STEADY STATE MECHANICS OF THE FALSE TWIST YARN TEXTURING PROCESS

by

RAYMOND Z. NAAR

Ing. A.I.V., École Superieure des Textiles de Verviers (1955)
S.M., Massachusetts Institute of Technology (1958)

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF SCIENCE at the MASSACHUSETTS INSTITUTE OF TECHNOLOGY FEBRUARY 1975

Signature of Author

Department of Mechanical Engineering, December 5, 1974

Certified by

Thesis Supervisor

Accepted by

Chairman, Department Committee on Graduate Students
ABSTRACT

In the false twist texturing process a moving threadline is subjected to temporary twist while heat is applied to part of the twist zone. The purpose of the process is to heat set the twist deformation so that after cooling and twist removal, the filaments will seek to return to their set helically crimped paths. This crimping action leads to bulk and/or stretch development such as is encountered in double knit suitings and/or stretch hosiery.

During processing, the entire twisted threadline is under constant tension and torque, but is not at constant temperature, since part of it is intentionally heated to effect the setting process. The threadline is thus divided into three contiguous zones, all under the same stress field but each with different mechanical properties. The threadline tension is a result of filament strain which arises from the difference between the denier of the exiting yarn (imposed by the feed and exit roll velocities) and the equilibrium denier for the unrestrained yarn at the heater temperature, while the torque is due to the twist imparted by the spindle. In the portion of the threadline before the heater, the torque is supported by components of the bending and torsional moments acting on the individual filaments, as well as the moment due to the tension of the filaments. At the heater, the bending and torsional moments become insignificant, and the torque is supported by tension components only; the twist, therefore, increases to maintain torque equilibrium. This uptwisting is assumed to occur without filament migration. As the properties of the filament change along the threadline due to temperature changes, so does the mechanism by which the yarn is deformed.

The pertinent filament properties and yarn twisting mechanism have been analyzed. Values of tension and torque have been calculated for a wide range of temperatures and over-feeds and are in agreement with experiment.
ACKNOWLEDGEMENTS

The author is indebted to the Phillips Petroleum Company and to the WESTVACO Corporation for their fellowship support in the early stages of his graduate program. Thanks are due also to Burlington Industries, Inc. and to the E. I. du Pont de Nemours & Company for fellowship and research assistantship support during the latter part of his doctoral research. Without this generous industrial backing and encouragement the completion of this program would not have been realized.

The author wishes to thank Professor S. Backer, Professor M. B. Bever, Professor E. Orowan and Professor I. V. Yannas who served on his Doctoral Committee. He greatly profited by contact and discussions with them on scientific and other topics. He also profited by discussions and collaboration with a number of classmates and office mates, among them A. Crugnola, D. Brookstein, A. Tayebi, S. Argyros, B.Y. Lee, C. Brogna.

He also greatly profited from contact, discussions and friendship with Drs. S. Batra and W. L. Yang, staff members of the Fibers and Polymers Laboratories. The support of Professor Yannas, who was always ready to help, listen and make suggestions is most gratefully acknowledged. Professor J. J. Thwaites was most helpful and gave unstintingly of his time. The writer greatly profited from discussions with a person of such critical and creative wit and is sincerely grateful to him.

The author wishes to acknowledge the moral support he received from his colleagues in the Department of Chemical Engineering at Tufts University, Professors van Wormer, Botsaris and Sussman. He is also grateful to Professor Gyftopoulos who never ceased to encourage him towards completion.

Among others, the writer wishes to thank his wife and his mother for their encouragement. Many thanks to Dorothy Eastman who typed many of the progress reports connected with the manuscript; to Joanna Larsen for typing the manuscript, and to Guddi Wassermann for her accurate drawings.
Finally, it would be unfitting to conclude without stressing the writer's deep indebtedness to Professor Stanley Backer, the Chairman of his Doctoral Committee, who helped, advised and encouraged the writer throughout his work. The writer considers himself lucky to have been associated with a person of Professor Backer's stature; he enjoyed and greatly profited from the experience of working under a man of uncompromising standards of quality and capability of seeing through complex situations to the simple physical essence of a problem.
## TABLE OF CONTENTS

Abstract  
Acknowledgements  
List of Tables  
List of Figures  
Chapter I.  
   Introduction  
Chapter II.  
   Plan of Work  
Chapter III. The False-Twist Process  
   (a) Description of the Process  
   (b) Twist Development in the Machine  
   (c) Controllable Machine Variables  
   (d) The False-Twisted Yarn  
   (e) Machine-Yarn Interaction  
Chapter IV. Yarn Geometry  
   (a) Model for Yarn Geometry  
   (b) Yarn Contraction  
Chapter V. Yarn Mechanics  
   (a) Classical Mechanics  
      1. Stress-Strain Curves of Yarns  
      2. Average Filament Strain  
      3. Yarn Torque  
      Bending Moment Contribution to Yarn Torque  
      Fiber Torsional Moment Contribution to Yarn Torque  
      Torque due to Tension  
   (b) Tension and Torque for the Case of Equal Tension on all Filaments  
      1. Tension: Stress-Strain Curve  
      2. Torque  
   (c) Overtwisting  
      1. Stress-Strain Curve
2. Torque in Overtwisted Yarn
3. Overtwisting from Zero Twist
4. Average Filament Strain on Overtwisting
5. Average Filament Strain on Overtwisting in Various Zones of the Yarn
6. Torque on Overtwisting in Various Zones of the Yarn

Chapter VI. Experimental Apparatus and Materials

(a) Apparatus
1. Texturing Machine
2. Torque Measuring Apparatus
3. Tensile Testing Machine

(b) Material Properties
1. Stress-Strain Curve of Feed Yarns
2. Stress-Strain Curve of Yarns Processed through the False-Twister without Twist
3. Stress-Strain Curve of Freshly Textured Yarns
4. Stress-Strain Curve of Yarns at Elevated Temperatures--The Contractile Stress

Chapter VII. Experimental Results: Twist and Tension

(a) Introduction
(b) Experimental Procedure
(c) Experimental Results
1. Twist in Exit Zone
2. Twist in Entrance Zone
3. Tension
4. Denier
5. Analysis of the Data and Implications for the Process

(d) Experimental Results: The Effect of Twist
(e) Experimental Data: The Effect of Draw Ratio
(f) Qualitative Model for the Process

Chapter VIII. Calculation of Tension

(a) False-Twister Operating, but Spindle Stationary
1. Analysis
2. Experimental Verification

(b) Determination of the Tension during False-Twisting
   1. Analysis
   2. Experimental Verification
   3. Discussion
   4. Example of Tension Calculation

Chapter IX. Measurement and Calculation of Torques

(a) Introduction
(b) Torque in the Entrance Zone
   1. Torque due to Filament Bending
   2. Torque due to Filament Torsion
   3. Torque due to Filament Tension
   4. Comparison of Calculated and Experimental Data for the Case of the Entrance Zone
(c) Torque in the Heater Zone
   1. Material Properties
   2. Torque from Classical Yarn Mechanics and from Equal Filament Tension Contributions
   3. Overtwisting
   4. Discussion of the Data
(d) Example of Calculation of Torque
   1. Entrance Zone
      (i) Torque due to Tension
      (ii) Torque due to Bending
      (iii) Torque due to Torsion
   2. Heater Zone
      (i) Torque due to Tension
      (ii) Torque due to Bending
      (iii) Torque due to Tension-Overtwisting

Chapter X. Discussions, Conclusions and Recommendations

(a) Discussion
(b) Conclusions
(c) Recommendations
APPENDICES

Appendix I. Additional Yarn Mechanics Derivations 161
(a) Stress-Strain and Torque-Strain Expressions for Yarns; Twist Varies with Extension 161
(b) Stress-Strain Curves of Yarns Assuming the Twist to Vary with Extension and the Filaments to Extend at Constant Volume 169
(c) Stress-Strain and Torque-Strain Curves of Yarns Assuming the Twist to Vary with Extension and the Filaments to Extend at Constant Volume (Alternate Derivation) 171
(d) Overtwisting at Constant Radius 173
(e) Mechanics of Yarn Shrinkage due to Fiber Shrinkage 175
(f) Shearing Strains on Fiber due to Torsion and Bending 179
1. Differential Geometry of Yarns 179
2. Local Shear Strain on Fibers as they lie in the Yarn 182
3. Local Shear Strain on Untwisted, False-Twist Fibers 186

Appendix II. Literature Review 191
(a) Interaction between Machine Variables 191
(b) Heating and Thermal Plasticization 198
(c) Crimp Rigidity 215
(d) Effect of Machine Settings on Yarn Quality 230
References 240
Biographical Sketch 243
LIST OF TABLES

TABLE I: YOUNG'S MODULUS OF YARNS PROCESSED THROUGH THE FALSE-TWISTER (BUT NOT THREADED AROUND THE SPINDLE)

TABLE II: YOUNG'S MODULI OF TEXTURED YARNS (GRAMS PER UNIT STRAIN)

TABLE III: STRESS-STRAIN CURVES FROM CONTRACTILE STRESS DATA

TABLE IV: RELATIVE LENGTH CHANGES IN YARN PROCESSED THROUGH THE FALSE-TWISTER (SPINDLE STATIONARY)

TABLE V: TWIST DISTRIBUTION - EFFECT OF TEMPERATURE AND TENSION

TABLE VI: TWIST DISTRIBUTION - EFFECT OF TWIST AT 210°C

TABLE VII: TWIST DISTRIBUTION - EFFECT OF YARN DRAW RATIO AT 228°C

TABLE VIII: TENSIONS ABOVE AND BELOW THE SPINDLE (210°C Heater Temperature)

TABLE IX: CALCULATED AND MEASURED THREADLINE TENSION (GRAMS) DURING FALSE-TWISTING (50 TPI BASIC TWIST)

TABLE X: MEASURED AND CALCULATED VALUES OF TORQUE (in-lb \cdot 10^{-5})

TABLE XI: COMPONENTS OF CALCULATED TORQUE (VIA OVERTWISTING) IN THE HEATER ZONE
LIST OF FIGURES

FIGURE 1  SCHEMATIC OF THE FALSE-TWIST APPARATUS
FIGURE 2  TENSION, TORQUE AND TWIST ALONG THE FALSE-TWIST THREADLINE
FIGURE 3  SCHEMATIC OF THE FALSE-TWIST APPARATUS SHOWING DIMENSIONS
FIGURE 4  PHOTOGRAPHS OF THE FALSE-TWIST APPARATUS
FIGURE 5  PHOTOGRAPHS OF THE TORQUE-MEASURING APPARATUS
FIGURE 6  STRESS-STRAIN CURVE (partial) OF FEED YARN
FIGURE 6'  STRESS-STRAIN CURVE OF FEED YARN
FIGURE 7  STRESS-STRAIN CURVES (partial) OF FRESHLY TEXTURED YARNS (210°C)
FIGURE 8  STRESS-STRAIN CURVES (partial) OF FRESHLY TEXTURED YARNS (134°C)
FIGURE 9  STRESS-STRAIN CURVES AT ELEVATED TEMPERATURES
FIGURE 10  TWIST IN THE ENTRANCE AND COOLING ZONES AS A FUNCTION OF TEMPERATURE AND OVERFEED
FIGURE 11  TWIST VARIATION WITH OVERFEED AT 191°C AND 228°C
FIGURE 12  RELATIVE CONTRACTED LENGTH VARIATION WITH OVERFEED AT 163°C, 191°C, and 228°C
FIGURE 13  TENSION vs. TEMPERATURE AT VARIOUS OVERFEEDS
FIGURE 14  VARIATION OF TENSION WITH TWIST AT 210°C
FIGURE 15  TWIST IN THE ENTRANCE ZONE AS A FUNCTION OF TWIST AT THE SPINDLE
FIGURE 16  EXPERIMENTAL AND CALCULATED VALUES OF TENSION (134°C)
FIGURE 16'  EXPERIMENTAL AND CALCULATED VALUES OF TENSION (210°C)
FIGURE 17  VARIATION OF $\varepsilon_2$ AND $E_2$ WITH TEMPERATURE AT -2.86% OVERFEED
LIST OF FIGURES (continued)

FIGURE 18  MEASURED TORQUE vs. TEMPERATURE AT VARIOUS OVERFEEDS

FIGURE 19  EXPERIMENTAL AND CALCULATED VALUES OF TORQUE (134°C)

FIGURE 19' EXPERIMENTAL AND CALCULATED VALUES OF TORQUE (210°C)

FIGURE 20  CALCULATED COMPONENTS OF TORQUE IN THE ENTRANCE ZONE AT 210°C (UPPER LIMIT)

FIGURE 20' CALCULATED COMPONENTS OF TORQUE IN THE ENTRANCE ZONE AT 134°C (UPPER LIMIT)

FIGURE 21  RELAXATION OF TORQUE OF STATICALLY TWISTED YARN

FIGURE 22  DIFFERENTIAL GEOMETRY OF YARNS

FIGURE 22' DIFFERENTIAL GEOMETRY OF YARNS

FIGURE 23  CONTRACTILE STRESS vs. TEMPERATURE
I. INTRODUCTION

Amongst the important developments which have occurred in the last 50 years in the textile industry, one must certainly include the invention and subsequent commercial use of texturing techniques for synthetic thermoplastic fibers.

Texturing is a broad term which covers a variety of industrial processes which may be quite different yet have as a common objective to modify the geometry of the straight cylindrical synthetic filaments; this makes the yarn composed of such filaments bulkier and fluffier and, in most cases, it also makes it stretchy and elastic. Of the numerous processes that have been developed by far the most important commercially is the so-called false-twist texturing process. It is estimated that 2/3 of all textured fibers are textured using this process (R1). In the course of the last 20 years or so that false twisting has been used in the textile industry, considerable development and improvements have been brought about in the false-twisting equipment; these have resulted in very significant improvements in productivity.

The process itself is described in the next section, and details of the various components of the equipment may be found in the literature and bulletins put out by various manufacturers (R2). As is often the case, art precedes science and although, as mentioned, much has been done to improve the equipment, very little work can be found in the published
literature which deals with the mechanism of the process, i.e., how a given yarn interacts with the equipment and in which way the process parameters (overfeed, temperature, rpm) affect the yarn in process. There exists no understanding or analysis which quantitatively or even qualitatively relates the process parameters to the observed yarn response (tension, torque, twist); there exists however, a considerable amount of experimental work, empirical in nature, which correlates changes in process parameters to changes in properties of the treated yarn ($R_1$, $R_3$).

The present research was undertaken with the objective of understanding and, if possible, quantifying the interaction between the machine variables alluded to above, and the corresponding response of the yarn being processed at steady state in the false twister. Given that the machine variables are of a mechanical and thermal nature, the yarn response will be determined by the laws of structural yarn mechanics and from the mechanical and thermomechanical properties of the filaments composing the yarn. Ideally then the problem would be solved if given the machine settings, and the appropriate material properties, one were able to predict the yarn response, which is (as we shall see) the twist distribution, the tension and the torque. The progress made in answering this question forms the body of this thesis.
II. PLAN OF WORK

The work done in this thesis will be presented in the following sequence:

(a) The false twist process will be described and its main features analyzed in some detail.

(b) Theoretical considerations on yarn geometry and appropriate equations for yarn mechanics will be described and/or derived.

(c) Material Parameters. The experimental techniques used in obtaining the material parameters will be described, and the values of the parameters thus obtained will be tabulated and graphed.

(d) Experiments on the false-twister. The experimental technique used in obtaining process data as well as data of the process effects on the yarn, will be described. The experimental data will be tabulated, graphed, and analyzed.

(e) Consideration of yarn mechanics in conjunction with the experimentally determined materials properties, will be used to explain the data obtained in the false-twist experiments; a model describing yarn behavior will be outlined, and using this as a basis calculations will be made of the important processing responses such as tension and torque which will then be compared to the values deter-
mined experimentally in the false-twisting experiments.

(f) Conclusions will be drawn and recommendations for future work made.

The Appendix contains literature review of articles on false twisting, as well as additional derivations of yarn mechanics equations.

III. THE FALSE TWISTING PROCESS

(a) Description of the Process

Fundamentally in the course of this process a yarn is highly twisted, heated to a high temperature to relax the stresses imposed by the twist and thus be "set", cooled to room temperature to improve the quality of set, and then untwisted and wound in a package. All these operations are performed continuously on a false-twist machine: untextured yarn enters the machine and textured yarn is wound on the package. The name "false-twist" derives from the fact that the twist the yarn sees is not permanent but is removed by virtue of the nature of the process itself.

A schematic of the process is given in Figure 1. With reference to Figure 1, the equipment fundamentally consists
of two sets of rubber covered rollers, the feed and exit rolls which serve to feed and move the yarn thru the machine. Between these rolls is interposed a spindle (a hollow cylinder with a pin across its center around which the yarn is "threaded" as per the sketch). The spindle rotates as shown and in so doing imparts twist in opposite sense to the yarn upstream and downstream. Between the feed rolls and the spindle is a heated plate whose temperature is carefully controlled and with which the yarn comes in contact as it travels through the machine. Downstream of the exit rolls there is a package into which the textured yarn is wound, often under some tension.

At steady state operation the experimentally observed situation is as follows: untwisted yarn (or yarn with only producer's twist, i.e., 1/3 to 1/2 tpi) enters the machine and is immediately twisted to some twist level (which is a function of the machine settings and material properties); as the yarn passes over the heater it gets heated and the twist is seen to increase (typically by 30%); between the heater and the spindle the yarn cools to room temperature while the twist remains substantially the same (a slight increase is often observed); finally immediately it exits the spindle, the yarn is untwisted and remains so as it moves through the exit rolls and gets wound up on the package.
(b) Twist Development in the Machine

That on operating at steady state there is only twist upstream of the spindle but not downstream is puzzling at first sight. However we can visualize how this comes about via the following thought experiment: imagine the feed and take up rolls stationary (no axial motion of the yarn) the heater removed but the spindle rotating. It will impart an equal number of turns upstream and downstream. Now allow, additionally, the yarn to move axially thru the machine and consider the number of turns entering and escaping in the zones upstream and downstream of the spindle, during a short period of time, \( \Delta t \). Assume that, at the start of this time interval, the zone upstream of the spindle contains \( x \) turns of twist \( Z \), and the zone downstream contains an equal number of turns of twist \( S \). During the time interval the following happens in the two zones:

<table>
<thead>
<tr>
<th>Upstream of the Spindle</th>
<th>Downstream of the Spindle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spindle inserts twist ( Z )</td>
<td>Spindle inserts equal number of turns ( S )</td>
</tr>
<tr>
<td>Yarn with ( Z ) twist escapes</td>
<td>Yarn with twist ( S ) escapes</td>
</tr>
<tr>
<td>through the spindle into the next zone</td>
<td>through the exit rolls of the zone into the package</td>
</tr>
<tr>
<td>Fresh untwisted yarn enters the zone through feed rolls</td>
<td>Yarn of twist ( Z ) enters the zone through the spindle and cancels some of the existing ( S ) twist</td>
</tr>
</tbody>
</table>
As can be seen from the above, yarn in the zone downstream, will receive fewer turns than yarn in the zone upstream when the yarn is made to run through the machine so that the twist in the downstream zone will decrease with time; as the twist approaches zero, the number of turns of twist $S$, escaping through the exit rolls also approaches zero, and, finally, at steady-state no twist is left in the downstream zone. At steady-state the turns of $S$ twist inserted in the downstream zone are exactly balanced by the turns of $Z$ twist escaping from the upstream zone, so that the zone downstream of the spindle remains free of twist. A simple mathematical derivation of the approach to steady state has been given by Denton $^{(R5)}$. 
(c) Controllable Machine Variables

For a given machine the controllable variables are, the velocities of the feed rolls $V_{\text{in}}$, and the exit rolls $V_{\text{out}}$, the heater temperature, and the spindle rpm $N$.

The twist level is controlled by the ratio of spindle rpm to outgoing velocity; the machine thus inserts at steady state a constant twist, \( \frac{N}{V_{\text{out}}} \), which is a basic twist (or flat twist) since it refers to a number of turns per untwisted length of yarn. From considerations of yarn geometry to be discussed later, one can calculate the actual twist (or number of turns per twisted length of yarn) given the basic twist; unfortunately, ideal yarn geometry is not obeyed because the tension in the process can vary considerably depending on the chosen machine settings, and more specifically, on (a) the "overfeed" \( \frac{V_{\text{in}} - V_{\text{out}}}{V_{\text{in}}} \) expressing the relative rates at which the yarn is fed in and withdrawn from the machine), and (b) the heater setting which generates shrinkage forces in the filaments.
The twist, temperature and overfeed are the major machine parameters that a processor has at his disposal; there are of course a number of other parameters (type of heater, length of heater or duration of contact time, length of cooling zone) that are decided upon by the machine manufacturer (R2). In our work residence time in the heater and in the cooling zone was such that thermal equilibrium was essentially attained, so residence time per se was not a limiting consideration.

(d) The False-twisted Yarn

The purpose of the sequence of operations occurring in the false-twister is to "set" the yarn while the latter is highly twisted; upon untwisting the yarn "remembers" its twisted configuration: if left free it tends to twist back (not all the way, of course, because the filaments interfere with each other); if it is prevented from twisting but allowed to retract, it will do so, the filaments buckling or collapsing into short helical segments and snarls.

This behavior can be explained by considering that when the yarn is heat set in the twisted configuration, the bending and torsional stresses built up on the filaments during twisting have been largely or completely relaxed and the filaments are thus stress-free, except for tensile stresses. Untwisting the yarn reimposes on the filaments considerable bending and torsional energy (R4).
This feature, i.e., the ability of the filaments to buckle into helices, whorls and snarls, results in a considerable shortening of the yarn and a very significant increase in diameter; these twin abilities of the yarn to retract (or conversely extend considerably under low stress) and to "bulk", are the two main properties which are of industrial importance and have allowed for the creation of "stretch" garments and hosiery.

(e) Machine-yarn Interaction

Restating the problem which forms the core of this thesis, the question is how do the processing parameters, overfeed, rpm and heater temperature interact with the yarn, that interaction being manifested by the yarn twist distribution, yarn tension and yarn torque? The first step in our analysis is to derive some basic "principles" about the operation of the machine, which will serve as a basis for further analysis of the interaction; these principles are either derived by direct consideration of the way the machine works or by experimental measurements. Referring to Figure 2 where a schematic representation of the machine is given along with plots indicating the yarn response, the following apply:

Twist

From experimental observation \((R_1,R_6,R_7,R_8)\) of steady state operation the yarn is twisted as it enters the machine and the twist remains substantially constant in the entrance
FIGURE 2

TENSION, TORQUE AND TWIST ALONG THE FALSE TWIST THREADLINE
zone. When the yarn reaches the heater, the twist suddenly increases (typically by 30%) as shown in Figure 2, and thereafter remains substantially constant until the spindle. At the spindle the basic twist must be such that \( \tau = \frac{N}{V_{\text{out}}} \), and the actual twist is, of course, the basic twist corrected for yarn contraction.

The transition at the heater is not a step function but more gradual (\( R8 \)), and often the twist is found to increase somewhat after the heater (\( R6, R7 \)). For purposes of this work, however, these refinements were neglected in order to deal with a more simple and tractable model.

One of the important things to explain is the origin and level of the twist increase at the heater or "twist jump", and to explain and predict if possible, the actual twist observed at the spindle, given the basic twist which is known and imposed by the machine. That last point should also be experimentally verified since a number of authors have on occasion claimed (\( R9 \)) that twist "escapes" from the spindle; this would then mean that the basic twist imposed by the machine is different from that given by \( \tau = \frac{N}{V_{\text{out}}} \).

**Torque**

The rotation of the spindle imposes a torque on the yarn and it is in response to that torque that the yarn twists on entering the machine. The torque below the spindle at
steady state is taken to be constant all the way to the entrance rolls. Now, in the absence of frictional restraints (neglecting air drag) that torque must remain constant from the spindle to the entrance. In actual industrial practice this may not be quite so because the yarn may be passed over various guides, the friction against which decreases the torque upstream. Also the friction against the heater will reduce the torque. However, in our experiments which will be described later, there were no guides to add friction to the yarn, and care was taken to minimize the tension against the heater. So, the hypothesis of constant torque is a realistic one.

Downstream of the spindle, where the yarn is untwisted, there is a small torque of opposite sense to the upstream one; as earlier explained, the untwisted yarn is a "torque" yarn, the untwisting having disturbed the yarn from its set, twisted minimum energy configuration.

**Tension**

For reasons similar to those previously expounded, the tension below the spindle will be constant along the thread-line. The tension above the spindle will be much higher since the yarn is wrapped around the spindle and is literally pulled by the exit rolls. The overall level of tension depends on the overfeed and the temperature.

We can summarize then by saying that the false-twisting
process is one where the machine imposes constant tension and constant torque; it is thus essentially a creep process but a non-isothermal one. Understanding and quantifying the reaction of the yarn to these particular creep conditions is, as already mentioned, the objective of this work.

IV. YARN GEOMETRY

In this section we shall derive the geometrical and mechanical relationships which determine the response of the yarn to various stress conditions; these equations will be used later to interpret the experimental data, and construct a model for yarn behavior.

(a) Model for Yarn Geometry

The model used in this work is that used by other investigators \(^{(R10)}\) as further refined by Gracie \(^{(R11)}\). In this model the yarn is assumed to be composed of filaments all of which lie along perfect concentric helices of identical pitch. The helix angle varies with the radial position of the filament. The packing density is constant across the cross-section. Given the packing density, this model predicts that the yarn volume remains constant on twisting \(^{(R11)}\). Gracie also assumed that the filaments are continuously distributed over the yarn cross-section; this makes it easy to handle mathematically any equations involving filament parameters, as one can integrate across the cross-section
to obtain quantities such as tension and torque.

The symbols used to describe the geometry are:

- \( W \) = number of fibers in the yarn
- \( p \) = yarn twist (actual) in rad/inch
- \( \tau \) = yarn twist (actual) in turns/inch
- \( k \) = packing factor
- \( f \) = filament radius
- \( r \) = radius at any point within the yarn
- \( b \) = outside yarn radius

Now, if at any radius \( r \) \(( r \leq b)\) of the yarn we "open" or unwind the yarn by one turn, the following geometrical relationships prevail (see sketch). The line \( AC \) is the unwound helical filament; its angle with respect to the yarn axis \( BC \) is \( q \). The length \( BC = \frac{2\pi}{p} \), since the yarn twist is \( \frac{p}{2\pi} \) turns per inch and we "opened" one turn which corresponds to \( 1/t \) inches.

The side \( AB \) is, of course, the circumference of the yarn at radius \( r \). Given thus \( AB \) and \( BC \) we obtain \( AC \) as shown. Given the sketch above, the following (identical) geometrical relationships also hold (see sketch on following page).
Thus to 1" of yarn we have
\[ \sqrt{1 + p^2 r^2} \] fiber length at radius \( r \), and \[ \sqrt{1 + p^2 b^2} \] fiber length at the outside radius.

Also,
\[ \cos q = \frac{1}{\sqrt{1 + p^2 r^2}} \]
\[ \sin q = \frac{pr}{\sqrt{1 + p^2 r^2}} \] (1)
where \( q \) is the local helix angle.

At the yarn surface
\[ \cos Q = \frac{1}{\sqrt{1 + p^2 b^2}} \] and \[ \sin Q = \frac{pr}{\sqrt{1 + p^2 b^2}} \] (1')
where \( Q \) is the outside helix angle.

With the symbols as defined, the following relationship can be established as shown by Gracie (R11) between the number of fibers and the yarn radius. Consider a cross-section of the yarn perpendicular to the yarn axis; within an annulus \( 2\pi r dr \) the occupied area is \( 2\pi kr dr \). If \( \pi f^2 \) is the cross-sectional area of a filament (perpendicular to its own axis) the cross-sectional area of a filament inclined at an angle \( q \) (the local helix angle) is \( \frac{\pi f^2}{\cos q} = \pi f^2 \sqrt{1 + p^2 r^2} \) in view of Equation (1). The number of filaments in the annulus is given by the ratio of the occupied area divided by the cross-
sectional area of an (inclined) filament

\[ dW = \frac{2\pi k r dr}{\pi f^2 \sqrt{1 + p^2 r^2}} \]  

(2)

The total number of filaments is obtained by integrating between the limits \( r = 0 \) and \( r = b \) and is

\[ W = \int_0^b dW = \frac{2k}{p^2 f^2} \left( \frac{\sqrt{1 + p^2 b^2}}{b} - 1 \right) \]

(3)

(The integration is easily accomplished by substitution of variables \( z^2 = \sqrt{1 + p^2 r^2} \)). Equation (3) can be rewritten with the substitution

\[ A = \frac{W f^2}{2k} \]

(4)

as

\[ \sqrt{1 + p^2 b^2} = A p^2 + 1 \]

(5)

whence

\[ b^2 = A (A p^2 + 2) \]

(6)

From Equations (5) and (1') we have \( \cos Q = \frac{1}{A p^2 + 1} \)

The quantity \( A \) is a quantity characteristic of the yarn
structure. \(2\pi A\) is the cross-sectional area of the same yarn with the same packing factor but with zero twist.

It is readily shown (by direct substitution) that the radius \(r_\phi\) of the yarn that contains a specified fraction \((0 \leq \phi \leq 1)\) of the total number of filaments \(W\), is given by

\[
\sqrt{1 + p^2 r_\phi^2} = A\phi p^2 + 1
\]

(7)

For \(\phi = 1\), \(r_\phi = b\). The cumulative fraction of filaments is thus not linearly related to the yarn radius \(r\) \((0 < r < b)\).

(b) Yarn Contraction

When a bundle of filaments is twisted into a yarn, it contracts lengthwise. This contraction, on twisting, has been calculated by a number of workers (R10, R11, R12). For the yarn model used here, which assumes the yarn to twist at constant volume, the contracted length (i.e., the ratio of the untwisted to the untwisted yarn length) can be calculated as follows: in an inch of yarn an annulus at radius \(r\) contains \(dW\) filaments, each of length \(\sqrt{1 + p^2 r^2}\) as described in the previous section. The sum of the lengths of all the filaments in the yarn cross-section divided by the total number of filaments, gives the average filament length which is the length of the untwisted yarn. So the average filament length is \(\int_0^b \frac{dW}{W} \sqrt{1 + p^2 r^2}\). The contracted length of the yarn, \(L_c\) (often referred to as "contraction" in this work) is the
ratio of lengths of contracted (twisted) to uncontracted (untwisted) yarn, and is

\[ L_c = \frac{1}{b} \int_{0}^{b} \frac{dW}{\sqrt{1 + p^2 r^2}} \]

which after substitution (using \( L = \frac{2}{Ap^2 + 2} \)) and integration (using \( z^2 = 1 + p^2 r^2 \)) gives \( L_c = \frac{2}{1 + \sec Q} \).

Equation (2)) and integration (using \( z^2 = 1 + p^2 r^2 \)) gives \( L_c = \frac{2}{1 + \sec Q} \).

The above describes the geometry of the yarn. We now turn briefly to describe some of the equations of yarn mechanics that give rise to tension and torque.

V. YARN MECHANICS

(a) "Classical" Mechanics

1. Stress-Strain Curves of Yarns

The mechanical behaviour of continuous filament yarns in tension has been extensively studied and reviewed \( (R10) \). In the simplest model, which was used here for the description of the mechanical behavior of the yarn in the entrance zone,
the yarn tension is the sum (in the axial direction) of the
individual filament tensions which arise when the yarn is ex-
tended while the twist is assumed not to change sufficiently
to materially change the inclination of the filaments. In
this derivation, which has been found to give a good repre-
sentation of actual behavior for small strains\(^{(R10)}\), the
effects of lateral fiber pressure are neglected (hence the
change in packing density) as is neglected the effect of the
Poisson ratio contraction of the filaments (hence of the
yarn). The derivation is very simple and is as follows, with
reference to the sketch. The number of fibers in an annulus
d\( \pi \) at distance \( r \) is

\[
dW = \frac{2\pi kr dr}{\pi f^2 \sqrt{1+p^2 r^2}}
\]

from Equation (2). Call \( \sigma_f \) the
stress on any fiber along the
fiber axis, the tension along the
fiber axis is \( t = \pi f^2 \sigma_f \). The ten-
sion contribution along the yarn
axis is (cf. Eq. (1))

\[
t_a = t \cos q = \frac{t}{\sqrt{1+p^2 r^2}} = \frac{\pi f^2 \sigma_f}{\sqrt{1+p^2 r^2}} \text{ per fiber. (9)}
\]

So the incremental tension along the annulus is:

\[
dT = dW \cdot t_a = \begin{pmatrix}
\frac{2\pi kr dr}{\pi f^2 \sqrt{1+p^2 r^2}} \\
\frac{\pi f^2 \sigma_f}{\sqrt{1+p^2 r^2}}
\end{pmatrix}
\begin{pmatrix}
dW \\
\sqrt{1+p^2 r^2}
\end{pmatrix}
\]

\[(9')\]
Further, from stress strain data on the filaments themselves

$$\sigma_f = E_f \epsilon_f$$  \hspace{1cm} (10)$$

where $E_f$ of $\epsilon_f$ are the filament Young's modulus and the filament strain. Finally, from geometrical considerations(R10):

$$\epsilon_f = \epsilon_Y \cos^2 q = \frac{\epsilon_Y}{1+p^2 r^2}.$$  \hspace{1cm} (11)$$

$\epsilon_Y$ is the yarn strain and $q$ the local helix angle. Replacing these quantities into Equation (9') we obtain

$$dT = 2\pi k E_f \epsilon_Y \frac{rdr}{(1+p^2 r^2)^2}$$  \hspace{1cm} (12)$$

and integrating with the usual substitution $z^2 = 1+p^2 r^2$ between $r = 0$ and $r = b$, we finally obtain for the yarn tension, also using Equation (5)

$$T = \pi k E_f \epsilon_Y \frac{A(Ap^2+2)}{(Ap^2+1)^2} = \pi k E_f (F-1) \frac{A(Ap^2+2)}{(Ap^2+1)^2}$$  \hspace{1cm} (13)$$

where $F = (\epsilon_Y + 1)$ is the extension ratio of the yarn.

Equation (13) corresponds to Equation (4.9) in (R10), and gives the relationship between the tension developed in the direction of the yarn axis, if we extend by $\epsilon_Y$ a yarn twist $p$ originally stress-free in all fibers.
2. Average Filament Strain

For a number of calculations to be discussed later, the concept of the average filament strain is desirable. Of course, one can define an infinity of averages, but two have been found useful in this work. They are defined below and refer to the yarn model under stress discussed in the preceding section, that is where the yarn prior to application of the stress is assumed to have been stress-free in all filaments.

**Filament Strain Averaged Over the Yarn Cross-section**

\[
\bar{\varepsilon}_f = \frac{\int_0^b \varepsilon_f dW}{W} \quad (14)
\]

Substitutions in terms of Equations (2) and (11) followed by integration gives

\[
\bar{\varepsilon}_f = \frac{\varepsilon_Y}{A p^2 + 1} \quad (15)
\]

**Filament Strain Averaged Across the Yarn Radius**

This is defined as

\[
\bar{\varepsilon}_f = \frac{\int_0^b \varepsilon_f dr}{b} = \int_0^b \frac{\varepsilon_Y}{p b} \left( \tan^{-1} \left( \frac{1}{p b} \right) \right) \quad (16)
\]

after substitution via Equation (11) and integration.
3. Yarn Torque

As various workers have shown \((R13, R14)\), yarn torque arises in continuous filament yarns from a variety of sources: the filaments in such a yarn are both twisted and bent, and that requires in turn (in the absence of setting) the existence of torsional and bending moments on the filaments, which have components in the axial yarn direction and hence, give rise to yarn torque. Further, if a twisted yarn is extended, there is a tension component in each filament (situated at \(r\) from the yarn center) perpendicular to the radius. These tension components multiplied by \(r\) constitute also a twisting moment.

We shall briefly describe the method used in deriving the equations of these torque components as used in our particular model.

**Bending Moment Contribution to Yarn Torque**

For beams in pure bending \((R15)\)

\[
M_{BE} = \frac{EI}{\rho}
\]  

(17)

where \(E\) is the Young's Modulus (assuming Hookean behavior in tension and compression), \(I\), the moment of inertia with respect to the neutral axis and \(\rho\) is the radius of curvature.

The filaments are assumed to lie in helices in the yarn, and from differential geometry of space curves \((R16)\) we have
The bending moment per fiber at radial position \( r \) is thus

\[
M = \frac{EI}{r} \left( \frac{p^2r^2}{1+p^2r^2} \right)
\]  

(19)

and the contribution of the bending moment to yarn torque is given by its component inclined by 90-\( q \) to the yarn axis \((R_{13})\)

\[
M_{\text{axial}} = M \cdot \sin q = \frac{EI}{r} \left( \frac{p^2r^2}{1+p^2r^2} \right)^{3/2}
\]  

(20)

per fiber, using Equation (1).

For an annulus containing \( dW \) fibers we have, \( dM_{BE} = dW \cdot M_{\text{axial}} \) and the total contribution is the integral over the whole cross-section:

\[
M_{BE} = \int_0^b dW \cdot M_{\text{axial}} = \frac{kEI}{fp^2} \left[ \ln(1+p^2b^2) - \frac{p^2b^2}{1+p^2b^2} \right]
\]  

(21)

using conditions (2) and (20). Substituting for the moment of inertia \( I = \frac{\pi f^4}{4} \), we get

\[
M_{BE} = \frac{\pi kEf^2}{4p} \left[ \ln(1+p^2b^2) - \frac{p^2b^2}{1+p^2b^2} \right] = \frac{\pi kEf^2}{4p} \left[ 2\ln(Ap^2+1) - \frac{Ap^2(Ap^2+2)}{(Ap^2+1)^2} \right]
\]  

(22)

using also Equations (5) and (6). This Equation corresponds
to Equation (10) in (R13).

**Fiber Torsional Moment Contribution to Yarn Torque**

The torsional moment per fiber in the direction of the fiber axis is (R15)

\[ M = \frac{G I}{p} p_f \]

where \( G \) is the shear modulus, \( I_p \) the polar moment of inertia, and \( p_f \) the fiber twist in radius per unit length. It was shown (R13) that

\[ p_f = p \cdot \cos^2 q = \frac{p}{1 + p^2 r^2} \]  

(23)

where \( p \) is the yarn twist and \( q \) the local helix angle. Moreover, the component of the fiber torsional moment to yarn torque is given (R13) by

\[ M_{\text{axial}} = M \cdot \cos q = \frac{G I_p k}{p} \left( \frac{p}{1 + p^2 r^2} \right)^{3/2} \]  

(24)

per fiber.

In an annulus, the incremental torque is \( dM_{TO} = dW \cdot M_{\text{axial}} \) and the total contribution to yarn torque is obtained by integration across the cross-section after the usual substitution:

\[ M_{TO} = \int_0^b dM_{TO} = \frac{G I_p k}{f^2} \left( \frac{p^2 b^2}{p(1 + p^2 b^2)} \right) \]  

(25)
Substituting for the polar moment of Inertia $I = \frac{\pi f^4}{2}$
we finally obtain, using Equations (5) and (6),

$$M_T = \frac{\pi Gk f^2}{2} \frac{p^2 h^2}{p(1+p^2 b^2)} = \frac{\pi Gk f^2}{2p} \frac{A_p^2 (A_p^2 + 2)}{(A_p^2 + 1)^2}$$

Equation (26) corresponds to Equation (32) in (R13).

As will be discussed below, the value of $G$ was not measured
directly. In this work for purposes of numerical computa-
tion, it was taken to be $G = \frac{E}{12}$ (R18).

**Torque due to Yarn Tension** (R14)

As was explained earlier, the components of filament tension
parallel to the yarn axis give rise to yarn tension; the
components of filament tension perpendicular to the yarn
axis give rise to fiber torque.

The derivation of the torque-tension or torque-strain curve
will now be given; it will follow the lines of the derivation
for yarn tension earlier described. In slightly different
form this derivation was originally given in (R14).

If $\sigma_f$ is the stress along the fiber axis, the tension per
filament is $\pi f^2 \sigma_f$. The tension contribution in the direc-
tion perpendicular to the yarn axis is (using Equation (1)):

$$t_{circ} = t \cdot \sin \theta = \pi f^2 \sigma_f \frac{pr}{\sqrt{1+p^2 r^2}}$$

per filament. The corresponding torque contribution per
fiber is \( t_{\text{circ}} \cdot r \), and the torque contribution of an annulus of \( dW \) fibers is (using Equations (2) and (27))

\[
dM_{\text{TE}} = dW \cdot t_{\text{circ}} \cdot r = \frac{2\pi kr}{\pi r^2 \sqrt{1+p^2r^2}} \cdot \pi f^2 \cdot \sigma_f \cdot \frac{pr}{\sqrt{1+p^2r^2}} \cdot r \quad (27)
\]

which, after the substitutions of \( \sigma_f = E_f \varepsilon_f \) (Equation (10)) and \( \varepsilon_f = \varepsilon_y / (1+p^2r^2) \), (Equation (11)) as for the case of derivation of tension, integrates (with the substitution of variables \( z^2 = 1+p^2r^2 \)) to

\[
M_{\text{TE}} = \frac{\pi kE_f \varepsilon_y}{p^3} \left[ 2\ln(Ap^2+1) - \frac{Ap^2(Ap^2+2)}{(Ap^2+1)^2} \right] \quad (28)
\]

If we express \( \varepsilon_y \) in terms of average filament strain, defined by Equation (15), we obtain for the torque,

\[
M_{\text{TE}} = \frac{\pi kE_f \varepsilon_y}{p^3} \left[ 2(Ap^2+1)\ln(Ap^2+1) - \frac{Ap^2(Ap^2+2)}{Ap^2+1} \right] \quad (29)
\]

Equation (29) corresponds to Equation ( ) in (R14).

From Equations (13) and (28) it is possible by simple substitution to derive an expression linking yarn tension to yarn torque due to tension:

\[
M_{\text{TE}} = \frac{T}{p} \left[ 2(Ap^2+1)^2 \ln(Ap^2+1) - \frac{Ap^2(Ap^2+2)}{Ap^2+1} \right] \quad (30)
\]

which shows that the torque is directly dependent upon yarn tension.

Restating the physical assumptions which served as a basis for the derivation of Equations (13) and (30), the yarn is assumed to be originally stress-free in all its filaments; it is then extended by an amount \( \varepsilon_y \), and the stresses gener-
ated in the filaments give rise to a tension (Equation (13)) and a torque (Equation (29)).

(b) Tension and Torque for the Case of Equal Tension on all Filaments

The physical situation in the entrance zone in the false-twister does not conform to the classical tension and torque model just described, which assumes the existence of an already formed yarn, which is then extended, giving rise to tension and to torque. Rather, in the case of false twisting, an untwisted yarn enters a constant torque-constant field and is twisted with free radial migration under the action of these force fields.

Free radial migration means that a given filament does not always occupy the same radial position in the yarn cross-section, but is free to migrate radially from the yarn core to periphery and vice versa, as the yarn is formed. Radial migration serves to eliminate differential strains, that would otherwise have been generated in the filaments, as the result of differences in path length. The subject of migration is extensively discussed in (R10).

For our purposes, therefore, a more realistic model for tension and torque is one where the filaments are under equal tension and the derivation for this case is given below.
1. Tension; Stress-Strain Curve

If all the filaments are under the same stress, $\sigma_f$ (corresponding to the same strain, $\varepsilon_f$), then exactly as in the case of yarn mechanics earlier described, the tension contribution per fiber in the axial direction is

$$t_a = \frac{\pi f^2 \sigma_f}{\sqrt{1+p^2 r^2}} \quad \text{(Equation (9))}$$

and the contribution to yarn tension in an annulus $dr$ is

$$dW \cdot t_a \quad \text{(Equation (9'))}, \text{ Integrating over the cross-section:}$$

$$T = \int_{0}^{b} dW \cdot t_a = \int_{0}^{b} \frac{2k \pi r dr}{\pi f^2 \sqrt{1+p^2 r^2}} \cdot \frac{\pi f^2 \sigma_f}{\sqrt{1+p^2 r^2}} = 2k \pi \sigma_f \int_{0}^{b} \frac{r dr}{1+p^2 r^2}$$

$$= \frac{2k \pi \sigma_f \ln(Ap^2+1)}{p^2} = \frac{2k \pi E_f \varepsilon_f \ln(Ap^2+1)}{p^2} \quad \text{(31)}$$

using Equations (2) and (9). Note that in this case $\sigma_f$ is a constant.

2. Torque

By a reasoning identical to that used in the derivation of the torque in the classical mechanics description, we arrive at the expression

$$dM_{TE} = dW \cdot t_{\text{circ}} \cdot r = \frac{2k \pi r dr}{\pi f^2 \sqrt{1+p^2 r^2}} \cdot \frac{\pi f^2 \sigma_f}{\sqrt{1+p^2 r^2}} \cdot \frac{pr}{r} \cdot r \quad \text{(27)}$$

which upon integration ($\sigma_f$ is now a constant) gives:
42

\[ \pi \kappa \sigma_f \frac{p^3}{p^3} \left( A_p^2 (A_p^2 + 2) - 2 \ln (A_p^2 + 1) \right) \]  \hspace{1cm} (32)

with, of course \( \sigma_f = E_f \varepsilon_f \).

As will be described later, this expression for the torque has been used to calculate the torque due to tension in the entrance zone. Equations (31) and (32) can be combined showing a direct relationship between tension and torque; both, of course, are zero when \( \sigma_f \) or \( \varepsilon_f \) are zero.

It may be said at this point, in anticipation of what will follow, that the values of torque calculated by this means are about 10% higher than the values of torque calculated using the "classical" expression (Equation (29)) at the same average fiber strain.

(c) Overtwisting

In the cases of yarn mechanics analyzed so far, migration was assumed to have taken place so as to equalize filament tension. There are cases of course, where migration is impeded: this is the case for instance, on uptwisting an already twisted yarn \((R10, R8)\). Since the situation during the false-twist operation as the yarn moves to the heater zone is one of uptwisting, it was necessary to study the yarn mechanics of uptwisting without migration, or "overtwisting".

1. Overtwisting - Stress Strain Curve

The analysis of overtwisting was modelled as follows: A yarn
is assumed to have been twisted to twist level $p_1$; it is then overtwisted (or uptwisted without migration) to twist level $p_2$ at constant radius, allowing no contraction to take place, and then it is extended or compressed by an amount $F$, ($F =$ extension ratio) at constant volume. The amount of overtwisting (uptwisting) is defined as $c \equiv \frac{p_2}{p_1}$ and the final twist is $p_3 = \frac{p_2}{F}$. The problem is to determine the stress strain curve and the torque-strain curve of this yarn.

The major difference between this deformation mode and the ones earlier described arises because on overtwisting in the absence of migration, the outside filaments of necessity must be extended and the inside filaments compressed, when a system is overtwisted under little or no applied external load. Thus the total tension across the yarn cross-section may be zero (the tensile and compressive components canceling each other) but the torque due to these tension components will not cancel out. Thus one could have a yarn under no net tension which could still exhibit torque due to tension components.

The derivation of the tension and torque relationships follows the reasoning employed earlier. The important difference lies in the relationship derived between the yarn strain or extension ratio, and the filament strain, (cf. Equation (11)). Moreover, this derivation takes into account the twist change brought about both by overtwisting and by the extension of
the yarn (this latter was neglected in the simple derivation of tension and torque earlier discussed).

To derive the desired expression, we imagine that we start with a yarn of twist \( p_1 \) which we uptwist at fixed length to twist \( p_2 \), and then we allow the length to vary so that the twist becomes \( p_3 \) at given extension ratio \( F \). We consider a yarn thus treated and focus on an annulus at \( r \) of thickness \( dr \); the contribution to yarn tension is

\[
dT = dW \cdot t_a = \frac{2krdr}{f^2 \sqrt{1+p_3^2r^2}} \cdot \pi f^2 \cdot \frac{E_f \varepsilon_f}{\sqrt{1+p_3^2r^2}} \tag{33}
\]

using Equations (2), (9) and (10).

We must now relate \( \varepsilon_f \) to \( F \) the yarn extension ratio (\( F = 1 + \varepsilon_y \)), and to do so we reason as follows: Identifying some fiber of fractional order (percentile rank) \( \phi \), we have, for 1" of yarn in the yarn before overtwisting, a fiber length

\[
l_1 = \sqrt{1+p_1^2r^2} = A \phi p_1^2 + 1
\]

where \( r \) is the yarn radius at which this filament is located (Equation (7)).

Since the overtwisting to the imaginary state \( p_2 \) is assumed done at constant radius (hence at constant volume), the relationship between the percentile rank of the filament and the yarn radius has not changed although all the filaments (save the central one) have been extended because their heli-
cal path has lengthened. Now after extension of the yarn, the same fiber of the same fractional order (percentile rank) \( \phi \), had a length

\[
\ell' = A \phi p^2 + 1 = \sqrt{1 + p^2 r^2},
\]

corresponding to 1" of the extended yarn; the filament length corresponding to \( F \) inches of yarn in the extended state is

\[
\ell = F \ell' = F(A \phi p^2 + 1) = F\sqrt{1 + p^2 r^2}.
\]

Therefore the extension ratio of a filament going from \( p_1 \) through \( p_2 \) to \( p_3 \) is

\[
\frac{\ell}{\ell_1} = 1 + \varepsilon_f = \frac{F(A \phi p^2 + 1)}{A \phi p^2 + 1}
\]

(34)

Now, since \( \phi \) by definition is

\[
\phi = \frac{\sqrt{1 + p^2 r^2} - 1}{A p^2} = \frac{\sqrt{1 + p^2 r^2} - 1}{A p^2}
\]

(7)

we have replacing it in Equation (34) and rearranging:

\[
\varepsilon_f = \frac{\ell}{\ell_1} - 1 = \left( \frac{F\sqrt{1 + p^2 r^2}}{\frac{\sqrt{1 + p^2 r^2}}{p_1} - \frac{p_1^2}{p_3} + 1} \right) - 1
\]

(35)

Replacing (35) in Equation (33) and integrating between \( r=0 \) and \( r=b \), with the usual substitution, \( z = \sqrt{1 + p^2 r^2} \), we obtain

\[
T = 2\pi k E_f \left( \frac{F}{p_1^2} \ln (A p^2 + 1) - \frac{1}{p_3^2} \ln (A p^2 + 1) \right)
\]

(36)
in terms of the final state of the yarn or, more explicitly

\[ T = 2\pi kE_f \left( \frac{F}{p_1^2} \ln(\frac{Ap_1^2 + 1}{c^2 p_1^2}) - \frac{F^2}{c^2 p_1^2} \ln \left( \frac{\lambda c^2 p_1^2 F^2}{F^2} + 1 \right) \right) \]  

(36')

in terms of the original state of the yarn, the amount of overtwisting, \( c = \frac{p_2}{p_1} = \frac{p_3^F}{p_1} \), and the imposed extension ratio, \( F \).

An interesting consequence of Equation (35), is that if we overtwist to \( p_3 \), and then extend the yarn by an amount \( F \) such that \( p_3 \) becomes equal to the original twist (\( p_3 = p_1 \)), then \( \varepsilon_f \) becomes simply equal to \( F - 1 \), i.e., the filaments are all under the same strain and their strain is equal to the yarn strain! Also, \( T = 0 \) in Equation (36) when \( F \) becomes

\[ F = \left( \frac{p_1}{p_3} \right)^2 \frac{\ln(Ap_1^2 + 1)}{\ln(Ap_3^2 + 1)} \]  

(37)

2. Torque in Overtwisted Yarns

The torque here is calculated as was done previously; we use Equation (27) and Hooke's law (Equation (10)), and the value of the strain in Hooke's law is given by Equation (35). We thus have:

\[ \frac{dM_{TE}}{dW} = \frac{2\pi krdr}{\pi f^2 \sqrt{1 + p^2 r^2}} \cdot \frac{p_2 r}{f^2} \frac{2}{\sqrt{1 + p_3^r}} \left( \frac{\sqrt{1 + p_3^r}}{p_3^r} \right. \left. - 1 \right) \frac{p_3^r}{\sqrt{1 + p_3^r}} \cdot r \]  

(38)

which is integrated from \( r = 0 \) to \( r = b \) with the substitution
\[ z = \sqrt{1 + p_3^2 r^2} \] to:

\[
M_{TE} = \frac{2\pi kE}{p_3^3} \left( \frac{1}{2} (H^2 - 2HN + 2N - 1) + \frac{N^2}{1-N} \left[ \frac{F}{m} + \left( \frac{1-H^2}{m} \right) \ln \frac{H-N}{H-N} + \ln H \right] \right)
\] (39)

with the abbreviations

\[
m = \left( \frac{p_1}{p_3} \right)^2 = \frac{F^2}{C^2}, \quad N = 1 - \frac{1}{m} = 1 - \frac{C^2}{F^2}, \quad H = \frac{A}{p_3^2} + 1 = \sqrt{1 + p_3^2 b^2}
\]

Equations (36) and (39) can be combined indicating a direct relationship between tension and torque; in this case however, when \( T = 0 \), which occurs for the value of \( F \) given by (37), \( M_{TE} \) is non zero.

Restating the model on which Equations (36) and (39) are based, it assumes a yarn of twist \( p_1 \) stressless in all filaments, which is first uptwisted to \( p_2 \) at constant length and constant volume, and then extended by \( F \) at constant volume so that its twist changes to \( p_3 \). During uptwisting migration is not allowed. The equations for the resulting tension and torque are given by (36) and (39).

3. Overtwisting from Zero Twist

A special case of the overtwisting model analyzed above is when the yarn is assumed to be twisted without migration starting from zero twist. This case is a simpler one to analyze as we can take as a starting point the case \( p_1 = 0 \). The stress-strain curve then simply becomes
where $\tau_{2}$ is the twist to which the yarn is brought up from zero prior to being deformed by $F$, and the torque equation becomes:

$$M_{TE} = \frac{2 \pi k E \left\{ \frac{(A_{p3})^2}{3} \cdot F (A_{p3}^3 + 3) - \frac{A_{p3}^2 (A_{p3}^3 + 2)}{2} + \ln(A_{p3}^3 + 1) \right\}}{p_{3}^3}$$

in terms of the yarn in the final state; to obtain the curve in terms of actual overtwisting twist, $\tau_{2}$, simply replace $p_{3}$ with $p_{3} = \frac{\tau_{2}}{F}$.

4. Average Filament Strain on Overtwisting

The average filament strain across the yarn cross-section in overtwisting, starting with a stress-free yarn of twist $\tau_{1}$ is defined as

$$\bar{\varepsilon}_{f} = \frac{1}{W} \int_{b}^{a} \varepsilon_{f} \, dW$$

Substituting $\varepsilon_{f}$ from (35) and $dW$ from (2) we have

$$\bar{\varepsilon}_{f} = \int_{0}^{b} \left\{ \frac{F \sqrt{1 + p_{3}^2 r^2}}{2 \pi \sqrt{1 + p_{3}^2 r^2 - m + 1}} - 1 \right\} 2krdr \cdot \frac{1}{F \sqrt{1 + p_{3}^2 r^2 - m + 1}} \frac{2krdr}{W}$$

This can be integrated with the usual substitutions to

$$\bar{\varepsilon}_{f} = \frac{F}{m} - \frac{F(1-m)}{m^2 A_{p3}^2} \ln(m A_{p3}^2 + 1) - 1$$

(42)
The average extension ratio of the fibers is defined as
\[
\Lambda_f = 1 + \bar{\varepsilon}_f \tag{43}
\]

5. Average Filament Strain on 2-Stage Over-twisting in Various Zones of the Yarn

It is sometimes necessary to calculate the average filament strain in annuli or concentric zones of the yarn cross-section; this is so because in calculating the torques it is important to associate the correct value of the modulus with the corresponding strain and it was found experimentally that the stress-strain curves at high temperatures are strongly non-linear.

To calculate, then, the average strain we proceed as follows:

We define the zone of interest via the cumulative fraction (percentile rank) of the filaments that correspond to it. Thus, suppose we want to break down the yarn in \( n \) concentric zones (of arbitrary magnitude). Suppose we are interested in the zone contained between the \( \phi_i \) and the \( \phi_{i+1} \) cumulative fraction of filaments; the difference
\[
\Psi_i, i+1 \equiv \phi_{i+1} - \phi_i \tag{44}
\]
gives the proportion of the total population contained in that zone. We can now define the strain in that zone as follows using Equations (2) and (35), with
\[
m \equiv \left( \frac{p_1}{p_3} \right)^2
\]
\[
\overline{\varepsilon}_{i,i+1} = \frac{1}{W_{\psi_{i,i+1}}} \int_{\phi_i}^{\phi_{i+1}} \varepsilon W^2 dW = \frac{1}{W_{\psi_{i,i+1}}} \int_{\phi_i}^{\phi_{i+1}} \left( \frac{F \sqrt{1+p_3^2 r^2}}{m \sqrt{1+p_3^2 r^2+1-m}} - 1 \right) \frac{2kr dr}{f^2 \sqrt{1+p_3^2 r^2}} \tag{45}
\]

which after integration, utilizing the relationships

\[z = \sqrt{1+p_3^2 r^2} = A \phi_i^2 + 1, \text{ and } y = mz + 1 - m = mA \phi_i^2 + 1\]

\[
\overline{\varepsilon}_{i,i+1} = \left( \frac{F}{m} - 1 \right) + \frac{F(m-1)}{\psi_{i,i+1} m^2 A \phi_i^2} \ln \frac{m A \phi_i^2 + 1}{m A \phi_i^2 + 1} \tag{46}
\]

6. Torque on Overtwisting in Various Zones of the Yarn

To calculate the torque due to overtwisting in any given zone of the yarn, we use again Equation (38), but, instead of integrating between \( r = 0 \) and \( r = b \), we integrate between

\[r_i = \sqrt{A \phi_i (A \phi_i p_3^2 + 2)} \text{ and } r_{i+1} = \sqrt{A \phi_{i+1} (A \phi_{i+1} p_3^2 + 2)},\]

these limits being taken directly from Equation (7), and the equation for torque in the zone \( \psi_i = \phi_{i+1} - \phi_i \) becomes

\[
M_{TE_{i,i+1}} = \frac{2 \pi kE}{p_3^3} \left\{ A p_3^2 (\phi_{i+1} + \phi_i) + 2 \right\} A p_3^2 (\phi_{i+1} - \phi_i) \left[ \frac{F-m}{2-m} \right] + \frac{m A \phi_i^2 + 1}{m A \phi_{i+1}^2 + 1} \cdot \frac{F(1-2m)}{m^3} + \frac{A \phi_{i+1}^2 + 1}{m^2 A \phi_i^2 + 1} \left[ \frac{F(1-m) A p_3^2 (\phi_{i+1} - \phi_i)}{m^2} \right] \tag{47}
\]
This equation was used to calculate the torque of the yarn over the heater. When the substitutions \( \phi_i = 0 \) and \( \phi_{i+1} = 1 \) are made, this equation reverts to Equation (39). The modulus \( E \), is the modulus corresponding to the strain \( \varepsilon_{i,i+1} \) given by (46) for the case of strongly non-linear stress-strain curves.

VI. EXPERIMENTAL APPARATUS AND MATERIALS

(a) Apparatus

1. Texturing Machine

The texturing machine used in this study is fundamentally a laboratory size machine operating at low spindle speeds, typically 1000 rpm, (whereas industrial machines operate at speeds of up to 500,000 rpm), but capable of operating under a wide variety of machine conditions. The feed rolls and exit rolls were taken from the front and back rolls of the MITEX Drafting Force Analyzer (R17). The motors used for driving the feed and exit rolls are hysteresis synchronous planetary gear motors and are independently controlled. The motor output is 18.75 rpm in both rotating directions which provides yarn speeds of the order of an inch per second. Such processing speeds were selected so as to make possible photographic recording of the moving yarn using ordinary photographic equipment (R8). In commercial machines the spindle rpm and the yarn velocity are so high as to make observation of the moving yarn impossible without the use of specialized high speed photographic equipment (R6).
The spindle on the laboratory apparatus is a Leesona Hi-Speed spindle which is driven by a direct current motor with a Variac speed controller. A power transmission belt passes over the hollow bore twist tube of the spindle to impart either S-twist or Z-twist to the yarn in the twisting zone. The spindle speed can be adjusted to give any twist level required. A laboratory hot plate is used as heater for setting the twisted yarn; it can be adjusted to any length corresponding to the required residence time.

The overfeed is controlled by varying a gear set on the exit rollers, and the range of overfeed and underfeed covers all the possible range within which the false twister can operate. The length of the cold zone, heating zone, cooling zone and the distance between the spindle and the exit rolls can easily be adjusted. For ordinary operation the yarn coming out of the exit rolls was aspired by a vacuum cleaner.

The yarn tension is measured by monitoring the deflection of the feed roller assembly. The feed rolls and their motor are mounted as an assembly on two pivots, about which the yarn tension is transmitted to a cantilever of a force transducer. The deflection of the cantilever is measured by a four-arm strain gage bridge input to a Sanborn Rectifier Amplifier/Indicator Model 311A. This device made it possible to detect the yarn tension in the twisting zone without introducing disturbances in the process as a result of contact between the yarn and a tension measuring device. By
exchanging positions of the heater and the spindle and feeding the yarn in the reverse direction the apparatus can be used to measure yarn tension in the take-up side.

The tension measuring device has some serious limitations. The mass of the motor roller head assembly is approximately 1 kg and though the strain gage arrangement was quite sensitive, the readings obtained at low processing tensions were poorly reproducible. For instance, the motors had to be kept cool and this was done by wrapping around them a piece of wet tissue paper: as the water evaporated when the motors were heated, (a matter of 3-4 minutes), the tension reading was affected. This necessitated great care in experimentation and considerable duplication to arrive at meaningful values of tension.

The twist distribution was measured by stopping the machine while operating at steady-state, removing the yarn and un-twisting it. Previous work on the machine by Dr. W.L. Yang had demonstrated that this gave a true distribution of the twist as it exists during actual steady state operation. He demonstrated this as follows: two multifilament yarns (Black Dacron 70/34 and White Dacron 70/34) were fed together through the machine. The machine was stopped and the twist was determined in a number of locations on the threadline from photographs taken through a low power microscope. The machine was operated again under identical settings and now photographs were taken through the microscope (at exposures of
54

$10^{-3}\text{sec}$) at the same locations, while the yarn was running. The twist was measured from the photographs and found to be identical to that measured previously. The twist distribution thus obtained was in agreement with that observed on commercial machines\(^{(R6)}\) and with that observed by other workers working with specially built lab equipment operating at low speeds\(^{(R7)}\).

A detailed scheme of the apparatus indicating the exact dimensions of the various zone is given in Figure 3. Figure 4 shows photographs of the apparatus.

2. Torque Measuring Apparatus

This was composed of a steel wire (piano wire) of known torsional rigidity (obtained from torsional pendulum experiments). The wire was suspended vertically and the yarn whose torque was to be measured was attached, through a special jaw arrangement in series with the wire itself. A device at the bottom part of the yarn prevented twist escape while permitting vertical movement of the yarn to allow for contraction (or lengthening). This same device permitted upto-twisting or detwisting the yarn under test. The torque was measured by following the deflection of a mirror permanently attached to the steel wire. A picture of the apparatus is shown in Figure 5. This apparatus was constructed by Mr. Danny Lee of the Fibers and Polymers Division.

3. Tensile Testing Machine

These were measured on a table model Instron Tensile Testing
FIGURE 3

SCHEMATIC OF THE FALSE TWIST APPARATUS SHOWING DIMENSIONS

- TENSION MEASUREMENT
- FEED ROLLS
- TEMPERATURE CONTROL
- EXIT ROLLS
- SPINDLE
- HEATER
- VACUUM

Roll diameter 1.5"
Motor speed 18.75 rpm
Gear reduction 1:4 on feed and variable on exit rolls
FIGURE 4

THE FALSE-TWIST APPARATUS
FIGURE 5
THE TORQUE MEASURING APPARATUS
Machine. Typical experimental conditions were 5" gage length
and 0.2" min crosshead speed.

(b) Material Properties

The yarns used in this study were supplied by the duPont Com-
pany and were all 150/34 Dacron yarns (150 total denier; 34
filaments). Dacron is duPont's trademark for its poly (ethy-
lene terephthalate) fiber. The yarns were identified as
High draw ratio, Medium draw ratio and Low draw ratio yarn,
but the actual draw ratios were not disclosed by the manu-
facturer. The Medium draw ratio yarn was chosen to perform
the majority of experiments on the false-twister (being the
yarn representative of commercial practice) while a few ex-
periments, confirming in nature, were performed on the High
and Low draw ratio yarns.

For purposes of this work, the important properties were the
mechanical properties of the yarns at room and at elevated
temperatures (i.e., at the temperatures of the heater). The
following measurements of mechanical properties were performed
on the Medium draw ratio yarn:

(1) The stress-strain properties of the untextured or
"feed" yarn were measured on the Instron Tensile Tester. The
Young's modulus (from the Hookean part of the curve) and se-
cant moduli at various strains were calculated. These quan-
tities are needed for the analysis and calculation of the
tension and torque in the entrance zone of the false-twister.
In this work the compressive modulus of the fiber was assumed
to be equal to the tensile modulus, hence equal to the bending modulus. The shear modulus $G$ was taken equal to $\frac{E}{12}$.

(2) The feed yarn was processed through the false twister with the heater on at various overfeeds, but was not threaded around the spindle. The stress-strain curves of yarns thus processed were determined on the Instron. The values of the moduli were calculated and used, as will be explained later, in the development of the theory for explaining the observed tensions in the actual false-twisting process.

(3) The stress-strain properties of freshly textured yarns were measured. The moduli were taken as representative of the mechanical properties of the yarn in the exit zone and were also used in the development of the theory explaining the tension in the false twisting process.

(4) The feed yarn was processed through the false-twister, with the heater on at various overfeeds, but was not threaded around the spindle, exactly as in case (b) above. The tension developed during processing (the so-called "contractile stress") was measured and was used to derive the stress-strain curves of the yarns at elevated temperatures (the so-called "hot" stress-strain curves) which were used in computing the tension and the torque of the yarns during the process.

1. Stress-strain Curves of Feed Yarns

The Medium draw ratio yarn was tested on the Instron Tensile Tester, (5" gage length, 0.2"/min cross-head speed). The
stress-strain curves thus obtained are shown in Figures 6 and 6'.

2. Stress-strain Curves of Yarns Processed Thru the False-twister without Twist

As mentioned, the yarns were processed thru the false-twister but were not threaded around the spindle; their mechanical properties were determined on the Instron as above. The Hookean moduli were calculated and are given in Table I. Notice that in general, heating lowers the modulus of the yarn even at the relatively low temperature of 134°C.

The modulus drop is more significant the higher the overfeed and the higher the temperature. One might have expected that processing at -6.67% overfeed and at a high temperature would have resulted in a higher modulus. Apparently the destruction of the molecular architecture (presumably orientation) which occurs under the influence of temperature cannot be remedied by additional extension at the levels used in these experiments.

3. Stress-strain Behavior of Freshly Textured Yarns

In the model used to predict the tension level developed during false-twisting, and which will be described later, it is necessary to know the modulus of the filaments in the cooling and exit zone. To obtain these values, measurements
FIGURE 6
STRESS-STRAIN CURVE (partial) OF FEED YARN

TENSION (grams)

EXTENSION %
FIGURE 6
STRESS-STRAIN CURVE OF FEED YARN

TENSION (grams)

EXTENSION %

500 400 300 200 100

0 1 2 3 4 5 6 7 8 9 10
### TABLE I

YOUNG'S MODULUS OF YARNS PROCESSED THROUGH THE FALSE-TWISTER (BUT NOT THREADED AROUND THE SPINDLE)

<table>
<thead>
<tr>
<th>Feed Yarn: 150/34 DACRON*</th>
<th>Modulus</th>
<th>Medium Draw Ratio</th>
<th>Overfeed</th>
<th>Temp</th>
<th>Grams/Unit</th>
<th>Grams Per Den</th>
<th>PSI</th>
</tr>
</thead>
<tbody>
<tr>
<td>16200</td>
<td>108</td>
<td>1.91 * 10^6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Processed At:**

- **134°C, +6.13% Overfeed**
  - 11910
  - 75.0
  - 1.32 * 10^6

- **134°C, 0% Overfeed**
  - 13850
  - 92.3
  - 1.63 * 10^6

- **134°C, -6.67% Overfeed**
  - 15350
  - 108.8
  - 1.92 * 10^6

- **210°C, +6.13% Overfeed**
  - 11790
  - 74.2
  - 1.31 * 10^6

- **210°C, 0% Overfeed**
  - 12650
  - 84.3
  - 1.49 * 10^6

- **210°C, -6.67% Overfeed**
  - 14410
  - 102.2
  - 1.81 * 10^6

* Limit of Hookean behavior for feed yarn 0.72% or 115 grams
must be performed on textured yarn. There are two main problems associated with measurements on textured yarn:

i) The filament modulus following heat-treating (contact with the heater) changes with time, as morphological changes (such as secondary recrystallization) go on for hours, or even days, following heat treatment.

ii) The mechanical handling of the textured yarn is very critical; the yarn comes through the spindle and up to the exit rolls under tension in the untwisted configuration. Beyond the exit rolls, depending on the level of tension, the yarn may retract (crimp may develop), and once this has happened, it becomes impossible to measure the moduli of the filaments because entanglements between fibers develop which are practically irreversible. In this case it is necessary, in order to remove the crimp, to apply forces which may be much higher than the forces to which the yarn was subjected while in the machine; this in turn may affect permanently the filaments (if the required stresses are in the non-Hookean region) so that a measurement of the modulus of interest may not be possible. It is thus important to measure the yarn properties on the yarn which has emerged from the exit rolls but which has not been allowed to contract so that the crimp has not been allowed to develop.

This was accomplished by collecting the yarn that exits the false-twister on a long ruler under 40 g tension and glueing its ends onto the ruler. Sections of the yarn were then mounted on a cardboard strip, and the ends of a given section
were glued onto the strip always under the same tension; each section was approximately 7" long. The cardboard strip was then securely mounted on the Instron at 5" jaw separation at which point the cardboard was cut across: a tension between 30 and 40 grams would register in the machine. The machine was then started and the stress-strain curve determined. The initial part of the curve was always found to be in the Hookean region and it was an easy matter to extrapolate to zero stress and back calculate the true gage length. In the course of these experiments the test was not allowed to proceed to the break but was stopped at given stress levels (100 grams) and the machine was reversed.

If, upon reversing, the machine was stopped at load levels of 20 to 30 grams, where crimp was not allowed to develop, then on reapplying the stress essentially the same value of the modulus (within ±5%) was obtained. If, however, the jaws were allowed to come close enough for crimp to develop, then the "modulus" obtained by extrapolating the straight part of the curve, after the crimp had been removed, was substantially lower than the value of the original modulus. Repeated cycling thereafter between 20 and 100 grams resulted in a progressive increase of the modulus, (note that by cycling at 20 grams, the yarn was not allowed to retract again) which approached the value obtained in the very first test; thus something like a progressive "combing" and disentanglement of the filaments occurred during this sequence of cycling. The results are shown in Table II, and Figures 7 and 8.
<table>
<thead>
<tr>
<th>LOADING HISTORY (CONSECUTIVE CYCLES)</th>
<th>134°C, -6.67%</th>
<th>134°C, +6.13%</th>
<th>210°C</th>
<th>-6.67%</th>
<th>210°C, +6.13%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hours after Texturing</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>18</td>
</tr>
<tr>
<td>1st CYCLE (40 to 100 gr)</td>
<td>11,900</td>
<td>9,740</td>
<td>10,800</td>
<td>11,400</td>
<td>12,650</td>
</tr>
<tr>
<td>2nd CYCLE (20 to 100 gr)</td>
<td>12,150</td>
<td>9,350</td>
<td>11,320</td>
<td>12,820</td>
<td></td>
</tr>
<tr>
<td>3rd CYCLE (20 to 100 gr)</td>
<td>11,000</td>
<td>9,440</td>
<td>11,250</td>
<td>12,800</td>
<td></td>
</tr>
<tr>
<td>4th CYCLE (20 to 100 gr)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>9,300</td>
</tr>
<tr>
<td>RELAXED TO:</td>
<td>2&quot;</td>
<td>2&quot;</td>
<td>1.7&quot;</td>
<td>2&quot;</td>
<td>2&quot;</td>
</tr>
<tr>
<td>1st CYCLE (to 100 gr)</td>
<td>9,250</td>
<td>7,600</td>
<td>7,100</td>
<td>6,350</td>
<td>7,350</td>
</tr>
<tr>
<td>2nd CYCLE (20 to 100 gr)</td>
<td>n.c.</td>
<td>n.c.</td>
<td>n.c.</td>
<td>n.c.</td>
<td>8,540</td>
</tr>
<tr>
<td>3rd CYCLE (20 to 100 gr)</td>
<td>11,250</td>
<td>11,100</td>
<td>8,600</td>
<td>n.c.</td>
<td>8,920</td>
</tr>
<tr>
<td>4th CYCLE (20 to 100 gr)</td>
<td></td>
<td></td>
<td>8,460</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5th CYCLE (20 to 100 gr)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>STRESSED TO 200 gr and RETESTED</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>8,420</td>
</tr>
<tr>
<td>RE-STRESSED TO 200 gr and RETESTED</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RELAXED TO 2&quot; and RETESTED</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
FIGURE 7
STRESS-STRAIN CURVES (partial) OF FRESHLY TEXTURED YARNS (210°C)

210°C -6.67%
210°C +6.13%

First Cycle
Yarn allowed to collapse, so as to develop crimp and reextended
FIGURE 8
STRESS-STRAIN CURVES (partial) OF FRESHLY TEXTURED YARNS (134°C)

TENSION (grams)

EXTENSION %

-6.67%

+6.13%
Comparison of the moduli given in Table II with those of the yarns that had been processed without twist through the false-twister (Table I) shows that the textured yarn moduli are considerably lower; as will be discussed later, for a given set of texturing conditions (temperature, overfeed) the tension will be lower the higher the twist; the yarns that are processed under zero twist are therefore processed under the higher tension.

4. Stress-strain of Yarns at Elevated Temperatures - The Contractile Stress

For purposes of calculating the tension and the torque of the yarn in the heater zone, the mechanical properties of the yarn at elevated temperatures must be known. This is a difficult problem because (a) at elevated temperatures the viscoelastic character of the filaments becomes pronounced and the experimental timescale will greatly affect the results and (b) the filaments shrink at elevated temperatures so that the reference state of the yarn (the equilibrium unstressed denier) is different at each temperature.

For these reasons it was decided to determine the stress-strain curves of the yarns at elevated temperatures, using data obtained on the false-twister itself (which would automatically give the right experimental timescale); this was accomplished by running the yarn through the machine but without threading it around the spindle. At a given temperature, the yarn was run at progressively increasing overfeeds.
and the tension developed at each overfeed (the "Contraction stress") was monitored and recorded. The tension, of course, decreased with overfeed, and at some value of the overfeed the tension developed was zero. The denier of the (shrunk) yarn, calculated on the basis of the equation of continuity, which gave rise to zero tension, was taken as the equilibrium unstressed denier at that temperature, \( d_e \). The tensions developed at lower overfeeds corresponded to an extension of the yarn from the equilibrium denier; by associating the tensions to the corresponding extensions, the stress-strain curves were constructed.

With reference to the appended sketch, which shows diagramatically the arrangement of the false-twister for these experiments, \( V_i \) and \( d_i \) are the velocity and denier of the feed yarn while \( V_3 \) and \( d_3 \) are the velocity and denier of the yarn exiting the machine. Note that the spindle has been omitted in the sketch since the yarn was not threaded around it (just as was the case for the experiments earlier described under heading (b)(2)).

Two assumptions are made which are subsequently verified experimentally: (a) The denier changes on heating, i.e., going from zone 1 to zone 2, but does not change as the yarn
moves from zone 2 to zone 3, the cooling zone; (b) the tension
developed does not depend on the length of the heater, as
long as thermal equilibrium was attained. Remembering that
in our experiments the controllable variables are the heater
temperature and the overfeed \( \frac{V_i - V_3}{V_i} \), and that \( V_3 \) was the
quantity that was varied, \( V_i \) having been kept constant for
all the experiments performed in this thesis, we have from
the equation of continuity that

\[ V_i d_i = V_3 d_3 \quad (48) \]

or

\[ d_3 = d_i \frac{V_i}{V_3} \quad (48') \]

For any given value of \( \frac{V_i}{V_3} \), a tension is registered by
the tension measuring device (earlier described) and recorded;
to that value of \( \frac{V_i}{V_3} \) there corresponds also a denier
given by Equation (48'). At some value of \( \frac{V_i}{V_3} \), say

\[ \frac{V_i}{V_3} = \epsilon \]

the tension registered is zero; the denier \( d_e \) corresponding
to that value of overfeed via (48'),

\[ d_e = \frac{V_i}{V_3} \]

is the equilibrium unstressed denier for the particular
heater temperature at which the experiments were performed.
In all cases where tension was registered (i.e., at overfeeds lower than \( \frac{V_i}{V_3} \)) the corresponding denier \( d_3 \) is the strained denier of a yarn whose equilibrium (unstrained) denier is \( d_e \). So \( d_e \) and any \( d_3 \) are related by

\[
d_e = d_3 (1 + \varepsilon)
\]

hence

\[
\varepsilon = \frac{d_e}{d_3} - 1
\]

The strains thus determined, with the corresponding tensions, were used to construct the stress-strain curves. These results are presented in Table III and Figure 9. Concerning the graph, note that at each temperature the starting denier is different, but that the yarn deniers corresponding to the same overfeed are necessarily equal. The stress-strain curves themselves are strongly non-linear; their non-linearity (concavity towards the y-axis) increases with temperature. This behavior is reminiscent of rubber, and, of course, the materials become more "rubbery" as the temperature is increased.

**Experimental Test of Hypothesis that** \( \text{den}_{\text{hot}} = \text{den}_{\text{cooling}} \)**

To check the hypothesis marks were made along the yarn prior to its passing through the machine. (The marks were actually two knots made about one inch apart). The length of these marks was measured with a ruler while they were in the entrance zone, the hot zone, and the cooling zone. The relative length changes are presented in Table IV.
<table>
<thead>
<tr>
<th>OVERFEED</th>
<th>95 Tens (g)</th>
<th>95 ε</th>
<th>114.5 Tens (g)</th>
<th>114.5 ε</th>
<th>134.5 Tens (g)</th>
<th>134.5 ε</th>
<th>163 Tens (g)</th>
<th>163 ε</th>
<th>191 Tens (g)</th>
<th>191 ε</th>
<th>210 Tens (g)</th>
<th>210 ε</th>
<th>228 Tens (g)</th>
<th>228 ε</th>
<th>245 Tens (g)</th>
<th>245 ε</th>
</tr>
</thead>
<tbody>
<tr>
<td>42.86</td>
<td>0</td>
<td>0</td>
<td>1.4</td>
<td>.223</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30.16</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>27.98</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25.60</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>22.86</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19.33</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16.20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15.14</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14.67</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13.96</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13.39</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11.11</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.56</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.41</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.13</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.76</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.22</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.79</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>27 .018</td>
<td>28 .050</td>
<td>37 .099</td>
<td>36 .172</td>
<td>30 .4 .240</td>
<td>28 .2 .296</td>
<td>26 .1 .309</td>
<td>21 .9 .750</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-2.86</td>
<td>128 .047</td>
<td>98 .080</td>
<td>81 .6 .130</td>
<td>70 .6 .205</td>
<td>57 .8 .275</td>
<td>50 .4 .333</td>
<td>43 .3 .428</td>
<td>35 .5 .850</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-5.60</td>
<td>204 .075</td>
<td>153 .109</td>
<td>123 .160</td>
<td><strong>106</strong> .238</td>
<td>85 .309</td>
<td>75 .369</td>
<td>63 .466</td>
<td>56 .848</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-6.67</td>
<td>218 .086</td>
<td>172 .120</td>
<td>147 .172</td>
<td><strong>121</strong> .250</td>
<td>9 .322</td>
<td>86 .383</td>
<td>81 .481</td>
<td>60 .867</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
FIGURE 9
STRESS-STRAIN CURVES AT ELEVATED TEMPERATURES
<table>
<thead>
<tr>
<th>TEMPERATURE °C</th>
<th>210</th>
<th>210</th>
<th>210</th>
<th>210</th>
<th>134</th>
</tr>
</thead>
<tbody>
<tr>
<td>OVERFEED, %</td>
<td>+21.78</td>
<td>-6.67</td>
<td>-6.67</td>
<td>0</td>
<td>-2.86</td>
</tr>
<tr>
<td>YARN THREADED ABOUT SPINDLE?</td>
<td>NO</td>
<td>NO</td>
<td>YES</td>
<td>NO</td>
<td>NO</td>
</tr>
<tr>
<td>TENSION (g)</td>
<td>0</td>
<td>73</td>
<td>59</td>
<td>24.7</td>
<td>63.7</td>
</tr>
<tr>
<td>(L_1/L_0)*</td>
<td>1.00</td>
<td>1.009</td>
<td>1.006</td>
<td>.995</td>
<td>1.010</td>
</tr>
<tr>
<td>(L_2/L_1)*</td>
<td>.800</td>
<td>1.046</td>
<td>1.029</td>
<td>1.000</td>
<td>1.015</td>
</tr>
<tr>
<td>(L_3/L_2)*</td>
<td>1.000</td>
<td>1.006</td>
<td>1.001</td>
<td>1.002</td>
<td>1.000</td>
</tr>
<tr>
<td>NUMBER OF EXPERIMENTS</td>
<td>2</td>
<td>7</td>
<td>4</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

\(L_0\) = yarn length before entering the false-twister  
\(L_1\) = yarn length in zone 1 (entrance zone) of the false-twister  
\(L_2\) = yarn length in zone 2 (heater zone) of the false-twister  
\(L_3\) = yarn length in zone 3 (cooling zone) of the false-twister  

* (The third decimal place arises from averaging length ratios; the lengths themselves were measured with a millimeter ruler.)
Experimental Verification of the Hypothesis that Tension is Independent of Heater Length

Experimentally, at 210°C and 0% overfeed, the tension was 24.55 g at 25 mm heater length, and 24.95 at 50 mm heater length. At 210°C and -6.67% overfeed the tension was 77.3 g at 24 mm heater length and 79.80 at 42 mm heater length.

It appears thus that both hypotheses are valid.

VI. EXPERIMENTAL RESULTS: TWIST AND TENSION

(a) Introduction

In what follows will be presented the results of the experiments performed on the false-twister with the objective of measuring twist distribution and tension. These experiments were performed with two objectives in mind:

(a) The first was to acquire experience and a feel for the behavior of the machine and the interaction with the thread-line, so as to provide necessary insight for construction of a model representing the process.

(b) The second objective was, of course, to have experimental results and data against which to compare the predictions of the model in order to assess whether the model completely represents reality.

For these reasons the experimental conditions chosen to carry
out the false-twist experiments were extremely broad, spanning a range very much broader indeed than are encountered in industrial practice. To give an example, typical machine conditions for processing 150 denier polyester yarns are 220 - 230°C heater temperature, 60 turns per inch twist at the spindle, and overfeeds of the order of 1-2%. Our experiments were conducted at temperatures ranging from 134°C to 245°C. The overfeeds used were varied from -6% to +6%, and the twist was varied from 40 to 60 turns basic twist, which corresponds to actual twist at the spindle from 46 to 84 turns per inch. Also, although most experiments were performed with Medium Draw Ratio yarn (as identified by the manufacturer), a number of experiments were performed with the so-called High Draw Ratio yarn (which exhibits higher shrinkage on heating or, equivalently exhibits higher contractile stresses when heated at constant length); the Medium Draw Ratio yarn is typical of commercial practice.

The quantities measured experimentally as the processing conditions above were varied were the twist distribution, the twist contraction, the tension and the torque. At least five experiments were performed in each condition tested (sometimes more). The experiments where torque measurements were carried out, were performed more than a year after the experiments where tension and twist were measured, random checks during the processing of the former showed good agreement (within the rather serious accuracy limitation imposed by the tension measuring device) with results obtained in
the earlier set of experiments.

(b) Experimental Procedure

Before starting the texturing experiments, components of the machine were tested and calibrated as follows:

The heater was brought up to temperature and checked. To check the heater temperature, substances of known melting point were sprinkled on the heater (at a point in the path of the yarn). Such substances are normally used for the calibration of hot stage microscopes. A thermocouple (attached to a potentiometer recorder) had been attached on the heater approximately 1" away from the path of the yarn. When the crystals melted indicating that the right temperature had been achieved, a reading was made of the voltage at the thermocouple and thereafter (usually for no more than 24 hours) the heater setting was maintained at that voltage reading. The accuracy of this reading was checked using these substances at least once daily (more often if the settings were changed in the course of a given day).

This was done because the heater was a simple laboratory hot plate and was not equipped with any controlling device that would maintain constant temperature; it was found that as line voltage fluctuated, temperature changes of 10-15°C would easily occur at the same Variac settings, hence the necessity for careful monitoring.
The tension device was calibrated next. A stand on which a light pulley was mounted was interposed between the feed rolls and the spindle; the pulley was exactly at the same level as the feed rolls. Yarn from the feed rolls was passed over the pulley and various weights were suspended at its free end. (See sketch) The Feed rolls were then rotated so as to let the weight descend; the rotation direction was then reversed. The potentiometer readings for the two cases were recorded and the average reading was used as calibration against the weight. This calibration also had to be performed daily as the apparatus showed considerable drift.

The spindle speed itself was adjusted at each experimental condition with the use of a Strobotac, and continuously monitored stroboscopically for the duration of a test.

For operation, the machine was started with the yarn slack (so as to take into account the contraction) and as the yarn twisted, it was positioned over the heater. The tension variation was followed in the potentiometer; it increased as the yarn contracted on twisting, and then decreased to a stable level, the stabilization requiring 2-3 minutes. Even when stabilized, the tension continuously fluctuated about a mean value and as many as 30 random readings of the potentiometer...
meter indicator were made to arrive at the mean; the mean as well as the range of variation observed are given in the tables with the experimental false-twist data. In spite of all precautions, the reproducibility of tension experiments otherwise identical, but performed weeks apart, remained unsatisfactory, especially at high values of overfeed which corresponded to very low tensions. Part of the problem is that the roller-strain gage arrangement was not built for accurately recording low tensions, and the potentiometer needle often did not return to zero when the machine was stopped and the tension released.

When the spindle rpm had been carefully adjusted and the tension stabilized as indicated above, the machine was allowed to run for about 5 minutes and then stopped, and the yarn was removed in order to measure its twist distribution, contraction, and, in later experiments, torque. The machine was stopped (both roller and spindle were stopped simultaneously) and the yarn was collected on an 18" long steel ruler equipped with alligator clips at its ends (the jaws of which were rubber lined).

The threadline was then cut beyond the edges of the ruler and the ruler with the yarn deposited on a table. Marks 5" apart had been traced on the ruler near each end so that a 5" interval bracketed yarn in the cold zone while another 5" interval bracketed yarn in the cooling zone. Both zones were at least 2" away from the ends of the heater zone;
red marks were made on the yarn at the edges of the 5" intervals.

For purposes of twist measurement, the yarn was then clamped with miniature clamps (one inch beyond the 5" gage length). It was then cut from the ruler, and, making sure that no twist escaped, it was mounted on the jaws of a twist meter set 5" apart, and untwisted under a tension of about 0.5 g/denier. The twist meter itself was mounted vertically so that the yarn would elongate as it untwisted without frictional constraints; a ruler mounted on the twist meter next to the yarn permitted measurement of length at the 0.1 inch level.

Torque measurements were performed as follows: With the yarn mounted on the ruler as removed from the machine, a thin strip of cardboard was slipped between the yarn and the ruler. The edges of the yarn just beyond, but contiguous with, the 5" gage marks were sandwiched and glued onto the cardboard using Eastman 910 contact cement. This gave a rigid bond which allowed no twist escape. The 5" specimens were then cut from the ruler and carefully mounted on the demountable jaws of the torque measuring device. The assembly was mounted on the machine; a weight was suspended on the lower jaw (corresponding to the tension measured during false-twist processing) and the torque measured by following the deflection of the mirror on the steel (piano) wire onto which the yarn was attached in series (see description of experimental ap-
paratus). In some experiments additional weights were hung, or weights removed, and the variation of torque with tension recorded.

(c) Experimental Results: The Effects of Temperature and Overfeed

The data collected are presented in Table V. The table classifies the results first according to heater temperature. Within each temperature classification the data are further broken down according to overfeed. For each experiment are then reported the spindle rpm, the theoretical machine basic twist (the ratio of the rpm to exit roller linear velocity), the actual twist, and the basic twist as measured on the twistmeter, as well as the contracted length (the twists' ratio) for both the entrance and the exit zone. Further, the ratio of the measured basic twist to that calculated (theoretically imposed by the machine) has also been computed. The following observations are pertinent to the data which are also presented in Figures 10-13.

1. Twist in the Exit Zone

(i) There is good agreement between the theoretical machine-imposed basic twist and the measured basic twist. While there is some systematic variation within each group, the variation over the whole range of overfeeds and the whole range of temperatures is only 8%. The fundamental hypothesis (that the false-twist machine inserts a constant basic twist) is a correct one. The systematic variation within each can
| TEMP. OVER- | RPM | THEORET. BASIC | RELATIVE T | "SET" BASIC | DENIER ENTRANCE | COOLING ZONE |
| FEED | | ACTUAL | T_B | ELASTIC | ACTUAL | CONTR'D TENSION |
| | | | | M | TWIST | LENGTH |
| | | | | | | |
| COOLING ZONE | | | | | | |
| 134°C | 1040 | 50.2 | 47.8 | 62.3 | 767 (B) | 35.9 | 41.4 | .867 | 7.1 | 6-8 | - | - |
| 0 | 1120 | 50.6 | 49.9 | 63.3 | .789 | .986 | 5.1 | - | - | - | - |
| -6.67 1180 | 50.0 | 50.6 | 60.7 | .832 | 1.012 | - | - | - | - | - | - |
| 163°C | 1040 | 50.2 | 48.3 | 62.8 | .771 | .962 | 1.3 | - | - | - | - |
| 0 | 1120 | 50.6 | 49.8 | 62.7 | .794 | .984 | 2.0 | - | - | - | - |
| -6.67 1180 | 50.0 | 49.8 | 60.0 | .830 | .996 | 2.2 | - | - | - | - | - |
| 191°C | 1040 | 50.2 | 49.4 | 65.2 | .757 | .984 | - | - | - | - | - |
| 0 | 1120 | 50.6 | 50.5 | 63.8 | .791 | .998 | - | - | - | - | - |
| -6.67 1180 | 50.0 | 50.2 | 62.5 | .819 | 1.022 | - | - | - | - | - | - |
| 210°C | 1040 | 50.2 | 49.4 | 63.2 | .782 | .984 | .45 | - | - | - | - |
| 0 | 1120 | 50.6 | 50.3 | 63.0 | .798 | .994 | .85 | - | - | - | - |
| -6.67 1180 | 50.0 | 50.8 | 61.6 | .825 | 1.016 | 1.40 | - | - | - | - | - |
| 228°C | 1040 | 50.2 | 49.6 | 65.0 | .762 | .988 | - | - | - | - | - |
| 0 | 1120 | 50.6 | 50.5 | 65.1 | .767 | .988 | - | - | - | - | - |
| -6.67 1180 | 50.0 | 50.6 | 64.1 | .769 | 1.000 | - | - | - | - | - | - |
| 245°C | 1040 | 50.2 | 49.6 | 65.5 | .755 | .988 | - | - | - | - | - |
| 0 | 1120 | 50.6 | 50.7 | 65.4 | .776 | 1.002 | - | - | - | - | - |
| -6.67 1180 | 50.0 | 51.0 | 62.8 | .812 | 1.020 | - | - | - | - | - | - |

* Measured at processing tension in hot zone

** Measured at 75 g

(B) Torsional buckling observed in threadline
be explained by considering that the measurement of the (basic) twist was performed under a stress of 0.5 g/denier or 75 grams; the yarn, however, was processed under variable tension, the tension in the exit zone being roughly twice that of the cooling zone (because of the tension jump across the pin). The theoretical machine twist, $T_M$, and the measured basic twist, $T_B$, should agree when the tension used to untwist the yarn is approximately the same as the tension in the exit zone: this should be so when the processing tension is in the range of 25-40 grams; reference to the table shows that $T_B$ and $T_M$ are in good agreement in this case. When the tension under which the twist was measured is higher than the processing tension, then the yarn was extended (relative to its state of strain in the exit zone) on measuring the twist, and the measured basic twist is thus lower than it was on the machine; the opposite is true for high processing tensions. Reference to Table V shows the data to be in agreement with the argument above.

(ii) The actual twist at the spindle is also roughly constant, varying between 60 and 65 tpi for the whole range of experiments. Within a given temperature group the twist decreases somewhat as overfeed decreases: this decrease is of the order of 3.5% and is no doubt due to the tension which elongates the yarn in the hot zone. Another way of looking at the data is to consider the contracted length or $T_B/T_A$. The contracted length increases as overfeed de-
creases and this must be a manifestation of tension which extends the yarn in the heater zone. See Figures 10, 11, and 12.

2. Twist in the Entrance Zone

The situation here is quite different from that in the cooling zone.

(i) Within a given temperature group the twist variation is quite pronounced, this being true of both the basic and the actual twist, the higher the underfeed, the higher the twist. (Figure 11)

(ii) The contracted lengths increase with tension and twist in contra-distinction to the results observed in the exit zone. No doubt this is because the contracted lengths in the entrance zone are less sensitive to tension than is the case in the heater zone (the yarn being much more deformable in the heater zone than in the entrance), and so the contraction increases with twist as predicted by yarn geometry. Also, of course, higher twist and higher tensions imply higher torques. (Figure 12)

(iii) The twist level distinctly decreases as the temperature is increased. (Figure 10)

3. Tension

(i) The tension within a given group varies with overfeed: the lower the overfeed the higher the tension.

(ii) In general, tension drops as temperature increases, especially at low overfeeds. (Figure 13). The prediction of tension
FIGURE 10
TWIST IN THE ENTRANCE AND COOLING ZONES
AS A FUNCTION OF TEMPERATURE AND OVERFEED
(50 tpi basic)

BASIC TWIST (tpi)

COOLING ZONE

ENTRANCE ZONE

+ +6.13%
○ 0%
△ -6.67%

TEMPERATURE °C
FIGURE 11
TWIST VARIATION WITH OVERFEED
AT 191°C (+), 228°C (0) and 245°C (△)
(50 tpi basic)
FIGURE 12

RELATIVE CONTRACTED LENGTH VARIATION WITH OVERFEED AT 163°C (△), 191°C (+) AND 228°C (○) (50 tpi basic)
FIGURE 13

TENSION vs. TEMPERATURE AT VARIOUS OVERFEEDS
(50 tpi basic)
will be analyzed in detail in a later section.

4. Denier

Values of yarn denier before and after texturing are reported. These values were obtained by weighing the untwisted entrance zone and exit zone specimens which had been used for the measurement of twist; the values of untwisted length were also used in the calculation of basic twist. All specimens were measured under a weight of 75 grams corresponding approximately to 0.5 g/denier. The deniers as reported have not been corrected for the extension of the filaments under the 0.5 g/den load. The correction in the entrance zone would be of the order of 0.5% and in the exit zone of the order of 0.8%

* The results show that the yarn denier follows rather well the values predicted by overfeed and the continuity equation.

5. Analysis of the Data and Implications for the Process

Summing up the data we observe that the machine inserts as a reasonable approximation a constant twist in the heater zone (as imposed by machine conditions) but a different value of twist in the entrance zone; since both the tension and the twist are seen to increase as the overfeed decreases, the torque must do the same. Also, since the final twist is constant, the magnitude of the driving torque is, therefore, related to the twist change at the heater, the so-called "twist jump". The existence of high torques at the negative

*due to the difference in moduli
overfeeds implies that a greater part of the twist at the spindle is elastic in nature, being the response of the cooled yarn to the high level of torque: therefore, at constant speed, at the spindle, the number of set turns of twist must be lower in the case of the high torque. This observation has been verified by allowing the cooled yarn to untwist (under zero tension and torque), measuring the number of turns thus lost, then completely untwisting the yarn in the twist-meter and expressing the number of elastic turns as a percentage of total turns. The results are shown in Table V and are in agreement with the argument above. Since it is the number of set turns of twist that determines the elastic and bulking characteristics of the yarn, the above imply that better bulking should be achieved at high overfeeds; this, however, is not the whole story, for the measured basic twist is lower at high overfeeds, and the quantity that matters for purposes of bulking is the set basic twist. When this is calculated it is seen (Table V) that the set turns of twist per untwisted length are essentially constant within each temperature grouping (they increase very slightly with underfeed), but that they increase with temperature.

It should be emphasized that the bulking or stretching quality is not merely determined by the set number of turns; other properties such as the quality or permanence of setting are quite important commercially but cannot be inferred from the data collected and reported above.
(d) Experimental Results: The Effect of Twist (Fig. 14)

Table VI gives the experimental results of the effect of twist variation at constant temperature over the same range of overfeeds as before. The same qualitative observations can be made from these data, as were earlier made, concerning the basic twist and the effect of overfeed on tension. The interesting phenomenon here, however, is that the tension is seen to drop systematically with twist at identical conditions of temperature and overfeed. The twist in the entrance zone at equivalent overfeeds and, therefore, the torque increases with twist as would be expected from the higher spindle rpm (Fig.15). We can therefore see here that as twist increases, the tension drops, and the torque increases; since the contractile stresses in the filaments generated at the heater are assumed to be the same irrespective of twist (and depending solely on overfeed and the equation of continuity), and since the inclination of the filaments (twist) is imposed by the machine, at high inclinations, these contractile stresses will have considerable tangential components which will be manifested as torque and smaller components (because of the high helix angle) in the axial direction of the yarn, which leads to lower tensions and vice versa.

(e) Experimental Results: The Effect of Draw Ratio

To assess the effect of yarn draw ratio, false-twist processing experiments were performed over a range of overfeeds at 228°C. The results are given on Table VII; for comparison, the data concerning the Medium Draw Ratio yarn under the same
FIGURE 14

VARIATION OF TENSION WITH TWIST AT 210°C
FIGURE 15

TWIST IN THE ENTRANCE ZONE AS A FUNCTION OF TWIST AT THE SPINDLE

ACTUAL TWIST (ENTRANCE ZONE)

- 6.67% OVERFEED
- 0% OVERFEED
+ 6.13% OVERFEED
### TABLE VI

**TWIST DISTRIBUTION - EFFECT OF TWIST AT 210°C**

<table>
<thead>
<tr>
<th>OVERFEED</th>
<th>RPM</th>
<th>THEORET. MACHINE TWIST, TM</th>
<th>BASIC TWIST TB</th>
<th>ACTUAL TWIST TA</th>
<th>RELATIVE CONTR'D TB/TA</th>
<th>% ELASTIC TURNS TB/TM</th>
<th>&quot;SET&quot; TURNS OF BASIC TWIST</th>
<th>BASIC TWIST LENGTH</th>
<th>CONTR'D TWIST LENGTH</th>
<th>TENSION g</th>
</tr>
</thead>
<tbody>
<tr>
<td>+6.13%</td>
<td>830</td>
<td>40.1</td>
<td>40.0</td>
<td>47.0</td>
<td>.852</td>
<td>.998</td>
<td>.47</td>
<td>39.8</td>
<td>17.9</td>
<td>.973</td>
</tr>
<tr>
<td>0</td>
<td>896</td>
<td>40.5</td>
<td>41.1</td>
<td>47.1</td>
<td>.873</td>
<td>1.015</td>
<td>.91</td>
<td>40.7</td>
<td>24.7</td>
<td>.953</td>
</tr>
<tr>
<td>-6.67%</td>
<td>944</td>
<td>40.0</td>
<td>41.4</td>
<td>46.2</td>
<td>.897</td>
<td>1.035</td>
<td>1.46</td>
<td>40.5</td>
<td>26.9</td>
<td>.947</td>
</tr>
<tr>
<td>+6.13%</td>
<td>1040</td>
<td>50.2</td>
<td>49.4</td>
<td>63.2</td>
<td>.782</td>
<td>.984</td>
<td>.45</td>
<td>49.2</td>
<td>29.8</td>
<td>.916</td>
</tr>
<tr>
<td>0</td>
<td>1120</td>
<td>50.6</td>
<td>50.3</td>
<td>63.5</td>
<td>.793</td>
<td>.994</td>
<td>.86</td>
<td>49.9</td>
<td>34.2</td>
<td>.886</td>
</tr>
<tr>
<td>-6.67%</td>
<td>1180</td>
<td>50.0</td>
<td>50.8</td>
<td>61.6</td>
<td>.825</td>
<td>1.016</td>
<td>1.41</td>
<td>50.1</td>
<td>35.1</td>
<td>.887</td>
</tr>
<tr>
<td>+6.13%</td>
<td>1250</td>
<td>60.3</td>
<td>56.5</td>
<td>84.0</td>
<td>.673</td>
<td>.937</td>
<td>.62</td>
<td>56.1</td>
<td>40.7</td>
<td>.810</td>
</tr>
<tr>
<td>0</td>
<td>1345</td>
<td>60.8</td>
<td>58.7</td>
<td>84.2</td>
<td>.697</td>
<td>.965</td>
<td>1.05</td>
<td>58.1</td>
<td>44.7</td>
<td>.783</td>
</tr>
<tr>
<td>-6.67%</td>
<td>1416</td>
<td>60.0</td>
<td>59.1</td>
<td>78.6</td>
<td>.752</td>
<td>.985</td>
<td>1.69</td>
<td>58.1</td>
<td>44.6</td>
<td>.752</td>
</tr>
<tr>
<td>OVER-FEED</td>
<td>RPM</td>
<td>THEORET. MACHINE TWIST, $T_M$</td>
<td>BASIC TWIST $T_B$</td>
<td>ACTUAL TWIST $T_A$</td>
<td>RELATIVE CONTR'D LENGTH $T_B / T_A$</td>
<td>COOLING ZONE</td>
<td>BASIC TWIST</td>
<td>ACTUAL TWIST</td>
<td>CONTR'D LENGTH</td>
<td>TENSION (g)</td>
</tr>
<tr>
<td>----------</td>
<td>-----</td>
<td>-------------------------------</td>
<td>------------------</td>
<td>------------------</td>
<td>-------------------------------</td>
<td>---------------</td>
<td>-------------</td>
<td>-------------</td>
<td>----------------</td>
<td>-------------</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MEDIUM</td>
<td>4.00</td>
<td>1070</td>
<td>50.5</td>
<td>49.9</td>
<td>65.1</td>
<td>.767</td>
<td>31.7</td>
<td>35.3</td>
<td>.898</td>
<td>(10-12)</td>
</tr>
<tr>
<td>DRAW</td>
<td>0</td>
<td>1120</td>
<td>50.6</td>
<td>50.6</td>
<td>64.1</td>
<td>.789</td>
<td>32.7</td>
<td>36.5</td>
<td>.894</td>
<td>(20-24)</td>
</tr>
<tr>
<td>RATIO</td>
<td>-5.60</td>
<td>1160</td>
<td>49.8</td>
<td>50.2</td>
<td>61.1</td>
<td>.822</td>
<td>33.6</td>
<td>37.8</td>
<td>.889</td>
<td>38</td>
</tr>
<tr>
<td></td>
<td>-6.67</td>
<td>1180</td>
<td>50.0</td>
<td>51.5</td>
<td>63.1</td>
<td>.816</td>
<td>34.9</td>
<td>39.5</td>
<td>.884</td>
<td>44</td>
</tr>
<tr>
<td>HIGH</td>
<td>4.00</td>
<td>1070</td>
<td>50.5</td>
<td>50.7</td>
<td>65.7</td>
<td>.772</td>
<td>33.2</td>
<td>37.5</td>
<td>.886</td>
<td>(16-18)</td>
</tr>
<tr>
<td>DRAW</td>
<td>0</td>
<td>1120</td>
<td>50.6</td>
<td>51.8</td>
<td>65.9</td>
<td>.786</td>
<td>35.8</td>
<td>41.1</td>
<td>.872</td>
<td>(28-30)</td>
</tr>
<tr>
<td>RATIO</td>
<td>-5.60</td>
<td>1160</td>
<td>49.8</td>
<td>52.1</td>
<td>63.9</td>
<td>.815</td>
<td>36.8</td>
<td>42.1</td>
<td>.876</td>
<td>(56-60)</td>
</tr>
<tr>
<td></td>
<td>-6.67</td>
<td>1180</td>
<td>50.0</td>
<td>52.2</td>
<td>63.2</td>
<td>.821</td>
<td>36.4</td>
<td>41.5</td>
<td>.876</td>
<td>(56-60)</td>
</tr>
</tbody>
</table>
conditions, already reported as part of Table V, are also included. There is a noticeable difference in the performance data between the two yarns; at comparable overfeeds the tension, as expected, is higher for the High Draw Ratio yarn and the twist in the entrance zone is also higher. That indicates, of course, that the corresponding torque is higher.

(f) Qualitative Model for the Process
On the basis of the data we have collected and studied, we can now put together a simple picture of the process. For a given rpm and exit yarn velocity, the spindle inserts a constant number of turns of basic twist to the yarn and to a reasonable approximation a constant number of actual turns of twist at the spindle. At constant twist two parameters are available and at disposal to vary: temperature and overfeed. A decrease in overfeed brings about an increase in tension, and that brings about a significant increase in torque, which is now manifested as an increase in the twist of the entrance zone; that must be the reason why the twist is found to increase in the entrance zone as the overfeed decreases.

The effect of a temperature change on tension is more difficult to predict; the reason can be seen by consultation of the stress-strain curves of the untwisted yarns at the high temperature: These curves are strongly non-linear with strain and are lower as the temperature increases. Now the origin of the tension in the process arises out of the tendency of the filaments to shrink when exposed to the heater,
to some equilibrium value at the given temperature, which can be characterized quantitatively by some equilibrium unstressed denier, \( d_e \). Of course, the higher the temperature, the higher the equilibrium shrinkage, and the higher the equilibrium denier; this means that, if at constant basic twist and overfeed we increase the temperature, we are in effect increasing the strain on the filaments in the hot zone, since their final denier is constant (controlled by the overfeed), but their equilibrium denier is now higher. At the same time the modulus at the high temperature is lower. It is the competing effects of modulus decrease and concomitant strain increase which decides the outcome: If one effect dominates over the other, then the tension will either increase or decrease or even not change. The dependence thus on temperature is a pure manifestation of the material behaviour and could be quite different for different materials; inasmuch as tension influences torque, the twist profile will also be different (though not the final twist) if the temperature dependence of the material properties is different. Finally, concerning twist, the comment made in the previous section apply: Higher twists leads, of course, to higher torques and higher twist values in the entrance zone while at the same time, because of the high inclination of the filaments, they lead to lower values of tension.

Qualitatively, thus we can explain the behavior observed experimentally. At the quantitative level, though, and in
a deeper sense we have left many fundamental questions unanswered as yet. For to fully understand a process, one should give quantitative answers which are in agreement with the experimental facts. Questions that need to be answered include the following:

(1) What kind of yarn twisting mechanism exists that generates an essentially constant twist under a variety of tension and temperature conditions?

(2) What mechanisms of tension and torque are operative in the yarn that result in the change of twist from the entrance zone to the heater zone? In other words, why should the twist be constant at the heater zone and not at the entrance zone; and why should the two differ?

(3) What kind of model, from the yarn mechanics standpoint, can account for the tension levels observed in the yarn?

(4) What kind of model, consistent with our qualitative observations can account for the torques experimentally measured?

We attack the problem first by discussing the tension behavior of the filaments and then going into some depth in the mechanism for torque generation.
VIII. CALCULATION OF TENSION

(a) False-twist Operating but Spindle Stationary

1. Analysis

We attack the problem of calculating the tension by analysing a case simpler than the actual false-twisting, namely, the case of the tension developed in the false-twister as the yarn is running through the machine and is threaded around the pin, but the pin itself is stationary (does not rotate). In this case, of course, no twist is developed, but just as for actual false twisting, the tension in the system will be higher downstream of the spindle and will be related to the tension upstream by the tension ratio \( r \); \( r \) was experimentally determined in this work by reverting the pin and heater position and the direction of motion of the yarn (since the strain gage which measures tension is attached on what serve as feed rolls during normal machine operation).

Fundamentally we seek an expression derived from the physics of the situation and the machine conditions, which will allow us a value of the strain of the yarn in zone 3 or zone 4 (whose tensions are related by the tension ratio \( r \)). Then through knowledge of the material properties we can relate strain to tension. The following relationships can be used to that effect: The **continuity equation** which expresses the constancy of mass at any point in the threadline:

\[
V_1 d_1 = V_3 d_3 = V_4 d_4
\]  
\[\text{Equation 48}\]
Relationship Between the Yarn Deniers

In zone 2

\[ d_e = d_2 (1 + \varepsilon_2) \]  \hspace{1cm} (49)

where \( d_e \) is the equilibrium denier to which the yarn would go if allowed to shrink freely at the heater temperature.

In zones 3 and 4 as a result of the tension difference

\[ d_{(unstr)} = d_3 (1 + \varepsilon_3) = d_4 (1 + \varepsilon_4) \]  \hspace{1cm} (50)

\[ \frac{d_3}{d_4} = \frac{1 + \varepsilon_4}{1 + \varepsilon_3} \]  \hspace{1cm} (50')

where \( d_{(unstr)} \) is the denier of the cooled yarn at zero stress.

In zones 2 and 3

\[ d_2 = d_3 \]  \hspace{1cm} (51)

postulated and experimentally verified as reported in an earlier section, \( \varepsilon_2, \varepsilon_3, \varepsilon_4 \) are the filament (and yarn) strains.

Equality of Tension and the Material Properties

\[ T_1 = T_2 = T_3 = \frac{T_4}{r} \]  \hspace{1cm} (52)

Assuming Hookean behavior,
from Equations (49) and (51).

If we now replace in (55) the value of \( d_3 \) from Equation (50'), and use (52)

\[
T_3 = E_2 \frac{d_3}{d_4} (1 + r \varepsilon_3) - 1
\]

This last equation, in conjunction with Hooke's law, Equation (53) forms a system of equations which can readily be solved to yield \( T_3 \). If the behavior of the material is non-Hookean, one needs to express the stress-strain curves analytically as \( T = f(\varepsilon) \) or alternatively one can express the moduli as functions of strain \( E = f(\varepsilon) \). This will complicate the computations but will not alter any of the arguments above. In any case two stress-strain relations are needed: one for the hot zone where the contractile stress is developed, and are for the cooling zone (the yarn upstream of the spindle having, of course, the same stress-strain curve as the yarn downstream).

2. Experimental Verification

To verify the analysis above, the threadline tension for
two different experimental conditions were calculated and compared to experimental values. The two cases are as follows:

**Laboratory False-twister Settings:**

\[ 210^\circ \text{C}, -6.67\% \text{ overfeed} \]

The measured tension was 71 grams and the tension ratio across the pin (measured as earlier described) was 2.1 with 150/34 Medium Draw Ratio Dacron yarn. The spindle was not stationary but was made to rotate slowly (\(\sim 100 \text{ rpm corresponding to } \sim 5 \text{ tpi}\)), to avoid snagging which occurs at the spindle if the yarn is entirely without twist. The tension level was high enough to be outside the linear range of the stress-strain curve, not only of the heated yarn but also of the cooled yarn. In order to avoid the complications of expressing analytically the stress-strain curves and then solving the tension equations, the moduli were taken from the stress-strain curves at the level of tensions experimentally determined (70 g.). The appropriate values were 225 grams/unit strain for the heated yarn and 12,000 for the cooled yarn (the linear part of the stress-strain curve has a modulus of 14400). Using these values, the predicted value of tension is 78 grams vs. 71 grams experimentally determined. Note that this testing condition is an extreme one, as yarns are usually processed under overfeed rather than underfeed conditions, so actual processing tensions are much lower.
Laboratory False-twister Settings:

210°C, 0% overfeed

The measured tension was 23 grams and the tension ratio was 2.3, measured as before. The machine rpm was 100 (corresponding to ~5 tpi). The hot yarn modulus (taken from the stress-strain curve at 23 grams tension) was 95.4 and the cooled yarn modulus was taken as 14400 grams/unit strain. The calculated value of tension was 28 grams. Thus the results for the two cases are reasonable and the model can be accepted as valid.

(b) Determination of the Tension During False-twisting

1. Analysis

The preceding simple analysis allows for determination of the strains developed in zone 2, zone 3 and zone 4, based on machine conditions, and on the a priori knowledge of the moduli. The moduli of importance were the "hot" modulus (and the concept of equilibrium denier) and the cooled yarn modulus. In the case of actual false-twist operation the picture is evidently more complex because the yarn in zones 1, 2, and 3 is twisted while in zone 4 it is untwisted. Consequently, in addition to material parameters, we must also consider and take into account yarn mechanics. Since the tension in zones 3 and 4 has been expressed in terms of filament strains that are then translated into tensions using the Young's modulus, it is useful to express the twisted yarn tension in terms of
filament strains rather than yarn strain. From yarn mechanics the yarn tension for small strains is given in terms of the average filament strain $\bar{\varepsilon}_f$ by combining Equations (13) and (15).

$$T = \pi k E_f \frac{A (Ap^2 + 2)}{(Ap^2 + 1)}$$

(57)

This equation, as explained earlier, assumes the yarn to have been originally stress free in all filaments; the yarn tension arises from the stress induced in the filaments upon extension of the yarn.

To derive the equations which will allow determination of yarn tension for the case of false-twisting, the following assumptions are made:

(1) The twist $p$ is known. Actually the basic twist is known, and if the contraction factor is known so is $p$.

(2) We assume the yarn denier to be the same in the heater and in the cooling zone. This was experimentally verified for the case of untwisted and very lightly twisted yarns (see Table IV); the validity of that assumption for the case of twisted yarns, is supported by the observation that little twist change occurs as the yarn moves from the heater to the cooling zone.
(3) We assume that the filaments obey Hooke's law in zones 2, 3 and 4 or, if the stress-strain curves are non-linear, they can still be expressed analytically. Alternatively, an appropriate value of the modulus must be chosen from the experimental data.

(4) We assume, in particular, that the shrinkage behavior of the filaments in the twisted yarn and so the "hot" stress-strain curves earlier determined on untwisted filaments are applicable to twisted yarn. We assume that the equilibrium denier \( d_e \) is only dependent on temperature; also the tension and torque in the heater zone arise from tension, which is developed on heating, because the yarn is not allowed to shrink to the value of equilibrium denier corresponding to that temperature.

(5) We assume that Equation (57) represents the mechanism of yarn tension buildup: the filaments are all stressed, with respect to some (hypothetical) unstressed state where they would have all been at their equilibrium unstressed level, and these strains give rise to yarn tension.

(6) We assume the tension ratio \( r \) to be known experimentally.
With these assumptions the equations to be used are:

**The continuity equation at steady-state**

\[ V_i d_i = V_4 d_4 \]  \hspace{1cm} (48)

**Relationship between the yarn (or filament) denier**

\[ d_e = d_2 (1 + \overline{\varepsilon}_2) \]  \hspace{1cm} (49)

where \( \overline{\varepsilon}_2 \) is now the average filament strain. Also

\[ d_3 (1 + \overline{\varepsilon}_3) = d_4 (1 + \varepsilon_4) \]  \hspace{1cm} (50)

with \( \overline{\varepsilon}_3 \) the average filament strain in zone 3. Also

\[ d_2 = d_3 \]  \hspace{1cm} (51)

**The equality of tensions**

This is expressed in terms of yarn mechanics

\[ T_2 = \pi k E_2 \overline{\varepsilon}_2 \frac{A (Ap^2 + 2)}{(Ap^2 + 1)} = T_3 = \pi k E_3 \overline{\varepsilon}_3 \frac{A (Ap^2 + 2)}{(Ap^2 + 1)} \]  \hspace{1cm} (58)

a consequence of Equations (52) and (57). Since the geometry (i.e., twist, denier, packing factor) are the same in zones 2 and 3, we can simplify (58) to

\[ E_2 \overline{\varepsilon}_2 = E_3 \overline{\varepsilon}_3 \]  \hspace{1cm} (58')

This, of course, assumes that the mechanism by which the yarn is supported is the same in both zones. Also

\[ T_3 = \frac{T_4}{r} \]  \hspace{1cm} (52)
which can be rewritten in full detail,

\[ T_3 = \pi k E_3 r_3 \frac{W f^2 (A p^2 + 2)}{2k (A p^2 + 1)} = T_4 \frac{4}{r} = \frac{1}{r} E_3 \epsilon_4 W f r^2 \]  

(52')

with all symbols previously defined. Note that the modulus in zone 3 and 4 is the modulus of the cooled yarn. Simplifying (52') and rewriting

\[ \bar{\epsilon}_3 = \frac{2(A p^2 + 1)}{A p^2 + 2} \cdot \frac{1}{r} \epsilon_4 \]  

(52'')

It will be seen later that the quantity \( \frac{2(A p^2 + 1)}{A p^2 + 2} \) is the tension ratio of an untwisted to a twisted yarn at the same average filament strain. We define, for brevity

\[ \frac{A p^2 + 2}{2(A p^2 + 1)} \cdot r \equiv r' \]  

(59)

Finally,

\[ \bar{\epsilon}_2 = \left[ \frac{d_2}{d_3} - 1 \right] = \left[ \frac{d_2}{d_3} - 1 \right] \]  

(55)

Replacing \( \epsilon_4 \) from (52'') in (50) we solve for \( \bar{\epsilon}_3 \); we then replace in the calculated expression \( d_3 \) from (55) and \( \bar{\epsilon}_2 \) from (58'). \( \bar{\epsilon}_3 \) is then given by the implicit equation

\[ \bar{\epsilon}_3 = \frac{d_2}{1 + \frac{E_3 \bar{\epsilon}_3}{E_2}} = \frac{1}{1 + \frac{E_3 \bar{\epsilon}_3}{E_2}} \frac{d_4}{d_e} \]  

(60)
The value of $\varepsilon_3$ is the average filament strain, which is translated into yarn tension through Equation (57); it is seen to depend on the ratios $E_3/E_2$ and $d_4/d_e$.

2. Experimental Verification

To check the validity of the model, the tensions for overfeeds of $+6.13\%$, 0% and $-6.67\%$ and for 2 temperatures ($134^\circ C$ and $210^\circ C$) were calculated via Equation (59) and compared to those actually measured. The various quantities in Equation (59) were calculated as follows:

Denier: $d_{eq}$ was taken directly from the contractile stress data; $d_4$ was calculated from the equation of continuity (48).

Moduli: $E_3$ was taken from Table II; the value of $E_3$ corresponding to 0% overfeed was not experimentally determined, but was interpolated from the $+6.17\%$ and $-6.67\%$ overfeed data which differ by less than 20%.

$E_2$ was taken from the contractile stress data. Since the stress-strain curves are non-linear, the experimental value of the tension was used (corrected for the twist from the equations of yarn mechanics) in conjunction with the stress-strain curves to determine $E_2$.

The alternative would have been to express $E_2$ analytically as a function of $\bar{\varepsilon}_2$ and to use an additional equation to determine $\bar{\varepsilon}_3$.

The tension ratio was taken as 2.3; $r$ was measured as
described earlier (Section VIII, (a), 1.); the results are shown on Table VIII and show considerable scatter. The value of \( \bar{\varepsilon}_3 \) from (60) was then calculated and was translated into yarn tension using Equation (57) and values of \( E_3 \) from Table II; the value of the twist used to relate filament tension to yarn tension was the one experimentally determined.

The results of the tension calculation are given in Table IX, Fig.16,16'. They apply to yarns textured to 50 tpi basic twist, whose twist distributions have been reported in Table V. The agreement between the calculated and experimental values, considering the miniscule values of the strains involved, and the assumptions and approximations used is good.

3. Discussion

The expression for \( \bar{\varepsilon}_3 \) depends on the ratios \( E_3/E_2 \), \( d_4/d_e \), and \( r' \), but this dependence cannot be expressed analytically in a simple way. To get insight into this dependence we consider first Equation (58') which was used to derive (59)

\[
\bar{\varepsilon}_3 = \frac{E_2}{E_3} \bar{\varepsilon}_2 \quad (58')
\]

This indicates that \( \bar{\varepsilon}_3 \) varies directly with \( E_2/E_3 \) and \( \bar{\varepsilon}_2 \). In our experiments the ratio \( E_3/E_2 \) was found to vary from 17 to 225 (Table IX) and this variation is almost entirely due to the variation in \( E_2 \) since \( E_3 \) varies over a range of only
<table>
<thead>
<tr>
<th>Overfeed</th>
<th>Contractile stress (yarn not threaded around the pin), grams</th>
<th>Standard machine configuration</th>
<th>Machine and yarn direction reversed</th>
<th>Average Tension Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>-6.7%</td>
<td></td>
<td>TEST 1</td>
<td>TEST 1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Tension at 100 rpm (4.2 tpi basic)</td>
<td>86.0</td>
<td>69.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Tension at 944 rpm (40 tpi basic)</td>
<td>59.8</td>
<td>149.2</td>
<td>2.09</td>
</tr>
<tr>
<td></td>
<td>Tension at 1180 rpm (50 tpi basic)</td>
<td>47.0</td>
<td>126.6</td>
<td>2.17</td>
</tr>
<tr>
<td></td>
<td>Tension at 1416 rpm (60 tpi basic)</td>
<td>46.0</td>
<td>101.7</td>
<td>2.11</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>72.8</td>
<td>1.66</td>
</tr>
<tr>
<td>0%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Contractile stress (yarn not threaded around the pin), grams</td>
<td>28.5</td>
<td>20.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Tension at 100 rpm (4.5 tpi basic)</td>
<td>-</td>
<td>52.7</td>
<td>2.27</td>
</tr>
<tr>
<td></td>
<td>Tension at 1120 rpm (50 tpi basic)</td>
<td>20.6</td>
<td>-</td>
<td>2.28</td>
</tr>
<tr>
<td>TEMPER.</td>
<td>OVER-</td>
<td>( \frac{T_{\text{unt}}}{T} )</td>
<td>( r' )</td>
<td>( \frac{E_3}{E_2} )</td>
</tr>
<tr>
<td>---------</td>
<td>-------</td>
<td>--------</td>
<td>--------</td>
<td>----------------</td>
</tr>
<tr>
<td>134°C</td>
<td>-6.67</td>
<td>1.171</td>
<td>2.69</td>
<td>17.6</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>1.191</td>
<td>2.74</td>
<td>29.0</td>
</tr>
<tr>
<td></td>
<td>+6.13</td>
<td>1.196</td>
<td>2.75</td>
<td>58.7</td>
</tr>
<tr>
<td>210°C</td>
<td>-6.67</td>
<td>1.173</td>
<td>2.70</td>
<td>68.1</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>1.192</td>
<td>2.74</td>
<td>116.1</td>
</tr>
<tr>
<td></td>
<td>+6.13</td>
<td>1.200</td>
<td>2.76</td>
<td>224.9</td>
</tr>
</tbody>
</table>

**TABLE IX**

CALCULATED AND MEASURED THREADLINE TENSION (GRAMS) DURING FALSE TWISTING (50 TPI BASIC TWIST)
FIGURE 16
EXPERIMENTAL AND CALCULATED VALUES OF TENSION (134°C)
FIGURE 16
EXPERIMENTAL AND CALCULATED VALUES OF TENSION (210°C)

TENSION (grams)

OVERFEED %
18% (from 9800 to 11900 grams/unit strain). Therefore, the higher the hot modulus the higher $\bar{\varepsilon}_3$.

Equation (58') also varies directly with $\bar{\varepsilon}_2$. To obtain insight into the dependence of $\bar{\varepsilon}_2$ itself with machine variables we note that the expression $d_4 / d_{\text{eq}}$ which appears in (59) can be rewritten as

$$
\frac{d_4}{d_{\text{eq}}} = \frac{d_4}{d_3} \frac{d_3}{d_4} = \frac{d_4}{d_2} \frac{d_2}{d_4} = \left(1 + \bar{\varepsilon}_2\right) \left(\frac{1 + r'\bar{\varepsilon}_3}{1 + \bar{\varepsilon}_3}\right) \quad (61)
$$

from Equations (51), (50) and (54). The second parenthesis in (60) for the highest value of strain encountered in the experiments ($r' = 2.69, \bar{\varepsilon}_3 = 9 \times 10^{-3}$) is 1.015. It can thus be neglected and Equation (60) can be rewritten

$$
\frac{d_4}{d_4} = 1 + \bar{\varepsilon}_2 \quad (62)
$$

or

$$
\bar{\varepsilon}_2 = \frac{d_4}{d_4} - 1 \quad (62')
$$

This equation says that $\bar{\varepsilon}_2$ is the "overall" yarn strain imposed by the machine ($d_2$ being controlled by the temperature and $d_4$ by the overfeed). Note that this affords us a simplified way of determining the tension, for given $\bar{\varepsilon}_2$, we have immediate access to $\bar{\varepsilon}_3$ through Equation (58'), remembering,
however, that $E_2$ is also a function of $\bar{E}_2$. Values of $\bar{E}_2$ calculated using Equation (62') are shown on Table IX; there is reasonable agreement between $\bar{E}_2$ calculated via Equation (62') and $\bar{E}_2$ calculated via Equation (58'), given $\bar{E}_3$ from Equation (60), which is also shown on Table IX. With the above in mind, we can now explore in more detail the predictions of Equation (59) concerning the effects of processing variables in $\bar{E}_3$.

Dependence of $\bar{E}_3$ on $r'$

A numerical calculation was made to assess how $\bar{E}_3$ depends on $r'$. Data were used from the experiment at 210°C and 0% overfeed. With reference to Table IX, the variables of interest in this case were $E_3/E_2 = 116.1, d_4/d_e = .7714$. Equation (59) was used to calculate $\bar{E}_3$ as $r'$ was given the values 2.1, 2.3, 2.5, and 2.7. The results, illustrating the effect on $\bar{E}_3$ of the variation of $r'$ are given below.

<table>
<thead>
<tr>
<th>$r'$</th>
<th>$\bar{E}_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>$2.52 \times 10^{-3}$</td>
</tr>
<tr>
<td>2.3</td>
<td>$2.52 \times 10^{-3}$</td>
</tr>
<tr>
<td>2.5</td>
<td>$2.51 \times 10^{-3}$</td>
</tr>
<tr>
<td>2.7</td>
<td>$2.51 \times 10^{-3}$ – actual case</td>
</tr>
</tbody>
</table>

So, at least in the range of interest, the dependence on $r'$ is not significant; in view of the difficulties in obtaining an accurate measure of $r'$, this observation lends further support to the validity of the data.
Dependence of $\varepsilon_2$ on Temperature at Constant Overfeed

As the temperature is increased, the equilibrium shrinkage modulus $d_e$ increases so Equation (62') predicts that $\varepsilon_2$ increases. However, as can be ascertained from Figure 7, at constant overfeed the modulus $E_2$ decreases with temperature. With reference to Equation (58'), the net result on $\varepsilon_3$ cannot be predicted as $\varepsilon_2$ increases and $E_2$ decreases. The relative change of material properties with temperature will therefore decide whether $\varepsilon_3$ increases or decreases with temperature at constant overfeed. Figure 17 illustrates the temperature variation of $\varepsilon_2$, calculated for the case of -2.86% overfeed, as well as that of the corresponding modulus $E_2$. Note that the calculated values of tension as a function of temperature given in Table IX are in agreement with experiment.

Dependence of Tension on Twist

The strain on the filaments, $\varepsilon_3$, is assumed to be independent of twist (cf. Equations (58) and (58')). However, the filament inclination increases the higher the twist, and therefore, the contribution to yarn tension decreases as the twist increases. This can be seen from Equations (57) and (31), and is in agreement with the experimental data (Table VI).

In summary, the origin of the tension force at zero or positive overfeeds is assumed to be due to the shrinkage tendency
VARIATION OF $\varepsilon_2$ AND $E_2$ WITH TEMPERATURE AT -2.86% OVERFEED
of the filaments under the influence of temperature; the
level of tension is controlled by (a) the temperature,
which determines the equilibrium denier to which the unre-
strained fiber would tend in the absence of machine con-
straints, and (b) the overfeed which imposes the denier of
the filament exiting the machine, hence, the overall fila-
ment strain (relative to the equilibrium denier). This
strain results in a filament stress which is manifested,
subject to the constraints of yarn geometry, as yarn tension.

In predicting the yarn tension, it is necessary to know the
material properties; therefore, the stress-strain behavior of
the untextured and textured yarns were determined experi-
mentally, as were the contractile stress curves which were
expressed as high temperature stress-strain curves; moreover,
the tension jump across the spindle was experimentally deter-
mined. The twist was also taken from the actual experimental
data though all twist values were very close, irrespective
of overfeed. The mechanical model chosen for expressing yarn
tension assumed the yarn to have been stress-free prior to
application of a strain. This model (discussed in
Section V (a) 1. of this work) assumes that as the yarn is
extended, the filaments are strained to an extent dependent
upon their distance from the yarn axis; the yarn tension was
expressed analytically in terms of the average filament strain.
The stress-strain curves of the fibers at elevated tempera-
tures are non-linear; use of the average filament strain also averages the modulus. The expression derived (Equation (60)), predicts that, irrespective of the type of material used, the tension will decrease as overfeed increases and as the twist increases; these predictions are in agreement with experiment. Equation (60) further predicts the tension to depend on the ratios $E_3/E_2$ and $d_4/d_e$; more specifically, the temperature variation of the tension depends on the quantities $d_e$ and $E_2$ which vary in opposite sense with temperature. The relative sensitivity to temperature of $d_e$ and $E_2$ will decide whether the tension will increase or decrease with temperature and cannot be predicted \textit{a priori}. Given the temperature dependence of $d_e$ and $E_2$, the model predictions are in agreement with experimental measurements of tension.

4. Example of Tension Calculation

To illustrate how the calculations pertinent to Table were performed, we go through the computation of tension for the case of $210^\circ$C and $-6.67\%$ overfeed.

We must solve Equation (59) to obtain $\varepsilon_3$. To translate $\varepsilon_3$ into yarn tension we use Equation (57)

$$T = \pi k E_3 \varepsilon_3 \frac{A (Ap^2 + 2)}{Ap^2 + 1}$$

or, more commonly, we calculate the tension ratio of the
untwisted to twisted yarn with the same average filament strain. The tension in the untwisted yarn is

\[ T_{\text{unt}} = \pi f^2 W \sigma_f = \pi f^2 W E_3 \bar{\varepsilon}_3 = 2\pi k A E_3 \bar{\varepsilon}_3 \]  

(63)

where all quantities have been previously defined. The ratio of Equation (63) to Equation (57)

\[ \frac{T_{\text{unt}}}{T} = \frac{2(Ap^2 + 1)}{Ap^2 + 2} \]  

(64)

is used to calculate the yarn tension in zone 3, given the average filament strain hence, the tension along the fiber axis in that zone.

The quantities we need to know or calculate are \( d_e, d_4, r, E_3, E_2, \bar{\varepsilon}_3, A, p \) and \( T \).

**Determination of \( p \)**

This is known experimentally and is 61.6 tpi or 387.044 radians per inch.

**Determination of \( d_4 \)**

This is given by the equation of continuity, \( d_4 = (V_i / V_4)d_i \).

The overfeed, \( (V_i - V_o)/V_i = -0.0667 \), so that \( V_4/V_i = 1.0667 \) and \( d_4 = 150/1.0667 = 140.625 \).
Determination of \( d_e \)

From the contractile stress table, \( d_e \), or zero contractile stress, corresponds to an overfeed \((V_i - V_{o1})/V_i = .2286\); hence, from continuity, \( d_4 = d_e = (V_i/V_4)d_1 = 150/7714 = 194.445 \)

Determination of \( A \)

By definition, \( A = Wf^2/2k \) with \( W = 34, k = 1 \). We must, however, compute \( f^2 \) which is the filament cross-sectional area in zone 3 (with \( d_2 = d_3 \)). We take the cross-sectional area to correspond to that of yarn of denier \( d_4 \). This is not quite true as the denier in zone 3 and zone 2 is somewhat higher than in zone 4 because of the tension difference across the spindle, but the assumption (especially at low tensions) is good. More exactly, \( d_3 \) and \( d_4 \) are related by

\[
\frac{d_4}{d_3} = 1 + \frac{\epsilon_4}{2.3};
\]

when \( \epsilon_4 \) is of the order of 5% (a very high strain), the ratio is 0.97.

From the definition of the concept of denier, we have

\[
\pi f^2 p (9 \times 10^5) = \frac{140.625}{34}
\]

for zone 3; hence, \( \pi f^2 = 1.643 \times 10^{-7} \text{ in}^2 \) and
A = 17f² = 2.793 \times 10^{-6} \text{ and } \Delta p = 0.4184

**Determination of \( E_3 \)**

We use the data on Young's modulus of textured yarns (Table II) since \( E_3 = E_4 \). For the case considered, the modulus averages to 11,100 grams per unit strain and

\[
E_3 = \frac{11,100/454}{1.5 f^2 \cdot W} = \frac{11,100}{454 \cdot \pi (1.643 \times 10^{-7})} = 1.39316 \times 10^6 \text{ psi} (34)
\]

**Determination of \( E_2 \)**

This was obtained from the experimental value of the tension; as mentioned, the "hot" stress-strain curves are highly non-linear; we can take this into account by mathematically expressing the relationship between \( E_2 \) and \( \bar{\varepsilon}_2 \) as \( E_2 = f(\bar{\varepsilon}_2) \) and adding another equation to the system we are solving. We can also use an alternative method; assume a value for \( E_2 \); solve the equations; determine \( \bar{\varepsilon}_3 \) (hence \( \bar{\varepsilon}_2 \)); and see whether the value of \( \bar{\varepsilon}_2 \) so obtained corresponds to the value of \( E_2 \) assumed. This is in essence what a computer would do if we were to feed it the experimental data for the stress-strain curves and ask it to solve the equations. It is the same procedure as the analytic technique, except that the steps in the computer are finite rather than continuous. The simplest step, however, for computational purposes, is to choose the right modulus to begin with rather than search
for it analytically or digitally; this is what was done here and the steps were as follows. The experimental tension on the twisted yarn was 47 grams so that the tension along the filament axis was (from Equation (62))

\[ 47 \times \frac{T_{\text{unt}}}{T} = 47 \times 1.1730 = 55.13 \text{ grams.} \]

From the contractile stress curves this corresponds to strain on .34. Thus, now we calculate \( E_2 \) as

\[ E_2 = \frac{55.13/454}{(.34)\pi f^2.W} = \frac{55.13/454}{(.34)\pi (1.643 \times 10^{-7})} = 20351.42 \text{ psi} \]

The ratio \( E_3/E_2 \) is therefore, \( E_3/E_2 = 68.46. \)

Notice that in both calculations of \( E_3 \) and \( E_2 \), the cross-sectional area, \( \pi f^2 \), with respect to which the modulus is calculated, is not the original (unstressed filament) area, but is the area corresponding to \( d_4 \); the justification is that the denier is the same in zones 2 and 3. We have, however, not corrected for the thermal expansion which would increase somewhat the cross-sectional area for the case of the filaments in zone 2.

The calculations were given here in detail to underline all the assumptions made; in practice, a short cut may be taken. Rather than expressing \( E_2 \) and \( E_3 \) in psi and since we only need their ratio, we can express the ratio directly in terms
of grams/unit strain, and obtain

\[ \frac{E_3}{E_2} = \frac{11,100}{55.13} = 68.46 \]

using the numerical values previously employed.

\[ \text{Determination of } \bar{\varepsilon}_3 \]

We substitute the proper numerical values into the equation for \( \bar{\varepsilon}_3 \), Equation (59), and we have

\[ \bar{\varepsilon}_3 = \frac{\frac{194.445}{1 + 68.46\bar{\varepsilon}_3} - 140.625}{2.7(140.625) - \frac{194.445}{1 + 68.46\bar{\varepsilon}_3}} \]

or,

\[ \bar{\varepsilon}_3^2 + 0.3778\bar{\varepsilon}_3 - 2.0768 \times 10^{-3} = 0 \]

Solving the equation, we obtain

\[ \bar{\varepsilon}_3 = \frac{-0.3778 \pm \sqrt{0.3778^2 + 0.3886^2}}{2} = \frac{0.0108}{2} = 5.4 \times 10^{-3} \]

The average strain in zone 3 is thus 0.54% which corresponds to a longitudinal fiber tension of 59.94 grams in that zone (modulus \( E_3 = E_4 = 11,100 \) g/unit strain) or an axial yarn tension through Equation (64) of 59.94/1.173 = 51.2 grams versus measured 47 grams (see Table IX).
IX. MEASUREMENT AND CALCULATION OF TORQUES

(a) Introduction

Calculating the torques is a more difficult task than calculating the tensions, since the material properties are known with less certainty. To illustrate, both the bending modulus and the shear modulus enter into the calculations of torque; the bending modulus was assumed equal to the tensile modulus, whereas in fact, it depends on both the tensile and the compressive modulus, the latter not having been measured in this work. Also, it was assumed that $G = E/12$, but the complete stress-strain curve in torsion is not known. Also, with the exception of Young's modulus, which was measured as earlier described, the values of the torsional and bending moduli at high temperatures are not known.

In the calculation of the torque, the tension in each filament is multiplied by the corresponding moment arm to give the torque. The contributions of the filaments to yarn torque are thus weighted, the outer filaments contributing more to yarn torque than the inner filaments.

The torques were determined experimentally following the procedure described in Section VI,(b); the objectives of this phase of the work were not only to measure the actual values of torque, but also to calculate them from the yarn geometry and the filament mechanical properties and to elucidate the phenomenon of yarn uptwisting, that is, the twist jump that is observed as the yarn enters the heater zone.
The results of the experimental measurements and the calculated values of torque are given in Table X and they shall be discussed, with reference to the Table by first examining the torques in the entrance zone and then the torques in the heater zone. (See also Figure 18)

(b) Torque in the Entrance Zone

The three components of torque (tension, bending, torsion) described in Section V, (a), 3. were calculated separately and then summed to give the total torque. These components were calculated as follows:

1. Torque Due to Filament Bending

The expression for this was derived following the method of Platt (R13), but adapting it to the constant packing yarn model, with filaments continuously distributed across the yarn cross-section; the torque is given by Equation (22). In our calculations the bending modulus was assumed to be equal to the tensile modulus, which was taken from the linear part of the stress-strain curves of the untextured yarns, determined on the Instron as described in Section VI, (b), 1. The twist values used in these calculations were those experimentally determined and reported in Table V. The results of these calculations are reported in Table X under the column "Entrance Zone/Upper Limit".

2. Torque Due to Filament Torsion

The expression for this was derived, as above, following the
<table>
<thead>
<tr>
<th>Table X</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>MEASURED AND CALCULATED VALUES OF TORQUE (in-lb 10^*)</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ENTRANCE ZONE</th>
<th>HEATER ZONE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured experimentally</td>
<td></td>
</tr>
<tr>
<td>130°C</td>
<td>210°C</td>
</tr>
<tr>
<td>+6.1% Overfeed</td>
<td>0% Overfeed</td>
</tr>
<tr>
<td>Upper Limit</td>
<td>Lower Limit</td>
</tr>
<tr>
<td><strong>Tension</strong></td>
<td></td>
</tr>
<tr>
<td>1.75</td>
<td>1.75</td>
</tr>
<tr>
<td><strong>Bending</strong></td>
<td></td>
</tr>
<tr>
<td>6.29</td>
<td>2.65</td>
</tr>
<tr>
<td><strong>Torsion</strong></td>
<td></td>
</tr>
<tr>
<td>4.77</td>
<td>2.01</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
</tr>
<tr>
<td>12.81</td>
<td>6.41</td>
</tr>
</tbody>
</table>

Calculated via "classical" mechanics. Tension component only:

| 1.30 | 7.02 | 22.04 | 2.06 | 5.03 | 12.10 |

Calculated via overtwisting model:

| 11.09 | 18.10 | 27.84 | 8.91 | 13.38 | 18.98 |

* The experimental determination was done on a yarn section taken from the cooling zone
n.c. = not calculated
FIGURE 18

MEASURED TORQUE vs. TEMPERATURE AT VARIOUS OVERFEEDS

TORQUE (in-lb x 10^5)

TEMPERATURE °C
method of Platt\textsuperscript{(R13)}, but adapting it to the constant packing yarn model; the torque is given by Equation (26). In our calculations the shear modulus was taken as $G = E/12$ with $E$ determined as outlined above. The twist values in the calculations were those experimentally determined and reported in Table V. The results of the calculations are reported in Table X under the column "Entrance Zone/Upper Limit".

3. Torque Due to Filament Tension

Two methods were considered in calculating the torque in the entrance zone.

"Classical" or Simple Yarn Mechanics

This model assumes that the yarn is formed under no tension and is then extended without change in twist; the extension generates a tension and a corresponding torque. This model was discussed in Section V(a), 31, but was not used in this work since the physical situation in the false twister does not conform to this model.

Torque Due to Equal Filament Tension

In the false twister untwisted yarn enters the twisting zone and is twisted under constant tension. A more realistic model, therefore, is one where the filaments of the yarn are under equal tension. The torque equation resulting from this situation was derived earlier; combining equations (31) and (32), this torque is given in terms of the tension by
For a given yarn tension and twist, the torque predicted by this model is higher than that predicted by the model of "classical" mechanics discussed in the previous paragraph and given by Equation (30). Also, the torques' ratio increases with the twist; it is typically 1.075 for 38 tpi and 1.13 for 48 tpi. (for 150/34 Dacron yarn).

In calculating the torque in the entrance zone, Equation (65) was used, and T, the tension, and p, the twist, were the actual experimental values and taken from Table V. Notice that the modulus does not appear in Equation (65); however, Equations (31) and (32), from a combination of which Equation (65) is derived, both assume Hookean behavior; the same is true of Equation (30), which links tension and torque for the case of simple (or "classical") yarn mechanics.

4. Comparison of Calculated and Experimental Data for the Case of the Entrance Zone

It is seen in examining Table X that the agreement between the experimental values of torque in the entrance zone and in the cooling zone is good; further, independent work, using a torque measuring device mounted on the threadline and recording torque while the machine was operating, confirms the results presented here. (R19).
In comparing the experimental and the calculated values of torque (reported under the column "Entrance Zone/Upper Limit") one sees the latter to be higher, the more so the higher the overfeed. The following are possible reasons for the variance between experimental and calculated results.

(i) The experimental values have been measured on the thread-line 15 – 20 minutes after the machine was stopped. In that period of time inevitably some stress relaxation has occurred in the sample. Even during the false-twist operation, while the yarn is still in the entrance zone, the bending and torsional moduli must relax, since any yarn segment remains in that zone for 25 – 30 seconds.

(ii) Mention should be made again of the uncertainty concerning the values of the bending and torsional moduli since the torques were calculated assuming the bending modulus to be equal to the tensile modulus and $G = E/12$.

(iii) Another significant effect is that due to the non-linearity of the stress-strain curve. The bending strains (i.e., the extreme tensile fiber strain or bending) for outside filaments at 30.2 tpe and 41.4 tpi of a 150/34 polyester yarn (typical of the values of twist encountered in the entrance zone) are 3.2% and 5.14% respectively. These are well beyond the elastic limit, and the associated secant moduli are 8500 and 6800 grams per unit strain, respectively. The Hookean Modulus is 16200 grams per unit strain ($\sim 108$ gpd) and the
elastic limit is approximately 0.72%.

To account for the non-linearity of the stress-strain curves, the bending and torsional contributions to torque were recalculated assuming now that the tensile modulus for all the filaments in the yarn was that corresponding to the strain experienced by the outer filaments. The strain depends on the twist and the experimentally determined values of twist, shown on Table V were used. The calculated values of torque gives us a lower bound for the torque values and are tabulated in Table X under the heading "Entrance Zone/Lower Limit". It is seen that the calculated values of torque bracket the experimental values. (Figures 19 and 19').

(c) Torque in the Heater Zone

The problem of calculating the torque in the heater zone is also complex: the material properties are not immediately accessible to experimental measurement, and the yarn twist suddenly increases as the yarn enters the heater zone. This suggests that the mechanism by which the yarn supports the torque changes at that point and that a new model must be sought in order to calculate the torque. The various elements of the calculations will now be discussed.

1. Material Properties

The critical material parameters are the moduli of elasticity in tension and in shear. These have been taken from the high temperature stress-strain curves as described earlier
FIGURE 19
EXPERIMENTAL AND CALCULATED VALUES OF TORQUE (134°C)
FIGURE 19
EXPERIMENTAL AND CALCULATED VALUES OF TORQUE (210°C)

TORQUE (in-lb $10^5$)

EXPERIMENTAL HOT ZONE
EXPERIMENTAL ENTRANCE ZONE

OVERFEED %
(Section V. (b) 4.). These curves are non-linear and the moduli are thus a function of the strain. The unstrained state for each processing temperature corresponds to the freely shrunk yarn at that temperature. The moduli of elasticity are much lower at the high temperatures than at room temperature. Again, for the pertinent calculations, the compressive modulus was taken equal to the tensile modulus and $G$ was taken as $E/12$.

Two points should be made at this time:

(i) The assumption $G = E/12$ may underestimate $G$, if $G$ depends on temperature only. As the temperature is increased, the orientation of the material decreases as shown in Table I, and its behavior becomes more rubberlike, so that $G$ should tend to $E/3$ as the melting point is approached. However, computation of the torque contributions due to filament bending and torsion (with $G = E/12$) will show them to be negligible compared to the contribution from tension. A tripling of the value of $G$ will not affect this conclusion. Even if stress relaxation does not intervene, the bending and torsional filament contributions play a minimal role in supporting the total torque, if the behavior of the yarn is elastic and remains so throughout the whole heater zone.

(ii) It is conceivable, however, that the mechanical properties, and, in particular, the bending and torsional rigidities, may change with time of heat treatment as new order
develops within the fiber. Arthur and Jones found(R20) and
Morris and Roberts(R21) confirmed the necessity of a minimum
residence time in the heater if the crimp rigidity and the
quality of crimp were to be at an acceptable level. The
present work does not take this possibility into account.

2. Torque from Classical Yarn Mechanics and from
Equal Filament Tension Contributions

These two methods were reviewed earlier in the discussion of
the calculation of the torque for the entrance zone; appli-
cation of Equations (30) and (65) to the experimental con-
ditions in the heater zone gives results (presented in Table \( \chi \))
which seriously underestimate the torque at high overfeeds.
This is not surprising as the physical phenomenon that occur
in the heater zone (namely, a sudden twist increase as the
temperature of the yarn is abruptly changed) cannot be re-
presented by the simple physical model on which this deriva-
tion is based. A more realistic physical model is necessary

3. Torque due to Overtwisting

The mechanics of overtwisting (or uptwisting of the yarn with-
out allowing for filament migration) have been analyzed in
Section V (c). From the physics of false-twisting, it is known
that yarns being processed in the false twister are twisted
to a high degree before reaching the heater and that consid-
erable friction exists between the filaments at that stage:
therefore, on uptwisting in the heater (where the yarn softens and the interfilament friction increases because of better wetting and more contact between the filaments) the filaments will not be able to migrate freely, and overtwisting will take place.

The torques due to overtwisting were calculated and are reported in Table X. The torque calculations require a number of steps: these are outlined in the paragraphs that follow, while a detailed example of calculations is given in a later section.

To calculate the torque, we use in principle, Equation (39) which, however, requires knowledge of $p_1$, the twist before overtwisting, $p_3$, the twist after overtwisting and extension, $F$, the amount of extension after overtwisting, and the tensile modulus, $E_2$. Now, $p_1$ and $p_3$ are the twists in the entrance and heater zone and can be obtained from the experimental data (Table V), but $F$ has to be calculated. To obtain $F$, we use the tension equation for overtwisting (Eq'n(36)) the experimental values of tension, $T$, and modulus, $E_2$, earlier determined in the section on the tension calculation (Table IX). We first relate $F$ to $\bar{\varepsilon}_f$, the average filament strain, via Equation (42), and then use Equation (36) relating $F$ to the yarn tension $T$ and to the modulus $E_2$, and solve for $F$. Having obtained $F$, we compute $\bar{\varepsilon}_f$, the average filament strain via Equation (42), and compare it to the value of $\bar{\varepsilon}_2$, corres-
ponding to the value of $E_2$, which was used to determine $F$ in the first place. If the two do not agree, a new value of $E_2$ is chosen and an iteration performed until the value of modulus used corresponds to the value of $\overline{A}_f$ or $\overline{e}_f$ obtained through Equation (42). An example of this calculation is given in the next section and it includes the various computational steps in full detail. It should be emphasized that this method of obtaining $F$ does not involve any approximations (other than any assumptions made in the derivation of the equations).

Having determined the value of $F$, we now proceed to calculate the torques and add the following refinement to the calculations; since the stress-strain curves are non-linear the moduli will vary depending on the filament portion in the yarn; since the component of yarn stress is multiplied by the moment arm, the external filament will contribute more to yarn torque by virtue of their position. To get the best possible estimate of the torque, the moduli corresponding to the appropriate value of strain should be used. To that effect, the yarn cross-section was divided into concentric layers, or zones, each the thickness of a filament diameter and each accommodating a number of filaments in accordance with the teachings of yarn geometry (R10): in our case, (150/34 yarn) the core accommodates a single filament, the second layer, 6, the third 12, and the last, or outside zone, 15. The zones are characterized by the cumulative percentage of the fibers con-
tained in each. Thus, the last zone contains $15/34=0.4412$ of the total filaments and in the cumulative notation (introduced earlier in Chapter II, Equation (7)), it is characterized by the range $\phi = 0.56$ to $\phi = 1$.

In Section V, (c) 5., Equation (46), it was shown that the strain in any part of the yarn (within any given $\phi$ limits) could be related to the yarn extension ratio $F$, and we use this equation, given $F$, to calculate the strain in each layer. Next, from the strain and the high temperature, stress-strain curves (Figure 6), we determine the modulus, and these 4 moduli (actually 3, as the core was ignored) were used to calculate the torque in each layer, or zone, using Equation (47). The sum of the torques of all the layers was then taken to be the torque of the yarn and was reported on Table X. While Table XI gives a detailed breakdown of the torque components, a detailed sample calculation is given at the end of this section. See also Fig. 20 and 20'.

4. Discussion of the Data

As earlier discussed, the agreement between the experimental values of the torque determined in the entrance zone, and in the cooling zone (a surrogate for the torque in the heater zone) is satisfactory. The values in the heater zone are consistently higher than in the entrance zone. This may be a consequence of the experimental technique of mounting the specimens on the torque measuring apparatus, which may affect much more the entrance zone segment (which is very twist-
<table>
<thead>
<tr>
<th></th>
<th>210°C</th>
<th>181°C</th>
<th>141°C</th>
<th>110°C</th>
<th>80°C</th>
<th>50°C</th>
<th>20°C</th>
<th>0°C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>21.1% OVERFED</td>
<td>20.8% OVERFED</td>
<td>20.5% OVERFED</td>
<td>19.8% OVERFED</td>
<td>19.0% OVERFED</td>
<td>18.1% OVERFED</td>
<td>17.8% OVERFED</td>
<td>17.6% OVERFED</td>
</tr>
<tr>
<td>TENSILE STRAIN</td>
<td>.3339</td>
<td>.1988</td>
<td>.6970</td>
<td>.0975</td>
<td>.3752</td>
<td>.2609</td>
<td>.1738</td>
<td>.2911</td>
</tr>
<tr>
<td>MODULUS</td>
<td>14445</td>
<td>5057</td>
<td>3466</td>
<td>84350</td>
<td>8336</td>
<td>4923</td>
<td>3381</td>
<td>59864</td>
</tr>
<tr>
<td>TURNOVER X13³</td>
<td>8.213</td>
<td>.652</td>
<td>.040</td>
<td>8.905</td>
<td>12.061</td>
<td>1.303</td>
<td>0.086</td>
<td>13.391</td>
</tr>
<tr>
<td>F = P₂/P₃</td>
<td>1.057</td>
<td>1.139</td>
<td>1.237</td>
<td>.947</td>
<td>1.014</td>
<td>1.065</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ΔN = P₃/P₁</td>
<td>.514</td>
<td>.609</td>
<td>.643</td>
<td>.665</td>
<td>.782</td>
<td>.789</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C = P₂/P₁</td>
<td>3.049</td>
<td>3.069</td>
<td>3.078</td>
<td>3.025</td>
<td>3.132</td>
<td>3.254</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P₂²/P₃²</td>
<td>64.6</td>
<td>72.33</td>
<td>74.34</td>
<td>58.9</td>
<td>64.22</td>
<td>64.95</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
FIGURE 20

CALCULATED COMPONENTS
OF TORQUE IN THE ENTRANCE
ZONE AT 210° C (UPPER LIMIT)

TORQUE
in-lb x 10^5

TOTAL TORQUE
(TENSION)
(TORSION)
(BENDING)

OVERFEED %
FIGURE 20'

CALCULATED COMPONENTS
OF TORQUE IN THE ENTRANCE
ZONE AT 134°C (UPPER LIMIT)

TORQUE
in-lb \times 10^5

(TENSION)

(TORSION)

(BENDING)

OVERFEED %

6 4 2 0 -2 -4 -6
lively) than the cooling zone segment (which, for all intents and purposes, is a solid rod). The test specimens are unavoidably disturbed, when, after mounting the yarn cardboard assembly on the jaws of the machine, the cardboard is cut and removed in order to allow the torque to register (see experimental technique in Section IV(b)). The difference could also be due to some additional relaxation that may take place in the entrance zone.

To illustrate, Figure 21 represents the torque decay curve of a yarn, twisted statically to the same basic twist, and under the same tension, as the yarn processed in the false twister at $134^\circ$C and $-6.67\%$ overfeed (Tables V & X). The twist insertion (240 turns in 5.8 inches of yarn under 75 g tension) took approximately 3 minutes, and the relaxation readings were started immediately after that, without removing the applied tensile stress; thus the relaxation is due to the bending and torsional components of the torque only. The data clearly shows that the relaxation rate is such that the torque decay not only occurs during the 15 minutes or so that elapse between stopping the machine and actually testing the specimen, but that it must also occur in the entrance zone in the course of false-twisting. The residence time in the entrance zone is of the order of 30 seconds in our equipment; in commercial equipment where speeds are considerably higher, the relaxation that occurs in the entrance zone is doubtless much less. One consequence of the long residence time in our equipment is that (since part of the components that
FIGURE 21
RELAXATION OF TORQUE OF STATICALLY TWISTED YARN

TORQUE
(in-lb x 10)

TIME, sec.

Experimental torque values
(134°C, -6.67%):
- heater zone
- entrance zone
support the torque relax so the twist must increase rather than remain constant along the entrance zone.

With reference to Figure 21, note that the torque is higher for the statically twisted yarn than for the false-twisted yarn. This is no doubt due to the fact that the twist insertion mechanism is different for the two yarns (R22); static twisting, which is really continuous uptwisting, impedes migration, and leads to higher torques than false-twisting where the yarn is formed in one step to its final twist.

In comparing the calculated with the experimental data, in Table X, the calculated values are in good agreement with experiment. The calculated values are in general higher than the experimental values; the one exception is the experiment at 134°C and -6.67% overfeed, where difficulties were encountered in measuring the torque. As earlier explained, undoubtedly some relaxation occurs in the 15 minutes that elapse between stopping the machine to collect the yarn specimen and the moment when the yarn torque is actually determined.

Another reason why the calculated torque in the heater zone is higher than that measured in the cooling zone is that, although the model used to calculate the torque excludes the possibility of any migration occurring at the heater zone, work by W. L. Yang (R8) demonstrated that some migration does indeed occur, though it was not possible to quantify the
amount. This additional migration undoubtedly mitigates somewhat the effects of overtwisting.
(d) Example of Calculation of Torques (210°C -6.67%)

To illustrate how the calculations pertinent to the values presented on Table X were performed, we go through the computation of the torques for the case of 210°C and -6.67%.

1. Entrance Zone

(i) Torque due to Tension

The relationship between tension and torque is for the case of equal tension on all filaments:

\[
M_{TE} = T \left( \frac{A_p^2(A_p^2 + 2)}{2 \ln(A_p^2 + 1)} - 1 \right)
\]

In our case, experimentally \( T = 47.5 \text{ g}, \ p = 40, \ 6 \times 2 \times \pi = 255.10 \text{ rad/inch} \) and \( A = \frac{Wf^2}{2k} \). We take \( W=34; \ f^2 \) is calculated from the denier definition \( \pi f^2 \times \rho \times 9 \times 10^5 = 150/34 \), whence \( f^2 = 1.1307 \times 10^{-6} \text{ cm}^2 = 1.7526 \times 10^{-7} \text{ in}^2 \), \( k=0.8 \).

Thus, \( A = 3.7242 \times 10^{-6} \) and \( A_p^2 = .24235 \). Replacing these quantities in Equation (65), we get \( M_{TE} = 10.20 \times 10^{-5} \text{ in-lb} \).

(ii) Torque due to Bending

The expression for this torque is

\[
M_{BE} = \frac{\pi ke f^2}{fp} \left( 2 \ln(A_p^2 + 1) - \frac{A_p^2(A_p^2 + 2)}{(A_p^2 + 1)^2} \right)
\]

Substituting the value of \( E = 1.91 \times 10^6 \text{ psi} \) for the modulus (Table I) and the other values as in the calculation just
above, we get \( M_{BE} = 6.75 \times 10^{-5} \). This is our upper limit as man fibers are bent probably beyond the elastic limit; we can calculate the strain on the surface of the outermost fibers in the yarn. We have from beam theory (R15)

\[
\varepsilon = \frac{f}{\rho}
\]

Here \( f \) is the filament radius and \( \rho \) the radius of curvature of the neutral axis, which we take to coincide with the filament centerline. From differential geometry of space curves (R16) and Equation (1)

\[
\frac{1}{\rho} = \frac{\sin^2 \varphi}{r} = \frac{p^2 r}{1 + p^2 r^2}
\]

where \( \varphi \) is the local helix angle and \( r \) the helix radius.

In our case, \( r = b \), since we consider the outermost helical filament of the yarn and, substituting Equation (67) in Equation (66), and also using Equation (5), we have

\[
\varepsilon = \frac{fbp^2}{(1 + p^2 b^2)} = \frac{fbp^2}{(Ap^2 + 1)^2}
\]

(66')

For 40.6, which is the twist observed in the entrance zone, we calculate through Equation (6), using the values of \( f \), \( A \) and \( p \) previously determined, \( b = \left[A(Ap^2 + 2)\right]^{1/2} = 2.8898 \times 10^{-3} \) inches, and \( \varepsilon \) through Equation (66') as 5.1\%.
This is clearly outside the Hookean range (which ends at about 0.7% extension) and the secant modulus at 5.1% is only 42% of the original value of $1.91 \times 10^6$. So, an absolute lower limit for the bending contribution is 42% that of the value originally calculated, or $2.84 \times 10^{-5}$. The range within which lies thus, the bending contribution is $2.84 \times 10^{-5}$ to $6.75 \times 10^{-5}$.

Note that we have assumed throughout that the bending modulus equals the tensile modulus. This is probably not true at larger strains, and we thus have an uncertainty about the value of the compressive modulus.

(iii) Torque due to Filament Torsion

This is given by Equation (26), which after the substitution $G = E/12$ becomes

$$M_{TO} = \frac{kGf^2}{24p} \frac{Ap^2(Ap^2 + 2)}{(Ap^2 + 1)^2} \quad (26')$$

Using the numerical values given above, Equation (26') gives $M_{TO} = 4.84 \times 10^{-5}$ in-lb for $E = 1.91 \times 10^6$ psi (upper limit) and $2.04 \times 10^{-5}$ for $E = 0.42 \times 1.91 \times 10^6$ (corresponding to the outer fiber (lower limit)). Note that we have always assumed $G = E/12$. Though we do not know the stress-strain curve of the filaments in torsion, the dependence of G on E hopefully, parallels the change of G with torsion or filament twist. The values calculated above are given in Table X.
2. Heater Zone

(i) Torque due to Tension—Equal Tension on all Filaments

We use the values of $A_p^2 = 0.4184$ with $p = 61.6$ tpi, or 387.044 radians per inch and tension $T = 47$ grams, given in the numerical example of the tension calculation (Section VIII, (b) 5.) We use Equation (65)

$$M_{TE} = \frac{T}{p} = \frac{A_p^2(A_p^2 + 2)}{2 \ln(A_p^2 + 1)} = 12.00 \times 10^{-5}$$

This equation implicitly assumes that the modulus is independent of strain; we know, of course, from Section IX, (c) 3., that this is a physically incorrect model.

(ii) Torque due to Bending

We use here the same values of $A$ and $p$ as above. We assume the modulus to be constant and equal to $E_2$ used in the tension calculations, and the same is true of $f^2$. However, here $k=1$. We use Equation (22)

$$M_{BE} = \frac{nE f}{4p} \left\{ 2 \ln(A_p^2 + 2) - \frac{A_p^2(A_p^2 + 2)}{(A_p^2 + 1)^2} \right\} = 6.7852 \times 10^{-6} [0.66906 - 0.50295] = 0.133 \times 10^{-5}$$

It is, thus, negligible compared to the actual torque. The torque due to filament torsion is equally small and is calculated the same way using $G = E/12$. Some values for $M_{BE}$ and $M_{TO}$
are reported on Table \( \text{X} \).

(iii) Torque due to Tension-Overtwisting

This is computed in various steps, and we shall illustrate them in their proper sequence.

1. Establish Relationship Between Average Filament Strain and Yarn Extension Ratio

That expression has previously been given as

\[ \bar{\Lambda}_f = 1 + \bar{\varepsilon}_f = \frac{F}{m} - \frac{F(1 - m)}{m^2 \Lambda_p^2} \ln(\Lambda_p^2 + 1) \quad (42) \]

with \( p = p_3 \), \( m = (p_1/p_3)^2 \). The quantities \( A \) and \( p \) are as above, \( m = (39.6/61.6)^2 = .413265 \); hence, we get after substitution, \( \bar{\Lambda}_f = 1.1119F \).

2. Determine the Actual Value of \( F \) Using the Over-Twisted Stress-strain Curve

The Stress-strain curve is

\[ \frac{T}{2\pi k E} = \frac{F}{p_1^2} \ln(\Lambda_p^2 + 1) - \frac{1}{p_3^2} \ln(\Lambda_3^2 + 1) \].

We use the experimental values of \( T \) and \( E_2 \) (from tension calculations, numerical example) as well as the experimental values of \( p_1 \) and \( p_3 \), and solve for \( F \); given \( F \), we immediately obtain \( \bar{\Lambda}_f \). We now check the modulus (from the hot stress-strain curve) corresponding to that value of \( \bar{\Lambda}_f \) (\( = 1 + \bar{\varepsilon}_f \)). It should be the same as the experimental value of \( E_2 \). If it differs,
we iterate, i.e., chose a different value of $E_2$ to solve the tension equations and obtain $F$ and $\bar{\Lambda}_f$. We do so until the value of $E_2$ chosen to use in the tension equation is the same as the value which corresponds to $\bar{\Lambda}_f$ obtained after solution of the tension equation. An alternate method would be to express analytically the dependence of $E_2$ on $\bar{\Lambda}_f$ (or $\bar{e}_f$) and to include this as an additional equation in the calculations. We would, thus, have

$$\bar{\Lambda}_f = f_1(F) = 1.119F$$

the first relationship we established

$$F = f_2(E_2, \text{ and } T, p_1, p_3)$$

the overtwisted stress-strain curve of the yarn

$$E_2 = f_3(\bar{\Lambda}_f)$$

the high temperature stress-strain curve of the filaments,

and with the three equations, we could solve for the three unknowns $\bar{\Lambda}_f$, $F$, and $E_2$. The reason this was not done is that the analytic expression, $E_2 = f_3(\bar{\Lambda}_f)$ would be a polynomial which would complicate the calculations considerably. The experimental curves could, of course, be fed into a computer, and the computer would then solve the three equations numerically by an iterative procedure as was done here. The important thing to notice is that these methods are equivalent, and that there is no simplifying assumption involved in solving the equations and determining $E_2$, $\bar{\Lambda}_f$ and $F$ by the method used here.

Note, however, that in a physical sense, this derivation uses the concept of the average strain just like the tension equa-
Returning now to the tension equation (stress-strain curve), we chose $T = 47.5$ and $E_2 = 20394$ (corresponding to $\varepsilon_2 = .34$) from the data on tension determination. Solving the stress-strain equation for $F$ with these values results in $F = 1.216$ and $\Lambda_f = 1.3524$. Notice that the agreement between the "starting" $\varepsilon_f (.34)$ and the computed $\Lambda_f = 1 + \varepsilon_f (1.3524)$ is good. Nonetheless, we shall seek a better value by iteration, for illustrative purposes. Let us choose $E_2$ corresponding to $\varepsilon = .345$ in lieu of .34. We now go to the high temperature stress-strain curves (which are expressed in grams vs. strain), and correct the value of $E_2$ (expressed in psi) by multiplying by the "modulus" $R_{.345}$ expressed in grams/strain corresponding to $\varepsilon = .345$ and dividing by the "modulus" $R_{.34}$ expressed in grams/strain corresponding to $\varepsilon = .34$; this is, thus, a dimensionless correction factor. This is a simpler computational technique than converting the "modulus" $R_{.345}$ in psi, but otherwise equivalent to it. We assume, of course, no change in the cross-sectional area of the fiber. Thus, the new $E_2 = 20394 \times R_{.345}/R_{.34} = 20394 \times 59/34.5 \times 34/55 = 21560$.

Using this value of $E_2$ in the stress-strain equation, we obtain $F = 1.199$ and $\Lambda_f = 1.334$, or $\varepsilon_2 = 33.4\%$. We can summarize the results as follows
Assumed $\bar{\varepsilon}_2$ and corresponding $E_2$ calculated by solution of stress-strain curves using values at left

<table>
<thead>
<tr>
<th>$\bar{\varepsilon}_2$</th>
<th>$E_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.34</td>
<td>20394</td>
</tr>
<tr>
<td>0.345</td>
<td>21560</td>
</tr>
</tbody>
</table>

Clearly, the sought after value is intermediate between the chosen ones. We choose $\varepsilon = 0.342$, corresponding to a modulus $E_2 = 20334 \times R_{342}/R_{34} = 20394 \times 57/34.2 = 21012$ psi. With that value of the modulus, the solution to the stress-strain equation yields $F = 1.2072$, or $\frac{F}{f} = 1.3423$ which we accept as a close enough approximation. Thus, the correct value of $F$ is 1.2072, which expresses the amount of extension the yarn has "suffered" after overtwisting at constant length.

Notice that the value of strain, $\bar{\varepsilon}_2$, and modulus, $E_2$, which satisfies the overtwisting stress-strain equation is very close, indeed, to the value which satisfied the tension equation; this lends further support to the consistency of the model.

(3) Calculate the Strain and Modulus in Four Concentric Layers of Yarn

As earlier described, the yarn was divided into four layers: a central core (first layer, 1 filament), the second layer (6 filaments), the third layer (12 filaments) and the outer layer (15 filaments). Each layer, thus, contains a given percentage of the yarn filaments. The strain in each layer has been calculated and is given by the expression
\[(\bar{\Delta}_i,i+1 = 1 + \bar{\varepsilon}_{i,i+1})\]

\[\bar{\varepsilon}_{i,i+1} = \left(\frac{F}{m} - 1\right) + \frac{F(m - 1)}{\psi_{i,i+1}m^2Ap^2} \ln\left(\frac{mAp^2\phi_{i+1} + 1}{mAp^2\phi_i + 1}\right)\] (46)

where \(\psi_{i,i+1} = \phi_{i,i+1} - \phi_i\) is the percentage of fibers in that layer, and \(\phi_{i+1}\) and \(\phi_i\) define the upper and lower limits of the layer in a cumulative way. Thus, for the outer layer, \(\psi_{3,4} = \frac{15}{34} = 0.44\), and \(\phi_4 = 1\), and \(\phi_3 = 1 - 0.44 = 0.56\).

Replacing these values in Equation (46), using the value of \(F\) calculated in the previous section (and the same values, of course, for \(m\), \(A\) and \(p\)), we obtain \(\bar{\varepsilon}_{3,4} = 41.34\%\) as the strain (relative to the equilibrium freely shrunk yarn at that temperature) in the outer layer. For the third layer, \(\psi_{2,3} = \frac{12}{34} = 0.35\), with \(\phi_3 = 0.56\) and \(\phi_2 = 0.21\); we obtain \(\bar{\varepsilon}_{2,3} = 31.54\%.\) We now group the central filament and the first layer together and calculate the strain \(\bar{\varepsilon}_{0,2} = 23.81\%\).

To each of the three strains there corresponds a modulus \(E_2\), and these are calculated exactly as in the previous case using as base the modulus \(E_2 = 20394\), corresponding to a strain of .34 and corrected by the ratio of the moduli \(R_{.4134/R_{.34}}\), \(R_{.3154/R_{.34}}\) and \(R_{.2380/R_{.34}}\). These values are:

- Outer zone \(\bar{\varepsilon} = 41.34\) \(E = 33181\) psi
- Middle zone \(\bar{\varepsilon} = 31.54\) \(E = 15931\) psi
- Inner zone \(\bar{\varepsilon} = 23.80\) \(E = 7107\) psi
(4) Calculate the Torque in Each Zone and Sum it over the Cross-section

We use Equation (47) and calculate, using the values of $A, p, m, \phi, \psi$ and $E$ derived above:

<table>
<thead>
<tr>
<th>Zone</th>
<th>Torque</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outer zone</td>
<td>$16.32 \times 10^{-5}$</td>
</tr>
<tr>
<td>Middle zone</td>
<td>$2.51 \times 10^{-5}$</td>
</tr>
<tr>
<td>Central zone</td>
<td>$0.14 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

**TOTAL TORQUE** $18.98 \times 10^{-5}$ inch-lb

The torque measured in the cooling zone was 18.74 in-lb and that measured in the entrance zone was 18.12 in-lb.

(5) Comment on the Strain Levels

Since the equilibrium denier (or freely shrunk denier) of the yarn is 194.445, the actual deniers in zone 2 and zone 3 of the overtwisted yarn are predicted to be

- **Outer zone**
  \[
  \frac{d_e}{(1 + \varepsilon_2)} = \frac{194.445}{1.4134} = 137.6
  \]

- **Middle zone**
  \[
  \frac{194.445}{1.3154} = 147.82
  \]

- **Central zone**
  \[
  \frac{194.445}{1.2380} = 157.06
  \]

Thus, relative to the starting denier of 150, the outer filaments would decrease in denier by 9%, while the central filaments would increase in denier by 4.5%. This strain, however, need not extend over long sections of the filaments,
as the filaments in the entrance zone have migrated radially and only a short section of a given filament may find its way at any given yarn position on the yarn surface. Thus, the denier variations may be over extremely short intervals and, the only way to discern then would be to make successive microtome sections of any given filament and measure the variability in cross-sectional area. Another possibility about which there are no published data is that when the yarn is overtwisted though radial migration is prohibited, longitudinal migration may still take place to minimize denier variation along a given filament.

X. DISCUSSION, CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

(a) Discussion

This work has focused on the operation of false twisting and had as an objective to understand the physical basis and to analyze the mechanism of steady-state false twisting, and to evolve a model for the process. There are actually a number of distinct aspects of the process one must understand which, however, are highly interdependent.

(1) The first, and most accessible, is the phenomenological analysis of the fundamental actions of the machine. Thus, for at least the simple machine we have in the laboratory, it is a good assumption that the tension and the torque are constant along the threadline, and that the machine inserts a number
of turns of basic twist, controlled and predicted by the ratio of the spindle rpm to the (untwisted) yarn take up velocity; a small correction need be applied to account for the yarn extension in the exit zone. Further, the actual twist at the spindle depends only on the basic twist and seems largely independent of tension under steady-state conditions.

(2) The next topic to be understood is how the yarn responds to the process which is one of torsional and tensile non-isothermal creep; in other words, what are the mechanics and mechanisms of yarn deformation in this stress field?

(3) Inextricably related to the mechanism of yarn response is the problem of the determination and knowledge of the material properties. The yarn, after all, responds to a stress field, while the machine actually imposes a strain (through overfeed, temperature and twist), which is "translated" into a stress through the material properties. To illustrate, under identical machine conditions, two yarns of the same count, but of even slightly different material properties (such as Medium and High Draw ratio PET) will respond differently.

(4) Finally, one must consider what is really the ultimate reaction of the yarn to the false-twist process; in other words, the setting quality of the yarn. The specifics of this interaction and the molecular mechanisms involved are, thus, another important part of false-twist texturing
The present work addressed itself to points (1), (2), and (3) but did not touch upon point (4). Other workers (R23, R24) have been concerned specifically with point (4). Concerning (3), the fact that many material properties (in bending and torsion) were not easily measured,

and that all material properties change along the threadline, made it impossible to assign unequivocal values to the various moduli; also, the properties that were measured (the high temperature stress-strain curves, and the stress-strain curves of the feed and textured yarns) were non-linear, which complicated the computations. Consider what happens during the course of false-twisting to the fiber structure along the threadline:

**Entrance zone:** Immediately after the yarn is formed, bending and torsional contributions to torque begin to relax, so that the twist should increase; the threadline tension leads to tensile creep which also leads to twist increase.

**Heater zone:** All moduli decrease significantly; the bending and torsional strains become negligible compared to the tensile strain; this last is controlled by the overfeed and the equilibrium denier which the unrestrained yarn would shrink to at the particular processing temperature. The modulus decrease and the shrinkage are due to the partial breakdown of the original structure and orientation. Concurrent with the disordering, there must be an ordering phenomenon, or gradual recrystallization in the new geometry, which must result in an
increase of the bending and torsional modulus; that, of course, is the essence of setting for a crystalline fiber.

**Cooling zone:** With the temperature decrease, there is a concomittant increase in the modulus in tension as well as bending and shear. Thermoelastic phenomenon due to the non-crystalline regions may become manifest at this point. Even if setting has not occured by rearrangement of the molecular structure in the heater zone, cooling below the glass transition would result in setting, albeit of a less permanent nature.

This continuous change of the material properties makes it very difficult to know the correct values of the moduli at various positions of the threadline; we have assumed in the calculations that the moduli measured experimentally correspond to the initial part of the entrance zone, the initial part of the heater zone and the last part of the cooling zone; we have also assumed that the twist does not change beyond the heater zone, and that it is constant in the entrance and in the heater and cooling zones.

(b) Conclusions

The conclusions we draw about the process are that it imposes a certain twist and a certain strain on the filaments; the strain level is controlled by the overfeed and the temperature, this last controlling the level to which the yarn would have shrunk if unrestrained. At constant overfeed, the higher the
temperature, the higher the strain; it does not necessarily follow that the stress is also higher because the modulus drops with temperature, while the stress-strain curves at elevated temperatures are non-linear. So, the tension level cannot be predicted without a priori knowledge of the material behavior at elevated temperatures. With the yarns used in this work, the tension was found to drop at constant overfeed, as the temperature was increased.

The tension level was calculated assuming simple yarn geometry, i.e., assuming that the yarn in the heater zone was formed stressless in all filaments and was then extended; the average filament strain in the heater zone equals the strain imposed by the machine. This strain then leads to a filament stress (the level of which depends on the high temperature modulus) and to yarn tension. The experimental results are in agreement with calculated values of tension.

The machine also imposes a torque on the yarn; that torque is constant from the entrance rolls to the spindle, and is supported in the entrance zones by contributions from filament bending moment, filament torsional moment and filament tension; at the heater zone, the contribution from bending and torsional moments becomes negligible, and the torque has to be supported by tensile contributions only. Since the modulus drops, the yarn must twist further to balance the torque, and this is the fundamental reason for the uptwist
observed at the spindle. Now, as the overfeed is decreased at constant temperature and constant (basic) twist, the tension, as earlier explained, must increase, and since the basic twist is the same at the higher stress, the torque is higher; this is manifested by an increase in twist at the entrance zone, which results in a smaller twist change at the heater.

The twist increase at the heater takes place by a mechanism referred to as "overtwisting", where migration is forbidden and the increase in twist occurs by increase in the helical path of the filaments in the yarn; at the same time, the yarn is extended so that the overall effect is one of uptwisting and extension. The equations for tension and torque on overtwisting have been derived, and the torque calculated on the basis of these equations, using experimentally determined values of tension and twist agrees well with the torque values measured experimentally (within a range of uncertainty due to imprecise knowledge of the material properties).

Thus, the elucidation of the mechanism of false-twisting has been achieved, and a model has been proposed to account for the measured tension and torque. The results of the calculations with the model are in reasonable agreement with experiment.
(c) Recommendations

There are, of course, many questions that remain unanswered and many aspects of false-twist texturing that have not been investigated. In the writer's opinion, among the topics that merit further research are the following:

1. Investigation of unsteady-state situations, such as a sudden change in threadline tension or spindle rpm. In view of the practical importance of these phenomena (they lead to barre effects in goods knitted from textured yarns), it is important to understand how the system adjusts to step changes and by what mechanism the yarn responds. Another type of transient disturbance occurs when real twist in the feed yarn, which had been stopped by a guide, escapes from the guide, and passes through the machine.

2. Even for the steady-state situation, a more detailed analysis may be in order; for instance, a careful analysis of the twist distribution and twist increase in the entrance zone is necessary in order to assess the effect of relaxation of the bending and torsional strains. Even then, the problem may be complicated by the fact that the torsional rigidity of the yarn is much higher immediately after it has been formed; so that a twist change, because of the inhibiting effect of the interfilament friction and absence of migration, may not accurately reflect the corresponding torque change.

3. Detailed studies and measurements of torque under different twisting conditions are needed to obtain better insight
into the twisting and uptwisting mechanisms at room and at higher temperatures; the interrelationship between torque level, twisting mechanism and migration should be investigated.

(4) The interaction between the pin and the yarn should also be examined more extensively, and the effect of the yarn finish assessed both on the tension level and on the ability of the spindle to transmit torque without twist escape.

(5) Better information concerning material properties is badly needed. The behavior of fibers in bending and torsion at various temperatures is not known; the effect of combined stresses on the behavior of the filaments is not known. Properties of the fibers in compression, both axial and lateral (the latter becoming important when the yarn is over the heater), at various temperatures must be determined. The breakdown and subsequent restructuring (at high temperatures) of the molecular architecture and its effect on the quality of setting should be investigated.

(6) No mention was made in this work of texturing processes such as draw texturing and friction twisting. These, because they offer economic advantages, are becoming increasingly important commercially and are more recent developments than false-twisting. The phenomena that occur in false-twisting also occur in draw texturing, and an analysis of the process, the elucidation of the drawing and flattening of the fibers would be of considerable merit.
APPENDIX I

This Appendix contains derivations on various topics of yarn mechanics, which, however, were not used in the modelling of the false-twist process presented in the main body of this work.
APPENDIX I

(a) Stress-strain and Torque-strain Expressions for Yarns; Twist Varies with Extension

This analysis is identical as to assumptions and method to that given in Sections V(a)2., and V(a)3., with the exception that it takes account of the twist change due to yarn extension; it still assumes, however, as did the earlier derivation, that the yarn radius remains constant in extension. The difference between the two methods arises in calculating the fiber strain, which is calculated as follows: In one inch of yarn before extension, we have a filament length at radius, $r$

$$\ell_1 = \sqrt{1 + p^2 r^2}$$

and to 1" of yarn after extension at constant radius, we have

$$\ell_2 = \sqrt{1 + p'^2 r^2}$$

where

$$p' = \frac{p}{1 + \varepsilon_y} = \frac{p}{F}$$  \hspace{1cm} (68)

where $F = 1 + \varepsilon_y$ is the extension ratio of the yarn. But one inch of unstressed yarn becomes $F$ inches after extension, and the length of filament corresponding to one inch of unstressed yarn becomes, after extension

$$\ell'_2 = F \ell_2 = F \sqrt{1 + p'^2 r^2} = \sqrt{F^2 + p'^2 r^2}$$  \hspace{1cm} (69)
Thus, the filament strain is

\[
\varepsilon_f \equiv \frac{l'}{l_0} - 1 = \frac{\sqrt{r^2 + p^2 r^2}}{\sqrt{1 + p^2 r^2}} - 1 \tag{70}
\]

Replacing Equation (70) and using Hooke's law, Equation (10), in Equation (9) and integrating with the substitutions

\[z^2 = 1 + p^2 r^2 \quad \text{and} \quad z^2 = F^2 + p^2 r^2,\]

we obtain for the tension:

\[
T = \frac{2 \pi k F^2 E_f}{\pi} \ln \left( \frac{\sqrt{1 + p^2 b^2}}{F^2 + p^2 b^2} + 1 \right) + \ln \left( \frac{F}{F + 1} \right) \tag{71}
\]

Replacing Equation (70) and using Equation (10) in Equation (27), and integrating with the usual substitutions, we obtain for the torque:

\[
M_{TE} = \frac{2 \pi k F F_f F}{2 p^3} \left[ \frac{(F^2 + p^2 b^2)(1 + p^2 b^2)}{\sqrt{F^2 + p^2 b^2 + 1 + p^2 b^2}} \right]^{\gamma_1} \]

\[
+ (1 + F^2) \ln \frac{1 + F}{\sqrt{F^2 + p^2 b^2 + 1 + p^2 b^2}}
\]

\[
+ F^2 \ln \left( 1 + \frac{p^2 b^2}{F^2} \right) - F - p^2 b^2 \tag{72}
\]

The symbols have all been defined above or in the Sections IV and V(a).
(b) Stress-strain Curves of yarns assuming the Twist to Vary with Extension and the Filaments to Extend at Constant Volume

The derivation is identical to the one that precedes this; the difference arises in calculating the filament strain: The filament strain after yarn extension $F$ is

$$\varepsilon_f = \frac{F\sqrt{1 + p'2r'^2}}{\sqrt{1 + p^2r^2}} - 1 \quad (73)$$

(cf. Equation (70)) where $p' = p/F$ (Equation (68)), and $r'$ is the radius of the yarn which locates a filament originally situated at radius $r$. To relate $r'$ to $r$ and $F$, we reason as follows: We express the constancy of volume of a filament as

$$\pi f^2 = \pi f'^2(1 + \bar{\varepsilon}_f) \quad (74)$$

with $f$ and $f'$ the radii before and after extension, and $\bar{\varepsilon}_f$ the average filament strain defined by Equation (16). Note that we assume that all filaments are extended by the average filament strain, and are, therefore, all reduced in diameter as follows

$$f' = \frac{f}{\sqrt{1 + \bar{\varepsilon}_f}} \equiv \alpha f \quad (74')$$

We also assume that the yarn radius changes as the filament radius, and that the packing factor remains constant, so that
Replacing Equation (74'') in Equation (73) and Equation (73) in Equation (9) and using Hooke's law, Equation (10), we get after integration, with the usual substitutions:

\[
T = 2\pi kE_f \left\{ \frac{F}{p} \sqrt{1 + p^2 b^2} - \frac{1}{p}\frac{1}{\psi} \ln \left( \frac{1 + p^2 b^2}{\psi} \right) \right. \\
\left. + \sqrt{(1 + p^2 b^2) + \frac{1 - \psi}{\psi}} \right) + \frac{1}{p^2 \psi} \ln \left( 1 + \sqrt{1/\psi} \right) \right\} \tag{75}
\]

with \( \psi = (\alpha/f)^2 \).

The corresponding torque has not been calculated.
(c) Stress-strain and Torque-strain Curves of Yarns

Assuming the Twist to Vary with Extension and the Filaments to Extend at Constant Volume

(Alternate Derivation)

As in case (b), we seek a suitable expression for $\varepsilon_f$. Here we make use of Equation (7) which identifies a filament of fractional order $\phi$, and we express the fact that the fractional order of a filament in the yarn remains, of course, unaltered by extension

$$\phi = \frac{2k}{Wf^2p^2} \left( \frac{\sqrt{1 + p^2r^2} - 1}{1 + p^2r^2} \right) = \frac{2k}{Wf^2p^2} \left( \frac{\sqrt{1 + p'2r'^2} - 1}{1 + p'2r'^2} \right) \quad (76)$$

Simplifying, and rearranging we obtain

$$\sqrt{1 + p^2r^2} = \left( \frac{fp}{f'p'} \right)^2 \frac{1}{1 + p'2r'^2} + 1 - \left( \frac{fp}{f'p'} \right)^2 \quad (76')$$

We abbreviate $D = fp/f'p'$ and using Equation (74') and Equation (68), we substitute in Equation (73). Then using Hooke's law and substituting in Equation (9), we obtain, after integration

$$T = 2\pi k E_f \left\{ \frac{F}{\theta^2p^2} \ln(Ap^2+1) - \frac{F^2}{p^2} \ln(Ap^2+F^2\theta^2) + \frac{F^2}{p^2} \ln F^2 \theta^2 \right\} \quad (77)$$

where $\theta^2 = D/F^2$ and $A$ is given by Equation (4).

All the terms in Equation (77) are in terms of the original
parameters, i.e., yarn parameters before extension.

To obtain the torque, we substitute the strain in Equation (27) also using Hooke's law, and we obtain for the torque, in terms of the original yarn parameters,

\[ M_{TE} = 2\pi kE \frac{F}{f} \frac{P^3}{P^3} \left( \frac{A_p^2 + 1}{2D^2} \right) \left( 2(1-D)(D-2F)+(F-D)(A_p^2+1) \right) \]

\[ + \frac{F(1-2D)}{D^3} \ln(A_p^2+1) + \ln \left( \frac{A_p^2}{D} \right) + 1 - \frac{D-3F-2D^2+4FD}{2D^2} \]  (78)
(d) Overtwisting at Constant Radius

This derivation differs from the one given in Section V(c), in that we assume that we start with yarn of twist $p_1$ which we overtwist without contraction to $p_2$ at constant radius and then extend by $F$ at constant radius to a final twist of $p_3$. To calculate the filament strain, we consider that a filament at radius $r_1$ in one inch of yarn prior to uptwisting has length

$$ l_1 = \sqrt{1 + p_1^2 r_1^2} $$

After uptwisting and extension by $F$, a filament at radius $r_1$ on one inch of yarn has length

$$ l_3 = \sqrt{1 + p_3^2 r_1^2} $$

But, the filament $l_1$ now has length (because of the yarn extension):

$$ l'_3 = l_3 F = F \sqrt{1 + p_3^2 r_1^2} $$

hence, the filament strain is

$$ \varepsilon_f = \frac{l'_3}{l_1} - 1 = \frac{F \sqrt{1 + p_3^2 r_1^2}}{\sqrt{1 + p_1^2 r_1^2}} - 1 \quad (79) $$

We replace that in Equation (33) and we obtain, after integration, for the tension
\[ T = 2\pi kE_f \left\{ \frac{F^2}{P_1P_2} \ln \left( \frac{\frac{p_2}{F} \sqrt{F^2 + p_2^2b^2 + Fp_1} \sqrt{F^2 + p_1^2p_3^2b^2}}{1 + \frac{p_2}{p_1}} \right) - \frac{F^2}{c^2p_1^2} \ln \sqrt{1 + \frac{cp^2b^2}{F^2}} \right\} \]

with \( p_2 = p_3F \).

The torque \( M_{TE} \) has not been calculated, but can be calculated by substitution of Equation (80) in Equation (27) and integration.
(e) Mechanics of Yarn Shrinkage Due to Fiber Shrinkage

We assume a yarn of twist $p_1$ which is suddenly exposed to an environment where the filaments would like to shrink (for instance, temperature or solvent). We examine the resultant relationship between filament shrinkage, yarn shrinkage, yarn tension and yarn torque. We assume that the yarn shrinks at constant volume as a result of filament shrinkage, and we examine the strain in a given filament.

With reference to the sketch at the left, the fiber length corresponding to 1" of yarn prior to shrinkage at radius $r_1$ is

$$l_1 = \sqrt{1 + p_1^2 r_1^2}$$

The fiber length corresponding to 1" of yarn in the shrunken state (state 2) is

$$l_2 = \sqrt{1 + p_2^2 r_2^2}$$

Because the volume of the yarn remains constant, the radii where a given filament is located before and after shrinkage are related by

$$r_2 = \sqrt{1 + \varepsilon_y} \ r_1 = \sqrt{F} \ r_1$$

where $F = 1 + \varepsilon_y$ is the yarn extension ratio, i.e., $F = L_1/L_2$. 
Now, the twists are related by $p_1 = p_2/F$, and replacing $r_2$ and $p_2$ in the expression for $l_1$ we get

$$l_1 = \sqrt{\frac{p_2^2 r_2^2}{F^3}} + 1$$

Now, since the yarn has shrunk from $F^\prime$ to $1^\prime$, the original fiber length is

$$l_1^\prime = \frac{F l_1}{1} = \sqrt{\frac{p_2^2 r_2^2}{F^3}} + 1$$

So, as the yarn shrinks from $L_1$ to $L_2$, a given filament will shrink from $l_1^\prime$ to $l_2$.

Now, if the fibers had been allowed to shrink freely, and if $t$ is the percent shrinkage, then

$$\frac{l_1^\prime}{l_{eq}} = 1 + t$$

where $l_{eq}$ is the (non-arrived at) equilibrium length of the filament after shrinkage. Therefore, the strain on the fiber, due to the fact that fibers have shrunk but have not attained their true equilibrium length is

$$\varepsilon_f = \frac{l_2}{l_{eq}} - 1 = \frac{\frac{l_2}{l_{1}}(1+t)}{l_1^\prime} - 1 = \frac{\sqrt{F(1+t)}\sqrt{p_2^2 r_2^2 + 1}}{\sqrt{p_2^2 r_2^2 + F^3}} - 1 \quad (81)$$

The "stress-strain" relation for the yarn is obtained by sub-
stituting Equation (81) in Equation (9), and we obtain after integration

\[ T = \frac{2\pi kE_f (1+t)\sqrt{F}}{p^2} \left( \ln \frac{1+p^2b^2}{1+\sqrt{F^3}} + \frac{\sqrt{F^3+p^2b^2}}{(1+t)\sqrt{F^3}} \right) \]  

(82)

and the torque is obtained by substitution of Equation (81) in Equation (27)

\[ M_{TE} = \frac{2\pi kE_f \sqrt{F}(1+t)}{2p^3} \left( \frac{\sqrt{F^3+p^2b^2}}{\sqrt{F^3+p^2b^2}} - \ln (1\sqrt{F^3+\sqrt{F^3}}) \right) \]

\[ + \frac{\sqrt{1+p^2b^2}}{(1+t)\sqrt{F^3}} \]  

(83)

In Equation (82) and Equation (83) \( p = p_2 \) and \( r = r_2 \) so that the expression for tension and torque are given in terms of final yarn parameters (after shrinkage). Note also that \( E_f \) is the modulus of the stress-strain curves of the filaments during shrinkage.

The "contractile stress" of a yarn which results if we keep the yarn length constant but generate the shrinkage force, is obtained by putting \( F = 1 \) in Equation (82) and is

\[ T = \frac{2\pi kE_f t}{t\ln (1+p^2b^2)} \]  

(84)

The contraction which ensues if we generate the shrinkage, allow yarn retraction but prevent any untwisting of the yarn, is obtained by putting \( T = 0 \) in Equation (82) and solving for \( F \); explicit solution in terms of \( F \) is not possible, but the expression is
\[
(1 + t) = \frac{\sqrt{F} \ln \frac{\sqrt{F^3 + p^2b^2}}{\sqrt{F^3}}}{\ln \frac{\sqrt{1+p^2b^2} + \sqrt{F^3 + p^2b^2}}{1 + \sqrt{F^3}}}
\] (85)

Equation (85) has been evaluated for the case of a 70/34 Nylon yarn and the actual yarn shrinkage for a given equilibrium fiber shrinkage are given below:

<table>
<thead>
<tr>
<th>( \epsilon_y = (F - 1)% )</th>
<th>( t% )</th>
<th>Yarn Shrinkage fiber shrinkage</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>1.5</td>
<td>1.33</td>
</tr>
<tr>
<td>3.8</td>
<td>2.9</td>
<td>1.31</td>
</tr>
<tr>
<td>6.5</td>
<td>5.2</td>
<td>1.25</td>
</tr>
<tr>
<td>9.1</td>
<td>7.2</td>
<td>1.26</td>
</tr>
<tr>
<td>13.0</td>
<td>10.5</td>
<td>1.24</td>
</tr>
<tr>
<td>16.7</td>
<td>13.7</td>
<td>1.22</td>
</tr>
<tr>
<td>20.0</td>
<td>16.7</td>
<td>1.20</td>
</tr>
<tr>
<td>33.3</td>
<td>29.3</td>
<td>1.14</td>
</tr>
<tr>
<td>50.0</td>
<td>45.5</td>
<td>1.10</td>
</tr>
</tbody>
</table>

Thus, the yarn shrinkage is higher than the fiber shrinkage, for fiber shrinkages up to 10\%, the yarn shrinkage is roughly 1.3 times the filament shrinkage.
(f) Shearing Strains on Fiber Due to Torsion and Bending

1. Differential Geometry of Yarn

Reference is made to the MIT Textile Division Notes\(^{(R25)}\) and Hildebrand\(^{(R16)}\) concerning differential geometry of yarns and space curves. For yarns obeying ideal geometry, the filaments are assumed to be in helices concentrically around the yarn axis as illustrated in Figures 22 and 22'\(^{(R25)}\).

The symbols have the following meaning: \(x_1, x_2, x_3\), are the system of axes to which the helix (filament path) is referred to; \(x_3\) is the yarn axis; \(r\) is the radius from the yarn axis to the centerline of a particular filament of radius, and \(S\) is the helix described by the centerline of the filament.

At any point \(P\) on \(S\), the unit tangent \(\hat{t}\), unit normal \(\hat{n}\) and binormal \(\hat{b}\)\(^{(R16)}\), can be defined and are shown on Figure 22.

Additionally, \(\hat{n}\) and \(\hat{b}\) are, of course, perpendicular to \(\hat{t}\), and their common plane intersects the filament as shown in Figures 22 and 22'. Any point \(R\) on the filament circumference can be defined by the angle \(\phi\) that the vector \(\hat{d}_{PR}\) makes with the normal \(|\hat{d}|=1\). Any point \(P\) or \(S\) is defined by the radius vector\(^{(R16)}\)

\[
\hat{x} = \overrightarrow{OP} = r\cos\theta \hat{i} + r\sin\theta \hat{j} + r\theta \cot\phi \hat{k}
\]

where \(\hat{i}, \hat{j}, \hat{k}\) are the unit vectors along \(x_1, x_2\) and \(x_3\).

Any point \(R\) or the fiber circumference can be defined by the radius vector
FIGURE 22

DIFFERENTIAL GEOMETRY OF YARNS
\[
\vec{\nu} = \vec{OR} = \vec{x} + \vec{\nu} = r \cos \theta \hat{i} + r \sin \theta \hat{j} + r \cot \phi \hat{k} \\
+ f \cos \phi \hat{n} + f \sin \phi \hat{b}
\] (87)

Substituting \( \hat{n} \) and \( \hat{b} \) in terms of the parameters of the helix itself (R16) and introducing into Equation (87), we get

\[
\vec{\nu} = (r \cos \theta - f \cos \phi \cos \phi + f \sin \phi \sin \phi \cos \phi) \hat{i} + (r \sin \theta - f \sin \phi \cos \phi - f \cos \phi \sin \phi \cos \phi) \hat{j} + (r \cot \phi + \cos \phi) \hat{k} \quad (87')
\]

2. Local Shear Strain on Fibers as They Lie in the Yarn

In a twisted yarn, any filament at radius \( r \) and helix angle \( \phi \) could have been brought to the same state of strain by "wrapping" around a cylinder of radius \( r - f \), at a helix angle \( \phi \). Since the fiber itself is a cylinder, each of the generators of the (as yet unwrapped) cylindrical fiber will correspond after wrapping to a given value of \( \phi \), which will moreover, be independent of \( \theta \). Thus, on the generator corresponding to the "inside" of the wrapped fiber, we will have \( \phi = 0 \), and on that corresponding to the outside, \( \phi = \pi/2 \).

The local shear strain on any point \( R \) of the fiber circumference will be given by \( \tan \alpha \), \( \alpha \) being the angle between the tangent to the generator through point \( R \) and the tangent to the corresponding point \( P \) on the fiber center line. But, \( \tan \alpha = \sqrt{1 - \cos^2 \phi} / \cos \phi \), and \( \cos \phi \) is, by definition \( (R25) \), the dot product of the unit tangents at \( R \) and \( P \). We seek \( \cos \phi \) as follows:
From differential geometry, the tangent is defined

\[ T = \frac{d\hat{x}}{d\theta} = \left( \frac{dx_1}{d\theta}\hat{i} + \frac{dx_2}{d\theta}\hat{j} + \frac{dx_3}{d\theta}\hat{k} \right) \]  

(88)

and the unit tangent is

\[ \hat{t} = \frac{\frac{d\hat{x}}{ds}}{\left(\frac{d\hat{x}}{ds}\right) \cdot \left(\frac{d\hat{x}}{ds}\right)} = \left( \frac{dx_1}{ds}\hat{i} + \frac{dx_2}{ds}\hat{j} + \frac{dx_3}{ds}\hat{k} \right) \]  

(89)

where \( \hat{x} \) is any radius vector. It follows that:

\[ \hat{t} = \hat{t} \frac{d\theta}{ds} = \frac{\hat{x}}{ds} \]  

(90)

Applying Equation (90) to Equations (86) and (87) we get,

\[ \hat{t} = \frac{d\hat{x}}{ds} = -\sin\theta \sin q \hat{i} + \cos\theta \sin q \hat{j} + \cos q \hat{k} \]  

(91)

\[ \hat{t}' = \frac{d\hat{y}}{ds} = G \left( \left[ (f \cos \phi - r) \sin \theta + f \sin \phi \cos q \cos \theta \right] \hat{i} \right. \]

\[ + \left. \left[ (r - f \cos \phi) \cos \theta + f \sin \phi \cos q \sin \theta \right] \hat{j} \right) + r \cot q \hat{k} \]  

(92)

with

\[ G = \frac{\sin q}{\sqrt{(f \cos \phi - r)^2 \sin^2 q + f^2 \sin^2 \phi \sin^2 q \cos^2 q + r^2 \cos^2 q}} \]  

(92')

As earlier mentioned, \( \hat{t} \cdot \hat{t}' = \cos \alpha \) and \( \tan \alpha = \sqrt{1 - \cos^2 \alpha} / \cos \alpha \).

Hence, we get, after suitable substitutions:
\[
\tan \alpha = \frac{f \cos \theta}{r - f \cos \phi \sin \theta \sin \theta}
\]

This indicates that the strain is non-homogeneous and depends on \( \phi \). From Equation (1) we have

\[
\tan \theta = p r
\]

\( (1') \)

\[
\sin \theta \cos \theta = \frac{p r}{1 + p^2 r^2}
\]

\( (1'') \)

Replacing Equations \( (1') \) and \( (1'') \) into Equation (93) and expressing \( \tan \alpha \) as a local fiber twist so that

\[
\tan \alpha = p_F \cdot f
\]

\( (1''') \)

where \( p_F \) is the local fiber twist in radians/inch, we obtain

\[
p_F = \frac{p}{1 + p^2 r^2 - p^2 r \cos \phi}
\]

\( (94) \)

which expresses the local fiber twist in terms of the yarn twist and the angle \( \phi \). A simplified version of Equation (94), obtained by omitting the term in \( f \) in the denominator (the quantity \( 1 + p^2 r^2 \) being usually much larger than \( p^2 r \cos \phi \)) has been given by Platt \( (R13) \) and was used earlier in the calculation of yarn torque due to fiber torque (Section V(a)3.)

\[
p_F = \frac{p}{1 + p^2 r^2}
\]

\( (23), (94') \)
Note that Equation (94') indicates the local fiber twist to be constant around the circumference of the fiber (homogeneous shear strain).

As was shown in the sketch in Section IV(a) which dealt with yarn geometry, the length of fiber of helix angle $q$, corresponding to one turn of twist of the yarn is $S_T = 2\pi \sqrt{1/p^2} r^2$. From the geometry of the "opened" yarn, we also have

$$S_T = \frac{2\pi r}{\sin q} \quad (95)$$

Replacing Equation (95) in Equation (94'), we get

$$\tan \alpha = \frac{f \cos q}{S_T - f \cos q \sin q} \quad (96)$$

referred to a constant fiber length corresponding to one turn of twist. This expression allows us to predict what happens to the strain if we collapse or straighten a filament. If we collapse the filament into a loop, $q = 90^\circ$, $\cos q = 0$. Hence, $\tan \alpha = 0$, i.e., no shear strain or twist, since the filament is now simply bent. Conversely, when we straighten the filament without allowing for end rotation, $q = 0$, $\sin q = 0$, $\cos q = 1$, and

$$\tan \alpha = \frac{2\pi f}{S_T} \quad (97)$$

or

$$p_F = \frac{2\pi}{S_T} \text{ radians} \quad \text{length} S_T \quad (97')$$
which corresponds to 1 turn of twist in the length $S_T$.

3. Local Shear Strain on Untwisted, False-twist Fibers

We consider a yarn made of filaments all of which already have some twist prior to being twisted into the yarn. In this case, the angle $\phi$ is no longer constant for a given filament, but is a function of $\theta$, $\phi = \lambda \theta$. Under these circumstances, $\tan \alpha$ is calculated as detailed in (R25) and its expression is identical to that given in (R25)

$$\tan \alpha = \frac{f\lambda + f \cos \theta}{r} - \frac{f \sin q \cos \lambda \theta}{\sin q}$$

Note that in (R25), Equation (98) gives the local helix angle of fibers in a single yarn as they lie in the ply.

If we now straighten the fiber without allowing for rotation ($q=0$), we have, using Equation (95), i.e., with reference to a fiber corresponding to one turn of twist

$$\tan \alpha = \frac{2\pi f}{S_T} (\lambda + 1)$$

or since $\tan \alpha = p_F \cdot f$,

$$p_F = \frac{2\pi (\lambda + 1)}{S_T} \text{, radians}$$

If we now untwist the filament by one turn ($2\pi$ radians), the
the twist becomes

$$p'_F = \frac{2\pi (\lambda + 1)}{S_T} - \frac{2\pi}{S_T} = \frac{2\pi \lambda}{S_T}$$

which can readily be shown to be the same twist as when the filament is bent into a loop ($q = 90^\circ$), using the simplified version of Equation (98); $p'_F$ is, of course, the original twist of the filaments.

Consider now a filament in a false-twist yarn, after setting but prior to untwisting; we assume the setting to be perfect so that by definition all strains on the filaments are nil so, $\tan \alpha = 0$. This means in Equation (98) that $\lambda = -\cos q$.

Now when we straighten the filament without allowing for rotation, we have, using Equation (99')

$$p_F = \frac{2\pi (1 - \cos q)}{S_T}$$  \hspace{1cm} (100)

and when we untwist it by one turn

$$p_F = -\frac{2\pi \cos q}{S_T}$$  \hspace{1cm} (101)

which is the twist in the untwisted filaments of the false-twist yarn; $q$ is, of course, the angle of the filament in the yarn prior to untwisting. The sign is negative because it is in the opposite sense than the twist in the (twisted) false-twist yarn. Clearly, the shear strain in an untwisted false-twist filament is
on the basis of which the shear strain energy can be calculated. The total strain energy must, of course, take into account the bending energy and the tensile energy (if any).

Equation (93) can be simplified, exactly as was done with Equation (94), by neglecting the term $f\cos\psi\sin\theta$ in the denominator.

$$\tan \alpha = \frac{f}{r} \sin \theta \cos \phi$$  \hspace{1cm} (93')

It thus becomes identical, except for the sign, with Equation (101'). Thus, Equation (101') says that the shearing stress on an untwisted and straightened false-twist filament equals the shear strain that the (originally untwisted) filament would experience if it were twisted (without relaxation) to the final twist that the yarn is subjected to prior to untwisting, assuming that all the twist is "set". The twisted yarn, of course, never fully experiences this shear strain, since the bending and torsional components relax at high temperature. Because the moduli are much lower, even if relaxation did not come into play the strain energy due to shear at high temperature is much lower than the strain energy corresponding to the false-twisted and straightened filament at room temperature.

$\tan \alpha$ in Equations (93') and (101') is directly related to the torsion of the filament centerline in the twisted yarn.
This Appendix contains review and analysis of various papers from the scientific literature relating to the false-twist process.
In their paper, "The Technology of the Production of False-Twist Textured Yarns", (R26,R3) these authors studied experimentally the effects on other processing parameters brought about by a change in the setting of one of them, as well as the yarn properties as a function of various machine settings. They conducted their experiments on a specially constructed false-twist unit, equipped with a Falspin spindle (made by Scragg) and a contact heater from the Scragg CS-1 machine. The unit was so constructed as to allow feeding the yarn at a controlled tension (by passing it through an adjustable tension gate) or feeding the yarn at a controlled rate (by interposing, after the tension gate, a pair of positively driven rollers). The various machine elements were set sufficiently far apart to permit insertion in the yarn path of a tensiometer to measure yarn tension. The machine rpm was kept constant at 45,000 rpm while the tension, the heater temperature and the speed were varied between wide limits. No provision was made for measuring yarn temperature. The yarn used in these tests was 70 denier nylon 6.6.

**Constant Tension Experiments**

In this series of experiments, yarns to be textured were fed into the machine at predetermined levels of tension; this was done by suitably adjusting the setting of the tension gate in the feed zone. The machine was then run, and tensions
at various positions in the yarn path were measured. The data shows that:

1. Higher pre-tensions, i.e., imposed tensions between the tension gate and the guide, lead, as expected, to higher tensions upstream of the guide.

2. For a given level of pre-tensions, the running tensions are almost always lower the higher the nominal twist. Some anomalies have also been observed which cannot be readily explained; for instance, the yarn tension is seen to drop across the heater; also the effect of the heater temperature on processing tension is equivocal.

**Constant Extension Experiments - Untwisted Yarn**

On operating the machine on a constant extension, by interposing a set of feed rollers between the tensioning gate (and above the guide) and the heater box, a number of experiments were performed. First, the authors determined (at negligible pretensioning, but at variable overfeeds) the tension developed due to yarn shrinkage, the so-called "contractile stress"; in these experiments, the false-twist spindle was removed and was replaced with a guide over which the yarn was made to pass; the yarn was thus run in the untwisted state. The tensions recorded are illustrated in Fig.23 and are seen to pass through a maximum as the temperature increases. The authors also found that:

1. the friction yarn-heater adds approximately 7 g to
FIGURE 23

CONTRACTILE STRESS vs. TEMPERATURE

TENSION (grams)

STRESS (gpd)

TEMPERATURE (°C)

overfeed (%)
the tension for the case of the untwisted yarn. Note that these results are the opposite of what they found in the constant tension experiment.

2. As the amount of overfeed increases, the stress maximum occurs at increasingly higher processing temperatures, until, at very high overfeeds, no maximum is observed before the sharp drop at the melting point. The authors anticipate this maximum to occur at even lower temperatures, and the higher in value for experiments run at negative overfeeds. At 2% underfeed, the tension was very high (35 g) and at higher underfeeds, the (untwisted) yarn breakage was excessive.

The effects of pre-tension were also investigated at room temperature and at 190°C and are as follows:

1. The higher the pre-tension the higher the tension upstream of the feed rolls; at the high temperature, however, this dependence is very slight.

2. The tension above the heater is higher than that below the heater. This would be expected because of the frictional drag of the yarn across the heater; as mentioned, however, it stands in contrast to the data found for constant-tension processing. The tension ratio is independent of the tension level. They also observed (and it is difficult to understand why) that the tension below the heater is often lower than the pre-tension.
Constant Extension Processing Using False-Twist

The authors find that:

1. The lower the overfeed, the higher the tension.
2. The tensions increase over the heater and over the spindle. This increase, however, is not proportional to the tension.
3. The temperature profile of the tensions is qualitatively similar to that found in the case of the untwisted yarn. However, at the same value of overfeed the tensions in the untwisted yarns are higher than the corresponding tension in the twisted yarns.

Conclusions

Although a great number of experiments were performed, some ambiguities and uncertainties exist which make it difficult to draw valid conclusions about much of the data presented. What is of interest to us in our work can be summarized as follows:

1. Higher pre-tensions or low overfeeds lead to high running tensions.
2. In constant extension processing, tension increases in the yarn path due to friction across the heater and around the spindle.
3. The effect of temperature on the running tension depends on overfeed.
4. Implicit in the previous discussion is the effect of draw ratio on the contractile stress. Lower draw ratios may lead to lower values of contractile stress. For undrawn yarns
which should exhibit no contractile stress, the processing dynamics may be quite different than for drawn yarns.
2. **Twist distribution:** The work of Sasaki et al

These workers (R7) measured the twist distribution along the false twist threadline in a specially constructed laboratory machine. A black and a white yarn were simultaneously fed to the machine, and the twist was measured directly from the photographs. Their apparatus differed in a number of ways from the apparatus used in this work: thus guides were interposed between the spindle and the heater and between the heater and the feed rolls. A tensiometer was also interposed between the heater and feed rolls. Thus neither the tension nor the torque were constant along the threadline. The twist levels in their experiments were comparable to ours: many of their experiments were conducted at approximately 50 tpi basic twist. Since the dimensions of their apparatus were different from ours (the heater dimension is 9 cm vs 4 cm for ours) different processing speeds were used: the speed ranged from 5.4 m/min to 12 m/min, whereas our (nominal) speed was 0.57 m/min. The authors measured the twist distribution as they varied the temperature, the tension, the yarn speed and the twist level. The twist distribution was found to be as earlier described in this thesis (Sect III (e)): low twist in the entrance zone and increasing twist over the heater. The twist, however, was not constant in the entrance zone but was found to increase gradually; the same gradual twist increase was observed over the heater.

The results of Sasaki's work, where comparable, are in agreement with the results obtained in this work: thus the twist
change at the heater is smaller the higher the tension. The twist sensitivity to tension, however, as measured by the actual twist at the spindle for a given basic twist (≈50 tpi) is higher than that observed in this work: at 230°C, for tensions varying between 75 and 30 grams they observe a twist varying from 71 to 63.5 tpi. Under comparable conditions experiments carried out in this work (Table V) show the twist to vary between 65 and 61.1 tpi.

Sasaki also performed crimp rigidity experiments and finds that the crimp rigidity depends almost exclusively on the actual twist at the spindle; the contact time at the heater was of the order of .75 sec, which is sufficient to impart good setting qualities to the yarn.
(b) HEATING AND THERMAL PLASTICIZATION

1. Theory

In all texturing processes where permanent deformation of the polymeric fibers is required, heating plays a most important role. Indeed, texturing usually requires that filaments be subjected to large deformations and remain at this level of deformation permanently. These deformations, if not beyond the breaking strain of the material at room temperature, would nonetheless require large forces for their realization; further, if realized at room temperature, they would be only partially set, since there always exists immediate and delayed recovery from deformation which becomes a more important component of the total deformation as the temperature is lowered. So, increasing the temperature facilitates the deformation of the filaments to the desired level and allows for good setting. Since fibers are usually crystalline polymers, heating to within 30 to 40°C of the melting point usually results in partial melting of the crystalline structure (and lowering of the modulus) which facilitates the deformation; this is probably followed by gradual recrystallization at the high temperature. When the fiber is cooled, crystallization occurs rapidly effectively immobilizing the chains and preventing molecular and macroscopic recovery.

In the original Helanca process, highly twisted yarn was wound on packages, and the setting operation took place in an autoclave. In the false twist process, the yarns are passed through heaters. These may be of various designs, though recent machines are all equipped with conduction-type heaters. These are
essentially metal plates (often curved) heated via various means, but most often electrically. The yarn comes in contact with the heater and so is heated primarily by conduction. The heater design provides for optimum insulation. The heater thermostatic controls are usually of very high quality as the constancy of the temperature level is all important for purposes of yarn uniformity. Earlier heaters heated the yarn by convection and radiation. This gave a very even heating, but the overall efficiency was lower than that of conduction heaters. The tremendous increase of speed of texturing within the last few years entails a decrease of the residence time in the heater. This resulted in poorly plasticized yarn. It was thus necessary to lengthen the heater and to increase the heat transfer efficiency. The latter was accomplished by abandoning the convection heaters in favor of the conduction heaters. Detailed descriptions of the heaters and their controls are given in R2 and R28.
Transient Conduction to a Cylinder

For an isotropic, homogeneous material, whose thermal conductivity is constant, the heat conduction equation in cylindrical coordinates is

\[
\frac{\partial T}{\partial t} = \alpha \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{1}{\rho c} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right) + \frac{Q'}{\rho c} \tag{102}
\]

\( T \) = Temperature
\( t \) = time
\( Q' \) = heat generated per unit of time and space within the differential element considered
\( \rho \) = density
\( \rho c \) = heat capacity
\( k \) = thermal conductivity
\( \alpha = k/\rho c \) = thermal diffusivity

For the case of a yarn (infinite cylinder) considered as a stationary system with no heat sources, equation (102) becomes, because of symmetry

\[
\frac{\partial T}{\partial t} = \alpha \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) \tag{103}
\]

For initial and boundary conditions we have

\[
T(r,0) = T_i \tag{104}
\]

that is uniform initial temperature across the yarn cross section and

\[
\frac{\partial T(0,t)}{\partial r} = 0 \tag{105}
\]
i.e., no temperature gradient at the center at any time because of cylindrical symmetry.

The physical meaning of these conditions is so general that they are applicable to any yarn heating system. The boundary condition concerning the yarn surface depends, however, on the heating method. If it were possible to heat the yarn so as to keep its surface temperature constant, then we would have

\[ T(b,t) = T_f \]  \hspace{1cm} (106)

More commonly in a convection heater we assume the heat flux transferred to the surface per unit area to be proportional to the temperature difference between the surface and the fluid \( q = q'/2\pi b L = h(T_f - T_s) \), and to be transferred by conduction to the center of the cylinder

\[ q = \frac{q'}{2\pi b L} = \frac{dT}{dr} \bigg|_{r=b} \cdot \frac{dT}{dr} < 0 \]

\[ \frac{\partial T(b,t)}{\partial r} = -\frac{h}{k} \left[ T(b,t) - T_f \right] \]  \hspace{1cm} (107)
Finally, if it were possible to have a specified value of \( q \) per unit area at the boundary,

\[
\frac{\partial T(b,t)}{\partial r} = -\frac{q}{k} = \text{const.} \tag{108}
\]

for instance, for radiation.

Equation (103) can be solved by separation of variables (R16), the constants and eigenfunctions involved in its solution being specified by appropriate boundary conditions. The final form of the equation is very unwieldy: it is usually graphed using dimensionless parameters

\[
\frac{T-T_f}{T_i-T_f} = \phi \left( \frac{\alpha t}{b^2}, \frac{h b}{k}, \frac{n}{b} \right) \tag{104}
\]

\( \alpha t/b^2 \) is known as the Fourier number \( F_o \), and \( h b/k \) is known as the Biot number \( B_i \) (R29, R30, R31).
Heat Transfer Equation for Large Values of $m$

A simpler case applies to systems where the ratio $k/hb = m$ is sufficiently large thus rendering the internal temperature gradient very small (R29,R31). Indeed $m$ is the ratio of the thermal conductivity to the heat transfer coefficient, and is called the "relative surface boundary resistance" expressing the ratio of the surface resistance (to passage of heat) to the internal (or bulk) resistance. When this ratio is small, the surface offers little resistance to passage of heat compared to the bulk: hence the surface temperature $T_s$ quickly or immediately goes from $T_i$ to $T_f$ and the temperature drop occurs along the radius of the cylinder. Conversely, if $m$ is large, then $T_s$ approaches $T_f$ only gradually, and the radial temperature gradient is then very small. This is generally taken to hold true for values for $m$ larger than 6. When this occurs, a simple heat balance on a cylinder of radius $b$, length $L$ over a time interval $dt$ yields:

$$dQ = 2\pi bLh(T_f - T)\, dt = \pi b^2 L \rho c \, dT$$

$$\frac{d(T_f - T)}{T_f - T} = -\frac{2h}{b \rho c} \, dt \quad \Rightarrow \quad \ln\frac{T_f - T}{T_f - T_i} = -\frac{2h t}{b \rho c}$$

$$\frac{T_f - T}{T_f - T_i} = e^{-\frac{2h t}{b \rho c}}$$  \hspace{1cm} (105)$$

All equations are, of course, applicable, given the right
boundary to either heating or cooling of the yarn.

Two types of heaters are used in false twist texturing. In the first (the "convection" heater) yarn passes through a heated chamber and is heated by the hot fluid. In the other (the "conduction" heater) which is now by far the most common, the moving yarn comes in contact with a heated plate; strictly speaking treating the heating and cooling of the yarn analytically, as if it were a stationary system is erroneous; the yarn is rotating and moving axially: in the convection heater it probably balloons and creates eddies so that the system is probably under turbulent conditions. In the conductive heater, the yarn does not balloon, but it probably is heated both by conduction and convection. On cooling, ballooning often occurs. The point has been made by Ziabicki (R32) that the problem of cooling of an axially moving wire has not been solved. Some work has been published on cooling of melt spun yarns (R32), but the knowledge of heat transfer coefficients is very poor, and widely different values have been reported. It appears thus that an exact treatment of the heating and cooling of the yarn is a very difficult one; hence the necessity of using approximate methods.
2. Experiments with a Convection Heater

Temperature Gradient across the Yarn

Arthur and Jones (R20) tackled the problem of heat transfer to nylon yarns moving in a convection heater. They considered the problem to be that of transient heat conduction in a stationary system previously discussed, with a boundary condition conforming to equation (107). With these assumptions they proceeded to calculate the material constants involved and specifically the diffusivity.

They found, experimentally, (a) that the diffusivities of dry and conditioned nylon are approximately the same (this merely signifying that the ratio $\frac{K}{\rho c}$ is the same in the two cases), (b) that the diffusivity decreases with temperature, (c) that the heat transfer coefficient is independent of yarn velocity in the range tested (200 - 1,000 ft/min).

They proceeded to calculate the temperature profile of the yarn, using numerical methods, since the variability of $\kappa$ does not allow the use of the published graphical solutions. For the conditions they used (convection heater at 500°C, and a 60/20 yarn of 75 tpi basic twist or 104 tpi actual twist at room temperature), they found that at 0.06 sec residence time the surface temperature $T_s = 119^\circ C$ and $T_s - T_c = \Delta T = 12.5^\circ$. At 0.13 sec $T_s = 180^\circ$ and $T = 9^\circ C$ and at 0.2 sec $T_s = 227^\circ$ and $\Delta T = 8^\circ C$. When $T_s = 232.5^\circ, \Delta T = 7^\circ C$, the "average" yarn temperature is 230°C and 75% of the volume of the yarn is within 5° of $T_s$. Thus $\Delta T$ decreases as $T_s$ increases, and
Arthur and Jones conclude that since in texturing practice the yarn surface temperature is typically 200°C at about 0.4 - 0.5 sec. residence time, the temperature is essentially uniform across the yarn cross section.

Next they utilize equation (105) valid for negligible temperature gradients (incorporating the temperature dependence of $C_p$ given in (R33)) to calculate $h$ which they find to be $3.7 \times 10^{-3}$ cal/sec/cm²/°C for 30 denier and to decrease to $3.15 \times 10^{-3}$ for 70 denier yarn. They do not offer any explanation for this change.

**Calculation of $m$**

With these data it is possible to obtain an approximate value for $m = \frac{h}{k_B}$. Since $K = \frac{m c_p}{P}$, using Arthur and Jones' data at two temperatures, 20°C and 220°C, we have

$$K_{20} = 0.95 \times 10^{-3} \times 0.345 \times 1.14 = 3.73 \times 10^{-4}$$

$$K_{220} = 0.58 \times 10^{-3} \times 0.582 \times 1.08 = 3.63 \times 10^{-4}$$

For 70 denier yarn we have, always using Arthur and Jones' data

$$m = \frac{3.68 \times 10^{-4}}{3.15 \times 10^{-3} \times 1.14 \times 10^{-2}} = 10.25$$

"critical" value of 6. With conditioned nylon, whose reported value of $C_p$ is higher, $m$ would be higher. On the other hand, increased values of $h$ would necessarily lower $m$. 
Conclusions:
These results answer the questions that prompted Arthur and Jones to undertake this research. They had observed that at $T_s = 230^\circ$ and 0.35 sec. residence time, the yarn properties were poor; at $T_s = 230^\circ$ and residence time 0.7 sec., these properties were optimum. They were uncertain as to the why, since it was possible that a big temperature gradient might have existed across the yarn radius at short times and, equally possible, that a certain minimum time was essential for setting. Their work indicates the latter to be the case, since the temperature gradient was found to be negligible.
3. Experiments with a Conduction Heater

Heat Transfer - Heating Step

Morris and Roberts (R25) studied the heating of the yarn and its effect on the resultant crimp properties. They used a Scragg Minibulk machine equipped with a standard conduction heater; this heater could be replaced by another of the modular type which allowed the temperature profile to be varied at will. Their experiments were analyzed assuming the temperature gradient to be negligible based on the results of Arthur and Jones. The governing equation is equation (105).

\[
\frac{T_f - T}{T_f - T_i} = \exp \left( - \frac{2h}{bpc} t \right) = \exp \left( -k_1 t \right)
\]

Morris and Roberts assumed the value of \(k_1\) to be constant for nylon. However, they could not calculate it (since they did not know the value of \(h\) for the conduction heater), so they proceeded to measure it, using equation (108) and the yarn surface exit temperature (which was close to the melting point); they found \(k_1\) to be 4.8. This is much higher than the apparent value of \(k_1\) obtained by Arthur and Jones at their highest reported temperature (this value for 227°C was 2.74 and extrapolated to 260°C it is 2.4, or exactly one-half the Morris and Roberts value). The Arthur and Jones data refer to a 60 and the Morris and Roberts to a 70 denier yarn; but Arthur and Jones found \(h\) for such yarns to differ by about 10%. These results show, therefore, that the heat transfer coefficient \(h\)
is considerably better (by a factor of 2) for the conduction heater.

However, Morris and Roberts assumed $k_1$ to be constant with temperature, on the basis of Jones and Porter's data \( \text{(R34)} \) on conditioned nylon. Arthur and Jones assume $k_1$ to be temperature-dependent (on the basis of Wilhoit and Jones' data \( \text{(R34)} \) which shows $C_p$ for dry nylon to be temperature-dependent). From Arthur and Jones' published data, it is possible to calculate $k_1$ via equation (105), and this is indeed found to be temperature-dependent. Multiplying $k_1$ by the corresponding value of $C_p$ yields the quantity $2h/b$ (cf Equation (106)) which is seen to be essentially constant; this lends support to Arthur and Jones' argument and, therefore, casts doubt on Morris and Roberts' argument concerning the constancy of $k_1$ during the heating of the yarn. This is shown on the Table below (data from Ref. 2a, Fig. 6, $T_f - T_i = 475^\circ$).

<table>
<thead>
<tr>
<th>Yarn Surface Temp. °C</th>
<th>Yarn Average Temp. °C</th>
<th>Residence Time from Eq'n \text{A&amp;J}</th>
<th>$k_1$ from ( \text{Eqn. (105)} )</th>
<th>$k_1 \times C_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 44</td>
<td>456 0.01</td>
<td>4.08</td>
<td>0.377</td>
<td>1.54</td>
</tr>
<tr>
<td>120 116</td>
<td>384 0.06</td>
<td>3.54</td>
<td>0.472</td>
<td>1.67</td>
</tr>
<tr>
<td>180 177</td>
<td>323 0.13</td>
<td>2.96</td>
<td>0.552</td>
<td>1.63</td>
</tr>
<tr>
<td>227 225</td>
<td>275 0.20</td>
<td>2.74</td>
<td>0.615</td>
<td>1.68</td>
</tr>
</tbody>
</table>
Further, if we accept Morris and Roberts' value of \( k_1 = 4.8 \), this means, as discussed, that the value of the heat transfer coefficient for the conduction heater is double the value found for the convection heater by Arthur and Jones. This, in turn, would halve the value of \( m = \frac{k}{h_b} \) to 5 which, in turn, implies that the temperature gradient across the yarn radius may no longer be negligible.

Heat Transfer in the Cooling Zone

Cooling data of the yarns in air can be represented by an equation analogous to the previous one

\[
\frac{T - T_{air}}{T_H - T_{air}} = \exp \left( -k_2 t \right) \tag{106'}
\]

In Morris and Roberts Fig. 2, plots of \( \ln (T - T_{air}) \) vs \( t \) result in straight lines; these correspond to \( k_2 \) values of 3.4 (for an untwisted yarn running at 324 ft/min) and 2.4 for a twisted yarn running at 59 ft/min, whose actual speed was 59/1.45 = 40.5 ft/min. \( k_2 \) appears to be constant over the temperature interval studied, and this supports the Morris and Roberts hypothesis. That \( k_2 \) is lower on cooling than on heating is also to be expected, since cooling occurs by convection only. The \( k_2 \) (hence \( h \)) dependence on speed probably is real (presumably the faster moving the yarn creates more turbulence), though the two yarns were not directly comparable, since one was twisted and the other was not.
If we accept the velocity dependence of $k_2$ as being real, then we see that faster moving yarns will be hotter when they reach the spindle, since the heat transfer coefficient increases in a slower fashion than the speed: to a 800% increase in velocity there corresponds only a 140% increase in heat transfer coefficient.
The objective of this work was "to provide a simple method for calculating the time-temperature history of a highly twisted yarn passing over a contact heater and cooling thereafter in bulk air". The authors start from the same theoretical premises as Morris and Roberts but use the equation

$$\ln \left( \frac{T_f - T_i}{T_f - T} \right) = \frac{2 \beta t}{\rho b} \quad \text{with} \quad \beta = \frac{h}{C_p}$$

the ratio of heat transfer coefficient to heat capacity.

The subscript H (\( \beta_H \)) is used to denote the heating step and the subscript C (\( \beta_C \)) indicates cooling. They performed their experiments on a "home-made" false twisting unit equipped with a conduction heater and operating with constant tension yarn feed; they used a device similar to that used by Morris and Roberts to measure the yarn temperature. The authors proceed to measure \( \beta_H \) and \( \beta_C \) by plotting

$$\ln \left( \frac{T_f - T_i}{T_f - T} \right) \text{ vs } 2 \frac{t}{\rho b}$$

making the following assumptions:

(a) The yarn denier in the twist zone is determined experimentally as a function of untwisted denier, twist and tension, hence the contraction is known. Given the take-up speed, the actual speed of the yarn is calculated, as are the residence times on the heater and in the cooling zone.

(b) The twisted density for nylon 6.6 is taken to be 1.04 (as determined by Arthur and Jones), and with that and the knowledge of the denier, the yarn radius \( b \) is calculated.
For speeds of the order of 60 ft/min (to take a point where Morris and Roberts have comparable results) and for 70 denier yarns, the equation becomes

\[ \ln \left( \frac{T_f - T_i}{T - T_f} \right) = 6.28 \cdot 10^6 \cdot 10^6 \cdot \frac{t}{T_f} = 0.2 \cdot \frac{t_{act}}{77} \]

or a value different by an order of magnitude from that calculated by Morris and Roberts. Thus the results of the two papers are in conflict. However, the heat transfer coefficient increases with speed as Morris and Roberts have found (but at a slower rate than the speed itself). If the present data are accurate, then cooling is considerably slower than anticipated from the (limited) data of Morris and Roberts.

Results for Other Fibers

Experiments similar to those performed on nylon were also carried out with other fibers: Terylene (PET), Ulstron (PPR), Lycra (elastomeric) and glass. The following results were obtained:

<table>
<thead>
<tr>
<th>Fiber</th>
<th>( h \cdot b \left( 10^6 \text{ cal/cm\text{sec}} \right) )</th>
<th>( C_p \left( \text{cal/g }^\circ \text{C} \right) )</th>
<th>( h \cdot b \left( 10^6 \text{ cal/cm\text{sec}} \right) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nylon 6.6</td>
<td>142</td>
<td>0.79</td>
<td>112</td>
</tr>
<tr>
<td>Terylene</td>
<td>251</td>
<td>0.45</td>
<td>113</td>
</tr>
<tr>
<td>Ulstron</td>
<td>184</td>
<td>0.70</td>
<td>129</td>
</tr>
<tr>
<td>Lycra</td>
<td>117</td>
<td>0.82</td>
<td>96</td>
</tr>
<tr>
<td>Glass</td>
<td>402</td>
<td>0.32</td>
<td>129</td>
</tr>
</tbody>
</table>

The \( C_p \) was estimated by measuring the energy input into the heater necessary to raise the temperature of a known mass of yarn by a known amount. The heat transfer coefficient seems
to be the same for all yarns and the average value of the quantity \( h \times b = 112 \times 10^{-6} \). These data strongly suggest, then, that the rate of heat transfer is a function of denier and a function of specific heat. A general equation can be used with this relation to cover all yarns studied, that equation being

\[
\ln \left( \frac{T_F - T_i}{T_i - T} \right) = \frac{703}{C_P} \times \frac{t_n}{D_n}
\]

Indeed, plotting the \( \log \) vs. \( \frac{t_n}{C_P D_n} \) results in a straight line for all yarns. In experiments on cooling the product \( h \times b \) was not constant, but found to be

\[
h_c \times b = \left[43.5 + 3.1 (Sb)^2\right] \times 10^{-6} \text{cal/cm sec } ^\circ\text{C};
\]

however, like its counterpart, it does not depend on the nature of the fiber. The heat transfer equation becomes:

\[
\ln \left( \frac{T_i - T_F}{T_i - T} \right) = \frac{6.28}{C_P D_n} \left[43.5 + 3.1 (Sb)^2\right] t_n
\]

and it covers all yarns (except Lycra, which could not be false-twisted).

Thus, the present authors extend the work of Morris and Roberts to other yarns; in experiments where conditions overlap, their results are in reasonable agreement with those of Morris and Roberts, as far as heating is concerned. However, the cooling rates they find are substantially lower than those found by Morris and Roberts.
1. Introduction

Crimp rigidity, crimp contraction, crimp intensity, crimp index are all terms that are used in practice to measure the extensibility of a textured yarn due to crimp removal. This turns out not to be a straightforward operation, because a textured yarn is a very delicate entity, and it is essentially impossible to reproducibly define an unstressed length for it; further, it is impossible to remove mechanically the crimp without at the same time straining in tension the filaments themselves. These problems have been obviated in practice by stressing the yarn, to remove the crimp, to a stress level $\sigma_2$ commonly 0.1 g/d which corresponds to a small and known extension of the filaments and using the length $l_2$ thus measured (which is reproducible) as the reference length for the test; to allow more or less reproducible measurement of the "unstressed" yarn length, one measures a length $l_1$, corresponding to some small level of stress $\sigma_1$.

The crimp rigidity is thus usually expressed as

$$CR = \frac{l_2 - l_1}{l_2} \times 100$$

(107)

and it has the significance of a compressive or contractive strain that a yarn of equilibrium length $l_2$ would undergo upon application of a compressive load (or removal of a tensile load) $\sigma_2 - \sigma_1$. It varies from 0 to 1 (or 0 to 100).
Had the crimp rigidity been expressed as extensibility in classical tensile terms, (for instance, as \( e = \frac{1_2 - 1_1}{1_1} \)) one could obtain values ranging from zero to infinity with reference, however, to an arbitrary and not easily reproducible gage length. (CR and \( e \) are related, of course, by \( e = CR/(1-CR) \)).

In addition to the plethora of synonyms for the term "crimp rigidity", a number of terms have also been used to describe the process (or processes) that regenerates the crimp, which is usually latent as a result of a setting process that occurs after the textured yarn has been put on a package. These include the terms "crimp development", "crimp release" and, sometimes, "relaxation".

2. Tests for Measuring Crimp Rigidity

In what follows will be given an outline of some of the tests used in practice.

**Development of Crimp in Hot Water: The Heberlein Test** \(^{R2,R28}\).

In this method, recommended by Heberlein for textured polyamide-fiber yarns, several skeins are first reeled off, the tension applied for removing the crimp during winding normally being about 0.1 g/den. The skeins are then placed in a relaxed state in water at 60°C for 10 min, after which they are centrifuged and dried without tension for 24 hours under normal atmospheric conditions.

The skeins are rewetted by immersion for 30 sec in water at 60°C containing about 2 g/l of wetting agent. The lengths
$l_f$ of wet stretched skeins under a load of 0.2 g/den are measured after 1 min. The skeins are then dried without tension in an oven for 60 min at 60°C and cooled in a conditioned room for 60 min. Their lengths under a load of 0.002 g/den are measured after 1 min. The crimp contraction is given by $C.C. = \frac{(l_f - l_o)}{l_f} \times 100$.

For polyester-fiber textured yarns, Heberlein's procedure is similar. The skeins are placed in water at 60°C, which is heated to the boiling point in 15 min. and then kept at the boil for a further 15 min. The table below gives typical values of crimp contraction for nylon yarns textured by various means (as determined by this test).

<table>
<thead>
<tr>
<th>Yarn Type</th>
<th>Percent (Crimp Contraction)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conventional Helanca stretch yarn (twist - heat-set - untwist)</td>
<td>about 70</td>
</tr>
<tr>
<td>False-twist type stretch</td>
<td>60 - 65</td>
</tr>
<tr>
<td>Modified stretch (post-treated, stabilized or set)</td>
<td>20 - 25</td>
</tr>
<tr>
<td>Stuffer-box textured</td>
<td>20 - 30</td>
</tr>
<tr>
<td>Edge-crimped textured</td>
<td>40 - 65</td>
</tr>
</tbody>
</table>

**Development of Crimp in Water at Room Temperature. The H.A.T.R.A. Test (R36).** In this test, a skein of unrelaxed yarn is immersed in water at room temperature under a tension of 0.1 g/den for 2 min. The major part of the tension is re-
moved, the yarn being under a tension of 0.002 g/den, and 2 min. later, the crimp rigidity (or percentage crimp contraction) is read directly from a scale.

Certain types of bulked yarn are first relaxed by immersion in hot water and subsequent drying, and the procedure for un-relaxed yarns is then followed.

**Development of Crimp in Steam** \( (R36) \). In this method, several skeins are reeled off under a tension of 0.1 g/den. Their lengths are measured after 1 min. under the same tension and then again after a further period of 1 min. under a tension of 0.2 g/den. The skeins are steamed for 1 min. under a tension of 0.002 g/den, and their lengths are remeasured.

**Development of Crimp in Water and Measurement of Skein Length as Function of Time** \( (R35) \). The skeins are completely immersed in water at 60°C for 10 min., dried in an oven at 60°C for 60 min. and cooled in a conditioned room for 1 hr. They are tensioned at 0.1 g/den for 1 min., and their lengths are measured. The load is reduced to 0.002 g/den, and the skein lengths are measured as a function of time over a period of 10 min. or 1 min.

**Further Variations in Procedure.** In addition to time, temperature, immersion medium, and tension as mentioned above, the fineness of skeins may be varied. Some additional variants on the tests described above are given in \( (R2) \).
Development of Crimp in Hot Air (R36). Skeins of 50 den are reeled off under a tension of 0.05 g/den; they are loaded to a tension of 0.001 g/den and treated in an oven in ventilated air at a given temperature for 5 min. They are then cooled for 5 min., and the skein length, \( l_1 \), is measured under 0.001 g/den tension. The load is increased to 0.1 g/den, and the length \( l_2 \) is measured after 30 sec. For each test, it is possible to obtain a result within 30 min; this time is rather short in comparison with that needed in other methods.

The Crimp Index Test (R37)

(a) Determination of yarn denier: Remove a length of yarn from the bobbin, apply a stress of 0.1 g/d; (based on un-textured denier) and measure the extended length; weigh the yarn and calculate the denier.

(b) Procedure: Wind skeins of 5000 total yarn denier. Load skeins with "light" load (3 loads are used: 0.5 mg/den, 15 mg/den and 50 mg/den). Develop crimp as per (c) below; cool and/or dry and condition (always under one of the loads above) and measure the length after conditioning (still under load), \( l_o \). Add a load of 0.1 g/den and measure length \( l_f \). Calculate crimp index as

\[
\text{C.I.} = \frac{l_f - l_o}{l_f}
\]

We obtain thus 3 values of crimp index each corresponding to a given "light" load.
(c) **Crimp Development:**

1. Hang skeins (under light load) in 100°C air for 10 min. (note that sticky finishes may interfere with crimp development).

2. Immerse skeins (under light load) in cold water (watch out for entrapped air bubbles which tend to float the skeins).

3. Immerse the skeins in cold water (as above) and heat water to the boil (or 98°C) at the rate of 2°C per minute. Use transparent container to allow viewing the skein.

**Crimp Permanence Test (R3).** After measuring the crimp index per above procedure, add to the lightly loaded skein a load of 0.1 g/den and leave weight on for 24 hrs., at the end of which remove the extra weight and measure the skein length after 10 sec., 15 min. and 24 hrs. From these data calculate percent recovery.

**Continuous Method for Measuring Crimp.** A continuous method has been developed by Luenenschloss (R38). The yarn is fed under high tension (sufficient to remove the crimp) through a first set of rolls, and it is kept under a lower constant tension thereafter. Under this low tension the yarn contracts and is taken up in the contracted (crimped) state by the second set of rolls (which rotate slower than the first and whose rotational speed is adjustable). A very light weight is suspended in the yarn between the rolls, and its position
(height) is monitored photoelectrically; the yarn path between the rolls is thus triangular. For a given value of crimp rigidity the weight and the takeup speed are suitably adjusted, and the machine is set in motion. If portions of the yarn have a different crimp rigidity, the position of the weight is affected: a servomechanism then adjusts the takeup roll velocity so as to bring the weight to its original position. The variation in the takeup roll velocity is continuously recorded. This leads to a definition of a "continuous crimp contraction"

\[ \text{CCR} = (1.00 - \frac{V_2}{V_1}) \times 100 \]

where \( V_2 \) and \( V_1 \) are the peripheral velocities of the takeup and feed rolls, respectively.

The significance of the variation of the takeup speed, in order to keep the portion of the weight unchanged, would be equivalent in the classical C.R. test to varying the gage length (or skein length) in order to get the same value of the contracted length of the skein (i.e., under the light load) as the crimp rigidity changes.

The information from the takeup roll can be fed directly into a computer which can then calculate averages, mean, and standard deviations and other pertinent statistical data, autocorrelation and spectrum functions.

In analyzing the effect of various parameters on the CCR, the author found that the optimum development temperature (in a convection oven) is 130°C and that about 1 minute is required to activate the crimp to a maximum. Typical results
for nylon 6.6, 70 denier textured at a theoretical twist of 65 tpi exhibits a continuous contraction value of the order of 70%, while the same yarn, tested by the Heberlein method exhibits a crimp contraction value of the order of 50%. 
3. Optimization of Crimp Development

Optimum Temperature for Crimp Development in Hot Air

Busch (R36), who proposed that crimp be developed in hot air (as previously described) carried out a series of tests wherein he developed the crimp at different temperatures and measured the crimp rigidity in order to define an optimum development temperature. He observed that the crimp rigidity vs. temperature curve for all fibers considered passed through a maximum value at some temperature: he chose this temperature, $T_E$, as the optimum development temperature, since it afforded the greatest possibility of discriminating between samples. For the samples he examined $T_E$ was as follows:

<table>
<thead>
<tr>
<th>Polymer</th>
<th>$T_E$, °C</th>
</tr>
</thead>
<tbody>
<tr>
<td>PET Stretch</td>
<td>100-110</td>
</tr>
<tr>
<td>PET Modified stretch</td>
<td>100-110</td>
</tr>
<tr>
<td>Nylon 6 Modified str.</td>
<td>100</td>
</tr>
<tr>
<td>Nylon 6.6 Modified str.</td>
<td>110</td>
</tr>
<tr>
<td>Nylon 12 Stretch</td>
<td>55-75</td>
</tr>
</tbody>
</table>

Busch observed that regular stretch yarns exhibit a more clearly defined maximum for $T_E$ than modified stretch yarns; the latter, of course, have already been heat relaxed, having been passed through a second heating cycle in the machine.
For these yarns essentially constant values of crimp rigidity can be obtained by developing the crimp over a broad range of temperatures.

Effect of Crimp Development Method on the Value of Crimp Rigidity

Brook, Eames and Munden (R 39) assessed the influence of the crimp development step on the resultant value of the crimp rigidity. Their experiments were performed on single yarns (rather than skeins) and they gauged the (developed) crimp in air, by measuring the length of the yarn successively under a load of 0.0002 g/den ($l_1$), under a load of 0.1 g/den ($l_2$) and, again, under a load of 0.02 g/den ($l_3$). Each load was applied for 1 min. They thus, calculate two values of crimp rigidity which they call, respectively, "yarn percentage collapse" = ($l_2$ - $l_1$) / $l_1$ x 100, and "yarn crimp rigidity" = ($l_2$ - $l_1$ / $l_1$) x 100%.

They evaluated a conventional nylon 6.6 twist-set-untwist yarn, 2 false-twisted nylon 6.6 yarns (one of which was a modified-stretch yarn which had been passed through two heaters to stabilize the crimp), 1 false-twisted PET yarn, and 1 edge-crimped nylon 6.6 yarn. The crimp development procedures were as follows:

(a) Yarn wet relaxed in water at 20°C for 2 hr.
(b) Yarn wet relaxed in water at 40°C for 2 hr.
(c) Yarn wet relaxed in water at 70°C for 2 hr.
(d) Yarn dry relaxed in an oven at 100°C for 2 hr.
(e) Yarn steam relaxed for 2 min. on the bottom bed of a Hoffman press.

For purposes of comparison, the crimp rigidity of each yarn was also measured via the standard H.A.T.R.A. test. Concerning crimp rigidity they find that:

(a) In all cases the H.A.T.R.A. crimp rigidity figure is lower than the results obtained under the other test.

(b) For regular stretch nylon yarns the temperature of the wet relaxing medium has a relatively small effect on the crimp rigidity; dry heat relaxation at 100°C is as effective as wet relaxation.

(c) For modified nylon stretch yarns higher wet temperatures (70°C and above) are needed to generate maximum crimp; moreover, dry heat relaxation at 100°C is less effective than hot wet relaxation.

(d) For PET the relaxation (i.e., crimp development) temperature plays an important role and high temperatures are preferred.
Discussion: Factors That Affect Crimp Rigidity Measurement

The Problem of Yarn Sensitivity

A textured yarn is very sensitive to handling because:
1) the crimped filament modulus is very low; 2) the filaments snag easily and also intermingle readily; the friction between filaments thus may critically affect the length, and is a factor contributing to an anelastic behavior on recovery from extension. These factors make it virtually impossible to assess an unstrained gage length. In false-twist yarns we have the additional problem of torque which tends to twist the unrestrained filaments. These factors are minimized if one deals with a skein of yarn and if one uses plied or balanced torque yarns.

Processing History

This, of course, is the prime parameter that determines crimp and a number of studies have been made to assess the influence of process variables on the crimp rigidity. These have been discussed elsewhere and will not be considered here.

Packaging

The conditions of packaging (and, in particular, the level of tension) affect the crimp by imparting to the yarn a "temporary" set. Both the overfeed to the package and the yarn position in the package are thus important. It is not known whether all of this set is temporary, but one may pre-.
sume that it does have some permanent effects since, structural changes (perhaps further crystallization or humidity conditioning, or both) are occurring in the fiber for a period of time after it has been packaged. That freshly made yarn is not "equilibrated" is known because crimp rigidity changes as function of time on the bobbin (R2, p. 270) and because freshly made yarn is even more sensitive to handling than "equilibrated" yarn. These structural changes will clearly be affected by the tension level and the filament geometry of the yarn on the package.

Crimp Development

Of the various means for developing the crimp, temperature (or temperature/plasticizer) is the most common and water is the usual immersion medium. Even here, however, a number of points are critical:

The temperature: water at about 100°C is usually recommended. Originally, when the nylons were the only synthetic fibers being textured, relatively low water temperatures were used (such as 60°C). Water, however, is not a good plasticizer for polyesters and higher temperatures were required for activation, hence, the use of 100°C water. Steaming and dry heat have also been practiced.

Another critical parameter is the stress under which the crimp is being developed. The crimp rigidity is affected by the level of stress.

In addition to the medium, the stress and the tempera-
ture, the length of time a specimen is subjected to the high temperature may be also critical.

The Crimp Rigidity Test Itself

A number of practical difficulties arise in the performance of the test:

In case of torque yarns, the skein is twisted under a light load, but tends to untwist under a heavy load.

On load removal, if the skein is prevented from twisting, filaments will tend to snarl upon retraction and when the skein is finally let free, its contracted length is different than that from a material which was allowed to contract while twisting.

The retraction of the skein is strongly time-dependent, presumably because of the interfilament friction (and, to a lesser extent, because of the viscoelastic properties of the filaments); successive loadings at relatively short time intervals lead to different values of the crimp rigidity because of the change in $l_1$. The greatest change occurs when a specimen is loaded for the first time, as this is when the maximum number of rearrangements takes place. The skein changes almost permanently with each measurement, so that the measurement is in a sense a destructive test.

Crimp rigidity tests attempt to measure the maximum extensibility and it is conceivable that this is not the quantity that is of most practical importance, as the yarn in a knit structure is not free to contract to an unstressed state, not
is it habitually extended to a fully uncrimped state. Therefore, for practical purposes, a test using loads intermediate between those used in practice may be useful.
EFFECT OF MACHINE SETTINGS ON YARN QUALITY

1. Crimp Rigidity, Crimp Contraction

The work of Burnip, et al.

In the course of the experiments described earlier (Appendix II,(a)) Burnip also measured the extensibility of the crimped Nylon yarns. They reported the extensibility as the percent extension between loads of 0.006 and 0.09g/den; from this data crimp rigidity (CR) values can be calculated.

The results of the constant-tension and constant-extension experiments show that CR depends strongly on temperature: the higher the temperature, the higher the crimp rigidity. Temperature by decreasing the crystallinity and increasing the segmental mobility of the noncrystalline regions, serves to relax the stresses imposed during twisting. Then, as recrystallization occurs, the new structure is locked in and is "set"; the higher the temperature, the higher the proportion of material which has melted and will recrystallize.

The effect of pretension and overfeed was also studied in both the constant-tension and constant-extension experiments. At very low tension levels (2.5g) the CR is low, but increases with tension up to a certain level (7 g) beyond which the CR decreases as the tension is further increased (to 16 g). In the case of yarns processed at very low tension, some degree of ballooning was noticed below the spindle; further, these
yarns exhibited a number of wild filaments; all this suggests an inadequate degree of threadline control during processing at very low tensions. Yarns processed under high tension conditions were uneven and exhibited "necking" (tight areas) in places.

It is also possible that when the yarn is kept under high tension, the fibers keep their orientation even at high temperatures. Perhaps when the molecules remain oriented in the longitudinal direction (as they do when the applied stress is high), they find it difficult to "set" in bending and in torsion.

The effect of twist has also been studied by Burnip in the constant-tension machine: stretch increases and, beyond a certain level, decreases with twist. While it is to be expected, of course, that low-twist levels should give little stretch, it is more difficult to understand why stretch should decrease beyond a certain level of twist. The authors, considering the highly twisted yarn, (which, after untwisting, still looks highly twisted with no bulk at all) suggest that "high twist binds the loops firmly into the core of the yarn and so prevents its easy removal on untwisting". An analogous suggestion is to be found in the literature. One must also note that, in Burnip's experiments, increased twist entails an increased residence time in the heater.
Other workers (R2 p. 74), using ARCT-FT-1 type equipment, have observed that the crimp contraction increases with temperature.

Experiments with ARCT-FT-3 (R2) indicate that an increase of overfeed from zero to 4% increases the crimp contraction in good agreement with Burnip's data.

The Work of Morris and Roberts
Morris and Roberts (R21) found that the crimp rigidity CR, at constant heater contact time, increased linearly with heater temperature, though a maximum is reached as we approach the melting point of the material. The CR maximum occurs at lower heater temperature and is higher in value as the residence time increases (i.e., yarn speed decreases).

The writer feels that Morris and Roberts have correctly identified the occurrence of the maximum with the "melting" of the yarn: he calculated from data presented in their Fig. 3 the yarn temperature corresponding to the point immediately following the maximum and found that the yarn temperature (using $k_1 = 4.8$) was of the order of 250°C which is the reported melting point for this yarn. The crimp rigidity maximum is lower, the higher the yarn speed, because high yarn velocities correspond to lower residence time in the heater and to appreciably higher yarn temperatures at the spindle (the yarn is hotter when untwisted).
Morris and Roberts also found, by crossplotting the data above, that at constant heater temperature the crimp rigidity increases with contact time, but the rate of increase gradually decreases to zero, i.e., the crimp rigidity becomes (asymptotically) constant at sufficiently long times (of the order of 2-3 seconds in their particular experimental setup). They also observe, significantly, that the rate of temperature increase of the yarn, with time, is considerably faster than the rate of increase of the crimp rigidity. This is in excellent agreement with the findings of Arthur and Jones, namely that for good setting the yarn must be kept at the high temperature for a sufficient amount of time for molecular changes to occur.

Further, since the variation of contact time above was achieved by varying the yarn speed (hence, among others, the cooling time), some further experiments were made at three different (but constant) yarn speeds (hence, three different cooling times) where the heater contact time was varied by varying the length of the heater (otherwise kept at constant temperature). These results show that at constant heater temperature and constant heating time, the longer the cooling time, the better the crimp rigidity. At "short" heater contact times (of the order of 0.5 to 1.0 sec.), the crimp rigidity increases by about 15% (from 25% to 40%), as the cooling time increases from 0.3 sec. to 1.3 sec. At "long" heater contact times, 2-3 sec., (where the crimp
rigidity has "plateaued"), it still increases, but apparently at a slower rate, with cooling time.

Summary of Temperature Effects

Morris and Roberts' works prove that the variables affecting the crimp rigidity are in order of importance:

(a) The yarn temperature. The higher the temperature, the better the crimp rigidity.

(b) The length of time the yarn has spent at the high temperature, for, since during the "contact time" the yarn is not at constant temperature, the longer and the more effective will be the heat treatment. Referring to the sketch depicting two hypothetical yarn heating rates, yarn B will have been subjected to a better heat treatment, even though at the exit from the heater yarns A and B have the same temperature (we assume that they move at the same velocity).

![Diagram of Yarn Temperature vs Distance along Heater]

<table>
<thead>
<tr>
<th>Yarn</th>
<th>Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>ENTRANCE</td>
<td>EXIT</td>
</tr>
<tr>
<td>Distance along heater</td>
<td></td>
</tr>
</tbody>
</table>
In practice, both yarn temperature and crimp rigidity increase exponentially with contact time, but the former increases faster and "plateaus" much earlier. For nylon 6.6 heating times of the order of 1 second are desirable.

(c) The cooling time also affects the crimp rigidity. Ideally, the yarn should have cooled to room temperature when it reaches the spindle. If it is hot when it reaches the spindle, the increase in tension that it experiences upon untwisting, while still plastic, decreases the level of crimp rigidity. Cooling times of at least 0.5 seconds and preferably as long as 1.5 seconds are desirable.

So maximum crimp rigidity is attained by operating at as high a temperature as possible and maximizing both heating and cooling time. In practice, of course, compromises are necessary, and there is always the possibility of some tradeoff between heating time and temperature; however, the least expensive way of increasing the crimp rigidity is simply to increase the cooling time by increasing the yarn path. Incidentally, trade opinion holds it that convection heaters give "better" and more uniform yarn than conduction heaters.
2. Yarn Breaking Strength

Effect of Temperature
The strength decreases as the temperature is increased, although this drop is small up to heater temperatures of the order of 210°C (R2, p. 67). Similar effects are reported by Burnip (R26) on both constant tension or constant extension equipment at residence times varying from 0.5 to 1.8 sec.

Effect of Twist
The data are not all in agreement, but suggest that a decrease in strength occurs as the twist is increased. Burnip et al (R26), experimenting with a very broad range of twist values (50 to 170 tpi) on the constant tension apparatus, observed "an overall trend towards reduction in strength as the twist is increased". However, "this was not statistically significant at the 95% confidence level".

The data on ARCT-FT-1 (R2, p. 76) which cover a much narrower twist range (60-80 tpi) suggest the existence of a maximum in the vicinity of 60-70 tpi.

In all of these experiments the basis of comparison is the breaking strength of the untextured yarn and, therefore, the effects of twist and temperature are confounded.

The Effect of Overfeed and/or Tension
The ARCT-FT-1 experiments indicate a decrease in breaking strength as overfeed is increased from 2 to 6% (R2 p. 345).
The ARCT-FT-3 experiments (overfeed 0 to 4%) and the Leeson 553 experiments (overfeed 2 to 4%) show either a slight increase (at low denier filaments) or no change (R2 p. 347).

Burnip (R3), operating on the constant extension apparatus, found a trend towards higher strengths at higher overfeeds.
3. Other Effects

These are listed, but only in general terms by El-Behery\(^{(R27)}\).

This list of material parameters that affect the properties of the textured yarns is as follows:

**Variation Between Supply Packages:** That this is a problem is shown by the work of Morris and Roberts\(^{(R21)}\) who state that, "under carefully controlled laboratory conditions, there is almost exact correspondence between dye uptake differences before and after texturing. Under production conditions, further variation is added by the bulking process." This is a problem which is very difficult to control, and no easy solution to it is seen.

**Type of Material to be Processed:** It is stated that if we assume the yarn to be composed of a number of identical helices of diameter \(A\), the retractive force of the yarn is

\[
F_Y = \frac{K E D d}{A^2};
\]

\(K\) is a constant, \(D\) and \(d\) are the denier of the total yarn and the filament, respectively, and \(E\) is the Young's modulus for the filament. So at constant \(F\) and \(D\) a change in \(E\) can be compensated by a change in \(d\).

**Filament Cross-section:** It is stated that textured yarns produced from multilobal cross-section have sparkle, glitter and a silk-like hand and appearance.
Total Denier: The author states that for yarns of different total denier, false-twisted with the same twist factor, the retractive force is independent of the total denier and depends on the filament denier. Also different migration patterns may be noticed.

Filament Denier: It has been found that the fewer the number of filaments in a stretch yarn of given denier, the greater its stretch and resistance to extension.

Effect of Yarn Structure: Plied yarns give better stretch and a more vigorous recovery in the resultant fabric than single yarns of equivalent total denier.

Effect of Spin Finish: The spin finish on the yarn before texturing will affect the amount of tension desired, the heat transfer rate, the amount of twist needed, and the yarn speed.
REFERENCES

R1 Comptes-Rendus of the International Study Session on Textured Yarns, J.E.T. (1969), Lyon


R6 Luenenschloss, J. & Weinsdoerfer, H., Chemiefasern 20, Nov. 1970


R9 Krause, H.W., "Bulk, Stretch & Texture", The Textile Institute 1966


REFERENCES (continued)


R19 Brookstein, D., Private communication


R21 Morris, W.J. & Roberts, A.S., "Bulk, Stretch and Texture", The Textile Institute, 1966

R22 Thwaites, J.J., Threadline Torsional Characteristics in the False-Twist Process; to be published


R25 Notes on Differential Geometry of Straight Yarns. M.I.T. Textile Division

R26 Burnip, N.S., Hearle, J.W.S., Man Made Textiles, October 1960, P40, November 1960, P54

R27 El-Behery, A., Clemson Review, 1971

R28 Bulletins by ARCT, Heberlein, Sotexa, Leesona, etc.


REFERENCES (continued)

R36  Busch, H., "Bulk, Stretch and Texture", The Textile Institute, 1966

R37  Backer, S., Private communication

R38  Luenenschloss, J. et al, Chemiefasern 21, 41, Jan. 1971

R39  Brook, G., Eames, J.R. and Munden, D.L., "Bulk, Stretch and Texture", The Textile Institute, 1966
The writer was born in Greece in 1933 where he completed his secondary education in 1951. His undergraduate studies were done in Belgium at the Ecole Superieure des Textiles de Verviers, and he came to M.I.T. after their completion in 1955, to what was then called the Textile Division of the Department of Mechanical Engineering. He obtained a Master's Degree in 1958. For about seven years he was directly involved in industrial research and development, and for four years thereafter he was a research administrator at Cabot Corporation. His research work originally dealt with fundamental polymer characterization via infrared, microscopy and X-rays, molecular weight distributions, stress cracking and rheology; at a later stage he became involved with process development, i.e., compounding and extrusion of crosslinked high density polyethylene pipe. He was then promoted to Research Manager in which capacity he continued to direct research; in addition, he was in charge of Customer Service. His last position with the Cabot Corporation was that of Research Manager, Cab-O-Sil (a pyrogenic silica of extremely large surface area) and Cab-O-Lite; he directed a Research group, a Technical Service group, and a Pilot Plant group.

The writer resigned his position at Cabot in order to return to M.I.T. for his doctoral program. Since 1972 he has been associated with Tufts University as an Adjunct Professor, lecturing on the Chemistry and Physics of Polymers.

