors in the final height estimation. We refer errors in unwrapped phase from the phase unwrapping process as "global errors". This is opposed to "local error", which we shall define as noise fluctuations errors that are not really affected by unwrapping. Note that global errors can be considered a consequence of high local error, by which the unwrapping process is unable to determine where a phase jump should occur. Global error also occurs for steep terrain, of which the slope limit can be derived like in [26]:

\[ \tau_{\text{max}} = \arctan \left( \frac{\lambda \cdot \rho \sin \theta}{2DB_{13} \cos(\theta - \alpha) \cdot URM} \right) \]  (30)

where \( \tau_{\text{max}} \) is the maximum local terrain slope, \( D \) is the ground range resolution (i.e. distance between 2 samples in the interferogram), \( URM = 1 \) and the remaining parameters as we have seen before. Hence, any slopes exceeding this limit seek to cause potential global errors in the simple phase unwrapping method proposed. However, we shall not be investigating the effects of steepness in this paper, though it should be duly noted that with the URM method together with the MLE, Equation (30) will be modified such URM will be an integer larger than unity, hence causing steeper slope sensitivity.

Having reiterated the potential problems that phase unwrapping technique can cause, it would seem best if the entire process of phase unwrapping could be minimized, in a bid to reduce the possibility of large-scale global errors. This makes way for the URM and MLE methods, which seek to reduce the number of phase unwrapping procedures...
performed. The aim of the URM is to utilize the difference in magnitudes of phase in phase unwrapping that occurs for the different baselines to predict a magnification value, called the *URM* constant, of which the wrapped $2\pi$ phase range is expanded to $2\pi^*URM$. This expanded range would be unambiguous since the different baselines, due to their non-symmetric configuration, will wrap at a different rate, as will be shown. This expanded unambiguous range would hence reduce the need for phase unwrapping to just outside that of the $2\pi^*URM$ phase range, hence effectively reducing global error throughout. Note that the URM and MLE methods do not attempt to improve the phase unwrapping process, but rather try to reduce the number of times in which phase unwrapping needs to be performed.

In short, one should note that the URM method merely demonstrates the possibility of using the 3-satellite configuration to determine the unwrapping constants within which a wider wrapped range can be determined. This by no means is equivalent to finding the exact phase value within this range in which a $2\pi$-wrapped value would be corresponding to; this instead is left for the MLE method. Hence, the URM method only sets up the range within which the MLE method will determine the wider wrapped phase value. Also, note that as opposed to the data averaging method, here we do not modify the results after height retrieval, but rather try to obtain the unwrapped phase values before the height retrieval process used as in Chapter 2.

### 5.2.2 URM Configuration

To introduce the URM method, one needs to re-examine the setup used for interferometry. Consider the setup used in Figure 27 in Section 5.1.

Here we have assumed the tilt of the cartwheel plane such that there is a perpendicular baseline equivalent that cuts the $\rho_3$ axis. Hence, with our original baselines $B_{12}$, $B_{13}$ and $B_{23}$, we now derive their equivalents by taking the cosine of the angle above the perpendicular.
Figure 28: Close-up of 3-antenna InSAR configuration for Figure 27.

Hence, we can show easily from Figure 28 that:

\[ B_{\perp,i} = B_{_0} \cdot \cos(\gamma) \]  

(31)

\[ \gamma = \alpha - \alpha_{_1} \]  

(32)

where \( \{i,j\} = \{1,2,3\}, i \neq j \).

We see here that the angle \( \alpha_{_1} \) is formed by the perpendicular projection onto \( \rho_3 \). Note that this angle is sensitive to the look angle \( \theta \), since as the SAR systems scan over the target area (in the range direction) the look angle increases.

Also, from Figure 28, we can obtain the following parameters: \( \Delta \Phi_{12}' \) and \( \Delta \Phi_{13}' \), which are the phase difference between SAR\(_2\) and its projected perpendicular position, and likewise for SAR\(_3\). These values will have a significant role in determining the URM constant which is crucial to this method. Also, it should be made clear that \( \Delta \Phi_{12}' \) and \( \Delta \Phi_{13}' \) should not be confused with the interferometric phase values \( \Delta \Phi_{12} \) and \( \Delta \Phi_{13} \) which can be found through Equation (7), for they represent different values altogether. Under the similar triangles relationship with angle \( \gamma \), we see that the phase difference values \( \Delta \Phi_{12}' \) and \( \Delta \Phi_{13}' \) follow the following relationships:
Next, the following far-range assumption can be deduced: The target position is far enough such that the range directions $p_1$, $p_2$ and $p_3$ are parallel. This assumption can hold easily for our space-borne parameters used, since $H$ and $X$ are sufficiently far (order 10$^5$) compared to the baseline values (order 10$^2$). Under this assumption, we can see that the look angles for each SAR satellite are approximately equal to $\theta$. Furthermore, the following deductions can then be made:

$$\alpha_\perp = \theta$$  \hspace{1cm} (35)

$$\Delta \Phi_{13}' = \frac{4\pi}{\lambda} (\rho_3 - \rho_1) = \Delta \Phi_{13}$$  \hspace{1cm} (36)

$$\Delta \Phi_{12}' = \frac{4\pi}{\lambda} (\rho_2 - \rho_1) = \Delta \Phi_{12}$$  \hspace{1cm} (37)

Note that Equations (35) – (37) have greatly simplified the problem setup. Also, Equations (36) and (37) imply that the interferometric phase due to the difference in positioning of the SAR systems is taken to be equal that of the phase difference between the SAR systems and their perpendicular images. Hence we modify Equation (34):

$$\tan \gamma = \frac{\Delta \Phi_{12}'}{B_{\perp 12}} = \frac{\Delta \Phi_{13}'}{B_{\perp 13}} = \frac{\Delta \Phi_{23}'}{B_{\perp 23}}$$  \hspace{1cm} (38)

We then consider the fact that the absolute interferometric phase values $\Delta \Phi_{12}$, $\Delta \Phi_{13}$ and $\Delta \Phi_{23}$ collected by the SAR systems are wrapped by modulo 2$\pi$. Therefore we have only the wrapped phase values $\Delta \varphi_{12}$, $\Delta \varphi_{13}$ and $\Delta \varphi_{23}$ instead of $\Delta \Phi_{12}$, $\Delta \Phi_{13}$ and $\Delta \Phi_{23}$. $\Delta \varphi_{12}$, $\Delta \varphi_{13}$ and $\Delta \varphi_{23}$ are given by the following:

$$\Delta \varphi_{12} = \text{mod}(\Delta \Phi_{12}, 2\pi)$$  \hspace{1cm} (39)

$$\Delta \varphi_{23} = \text{mod}(\Delta \Phi_{23}, 2\pi)$$  \hspace{1cm} (40)

$$\Delta \varphi_{13} = \text{mod}(\Delta \Phi_{13}, 2\pi)$$  \hspace{1cm} (41)

Equations (39) – (41) can be rewritten as:

$$\Delta \Phi_{13} = \Delta \varphi_{13} + r \cdot 2\pi$$  \hspace{1cm} (42)

$$\Delta \Phi_{23} = \Delta \varphi_{23} + s \cdot 2\pi$$  \hspace{1cm} (43)
\[ \Delta \Phi_{12} = \Delta \phi_{12} + t \cdot 2\pi \] (44)  

where \( r, s \) and \( t \) are unknown integers. Note that as seen in Chapter 2, it is impossible to obtain the values of \( a, b \) and \( c \), since otherwise the absolute phase would have already been obtained.

With only the wrapped phase values given in Equations (42) – (44), we now set out to make use of these phase values to obtain the URM constant, as well as to show how this value can help magnify the wrapped phase range. To do so, we need to first set the following condition: that the ratio between \( B_{123} \) and \( B_{13} \) is smaller than unity. Hence, we rearrange Equation (38) to obtain the following relationship between \( \Delta \Phi_{13} \) and \( \Delta \Phi_{23} \):

\[
p = \frac{B_{123}}{B_{13}} = \frac{\Delta \Phi_{23}}{\Delta \Phi_{13}} = \frac{\Delta \varphi_{23} + n \cdot 2\pi}{\Delta \varphi_{13} + m \cdot 2\pi} = \frac{\rho_2 - \rho_3}{\rho_3 - \rho_1} \] (45)

where the value \( p < 1 \). Also note here that this would mean that SAR2 would be closer to SAR3 than SAR1, which is the same configuration we assumed for the 2-component data averaging method. As will be shown, because of the closer proximity of SAR2 to SAR3, the value of \( \Delta \Phi_{12} \) will not play a significant role in obtaining the URM-maximized phase range because its wrapping effect is closer to that of \( \Delta \Phi_{13} \) than \( \Delta \Phi_{23} \), and hence its results will be overshadowed in significance by \( \Delta \Phi_{13} \).

We now have a relationship between the perpendicular baselines and the wrapped phase data. From Equation (45), we can see that for \( p < 1 \), it is clear that the absolute interferometric phase \( \Delta \Phi_{23} \) is definitely smaller than that of \( \Delta \Phi_{13} \). This means that when wrapped over \( 2\pi \), it would take a longer range distance covered (or equivalently a wider look angle shift) by \( \Delta \Phi_{23} \) to have a single \( 2\pi \) phase jump than it would \( \Delta \Phi_{13} \). Hence, we can apply this behavior of phase unwrapping to uncover a wider phase region up to as much as \( \Delta \Phi_{23} \) can offer before a phase jump occurs for this value.

A good way of demonstrating the widened range would be to rearrange the integer \( r \) values in Equation (45) in terms of \( s \) and \( p \), as shown here:

\[
r = s \cdot \frac{1}{p} + \frac{1}{2\pi} \left( \frac{1}{p} \Delta \varphi_{23} - \Delta \varphi_{13} \right) = s \cdot URM + \frac{1}{2\pi} \left( URM \cdot \Delta \varphi_{23} - \Delta \varphi_{13} \right) \] (46)

Here, we have assigned the URM constant to be the inverse of \( p \), i.e. \( URM = 1/p > 1 \). Also, note that since \( r \) and \( s \) are integers, hence with an integer value assigned for \( URM \), this would mean that the right-hand term of Equation (46) would have to be an integer too.
We also know the following:

\[ 0 < \frac{\Delta \varphi_{23}}{2\pi} < 1 \quad (47) \]

\[ 0 < \frac{\Delta \varphi_{13}}{2\pi} < 1 \quad (48) \]

for positive \( \Delta \Phi_{13} \) and \( \Delta \Phi_{23} \) values. We use Equations (47) and (48) to obtain the inequality in Equation (49) for the right-hand term of Equation (46).

\[ -1 < 0 \leq \frac{1}{2\pi} \left( URM \cdot \Delta \varphi_{23} - \Delta \varphi_{13} \right) \leq URM - 1 < URM \quad (49) \]

Equation (49) implies the following: For any integer value \( s \), the maximum range of values \( r \) can adopt are \( 1/p \) different integer values, assuming \( URM \) is an integer. If we plot the possible \( r \) values for each \( s \) on a number line, we see that the \( URM \) different integer values of \( r \) are spread out to cover all \( s \) space, such as that shown in Figure 29.

Figure 29: Possible solutions for \( r \).

The expansion (or restriction) of possible \( r \) values to within \( 1/p \) possibilities for each \( s \) translates to the following conclusion: That for each integer \( s \) value produced by the wrapping of \( \Delta \Phi_{23} \), we can obtain a corresponding value \( r \) value which is wrapped to \( 1/p \) (or \( URM \)) possible solutions for \( \Delta \Phi_{13} \). Equivalently, this means that while \( \Delta \Phi_{23} \) is wrapped by modulo \( 2\pi \), we can use to relations derived in Equation (49) to expand our wrapped \( \Delta \Phi_{13} \) phase value to modulo \( URM \cdot 2\pi \) instead.

This promised expansion however involves the unknown \( s \) value in which to derive \( r \), since here we assume we know the values \( p, \Delta \varphi_{23} \) and \( \Delta \varphi_{13} \) as seen before in Chapter 2. One method to solve this problem would be simply to guess back at the values of \( s \) to obtain the \( r \) value relations. This would then convert to a uniqueness problem, which has to be solved using a single ground control point. Here, we see that the GCP issue is hence still not resolved, unlike in Chapter 4, and we still need a GCP in order to obtain the expanded and hence absolute phase values. However, if we consider the special case in which the URM value is such that the expanded phase range covers the entire scanned
swath, this would hence mean that there would be an exact \( s \) value in which the absolute phase can be obtained, and hence a GCP would be obsolete.

Nonetheless, we shall still demonstrate the results of \( r \) for guessing at the values of \( s \) by scanning over a target using the parameters given in Table 4. An example of such is shown in Figure 30, with \( URM = 4 \) (i.e. \( p = 1/4 \)) which is consistent with our setup.

![Plot of \( m \) against \( n \) values for Noise Level = 0°](image)

**Figure 30:** Demonstrating the expanded wrapped \( r \) values for varying \( s \) values.

We see from Figure 30 that \( r \) is wrapped as multiples of \( URM \) for increasing integer values of \( s \). It is a coincidence though that the 2 points start at zero. We should hereby disclaim that it is entirely possible for \( r \) and \( s \) to be non-integer values. This is because we have initially made the far-range assumption that \( p_1, p_2 \) and \( p_3 \) are parallel, which in reality may not be the case. This results in small discrepancies in wrapped phase values, but will be absorbed into \( r \) and \( s \) values since the discrepancies are fixed constants invariant to the interferometric phase values.

Another observation made is that since \( r \) is dependent on the \( URM \) (or \( 1/p \)) value, hence if \( p \) decreases, the expanded range would hence increase. Note here that according to our investigations of data averaging and the optimum satellite configuration, our optimum configuration derived was such that SAR_2 and SAR_3 positions overlap in space. If this was applied to our URM method, it would seem then that as \( p = 0 \), then \( URM \rightarrow \infty \). This would mean that for any \( s \), then our value of \( r \) would be infinite, hence our URM method would promise an infinite expanded range. This conclusion would have to be approached with caution, since it seems unlikely that \( r \) could promise such a result.
It would hence be wise to go back to the initial setup, and make reasonable observations in such a limit. We observe that when \( B_{23} = 0 \), this causes the phase difference to between \( \rho_2 \) and \( \rho_3 \) to be zero as well, since they really are the same length. This then means that Equation (45) would hence become singular, and therefore the derivation would not hold. Hence, we deduce that the URM method is invalid for the case \( p = 0 \). Furthermore, we shall see that although the URM method decreases the global errors due to reduced phase unwrapping trials, the tradeoff in this would be an increase in local error in the presence of noise. This increase in local error increases with the URM constant, which poses another limitation to this method.

5.2.3 Back-up to the URM method derived

A careless reader may suggest that instead of solving the URM method as proposed in Section 5.2, one can directly use the angle \( \gamma \) to solve for the absolute interferometric phase values \( \Delta \Phi_{12}, \Delta \Phi_{13} \) and \( \Delta \Phi_{23} \).

\[
\Delta \Phi_{ij} = B_{ij} \sin(\gamma)
\]  

(50)

where \( \{i,j\} \in \{1,2,3\}, i \neq j \).

A more accurate \( \alpha_\perp \) value can also be found:

\[
\alpha_\perp = \theta - \beta
\]  

(51)

\[
\beta = \arccos\left(\frac{\rho_1^2 + \rho_3^2 - B^2}{2\rho_1 \cdot \rho_3}\right)
\]  

(52)

As cogent as this argument may seem, it however rests upon the fact that both the values \( \rho_1 \) and \( \rho_3 \) are known. Even with GCPs, at most only 1 of these 2 values will be known but not both, and therefore there can be no way of solving Equation (52). Therefore the next best approach would be to use our derived URM method.
5.3 Simulations and Results for URM

5.3.1 URM without Noise

Nonetheless, we shall demonstrate the use of the URM method in the absence of noise. Figure 31 (a) and (b) illustrates the phase plot variations for $URM = 2$ and $URM = 4$. The terrain used is that of the sine function described in Equation (12).

Figure 31(a): Phase plots for the URM method for noiseless scenario, $URM = 2$
In Figure 31, we see that for the terrain described, the expanded phase estimate that we predict by guessing the values of \( s \) still result in the wrapped phase being expanded to \( URM \times 2\pi \). Note that we plot the phase values against \( \rho_1 \) length in order to illustrate the order of arrival to the SAR systems. Note also that the 'x' plot is our expanded range, the '□' and '○' plots are the phase values \( \Delta\phi_{23} \) and \( \Delta\phi_{13} \) respectively, and the 'o' plot is the absolute phase \( \Delta\phi_{13} \) wrapped by \( URM \times 2\pi \) to illustrate the expanded phase range accuracy. Also, from Figure 31(a), we can deduce that if \( URM = 1 \), then naturally there would be no expansion from the \( 2\pi \) range.

More important from Figure 31 is the demonstration that phase unwrapping processes can be reduced, by a total of \( URM - 1 \) times. Yet, despite showing these possible reductions in number of phase unwrapping trials needed with the expanded unambiguous phase ranges, one thing to note is that in the noiseless case that we consider here, there is really no need for URM methods, since phase unwrapping using Matlab’s \textit{unwrap} function will give the exact absolute phase and hence height. However this also debunks the need for more complicated phase unwrapping procedures. Yet, it is the possible improvements and limitations in the presence of noise (or varied terrain) that becomes the focus of interest, which we will introduce next.

5.3.2 URM Method with Noise

We have seen that the URM method by virtue of expanding the wrapped phase range reduces the likely global errors that will occur when unwrapping. However, in this
section we shall see that in the presence of noise, the reduced global errors come at the expense of increased local errors. This happens because $B_{23}$, being the shortest baseline, becomes most sensitive to noise changes, as we have also seen in Chapters 4. This causes the height errors for $B_{23}$ to spike by larger extents than with the longer baseline lengths. However, because the URM method still involves the $\Delta\varphi_{23}$ value, it is inevitable then that the method will become sensitive to noise too. Yet, as we shall see, this corresponds to an increase in local error only, but as a result we have a wider unambiguous phase range.

Figure 32 (a) – (c) shows the effects of the URM methods for a single simulation, in the presence of increasing noise and $URM = 4$. The noise function we used here is the same as that of Equation (13).

![Phase Plots for URM Method, URM = 4, noise level = 5°](image)

Figure 32 (a): Phase Plot Variations for rising noise levels, noise = 5°
Phase Plots for URM Method, URM = 4, noise level = 10°

Figure 32: Phase Plot Variations for rising noise levels, (b) noise = 10°, (c) noise = 15°
From the plots we conclude that even though there are less occurrences of phase unwrapping, the local phase error per pixel as predicted goes up drastically, rendering the plot completely “unwrappable” by $n = 15^\circ$ even though the range has increased.

Following the measurement steps mentioned in Chapter 2, we now wish to find the effects of noise on the height profiles. We can simultaneously obtain the effects of URM on height as well. We start off with measuring the effects on a single simulation. Figures 33(a)—(c) show the retrieved height profiles of Figure 32 using the methods of unwrapping and plot alignment as we saw in Chapter 3, $URM = 4$. (Note that the Matlab function unwrap here adds $URM*2\pi$ instead of just $2\pi$, with the tolerance level half of the new additive value, i.e. $URM*\pi$). Also, we have used a single ground truth point to align the wrapped phase values for retrieving height.

Figure 33 (a): Retrieved Height Profiles for rising noise levels, noise = $5^\circ$
Figure 33: Retrieved Height Profiles for rising noise levels:
(b) noise = 10°, (c) noise = 15°
However, as seen in Figure 33 and Chapter 4, having a single trial simulation in the presence of noise creates the problem of random error in predicting the effects of noise on RMS height error. We also see from Figure 33 (c) that the random error for $n = 15^\circ$ is large enough to cause the unwrap algorithm to be unable to unwrap correctly the expanded phase value. The large height jump in this Figure also serves as a demonstration to what it means by 'global error', as compared to the small fluctuations we coin as 'local error'. Thus, to avoid random error as well as to minimize the effects of global error due to phase unwrapping, we instead conduct 100 simulations, and compare the mean RMS errors as explained before. Figures 34 (a) and (b) show the average RMS height error against noise and URM relationships. The data has been obtained for URM values ranging from 1 to 11. Note that $URM = 1$ simply refers to the scenario that 2 satellites are being spatially atop each other, like in Chapter 4.5a. Numerical results are provided in Table 14.

<table>
<thead>
<tr>
<th>Noise Level [°]</th>
<th>RMS Error [m]</th>
<th>URM 1</th>
<th>URM 2</th>
<th>URM 3</th>
<th>URM 4</th>
<th>URM 5</th>
<th>URM 6</th>
</tr>
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<tr>
<td>5</td>
<td>0.4339</td>
<td>0.7654</td>
<td>1.1160</td>
<td>1.4431</td>
<td>1.7952</td>
<td>2.1275</td>
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<td>10</td>
<td>0.8916</td>
<td>1.5325</td>
<td>2.2052</td>
<td>6.2959</td>
<td>5.0088</td>
<td>4.3187</td>
<td></td>
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<tr>
<td>20</td>
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<td>6.8882</td>
<td>6.1377</td>
<td>18.8807</td>
<td>10.8514</td>
<td>10.8212</td>
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<tr>
<td>40</td>
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<td>27.9453</td>
<td>45.1122</td>
<td>42.4816</td>
<td>50.4023</td>
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<tr>
<td>45</td>
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<td>25.1493</td>
<td>34.6364</td>
<td>51.6157</td>
<td>55.6349</td>
<td>68.4706</td>
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<tr>
<td>50</td>
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<td>30.5264</td>
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<td>75.2265</td>
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<table>
<thead>
<tr>
<th>Noise Level [°]</th>
<th>RMS Error [m]</th>
<th>URM 7</th>
<th>URM 8</th>
<th>URM 9</th>
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<td>120.4127</td>
<td>147.6596</td>
<td></td>
</tr>
</tbody>
</table>

Table 14: Numerical RMS Height Error Values for URM and Noise Comparisons
Figure 34: Demonstrating the increase in mean RMS height error with URM and noise
(a) for URM, (b) for noise
From both the Table and Figures, we can deduce quite easily that the mean RMS height error increases for 2 factors; increasing noise level as we have seen in Chapter 3, and increasing URM value which indicates the closeness of distance between SAR2 and SAR3. Another observation of the data values is that even though from Equation (12) the terrain spans only between -5 to 5m in height, for large URMs we get errors that reach up to over a hundred meters. This already exceeds the maximum range of height of the terrain by 15 times, which illustrates the extreme sensitivities of the URM method to noise. Obviously this shows that the URM method is not a viable method to perform under noisy conditions, if we were to directly calculate the \( r \) values for a guessed \( s \) values. It follows that only under very low noise conditions (<5°) can this method provide better height estimation through lesser unwrapping steps needed. The results of the URM analysis can be summarized as follows:

\[
\text{[Mean RMS Error]}_{\text{URM}} = f(n, \text{URM})
\]

where \( f \) is a function of the respective components \( n \) and \( \text{URM} \). We will not seek to find this function, as this is beyond our scope of investigation.

Note that the non-linearity of the curves seen in Figure 34 are an exemplification of the global errors occurring randomly, making it hard to exactly produce a good estimation. Also, one must take caution when observing the URM effects on the height profile. Since we rely on \( B_{23} \)'s length to expand the phase instead of \( B_{12} \)'s as we saw in Chapter 4, this would mean that as \( \text{URM} \rightarrow \infty, B_{23} \rightarrow 0 \). From Equation (16) the inverse relationship of \( B \) length would result in infinite local error for \( B_{23} \rightarrow 0 \). Hence, we see that by increasing the \( \text{URM} \) value to get a wider expanded range (and hence less global error for unwrapping steps), we in fact increase the local error sensitivity of the method to noise.

We have thus explored the effects of noise on the URM method. Even though we have indeed obtained an expanded unambiguous phase range in the attempt of reducing global errors, we instead trade the global errors for local ones instead. Furthermore, if we intend to get “greedy” with the \( \text{URM} \) values, we further trade global error for more local error, hence effectively canceling any improvement. As a result, the combined height error effect will depend on the \( \text{URM} \) and noise values used, whereby a low noise level as well as a moderate \( \text{URM} \) chosen may provide improvements, though this was not demonstrated here.

### 5.4 Maximum Likelihood Estimation (MLE)

Based on the limited use of the URM method, it seems that unless a different approach towards the height retrieval method is proposed, the effect of expanding the wrapped phase range may hardly seem useful when compared to directly using the data averaging method of Chapter 4. Hence, the maximum likelihood estimation (MLE) method comes into play. Like the URM, this method (when applied to SAR interferometry) was first proposed by F. Lombardini in his paper *Absolute Phase Retrieval in a Three-Element*
Synthetic Aperture Radar Interferometer [24], as was the URM magnification (briefly mentioned). This method is asymptotically efficient, i.e. the results improve if more data samples (or looks) are taken of the same image area. As we shall demonstrate, this method will perform badly for a single data set, but with more sets the results improve tremendously. Through a step-by-step procedure, we shall identify the reasons for choosing this statistical method, starting with passing reference to the Minimum Variance Unbiased (MVU) estimator, followed by the model adopted by Lombardini (and subsequently modified to suit our own model as in Chapter 2), before finally arriving at the simulations obtained for our model. Comparisons of this method with the 2 other methods previously proposed will be presented in the next chapter.

5.4.1 Minimum Variance Unbiased (MVU) Estimator

According to Steven Kay's Fundamentals of Statistical Signal Processing I [4], the MVU estimator "is the one whose variance for each component becomes minimized among all unbiased estimators". This means that of the many unbiased estimators made possible through different statistical methods, the MVU estimator provides the result which most closely approximates the true value of a certain component. This MVU estimator is said to have achieved the Cramer-Rao Lower Bound, or CRLB as it is commonly known [4]. With respect to our case, the estimator we wish to find would be the absolute phase value. However, because of the inability to recover the number of cycles the measured phase has lost due to wrapping, it would be literally impossible to recover this absolute phase estimator. This is where the URM expanded range comes in. Because of the relationships in phase wrapping amongst satellites as we saw earlier in the URM method description, this allows for an estimator to be determined such that this value would be within the URM expanded range. Of course, this would not have removed the need for phase unwrapping altogether, since it is likely that the URM expanded range may not encompass the entire region for large scale maps. Conversely, phase unwrapping may also be removed for the opposite reason. Nonetheless, one can argue that in general phase unwrapping is still required, even though the number of such procedures have been reduced by URM - 1 times. This conclusion is not new as we already determined this earlier on. What is remarkable, however, is determining the MVU estimator within this range through the MLE. The MLE provides an asymptotically efficient method of determining the MVU estimator such that the MVU can be achieved eventually, given the appropriate number of looks.

Nonetheless, the MLE will be the MVU estimator for $N \to \infty$, where $N$ is the number of looks [4]. (Note that it is not our interest in showing that the MLE will produce the MVU, and also that the CRLB will give the MVU. These derivations can be found in [4]).

5.4.2 Determining the MLE

Now, we wish to first determine a formulation for the MLE. We know that for a Gaussian white noise model, the probability of obtaining a measured value $x$ is given by the following:
\[
p(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}
\]  

(54)

where \( \mu \) is the sample mean and \( \sigma \) the sample variance. The estimator value is also the sample mean here, i.e. \( \text{MLE} = \mu \). The MLE estimator is given simply by taking the natural logarithmic of Equation (54), differentiating it and obtaining the estimator value for which the differentiated logarithmic function is set to zero. In other words,

\[
\frac{\partial \ln[p(x)]}{\partial x} = 0
\]

(55)

for which solving Equation (55) will produce \( \text{MLE} = \mu \). This shall hence be the underlying steps in deriving the MLE value for the 3-satellite SAR model.

### 5.4.3 MLE in Lombardini’s 3-Satellite Model

Applying the MLE to 3-satellite InSAR first requires adopting a suitable model. Lombardini’s modeling of the SAR signals measured is somewhat similar to ours; though his takes a more in-depth approach by considering the SAR amplitude values with its relation to the signal-to-noise ratio. We shall hence adopt his model here in order to demonstrate the mathematical derivations for the MLE. We assume here a single look collection of data first, before extending it to \( N \) trials. The setup of the model will be the same as we have seen in describing the URM method.

Since we are obtaining 3 sets of SAR signals per look, one for each satellite, we can group the amplitudes and phases of the signals measured by the 3 satellites together into a single column vector.

Let our single look column vector be:

\[
\overline{S} = \begin{bmatrix}
\hat{S}_1 \\
\hat{S}_2 \\
\hat{S}_3
\end{bmatrix}
= \begin{bmatrix}
A_1 e^{\frac{j4\pi}{\lambda} \rho_1} \\
A_2 e^{\frac{j4\pi}{\lambda} \rho_2} \\
A_3 e^{\frac{j4\pi}{\lambda} \rho_3}
\end{bmatrix}
\]

(56)

where \( \hat{S}_i \) is the measured complex signal value of each satellite, \( i \in \{1, 2, 3\} \). The right hand side of Equation (56) is such that the interferometric phase values can be obtained as before.
That is,

\[
\angle(\delta_2, \delta_1) = \frac{4\pi}{\lambda} (\rho_2 - \rho_1) = \frac{4\pi}{\lambda} \delta_{12} = \Phi_{12}
\]  \hspace{1cm} (57)

\[
\angle(\delta_3, \delta_1) = \frac{4\pi}{\lambda} (\rho_3 - \rho_1) = \frac{4\pi}{\lambda} \delta_{13} = \Phi_{13}
\]  \hspace{1cm} (58)

Equations (57) and (58) are synonymous with the absolute interferometric phase of Equation (7). However, it is also clear that Equations (57) and (58) are really not the values measured, since phase wrapping once again comes into play. Thus this implies that the column vector SAR signal values are again wrapped. Hence, Equations (42) – (44) take over Equations (57) and (58) instead, and Equation (56) should be replaced by Equation (59), as shown below.

\[
\tilde{S}' = \begin{bmatrix}
\delta_1' \\
\delta_2' \\
\delta_3'
\end{bmatrix} = \begin{bmatrix}
A_1 e^{\frac{4\pi}{\lambda} \rho_1'} \\
A_2 e^{\frac{4\pi}{\lambda} \rho_2'} \\
A_3 e^{\frac{4\pi}{\lambda} \rho_3'}
\end{bmatrix}
\]  \hspace{1cm} (59)

where \(\rho_1', \rho_2', \rho_3'\) are all less than \(\lambda/2\). We have also ignored the effects of noise so far, and this must be incorporated into our model later on.

Ideally speaking, Equation (59) would have amplitudes \(= 1\) and \(\rho_1', \rho_2', \rho_3'\) to be noise-free. However, this is not realistic towards the MLE formulation, and instead Lombardini assumes that the 3 sets of amplitude and phase noise are correlated complex Gaussian random variables through the following Equation:

\[
\tilde{S}' = \tilde{L}\tilde{I}
\]  \hspace{1cm} (60)

where \(\tilde{L}\) is the lower triangular matrix determined by the Cholesky factorization of the covariance matrix given in Equation (67) below, and \(\tilde{I}\) is given by 3 independent complex Gaussian random variables with zero mean and variance \(= 1\), as in Equation (61).

\[
\tilde{I} = \begin{bmatrix}
\tilde{I}_1 \\
\tilde{I}_2 \\
\tilde{I}_3
\end{bmatrix}
\]  \hspace{1cm} (61)

\[
\tilde{I}_i = u_i + jv_i
\]  \hspace{1cm} (62)

\[
E(\tilde{I}_i) = E(u_i) + jE(v_i) = 0
\]  \hspace{1cm} (63)
\[ \text{var}(\tilde{I}_i) = E\left( |\tilde{I}_i|^2 \right) - |E(\tilde{I}_i)|^2 = 1 \]  
\[ E\left( |\tilde{I}_i|^2 \right) = E(u_i^2) + E(v_i^2) = 1 \]

where \( u_i \) and \( v_i \) are real and \( i \in \{1, 2, 3\} \). Assuming \( \sigma(u_i)^2 = \sigma(v_i)^2 = \frac{1}{2} \), hence each variable follows a Gaussian distribution similar to Equation (54).

The distribution for \( \tilde{I}_i \) would therefore be:

\[ p(\tilde{I}_i) = \frac{1}{\pi \sigma^2} e^{-\frac{|\tilde{I}_i|^2}{\sigma^2}} \]  
\[ (66) \]

where \( \sigma^2 = 1 \).

Equation (66) describes the independent complex Gaussian random variable of the noise in each signal. To correlate the noise in all 3 pixels measured, we require the lower triangular matrix \( \overline{L} \). The normalized correlation matrix \( \overline{\eta} \) of the noise for the signal set \( \overline{S_n} \)' can be found through Rodriguez's Theory and design of Interferometric Synthetic Aperture Radars [27]. This matrix is given as:

\[
\begin{pmatrix}
1 & k(1-b_{12}) & k(1-b_{13}) \\
k(1-b_{12}) & 1 & k(1-b_{23}) \\
k(1-b_{13}) & k(1-b_{23}) & 1
\end{pmatrix} = 
\begin{pmatrix}
\rho_{11} & \rho_{12} & \rho_{13} \\
\rho_{21} & \rho_{22} & \rho_{23} \\
\rho_{31} & \rho_{32} & \rho_{33}
\end{pmatrix}
\]

\[ (67) \]

\[ k = \frac{\text{SNR}}{\text{SNR} + 1} \]  
\[ (68) \]

where \( b_{12}, b_{13}, b_{23} \) are the baseline values \( B_{12}, B_{13}, B_{23} \) normalized by the critical baseline value \( B_{\text{critical}} \) and \( \text{SNR} \) is the signal-to-noise ratio. To find \( \overline{L} \) from Equation (67), we can easily perform a Cholesky matrix factorization of \( \overline{\eta} \).

With \( \tilde{I} \) and \( \overline{L} \) characterized now, we wish to find the distribution of \( \overline{S_n} \)' . Assuming the 3 measured values are correlated, we get:

\[
p(\overline{S_n}') = \frac{1}{\pi^3 \text{det}(\overline{C_n})} e^{-\frac{1}{2} \overline{S_n}' \overline{C_n}^{-1} \overline{S_n}}
\]

\[ (69) \]

where \( \overline{C_n} \) is the covariance matrix for the multivariate complex Gaussian PDF of Equation (69). \( \overline{C_n} \) is given by Equation (70) in the most general case, and can be found from the
correlation matrix \( \tilde{\mathbf{Y}} \) by multiplying each component by the corresponding \( \sigma, \sigma_i \) values. In our case, since we assumed that \( \sigma^2 = 1 \), hence the correlation matrix would be the same as the covariance matrix.

\[
\tilde{\mathbf{C}}_n = \tilde{\mathbf{Y}} = E[(\tilde{\mathbf{S}}_n - E(\tilde{\mathbf{S}}_n))(\tilde{\mathbf{S}}_n - E(\tilde{\mathbf{S}}_n))^H] \\
= \begin{bmatrix}
\text{var}(\tilde{S}_{n1}) & \text{cov}(\tilde{S}_{n1}, \tilde{S}_{n2}) & \text{cov}(\tilde{S}_{n1}, \tilde{S}_{n3}) \\
\text{cov}(\tilde{S}_{n2}, \tilde{S}_{n1}) & \text{var}(\tilde{S}_{n2}) & \text{cov}(\tilde{S}_{n2}, \tilde{S}_{n3}) \\
\text{cov}(\tilde{S}_{n3}, \tilde{S}_{n1}) & \text{cov}(\tilde{S}_{n3}, \tilde{S}_{n2}) & \text{var}(\tilde{S}_{n3})
\end{bmatrix}
\] (70)

where \( \text{cov}(\tilde{S}_{ni}, \tilde{S}_{nj}) = E(\tilde{S}_{ni} - E(\tilde{S}_{ni}))(\tilde{S}_{nj} - E(\tilde{S}_{nj})) \), \( i \neq j \) (71)

If the three \( \tilde{S}_{ni} \) values are independent, Equation (70) reduces to zero for all cases. Hence, Equation (70) reduces to:

\[
\tilde{\mathbf{C}}_n = \begin{bmatrix}
\text{var}(\tilde{S}_{n1}) & 0 & 0 \\
0 & \text{var}(\tilde{S}_{n2}) & 0 \\
0 & 0 & \text{var}(\tilde{S}_{n3})
\end{bmatrix}
\] (72)

Equations (60) – (72) were meant to describe the behavior of the noise measurement in the signal sets only. We wish to include now the effect of the different positioning of the 3 satellites along a collinear baseline. To do so, we simply add a positioning matrix \( \Phi \) as follows:

\[
\tilde{\mathbf{S}} = \Phi \mathbf{S}_n = \Phi L \mathbf{I}
\] (73)

\[
\Phi = \begin{bmatrix}
1 & 0 & 0 \\
0 & e^{j \phi_{12}} & 0 \\
0 & 0 & e^{j \phi_{3}}
\end{bmatrix}
\] (74)

Quite simply, the covariance matrix \( \tilde{\mathbf{C}} \) would be characterized by:

\[
\tilde{\mathbf{C}} = \langle \tilde{\mathbf{S}} \tilde{\mathbf{S}}^{HH} \rangle = \Phi \mathbf{C}_n \Phi^*
\] (75)

If \( N \) independent looks are present, Equation (69) will again be modified to obtain the final characterization of the multivariate complex Gaussian estimator \( \tilde{\mathbf{S}}' \).

\[
p(\tilde{\mathbf{S}}') = \sum_{i=0}^{N} \frac{1}{\pi^3 \det(\tilde{\mathbf{C}})} e^{-\frac{\mathbf{C}_n^{i} \mathbf{C}_n^{i}}{\pi^3 \det(\tilde{\mathbf{C}})}}
\] (76)
Finally, we can apply the MLE Equation (55) to Equation (76) to finally obtain our expression for the maximum likelihood estimator. To obtain the estimator \( \tilde{\phi} \), we apply the following expression:

\[
\text{MLE} = \max_{\phi \in [-URM \cdot \pi, URM \cdot \pi]} \left( \rho_{12} - \rho_{23} \rho_{13} \right) \text{Re} \left[ e^{-j\phi \left( \frac{1}{URM} \sum_{i=0}^{N} \tilde{S}_1^{(i)} \cdot \tilde{S}_2^{(i)} \right) + \left( \rho_{13} - \rho_{12} \rho_{23} \right) \text{Re} \left[ e^{-j\phi \sum_{i=0}^{N} \tilde{S}_1^{(i)} \cdot \tilde{S}_3^{(i)} \right] + \left( \rho_{23} - \rho_{12} \rho_{13} \right) \text{Re} \left[ e^{-j\phi \left( \frac{1}{URM} \sum_{i=0}^{N} \tilde{S}_2^{(i)} \cdot \tilde{S}_3^{(i)} \right) \right]}
\]

(77)

The value of \( \tilde{\phi} \) within the \( URM \cdot 2\pi \) range that maximizes the estimator expression given in Expression (77) is the predicted value of \( \phi \) within the expanded range. It can be shown that as the value \( N \) increases, the statistical fluctuations of the measured \( P \) values in the real parts of (77) will steadily decrease, hence resulting in a more accurate \( \phi \) within the expanded range. Furthermore, it will also be shown that as the resolution between \( URM \cdot 2\pi \) of test \( \phi \) improves, so does the estimator value. The next section will demonstrate the capabilities of the MLE method.

### 5.5 Simulations and Results for MLE

We now wish to explore the performance of the MLE. Like before, we shall perform the MLE for a single simulation and single look first, with and without noise effects using the noise model of Chapter 2. We then wish to see the effects of increasing asymptotic efficiency. Hence, we shall perform simulations for the MLE with an increasing number of looks \( N \) for a fixed level of noise. Next, we shall explore the effects of increasing noise level with a fixed look number. Lastly, we will find any possible effects of the baseline lengths and \( URM \) values on the MLE. Once again comparisons with the other methods introduced will be left for the next Chapter.

Note that we wish to make noise comparisons which have been consistent with Lombardini’s model. Hence we adopt the \( SNR \) as a means of comparison instead of just simply the noise angle measured in [°]. The \( SNR \) would thus enable us to make noise comparisons of both amplitude and phase angle at the same time.

#### 5.5.1 Single Trial Simulations

Naturally, the most assuring method of determining if a method should work is to test a single simulation for the noiseless scenario, of which the height retrieved is expected to be the ideal case. However, this is not the case for Lombardini’s model. This is because the noise model through \( \tilde{I} \) as a complex random Gaussian vector is created such that there will always be random error even as \( SNR \to \infty \). Of course, a more suitable model
could be reformulated instead, but nonetheless Lombardini’s derivations will prove effective for the true region of interest, i.e. $N \to \infty$. Figures 35(a) and (b) show the phase vs. estimator and retrieved height diagrams for the single trial case of $SNR \to \infty$. Our setup has been the same as earlier described in this Chapter, and like before we perform any unwrapping procedures needed with the Matlab `unwrap` function after which we perform plot alignment with the assumption of ground truth. The $URM$ value is again set at 4, with $B_{13} = 200m$ and $B_{23} = 50m$. For convenience, the remaining plots will all have $B_{13} = 200m$ and $B_{23} = 50m$, where we write $URM = 4$ to imply that this set of baseline lengths have been adopted. The reason to emphasize this is that in truth the $URM$ value will not affect the results for the MLE as do the baseline lengths, as we shall see towards the end of the chapter.

Figure 35 (a): The estimator value against the test $\phi$ by which to find the best $\phi$ that can produce the maximum estimator. These are results of a single simulation in the absence of noise, for interval width = 0.001, and $URM = 4$. 

99
Figure 35 (b): The zero-centered retrieved height profile of a single simulation in the absence of noise, for interval width = 0.001, and $URM = 4$.

From Figure 35, it is clear that even at infinite $SNR$ value, it seems impossible to avoid random error for a single-look trial using Lombardini’s method. The RMS height errors for the single-look single trial are given in Table 15, as shown.

<table>
<thead>
<tr>
<th>$SNR$ [dB]</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>$\infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean RMS Height Error [m]</td>
<td>94.3589</td>
<td>48.2866</td>
<td>26.6267</td>
<td>11.1784</td>
<td>6.4114</td>
<td>4.8662</td>
<td>5.6838</td>
</tr>
</tbody>
</table>

Table 15: Measuring the mean RMS height error [m] against the $SNR$ value [dB] for a single-look simulation trial

We see that random error causes the values in Table 15 to be fluctuating and random. These errors are further propagated when $SNR$ is not infinite. These are displayed in Figures 36 (a) – (c).
Figure 36: Single Simulation Trial for: (a) SNR = 20 dB, (b) SNR = 10 dB.
Figure 36 (c): Single Simulation Trial for SNR = 0 dB.

Figure 36 shows how the random error fluctuations become so high for decreasing SNR such that phase unwrapping global errors propagate throughout the retrieved height profile. Since random error is unavoidable with the model proposed by Lombardini, we proceed to measure the average random error for the model instead. We do this by performing $T = 100$ simulations and measuring the average RMS height error as we have done before. This is not to be confused with using 100 looks of data for Expression (77) of the MLE method, i.e. $N \neq 100$, but rather we are directly taking the average of 100 single trial results. The results are shown in Table 16 below, along with the results for different SNR values.

<table>
<thead>
<tr>
<th>SNR [dB]</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>∞</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean RMS Height Error [m]</td>
<td>61.0105</td>
<td>43.5484</td>
<td>21.3677</td>
<td>9.8559</td>
<td>6.6551</td>
<td>5.1482</td>
<td>4.3241</td>
</tr>
</tbody>
</table>

Table 16: Measuring the mean RMS height error [m] against the SNR value [dB] for $T = 100$ simulations

From Table 16 and Figure 36, we see that when SNR value increases (i.e. the noise level decreases), then the mean RMS height error decreases as well. This is not beyond expectations, even though the noise model is dissimilar to ours. Nonetheless we conclude that mean RMS height error is proportional to the SNR. Note here that unfortunately because Lombardini’s model is different from ours, we are unable to provide a comparison with the 2-satellite model as yet. This will be done in Chapter 6, where we
convert the results obtained by the previous methods to a common denominator to enable comparison.

The RMS height error for the retrieved height value is a function of the interval width of test $\phi$ values between $URM*2\pi$ (the finer the spacing, the lower the RMS height error). Table 17 shows the mean RMS errors for alignment against the spacing size.

<table>
<thead>
<tr>
<th>Interval Size/ (URM*2$\pi$) [°]</th>
<th>1</th>
<th>0.1</th>
<th>0.01</th>
<th>0.001</th>
<th>0.0001</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMS Height Error [m]</td>
<td>16.4906</td>
<td>5.1231</td>
<td>4.9441</td>
<td>4.8115</td>
<td>4.7172</td>
</tr>
</tbody>
</table>

Table 17: Observing the relationship between mean RMS height error and spacing size, $SNR \rightarrow \infty$, for 100 simulations

Hence, from Table 17, one can tell that the accuracy of the ML estimator will depend on the interval width. Because of considerations of time needed for each simulation as well as memory space for the CPU, we shall assign a fixed interval size for the remainder of our simulations. Unless stated otherwise, our normalized interval spacing will be at 0.001 [°].

5.5.2 Varying Look Number for a Fixed Noise Level

When multiple looks of data are collected, the maximum likelihood estimator will significantly improve the accuracy of the height profiles retrieved. Figure 37 shows the variations in mean RMS height error with different numbers of looks used in the estimator, at $SNR = 20$ dB. To make the comparison a fair one, we keep the total number of simulations $T$ constant, say at 100. Hence, when we vary the look number $N$ while keeping the value $T$ constant, the factor difference $M$ (given by $T = M \cdot N$) will be the number of repetitions in which to repeat the experiment, after which we take the mean of the $M$ repetitions. Simply put, for a given $N$, we also vary $M$ such that the total number of simulations performed is $T = 100$. For example, when $N = 1$, $M = 100$ and this means we are repeating the single-look simulation 100 times and averaging, as we saw in the previous section 5.5.1. Likewise when $N = 100$, $M = 1$, this implies we do not perform averaging at all. Table 18 and Figure 37 depict this.

<table>
<thead>
<tr>
<th>Number of looks, $N$</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>10</th>
<th>20</th>
<th>50</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of repetitions, $M$</td>
<td>100</td>
<td>50</td>
<td>20</td>
<td>10</td>
<td>5</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Mean RMS Height Error [m]</td>
<td>6.6129</td>
<td>1.2815</td>
<td>0.5226</td>
<td>0.3216</td>
<td>0.2421</td>
<td>0.1531</td>
<td>0.1041</td>
</tr>
</tbody>
</table>

Table 18: Comparing the results of changing $N$ while fixing $T$ at 100, $SNR = 20$ dB
It is clear that the improvements made by the MLE by increasing $N$ clearly exceed random error removal through averaging. As we shall see in the next Chapter, these conclusions will hold also for the MLE against the data averaging method, provided $N > 1$.

5.5.3 Varying Noise Level for a Fixed Look Number Greater Than 1

Through the previous section, we see that the more looks are incorporated into the MLE, the better the result. We now wish to explore the effect of increasing the SNR while keeping the MLE look number $N$ constant. We choose a reasonable look number for this, say $N = 5$ since realistically this can be achievable within a relatively short period. The results we expect are similar to that of Table 15. Note here that only $M = 20$ simulation trials were performed and averaged for these results, to demonstrate its improvement over the averaged single-look $M = 100$ trials in Table 16.

<table>
<thead>
<tr>
<th>SNR [dB]</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>$\infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean RMS Height Error [m]</td>
<td>21.1509</td>
<td>5.3615</td>
<td>0.8689</td>
<td>0.5047</td>
<td>0.53469</td>
<td>0.2284</td>
<td>0.1263</td>
</tr>
</tbody>
</table>

Table 19: Comparing the improvement in results for $N = 5$ for changing SNR, $M = 20$

To demonstrate the improvement in SNR against $N$ used, we plot the values of Table 16 and Table 19 on the same plot, as shown in Figure 38. The values at $SNR \to \infty$ have been omitted from the graphs to limit the axes. Also included are the values for $N = 2$ ($M = 50$),

Figure 37: Plot of mean RMS height error [m] against an increasing $N$ value.
$N = 10 (M = 10), N = 20 (M = 5), N = 50 (M = 2)$ and $N = 100 (M = 1)$, as we saw in the previous section.

Figure 38: Showing the improvements in mean RMS height error [m] with increasing look number $N$ and increasing $SNR$ [dB]

The numerical results for Figure 38 are displayed in Table 20.

<table>
<thead>
<tr>
<th>SNR [dB]</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>10</th>
<th>20</th>
<th>50</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>61.5850</td>
<td>54.3194</td>
<td>26.3579</td>
<td>6.3597</td>
<td>2.1482</td>
<td>0.6625</td>
<td>0.4783</td>
</tr>
<tr>
<td>5</td>
<td>43.1011</td>
<td>18.3716</td>
<td>3.9413</td>
<td>0.9766</td>
<td>0.5375</td>
<td>0.3252</td>
<td>0.2367</td>
</tr>
<tr>
<td>10</td>
<td>21.1509</td>
<td>5.3615</td>
<td>0.8689</td>
<td>0.5047</td>
<td>0.3469</td>
<td>0.2284</td>
<td>0.1263</td>
</tr>
<tr>
<td>15</td>
<td>9.6669</td>
<td>2.3588</td>
<td>0.5815</td>
<td>0.3784</td>
<td>0.2622</td>
<td>0.1670</td>
<td>0.1142</td>
</tr>
<tr>
<td>20</td>
<td>6.4932</td>
<td>1.3477</td>
<td>0.5195</td>
<td>0.3476</td>
<td>0.2396</td>
<td>0.1424</td>
<td>0.1021</td>
</tr>
<tr>
<td>25</td>
<td>5.5456</td>
<td>1.1883</td>
<td>0.4929</td>
<td>0.3303</td>
<td>0.2334</td>
<td>0.1409</td>
<td>0.1017</td>
</tr>
<tr>
<td>∞</td>
<td>4.6685</td>
<td>1.1079</td>
<td>0.4809</td>
<td>0.3278</td>
<td>0.2200</td>
<td>0.1387</td>
<td>0.0927</td>
</tr>
</tbody>
</table>

Table 20: Numerical results showing the improvements in mean RMS height error [m] with increasing look number $N$ and increasing $SNR$ [dB]

Again, it is clear from Figure 38 (and Table 20) that at every fixed $SNR$ value, the mean RMS height error is proportionately lower for increasing $N$ looks, and likewise for every fixed $N$ value, there is a proportionate decrease in mean RMS height error with increasing $SNR$ value.
5.5.4 Effects of Varying the URM by Changing Baseline Length

Like in the data averaging method, we wish to investigate how the baseline lengths selected will affect the accuracy of the MLE method. We do so by changing the length of only one baseline length, say $B_{13}$, and thus changing the URM value, the results are displayed below in Table 21 and Figure 39. (Recall that $URM = B_{13}/B_{23}$.) We introduce a multiplication factor $k$ which multiply $B_{13}$ to, although keeping $B_{23}$ unchanged. (Note that $k \cdot B_{13}$ must still be greater than both $B_{12}$ and $B_{23}$ for proper comparison). Since $B_{23}$ is unchanged and $B_{13}$ changes, we note also that $B_{12}$ will change proportionately, since $B_{12} + B_{23} = B_{13}$. Of course, it would be necessary for the baseline changes are such that they do not exceed the critical baseline value given by Equation (3), which gives about $B_{critical} \approx 3000m$ for our setup. Also, the simulations were performed for $N = 5$, $M = 20$ and SNR = 20 dB.

<table>
<thead>
<tr>
<th>Multiplication Factor, $k$</th>
<th>$\frac{1}{2}$</th>
<th>$\frac{3}{4}$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k \cdot B_{13}$ [m]</td>
<td>100</td>
<td>150</td>
<td>200</td>
<td>400</td>
<td>600</td>
<td>800</td>
<td>1000</td>
</tr>
<tr>
<td>$B_{23}$ [m]</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>$B_{12}$ [m]</td>
<td>50</td>
<td>100</td>
<td>150</td>
<td>350</td>
<td>550</td>
<td>750</td>
<td>950</td>
</tr>
<tr>
<td>URM</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>8</td>
<td>12</td>
<td>16</td>
<td>20</td>
</tr>
<tr>
<td>Mean RMS Height Error [m]</td>
<td>0.750</td>
<td>0.595</td>
<td>0.540</td>
<td>0.406</td>
<td>0.663</td>
<td>0.856</td>
<td>1.084</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Multiplication Factor, $k$</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k \cdot B_{13}$ [m]</td>
<td>1200</td>
<td>1400</td>
<td>1600</td>
<td>1800</td>
<td>2000</td>
<td>2200</td>
<td>2400</td>
</tr>
<tr>
<td>$B_{23}$ [m]</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>$B_{12}$ [m]</td>
<td>1150</td>
<td>1350</td>
<td>1550</td>
<td>1750</td>
<td>1950</td>
<td>2150</td>
<td>2350</td>
</tr>
<tr>
<td>URM</td>
<td>24</td>
<td>28</td>
<td>32</td>
<td>32</td>
<td>32</td>
<td>32</td>
<td>32</td>
</tr>
<tr>
<td>Mean RMS Height Error [m]</td>
<td>1.395</td>
<td>1.465</td>
<td>1.315</td>
<td>1.485</td>
<td>1.274</td>
<td>1.652</td>
<td>1.265</td>
</tr>
</tbody>
</table>

Table 21: Comparing the effects of changing $B_{13}$ without changing $B_{23}$, at $N = 5$, $M = 20$, SNR = 20 dB, $B_{13}|_{k=1} = 200m$, $B_{23}|_{k=1} = 50m$
The results in Table 21 and Figure 39 reveal a somewhat surprising result. Contrary to what one might expect, it seems that there is a decrease in mean RMS height error when the $k$ increases from $\frac{1}{2}$ towards 2, but above 2 the mean RMS height error increases. This indicates a limited region within which only good results can be expected. This phenomenon can be understood by the following explanation: Using Lombardini’s model, it is expected that as $B_{13}$ increases, the corresponding correlation values for $B_{13}$ (as determined in Equation (67)) start decreasing. When $k$ is low, this corresponds to a high correlation result for $B_{13}$ while the reliance of $B_{23}$ values is big. Hence, when $k$ is increasing towards 2, the decrease in high correlation for $B_{13}$ as a result of increasing $B_{13}$ is offset by the decrease in reliance of $B_{23}$. Similarly, once $k$ exceeds 2, the decrease in correlation of $B_{13}$ is too great, hence offsetting the decrease in reliance of $B_{23}$, after which the mean RMS value becomes hard to predict since the correlation becomes so low. This results in the curve as we see in Figure 39. (Note that the value at $k = 1$ can be used as a check to compare with previous Tables).
Likewise, we can change $B_{23}$ instead by multiplying the same factors while keeping $B_{13}$ constant. (Note that $k \cdot B_{23}$ must still be less than $B_{13}$). The results are displayed below:

<table>
<thead>
<tr>
<th>Multiplication Factor, $k$</th>
<th>$\frac{1}{6}$</th>
<th>$\frac{1}{2}$</th>
<th>$\frac{1}{6}$</th>
<th>$\frac{1}{2}$</th>
<th>$\frac{1}{4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_{13}$ [m]</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>$k \cdot B_{23}$ [m]</td>
<td>6.25</td>
<td>7.14</td>
<td>8.33</td>
<td>10</td>
<td>12.5</td>
</tr>
<tr>
<td>$B_{12}$ [m]</td>
<td>193.8</td>
<td>192.9</td>
<td>191.7</td>
<td>190</td>
<td>187.5</td>
</tr>
<tr>
<td>$URM$</td>
<td>32</td>
<td>28</td>
<td>24</td>
<td>20</td>
<td>16</td>
</tr>
<tr>
<td>Mean RMS Height Error [m]</td>
<td>5.902</td>
<td>4.334</td>
<td>3.534</td>
<td>2.078</td>
<td>1.374</td>
</tr>
</tbody>
</table>

Table 22: Comparing the effects of changing $B_{23}$ without changing $B_{13}$, at $N = 5, M = 20$, $SNR = 20$ dB, $B_{13}|_{k=1} = 200$ m, $B_{23}|_{k=1} = 50$ m

![Plot of mean RMS height error [m] against multiplication factor, $k$, for changing $B_{23}$](image)

Figure 40: Comparing the effects of changing $B_{23}$ without changing $B_{13}$
Unlike in the first case where $B_{13}$ varies, the variation of $B_{23}$ leads to a less surprising trend. We see from Figure 40 that as $k$ increases while $B_{13}$ is kept unchanged, the effect of increasing $B_{23}$ is that mean RMS height error decreases (i.e. an improvement). From this, we can reasonably argue that the cause of this improvement is because of increasing $B_{23}$ value, which decreases the noise reliance of the overall result. Furthermore, since $B_{13}$ is kept constant, there is no influence from its correlation value; hence the curve is an asymptotic one. Furthermore, we see that the mean RMS height error in fact seems independent of the $URM$ value, since conflicting $B_{13}$ and $B_{23}$ changes bring about different mean RMS error trends while $URM$ can remain unchanged.

**5.5.5 Demonstrating the Independence of RMS Error on the $URM$ Value**

From 5.5.4, we see that conflicting actions in changing $B_{13}$ and $B_{23}$ result in changes not consistent with the $URM$ value changes. This implies that the mean RMS height error value is in fact independent of the $URM$ value. To show this, we change the multiplication factor $k$ simultaneously for $B_{13}$ and $B_{23}$ such that the $URM$ value remains constant at a value 4. The results against the baseline lengths are shown in Table 23 and Figure 41. Hence we vary $B_{13}$ and $B_{23}$ lengths by multiplying them by a factor $k$ to both values, hence keeping $URM$ unchanged.

<table>
<thead>
<tr>
<th>Multiplication Factor, $k$</th>
<th>$\frac{B_{13}}{k}$</th>
<th>$\frac{B_{23}}{k}$</th>
<th>$\frac{B_{12}}{k}$</th>
<th>$URM$</th>
<th>Mean RMS Height Error [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k \cdot B_{13}$ [m]</td>
<td>20</td>
<td>22.22</td>
<td>25</td>
<td>28.57</td>
<td>33.33</td>
</tr>
<tr>
<td>$k \cdot B_{23}$ [m]</td>
<td>5</td>
<td>5.57</td>
<td>6.25</td>
<td>7.14</td>
<td>8.33</td>
</tr>
<tr>
<td>$k \cdot B_{12}$ [m]</td>
<td>15</td>
<td>16.65</td>
<td>18.75</td>
<td>21.43</td>
<td>25</td>
</tr>
<tr>
<td>$URM$</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Mean RMS Height Error [m]</td>
<td>2.223</td>
<td>2.120</td>
<td>2.000</td>
<td>1.728</td>
<td>1.527</td>
</tr>
</tbody>
</table>

Table 23: Comparing the effects of changing the baseline lengths without changing the $URM$, at $N = 5$, $M = 20$, $SNR = 20$ dB, $URM = 4$, $B_{13} \mid_{k=1} = 200$ m, $B_{23} \mid_{k=1} = 50$ m
Figure 41: Comparing the effects of changing the baseline lengths without changing the

Again, from Figure 41, we see the effects of both correlation of $B_{13}$ coming together with
the reliance of $B_{23}$'s length. Hence, we see that while $URM$ is kept fixed, the mean RMS
height error still varies, and does so according to the phenomenon described in the
previous Section. Thus, we conclude that while the mean RMS height error is more
reliant on the correlation of $B_{13}$ and the noise dependence of $B_{23}$, the baseline ratio $URM$
does not play a significant role in the mean RMS error determination.

Thus, we conclude the following about the RMS error for the maximum likelihood
estimator:

$$[\text{mean RMS error}]_{MLE} = g\left(\frac{1}{N}, \frac{1}{SNR}, \frac{B_{13}}{B_{23}}\right)$$

(78)

where $g$ is a function of the respective components $N$, $SNR$, $B_{13}$ and $B_{23}$. Like before, we
will not seek to find the exact function here.
Chapter 6

Method Comparisons

6.1 Comparing the Methods

6.1.1 Converting Results to Achieve Uniformity for Comparison

We are finally ready to compare the performance of all the methods introduced before for the 2- and 3-satellite methods. Because the MLE method was formulated using Lombardini’s model, where the noise model was different (and signal amplitudes were not equal to 1), there becomes a need to convert the results of either the MLE (in terms of $SNR$ (in [dB]) to [°]) or vice versa. However, because the signal amplitudes of Lombardini’s model are hard to predict, we choose the simpler path of conversion: By assuming signal amplitudes for the 2-satellite, 3-satellite data averaging and 3-satellite URM methods to be equal to 1, we convert the noise angle (in [°]) to $SNR$ (in [dB]) instead. To do so, we first observe the diagram given in Figure 42 to see what it means to have noise for the angle only.

![Figure 42: Plot of complex signal with amplitude $A$ and phase $\phi = 0$, with noise angle.](image)

Figure 42 shows an arbitrary example of a signal measured from the SAR, with amplitude $A$ and no initial phase angle, i.e. $\phi = 0$, but with a phase noise angle $\phi$. Applying this Figure to our model, when we specify a noise angle $\phi$ as we have done in previous chapters (with the exception of the MLE), we assumed that with reference to Figure 42 the amplitude $A = 1$, and the noise angle $\phi = \phi$. Hence the noise amplitude for Figure 42 would be given as the radius of the arc with angle $\phi = \phi$. Since we wish to
convert this noise angle (in [°]) to SNR form (in [dB]), we use the following conversion formula:

$$
SNR \ [dB] = 10 \log \left( \frac{Signal \ Power}{Noise \ Power} \right) = 10 \log \left( \frac{A}{\phi \cdot \pi} \right)^2 = 20 \log \left( \frac{180}{\phi \cdot \pi} \right)
$$

(79)

where we assumed $A = 1$.

Hence, a noise angle of $n = 1^\circ$ would correspond to $SNR = 35.16 \ dB$. Table 24 shows the conversion for other values, while Figure 43 shows the pictorial view.

<table>
<thead>
<tr>
<th>$n \ [^\circ]$</th>
<th>0</th>
<th>1</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SNR \ [dB]$</td>
<td>$\infty$</td>
<td>35.163</td>
<td>21.183</td>
<td>15.163</td>
<td>11.641</td>
<td>9.142</td>
<td>7.204</td>
</tr>
</tbody>
</table>

Table 24: Some noise-SNR conversion values using Equation (79)

Figure 43: Pictorial representation for Equation (79)

Therefore, we now have a common representation by which we can compare the values of the MLE/URM/Data Averaging/2-satellite methods. We convert the noise values of the 2-satellite, data averaging and URM methods into $SNR$ form, while keeping their
mean RMS height errors. Likewise, to match the \( SNR \) values, we perform the MLE simulations again, using the \( SNR \) values specified by the noise values for the other methods, and finally plot them on the same plot. The recalculated results for the MLE are given in Table 25.

<table>
<thead>
<tr>
<th>( SNR ) [dB]</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>10</th>
<th>20</th>
<th>50</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.204</td>
<td>32.0903</td>
<td>11.3605</td>
<td>1.4347</td>
<td>0.6325</td>
<td>0.4333</td>
<td>0.2811</td>
<td>0.2153</td>
</tr>
<tr>
<td>9.142</td>
<td>25.4099</td>
<td>8.7043</td>
<td>1.0997</td>
<td>0.5497</td>
<td>0.3513</td>
<td>0.2434</td>
<td>0.1735</td>
</tr>
<tr>
<td>11.641</td>
<td>15.3185</td>
<td>4.1725</td>
<td>0.7007</td>
<td>0.4394</td>
<td>0.3117</td>
<td>0.2112</td>
<td>0.1424</td>
</tr>
<tr>
<td>15.153</td>
<td>9.4376</td>
<td>2.6450</td>
<td>0.5977</td>
<td>0.3919</td>
<td>0.2554</td>
<td>0.1546</td>
<td>0.1228</td>
</tr>
<tr>
<td>21.183</td>
<td>5.8957</td>
<td>1.2115</td>
<td>0.5210</td>
<td>0.3353</td>
<td>0.2317</td>
<td>0.1488</td>
<td>0.1166</td>
</tr>
<tr>
<td>( \infty )</td>
<td>4.6685</td>
<td>1.1079</td>
<td>0.4809</td>
<td>0.3278</td>
<td>0.2200</td>
<td>0.1387</td>
<td>0.0927</td>
</tr>
</tbody>
</table>

Table 25: Recalculated Values for MLE method, to suit the \( SNR \)-converted noise angles.

6.1.2 Comparisons for Varying \( SNR \) and Fixed Number of Looks

The parameter setup will be the same as before (see Table 4). We first perform our \( SNR \) comparison for a fixed number of looks, starting with a single simulation (averaged over a 100 trials, or \( T = 100 \)) for all the methods. For the MLE, \( N = 1, M = 100 \). This is depicted in Figure 44(a) and (b). In the plot, we also include the MLE results for \( N = 2, M = 50; N = 5, M = 20 \); as well as \( N = 10, M = 10 \). Figure 44(a) includes all methods, while Figure 44(b) ignores the URM method and MLE for \( N = 1 \).
Figure 44: Comparing the Capabilities of Different Methods by varying SNR
(a) For all methods,
(b) Magnified version of (a) (Ignores URM and MLE at $N = 1$ and $N = 2$)
<table>
<thead>
<tr>
<th>Method Used</th>
<th>Mean RMS Height Error [m]</th>
<th>SNR [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>7.204</td>
<td>2.1011</td>
</tr>
<tr>
<td></td>
<td>9.142</td>
<td>1.6737</td>
</tr>
<tr>
<td></td>
<td>11.641</td>
<td>1.2603</td>
</tr>
<tr>
<td></td>
<td>15.163</td>
<td>0.8418</td>
</tr>
<tr>
<td></td>
<td>21.183</td>
<td>0.4246</td>
</tr>
<tr>
<td>2-satellite</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Data Average</td>
<td></td>
<td></td>
</tr>
<tr>
<td>URM</td>
<td>20.9166</td>
<td>18.8807</td>
</tr>
<tr>
<td></td>
<td>13.6686</td>
<td>6.2959</td>
</tr>
<tr>
<td></td>
<td>1.4431</td>
<td>0</td>
</tr>
<tr>
<td>MLE, N = 1</td>
<td>32.0903</td>
<td>25.4099</td>
</tr>
<tr>
<td></td>
<td>15.3185</td>
<td>9.4376</td>
</tr>
<tr>
<td></td>
<td>5.8957</td>
<td>0.4246</td>
</tr>
<tr>
<td></td>
<td>4.6685</td>
<td>0</td>
</tr>
<tr>
<td>MLE, N = 2</td>
<td>11.3605</td>
<td>8.7043</td>
</tr>
<tr>
<td></td>
<td>4.1725</td>
<td>2.6450</td>
</tr>
<tr>
<td></td>
<td>1.2115</td>
<td>0.4246</td>
</tr>
<tr>
<td></td>
<td>1.1079</td>
<td>0</td>
</tr>
<tr>
<td>MLE, N = 5</td>
<td>1.4347</td>
<td>1.0997</td>
</tr>
<tr>
<td></td>
<td>0.7007</td>
<td>0.5977</td>
</tr>
<tr>
<td></td>
<td>0.5210</td>
<td>0.4246</td>
</tr>
<tr>
<td></td>
<td>0.4809</td>
<td>0</td>
</tr>
<tr>
<td>MLE, N = 10</td>
<td>0.6325</td>
<td>0.5497</td>
</tr>
<tr>
<td></td>
<td>0.4394</td>
<td>0.3919</td>
</tr>
<tr>
<td></td>
<td>0.3353</td>
<td>0.3278</td>
</tr>
</tbody>
</table>

Table 26: Comparing mean RMS height error against varying SNR values, 
$T = 100$, $URM = 4$

From Figure 44(a), we clearly see the limitations of some of the methods proposed. The worst performing method is the MLE at $N = 1$, followed by the URM method. Following this would be the 2-satellite method from Chapter 3, followed by the 2-component data averaging method from Chapter 4 (see Figure 44(b)). The MLE at $N = 5$ has some interesting results, since it does not outperform the 2-component data averaging method and 2-satellite method for high SNR values. However, the MLE at $N = 10$ clearly outperforms all the other methods. Table 26 above provides the numerical results obtained for the plots of Figure 44.

Indeed then, from Figure 44, we can conclude that the MLE method greatly improves with increasing number of looks incorporated into Equation (77). Even with just 2 looks, the improvement made by the 2$^{nd}$ set of data is more than twice that of when only 1 look was available, surpassing the URM method. When $N$ is increased to 5, the MLE outperforms both the data averaging and 2-satellite methods in the higher noise cases (i.e. low SNR), but is overtaken by both methods when only very low noise is present (high SNR). A possible reason for this is because of the different noise models considered, since Lombardini’s model gives noise even at $SNR \rightarrow \infty$ (see Table 26), which has zero noise for the other methods. At $N = 10$, the MLE performs better than all the other methods proposed within the sampling range (but obviously not so outside, since at $SNR \rightarrow \infty$ there is residue error), demonstrating the asymptotic efficiency of the method over simple averaging. Of course, if a different model was adopted instead of Lombardini’s for the MLE, the MLE exhibits the potential to perform better than all methods even for $SNR \rightarrow \infty$, though this is not demonstrated in our work.

Also, the reason why MLE at $N = 1$ performs so poorly should be partly attributed to the different noise model used. However, there should be a mention that this poor performance could be also due to the heavy dependence of $B_{23}$ for its results. Since we already know that the short baseline $B_{23}$ is more sensitive to noise changes, hence this would cause the MLE to spike up if only a single look were present. When unwrapped, the extreme noise changes in phase cause the height profile to fluctuate tremendously. This would then explain for the performance for the URM method as well, since it also exhibits large errors which exceed the maximum range of height.
6.1.3 Comparisons for Varying Number of Looks and Fixed SNR

Plot of Different Method Performance versus Number of Looks, N, with total number of simulations fixed at 100

![Plot of Method Performance](image)

Figure 45: Determining the point at which MLE exceeds all methods for increasing N, \( SNR = 15.163 \) dB, \( T = 100 \), \( URM = 4 \)

<table>
<thead>
<tr>
<th>Mean RMS Height Error [m]</th>
<th>Number of Looks, N</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Method Used</strong></td>
<td>1</td>
</tr>
<tr>
<td>2-satellite</td>
<td>0.8418</td>
</tr>
<tr>
<td>Data Average</td>
<td>0.7401</td>
</tr>
<tr>
<td>MLE</td>
<td>9.4376</td>
</tr>
</tbody>
</table>

Table 27: Numerical results to demonstrate the MLE's performance over increasing N

Figure 45 and Table 27 above show the results of increasing the number of looks \( N \) while keeping the total number of simulations \( T \) fixed at 100 and \( SNR = 15.163 \) dB. On the one hand, since the other methods do not depend on \( N \) but rather just \( T \), hence their values remain unchanged throughout. On the other hand, the MLE performance improves for increasing \( N \) (and hence decreasing \( M \)), thus we see a steep decreasing slope as shown in Figure 45. It would not be unreasonable to expect that when \( N \) is infinite, then the MLE would have zero noise altogether (though limitations of computability prevent us from proving this). Therefore, from the Figure at \( SNR = 15.163 \) dB, we see that the MLE performs better than the URM at \( N \geq 2 \), better than both averaging and 2-satellite
methods for $N \geq 5$. Note that again these conclusions apply only for $SNR = 15.163$ dB and separate conclusions will have to be made for different $SNR$ levels.

Thus, we summarize the following results obtained for the comparison of the different methods used: If only one or 2 looks are present, the best performing method would be the averaging method, since it provides definite improvements independent of look numbers. Furthermore, this method is simple to introduce and is computationally efficient.

However, if multiple looks were present, then the MLE method would take precedence in performing well. In fact, the performance would be much better than the averaging method in the presence of large $N$, performing up to many times better than the averaging method for low $SNRs$. Also, in low $SNR$ cases (i.e. high noise power), then the MLE method works better than the averaging method as well for a reasonable number of looks.

### 6.2 Limitations of comparison

#### 6.2.1 Noise Models

Naturally, the comparisons made in Section 6.1 can only be considered partially fair, since the noise model used for the MLE against the one for the other methods are fundamentally different. The model proposed in Chapter 2 ignores the signal amplitudes, and the noise is additive only to the interferometric phase values. On the contrary, Lombardini’s method affects both signal amplitude and interferometric phase. This makes comparison difficult, since this poses a problem on how to correctly account for the signal amplitude values for Chapter 2’s model. Furthermore, Lombardini’s model is such that even at $SNR \to \infty$ some residue error still exists, which is somewhat unrealistic from our interpretations. Hence, if a fairer comparison were demanded, a different MLE could be re-derived using a similar procedure as Lombardini such that the same noise model could be adopted. Nonetheless one can expect that the conclusions that the MLE method would work more efficiently in the asymptotic limit will remain unchanged. Another point to note here would be the fact that our comparisons were based on a fixed range of $SNR$ values; in truth the conclusions derived between methods may starkly contrast other $SNR$ regions adopted.

#### 6.2.2 Phase Unwrapping

Phase unwrapping remains mostly unaccounted for the simulation results we obtained, where we conveniently used Matlab’s $unwrap$ function to unwrap phase jumps that occur within $\frac{1}{2}$ of the maximum phase range. However, this assumption may overlook many important steps required to properly unwrap phase, such as determining the starting phase unwrap value, the tolerance level to add phase for different terrain slopes, and in particular accounting for the number of phase unwrapping steps performed (ignored in our simulations). Nonetheless our methods have been somewhat consistent in using the $unwrap$ formula, thus the conclusions made can considerably ignore the effects of phase unwrapping for lower noise ranges. At higher noise ranges, the $unwrap$ formula will be
unable to account for more fluctuations, and hence result in fluctuating random errors. As before, in order to better separate the phase unwrapping methods, it would be recommended to perform repeated trials and taking their mean, much like we have done before.

6.2.3 Ground Truth And Plot Alignment

One of the major problems left unresolved is still the loss of the quotient value when the absolute interferometric phase gets wrapped. Even though we attempt to resolve this problem with the data averaging method (see Chapter 4), the method introduced is again limited by the presence of noise, and hence ground truth is still largely assumed for many of the methods we introduce. This translates to having GCPs or existing DEMs in order for correct retrieval, but this would indeed be unrealistic in terrain which has been left unexplored.

Also, in many of the simulations the presence of noise may cause the retrieved height profiles to become consistent only at the ground truth point and inconsistent at all other points; hence we fine-tuned our results using the plot alignment method. This method aligned and interpolated the plots against one another, many times with an assumed complete DEM, and we compared the RMS results taking the difference of the retrieved height with again the assumed complete DEM. This again becomes unrealistic in many practical scenarios. Yet, since our work was based more on interpreting the performance of the different methods proposed, therefore assuming this complete DEM is perhaps an acceptable reference by which to base performance on.

6.3 Limitations of Method Use

6.3.1 Satellite Orientation

The methods introduced in the previous chapters all assume a common configuration for the 3 satellites; i.e. that all 3 satellites were collinear against one another. However, in many practical scenarios this may not be the case, and hence reformulations may have to be performed to fit a more generic case. Nonetheless, it seems safe to assume that the 2-component data averaging method should work for other configurations as well, since it merely involves averaging the overall retrieved DEMs of the interferograms, provided 2 satellites are closer to one another than the 3rd. However, the same may not apply for the URM and MLE methods, of which the expanded phase seems heavily reliant on the fact that the similar triangle assumption can hold. Hence, it is not unreasonable to conclude that the data averaging method seems more flexible in terms of satellite orientation than do the other 2 methods. It is clear that the 2-satellite model will not have these orientation problems.
6.3.2 Single and Multiple Look Events

As we have seen in the earlier part of this Chapter, single and multiple look events change the decision at which to apply the methods used. In the event of a single look event, the data averaging method would be simple to use and yet provide accurate results. However, when confronted with multiple looks, the wiser choice would be the MLE method, by which it helps stem out a significant portion of random error efficiently with only a handful of looks, more so than data averaging can provide. The URM method performs badly in both single and multiple look scenarios, and hence should not be a viable option to adopt.

6.3.3 Noise Events

The different degrees of noise also affect the decision of height retrieval method. For high SNR (or low noise) scenarios, it seems that the data averaging method would be a good option to adopt if only a few looks are available. With more looks, this will be preceded by the MLE method, which will be able to filter out noise more efficiently. However, in low SNR (or high noise) scenarios, the preferred choice would be the MLE method. It seems that the URM method is severely limited in its ability to filter noise, and perhaps the only advantage it provides is setting up an expanded phase range by which to use the MLE method.

6.3.4 Baseline Length

Though a formal comparison between methods was not performed on the effects of varying baseline length, one can make some logical deductions about this effect. Since the data averaging method involves merely an application of the 2-satellite model’s results, thus naturally we expect that the baseline length is inversely proportional to the data averaging method’s results (see Equation (16)). Furthermore, because we made the assumption of perfect coherence (i.e. SAR signal amplitude = 1), hence we do not expect any effects from coherence change due to varying baseline length. Thus Equation (16) will hold for the data averaging method, i.e. as the baseline length increases (\(B_{13}\) and/or \(B_{23}\)), the results for the data averaging method will improve (see Chapter 4.5). That is, we postulate:

\[
[\text{Mean RMS Error}]_{\text{2-component Data Averaging}} = f\left(n, \frac{1}{B_{13}}, \frac{1}{B_{23}}\right) \quad (80)
\]

On the other hand, as shown in Chapter 5.5.4, we have seen that the effect of baseline changes (\(B_{13}\) and/or \(B_{23}\)) give varying results to the overall RMS height error. For instance, too small a baseline length results in a heavy reliance of \(B_{23}\) and hence an increase in overall RMS error, while too large a baseline results in a large decrease in the correlation value, hence increasing RMS error as well. Hence, only within a definite range can the MLE method perform well (and better than the data averaging method).
Likewise, a decrease in baseline length for $B_{23}$ can raise the $URM$ constant, which ultimately results in a worse-off RMS error result, and vice versa.

6.3.5 Computational Speed

As mentioned before, it seems almost certain that the computational speed of the data averaging method would be less than the MLE method, since it is less involved as the MLE method in determining the RMS height error (after retrieval, it just adds all the data up and averages). On the other hand, the MLE method requires finding the correlation values, after which to predict the maximum value for the estimator (which in turn relies on the resolution of the test phase range), followed by unwrapping before finally retrieving the height.
Chapter 7

Summary

7.1 Summary

The following summarizes the results of the previous 6 chapters.

For the 2-Satellite Model:

- The mean RMS height error for a single baseline increases with increasing noise, and decreases with increasing baseline length (up to the critical baseline condition).
- The accuracy of the setup parameters used for height retrieval is crucial in determining the accuracy of the DEMs retrieved. It has been shown that even a small offset in the baseline parameter, i.e. baseline uncertainty, can cause severe misalignment in the retrieved height profile with the original DEM. For $B' < B$, this causes the right-side terrain of the center to be higher it should be, while the left-side terrain appears lower. This appears reversed for $B' > B$.

For the 3-Satellite Models:

- For Data Averaging:
  - Data averaging is a simple method of applying weighting functions to the 3 sets of heights retrieved from the 3-satellite configuration, resulting in a better result than the 2-satellite case. The choice of weighting functions will determine the performance of the data averaging method.
  - For simplicity, we adopted a collinear orientation for all 3 satellites. Likewise for the other methods.
  - The 2-component data averaging method, which averages the retrieved height values for the 2 longer baselines of a 3-satellite configuration, results in lower RMS height error than the best (i.e. longest) single baseline case for the same configuration, given certain conditions. The results for this method also bring about a theoretical baseline result which will be longer than maximum baseline length for the 3-satellite setup. Note that this theoretical result should not be used against the critical baseline condition, since it bears no physical equivalence.
  - There is a minimum threshold length for the $B_{12}$ component of the data averaging method, of which below this length the data averaging method does not provide better results than the single longest baseline $B_{13}$ setup. This generates a physical bound within which the baseline values should
be: \( B_{12} \) must exceed the minimum threshold length while both \( B_{12} \) and \( B_{13} \) must not be longer than the critical baseline length.

- Adopting the 2-component data averaging method with the slow cartwheel motion of the 3 satellites, an optimum arrangement for the satellites can be obtained to minimize the achievable RMS height error. This optimum arrangement occurs when \( B_{12} = B_{13} \).
- For a multiple baseline mismatch (with \( B_{13}' \) and \( B_{12}' \) uncertain), there is only 1 point in which the RMS height error for the averaged method nullifies, and this is where \( B_{13}' = B_{13}, B_{12}' = B_{12} \). This point can be used as a guideline to determine the accuracy of the baseline length predictions, and also to remove the need of GCPs. This however applies only with the absence of noise, and with the presence of the absolute true DEM.
- If this DEM were not present, then one can use the results of the maximum baseline length \( B_{13}' \) as a reference instead.
- If noise is present, then using the maximum baseline length as a reference has its limitations, since it is cluttered with noise as well. The ability to remove the GCPs then becomes unclear, but with repeated trials it may still be possible to eliminate GCPs.

- **For URM:**
  - The purpose of the URM method is to introduce a method to reduce the number of phase unwrapping steps necessary for a given phase function.
  - The setup it assumes is collinear; with the center satellite asymmetrically placed (i.e. it is nearer to one end than the other).
  - Based on the difference in magnitudes of absolute phase for baselines with different lengths, it is possible to retrieve an unambiguous expanded phase range which is wrapped by \( URM \times 2\pi \), where \( URM > 1 \). This corresponds to a reduction in phase unwrapping by \( URM - 1 \) times than per normal.
  - Because this method is based on the far-range assumptions, several sensitivities need to be resolved before applying this method. Amongst others they include the infinite \( URM \) case, the reasoning behind using the shortest baseline \( B_{23}' \)'s phase variations, as well as the uniqueness of solution with respect to absolute phase and GCPs.
  - In the presence of noise, this process effectively reduces global error by reducing the count for phase unwrapping techniques, but at the same time sacrifices for local error instead.
  - The second tradeoff is that as URM increases indicating a wider phase range, local error increases as well, due to the dependence on \( B_{23}' \)'s phase variations.
  - Even though this method is the worst faring in determining the RMS height error, its main advantage is to set up the possibility of an expanded range for the MLE method.

- **For MLE:**
  - The purpose of the MLE is to determine accurately the phase value within the expanded phase range provided by the URM method.
- Contrary to the noise model used for the other methods, we used F. Lombardini's noise model instead, to maintain constancy in determining the maximum likelihood estimator as Lombardini has done.
- The noise model makes many assumptions which have stark differences from the noise model used before. These include, for instance, noisy signal amplitudes, and the presence of error even at $SNR \to \infty$. Hence, one has to be cautious when comparing the results obtained between methods.
- Like for the other 2 methods, it was shown that the mean RMS height error increases with noise (i.e. decreasing $SNR$).
- Furthermore, accuracy also depends on the resolution of test phase values, which are in turn used to determine the estimator values, as well as the number of computations needed.
- This method is asymptotically efficient, which means results improve as the number of looks increase.
- Contrasting effects come in when comparing the effects of the baseline length on the MLE. For exceedingly low baseline lengths, the RMS results are cluttered by noise due to the short $B$ length, since the noise is inversely proportional to the baseline length. At exceedingly high baseline lengths, the RMS values again rise, this time because of the decorrelation that occurs when $B$ becomes large.
- Thus, there is only a suitable range of baseline lengths by which the MLE will perform well.
- It was also shown that the MLE results seem less dependent on the $URM$ value (i.e. the baseline ratio) than they do the actual baseline values.

**For Comparisons between the Methods:**
- Due to limitations in comparison such as different noise models, phase unwrapping procedures and the use of ground truth, it can be said that the conclusions made in comparing the methods is only partially fair.
- Nonetheless, the conclusions made are given in Table 28, indicating also the scenarios preferred for each method.
<table>
<thead>
<tr>
<th>Method</th>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-Satellite</td>
<td>Computationally fast; Assumes no satellite orientation;</td>
<td>Inaccurate results</td>
</tr>
<tr>
<td>Data Averaging</td>
<td>Computationally fast; Results are more accurate than 2-satellite methods in all cases; Works best for low look number; Results are independent on signal amplitudes and correlation;</td>
<td>Assumes fairly lenient satellite orientation; Works poorly for high noise levels; Results critical of choice of weighting functions</td>
</tr>
<tr>
<td>URM</td>
<td>Fairly computationally fast; Results are independent on signal amplitudes and correlation;</td>
<td>Assumes strict satellite orientation; Least accurate of all methods; Works poorly for all noise levels; Works poorly for all look numbers</td>
</tr>
<tr>
<td>MLE</td>
<td>Works best for high look number; Works best for high noise;</td>
<td>Computationally slow; Assumes strict satellite orientation; Results are dependent on signal amplitudes, test phase resolution, and correlation;</td>
</tr>
</tbody>
</table>

Table 28: Weighing the performance of the different methods examined

5.2 Comments and Future Work

Without a doubt, the inclusion of a third satellite most definitely introduces more options by which one can retrieve height information. We have seen the advantages and disadvantages of a few methods, namely the data averaging, URM and MLE methods, and we've also shown their performance and limitations of use. These methods are by no means exhaustive, and neither will be the studies conducted on them. Nonetheless, the conclusions of these methods are clear: When using the appropriate method for the appropriate situation, the third set of data measured allows for a most definite improvement in results as compared to the 2-satellite case.

Yet, though this holds true, there have been some hiccups in the method presentation which poses as future challenges. The first would naturally be finding a common noise model between the data averaging and URM/MLE methods such that a better comparison can be achieved. Secondly, the problem of eliminating ground truth has been partially solved through the data averaging model, though it becomes somewhat problematic in the presence of noise. Further study needs to be conducted to find a way to better eliminate, or at least suppress the noise effects on the proposed solution. Thirdly, even though the URM and MLE methods aim to reduce the amount of phase unwrapping processes needed, they inevitably cannot remove all of these processes. A possible solution might be to overlap the resulting DEMs by finding another MLE such that phase unwrapping can be eliminated [36], but this again has assumed more than 1 look of data collected. Lastly, we have only studied the effects of 3 satellites as opposed to 2. If we instead had 4 or more satellites present, the presence of the additional satellites may perhaps provide better solutions to our ultimate goal of obtaining perfect DEMs.
Bibliography

Books


Journal Articles and Reports


