SQUEEZE-FILM DAMPING FOR MEMS STRUCTURES

by

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Chair, Department Committee on Graduate Students
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ABSTRACT

This thesis presents the study of the isothermal compressible squeeze-film damping effects for MEMS devices. The squeeze-film damping governing equation, the isothermal Reynolds equation, is the fundamental equation in this work. For small amplitude sinusoidal motions, which are governed by the linearized form of the Reynolds equation, both damping and compressibility effects are modeled numerically. Analytical solutions of the linearized Reynolds equation for structures with various simple geometries are summarized; comparison and discussion of different slip-flow models from previous studies are also provided. A procedure of solving the linearized model, which is leveraged by using typical commercial finite-element fluidic packages, is demonstrated. A numerical example of dynamical macromodel for a capacitive accelerometer indicates that viscous damping dominates at low frequency, and air-spring effect gives rise to a shifting in resonance at high frequency. The theory and method of estimating damping effects for flexible devices with small amplitude motions are also presented. The theory is derived from the structural dynamics modal analysis and the simulation of the linearized Reynolds equation. Experimental results show excellent agreement in quality factor and resonance shifting. For large amplitude motions, the Reynolds equation is simulated by finite-difference codes. Simulation shows that the nonlinearity of the Reynolds equation results in the nonlinear effect of pressure force with respect to motions. A multi-energy-domain numerical model, which consists of fluid dynamics, structural dynamics and electrostatics, is formulated and simulated for the pull-in dynamics of a fixed-fixed beam microrelay. With appropriate adjustment for stress-stiffening effect due to nonlinear bending of the beam, experimental pull-in-time results agree with the simulated results. Perforated devices are common in MEMS, but notorious for the complexity in damping modeling. An efficient numerical model with acceptable accuracy is proposed. This model uses pipe-flow resistance elements to account for the pressure leakage effects through perforations. Simulation and experiment results for a perforated accelerometer indicate that the model gives significant improvement in damping estimation compared with the results given by the methods suggested in previous studies.

Thesis Supervisor: Stephen D. Senturia
Title: Barton L. Weller Professor of Electrical Engineering
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CHAPTER 1

INTRODUCTION

1.1 Background

Recent progress in integrated circuit technology has enabled the fabrication of microelectromechanical systems (MEMS), such as micro accelerometers, pressure sensors, gyroscopes, switches, mirrors and resonators [1]. MEMS devices often use effectively parallel-plate electrodes as the capacitive sensing and electrostatic actuation mechanisms, and many of these devices are operated in air, in some cases for cost reasons, and in other cases to achieve a specific damping of the motion. Figure 1.1.1 shows a typical schematic of an accelerometer, whose top electrode is a capacitive sensing element, and bottom electrode, the electrostatic actuation element. In order to increase sensitivity for sensing, and to increase force for actuation, MEMS designers usually try to increase the electrode area and decrease the gap between electrodes. Since the plate-length to gap-thickness ratio is very large, a fairly small displacement of the plate in normal direction will squeeze (or suck) a significant amount of air flow out of (or into) the very narrow gap. However, the viscosity of the air film limits the flow rate along the gap, and thus gives rise to a pressure distribution against the plate. The total pressure force, which affects the dynamics of the moving plate, is known as squeeze-film damping. Figure 1.1.2 shows a schematic view of squeeze-film damping for a rigid-plate structure.
The theory of squeeze-film damping has been developed decades ago [2][3]. For small-amplitude oscillating devices, Blech [4] and Griffin [5] linearized the squeeze-film damping governing equation, the Reynolds equation, into a compact form and showed that the viscous damping effect of the air film dominates at low frequencies, but spring-like behavior takes over at higher frequencies. The experimental studies in general agree with the theory [6][7][8]. Observation of slip-flow effects due to small gap thickness were also reported in these papers, and different models of effective viscosity have been proposed to account for slip-flow effects. Note that most of these studies were focused on the behavior of small amplitude oscillation for rigid rectangular or circular parallel plates. Starr [9] further simplified the linearized Reynolds equation into an incompressible-flow form, which is the same as a time independent diffusion equation, and showed that the damping coefficient due to an air film can be obtained numerically for moving plates with arbitrary geometries. However, the spring effect due to the compression of the squeezed film was not taken into consideration, and the numerical scheme is not applicable for perforated plates whose hole size is relatively small compared with the plate thickness.

![Figure 1.1.1](image_url)  
Figure 1.1.1 A MEMS device (an accelerometer) with squeeze film damping. (a) is the top view of the moving part, and (b) is the cross-section view.
1.2 Thesis Goal

This thesis is primarily focused on the numerical approach for analyzing squeeze-film damping, which will serve as a tool for MEMS modeling. Basically, this study classifies the types of devices with squeeze-film damping as (1) rigid structures, (2) flexible structures, and (3) perforated structures. Theories and numerical models will be derived and presented to deal with each type of device. Furthermore, for small-amplitude-motion devices, in general, the emphasis is on the characteristics of the frequency response, such as quality factor or resonance frequency. For large-amplitude-motion devices, the focus is on the behavior of transient response, such as rise time or pull-in time. Therefore, proper simulation approaches are needed for the analyses of interest. Experimental data are of importance for verification of numerical results. Simulated and experimental results for various devices will be presented.
1.3 Thesis Outline

In Chapter 2, the squeezed film governing equations, the Reynolds equation and the linearized Reynolds equation, will be discussed. The linearized model is used for structures undergoing small amplitude vibration. Some analytical solutions for the linearized model will be presented. Chapter 3 first introduces a numerical scheme to solve the linearized Reynolds equation. This scheme uses a commercial diffusion equation solver by suitable mapping of the variables onto the linearized Reynolds equation. A home-built code for solving the Reynolds equation for large amplitude motion is also demonstrated in this chapter. Finally, a pipe-flow model for calculating the pressure leakage through perforations is presented. This model is coupled with the Reynolds equation by applying the continuity equation to match the flow rate at each hole. In Chapter 4, an accelerometer macromodel with squeeze-film damping will be presented. The perforation of the proof mass will be modeled by using the pipe-flow model. Simulation shows that the Reynolds equation with the new pipe flow model predicts higher damping ratio values than the value provided by previous studies, and agrees with the experimentally determined damping ratio to within 14 %. Given the strong dependence of damping on geometry and expected dimensional uncertainty, this agreement is acceptable, although not perfect. Furthermore, simulation and testing results of a flexible fixed-fixed beam microrelay are presented in this chapter. For small-amplitude first-mode oscillation, the numerical model provides excellent estimation for both quality factor and resonance frequency; for large amplitude motion, the results of pull-in time simulated from a coupled-energy-domain model match the experimental results very well. The summary and conclusion of this work will be presented in Chapter 5. A table of comparison of simulated and experimental results is provided.
CHAPTER 2

THEORY

In this chapter, two different forms of the squeeze film governing equation, the Reynolds equation, will be introduced: the (isothermal) Reynolds equation and the linearized Reynolds equation. The Reynolds equation is valid for both large amplitude motions and non-uniform deformations. The linearized model is more compact but only useful for small amplitude motion. Some useful analytical solutions for the linearized Reynolds equation will be presented. The solutions show that the spring and damping forces due to squeezed films have a frequency dependence. Also, the slip flow effect is very important in microscale. Some useful effective viscosity models, which account for slip-flow effects, will be discussed in the end of the chapter.

2.1 Isothermal Reynolds Equation

Parallel rigid plates in relative normal motion experience a back-force from the time-dependent pressure distribution in the gas film. The product of the plate velocity and this back-force is the power transferred into the gas film by the plate motion, some of which appears as damping, the rest as stored energy in the compressed gas. The governing equation, the compressible isothermal Reynolds equation [2][3], is a time-dependent non-linear parabolic
partial differential equation. The non-dimensional form of this equation for normal relative motion is

\[(2.1.1) \quad \nabla \cdot (PH^3 \nabla P) = \sigma^* \frac{\partial (PH)}{\partial t}\]

where \(H = \frac{h}{h_0}\); \(P = \frac{P}{P_a}\); \(\sigma^* = \frac{12\mu^*L^2}{P_a h_0^2}\), \(h\) is gap thickness, \(h_0\) is the initial gap thickness, \(P\) is pressure, \(P_a\) is ambient pressure, \(L\) is plate length, \(\mu^*\) is effective viscosity, and \(\sigma^*\) is a characteristic time [2].

The Reynolds equation is a combination of the Navier-Stokes equation, the continuity equation and the equation of state. The derivation of the isothermal Reynolds equation is as follows: we first integrate the Navier-Stokes equation, whose inertia term is neglected, in the \(z\)-axis (see Figure 1.1.2), then obtain mass flow equations which are functions of the pressure gradient in \(x\) and \(y\) directions. Then we substitute the results into the continuity equation and obtain a primitive form of the Reynolds equation, which is a function of density and, of course, has temperature dependence. Under the assumption of isothermal conditions, we can further eliminate the density term by using the equation of state and thus obtain the isothermal Reynolds equation.

The Reynolds equation is actually a \textit{two dimensional} partial differential equation for pressure and assumes that the pressure distribution across the gap is uniform and the fluid velocity component perpendicular to plate surface is negligible. Therefore, the Reynolds equation is only valid for structures with narrow gap thickness \((h<<L)\).

\section{2.2 Linear Model}

Since the isothermal Reynolds equation is a non-linear partial differential equation, in general it has to be solved numerically. However, a lot of MEMS devices, such as accelerometers and gyroscopes, are operated in small amplitude oscillation motions for the sake
of signal sensing or optimal performance. Under such circumstances, the Reynolds equation can be linearized into a diffusion-like partial differential equation [4][5] by assuming (i) small motion amplitude, and (ii) small pressure variation. The non-dimensional form of the linearized Reynolds equation is

\[ \nabla^2 \Theta - \sigma \frac{\partial \Theta}{\partial \tau} = \sigma \frac{\partial e}{\partial \tau} \]  

(2.2.1)

\[ \sigma = \frac{12 \mu^* t^2 \omega}{P_a h_0^2} \]  

(2.2.2)

where \( \tau = \omega t \); \( e = \frac{\delta}{h_0} \cos \omega t \); \( \Theta = \frac{\Delta p}{P_a} \); \( \omega \) is oscillation frequency, \( \tau \) is non-dimensional time, \( \delta \) is amplitude of oscillation (\( \ll h_0 \)), \( \Delta p \) is small variation of pressure (\( \ll P_a \)), \( \Theta \) is the linearized non-dimensional pressure, and \( \sigma \) is the squeeze number, which is proportional to frequency.

The key parameter in the linearized Reynolds Equation is the squeeze number \( \sigma \), which measures the degree of compression of the fluid in the gap. If \( \sigma \) is close to 0 (low speed or frequency), the air film obeys nearly incompressible viscous flow; if \( \sigma \) goes to infinity (at very high speed or oscillation frequency), the fluid is essentially trapped in the gap and behaves like a spring.

### 2.3 Analytical Solutions for Linear Model

For rigid rectangular parallel plates oscillating in normal direction, the analytical solutions of pressure force components [4] of the linearized Reynolds equation are shown in Eqn. (2.3.1). Note that for this case the pressure boundary conditions on the edges of the plates are equal to ambient pressure.
\[
\frac{f_d}{\delta} = \frac{64\sigma}{\pi^6} \sum_{m,n_{\text{odd}}} \frac{m^2 + (n/\beta)^2}{\left(\left(m^2 + (n/\beta)^2\right)^2 + \sigma^2/\pi^4\right)}
\]

\[
\frac{f_s}{\delta} = \frac{64\sigma^2}{\pi^8} \sum_{m,n_{\text{odd}}} \frac{1}{\left(m^2 + (n/\beta)^2\right)^2 + \sigma^2/\pi^4}
\]

(2.3.1)

where \(f_d\) and \(f_s\) are non-dimensional linear damping and spring force amplitudes of the total pressure back-force, respectively, \(\delta\) is the non-dimensional oscillation amplitude, which has been normalized to the nominal gap thickness, and \(\beta\) is the rectangular plate's length/width ratio. Note that \(f_d\) and \(f_s\) are normalized to \(P_0 \beta L^2\).

The solution has two parts: a linear damping-force amplitude and a linear spring-force amplitude. By definition, the linear damping force is always in phase with the plate velocity, and the linear spring force is in phase with the plate displacement. These two components are also functions of the squeeze number. Figure 2.3.1 shows the relationship of two force components to the squeeze number. The solutions are derived by assuming that the total pressure is a superposition of the spatial sinusoidal pressure shapes with required boundary conditions. The analytical solutions converge rapidly; only the first few pressure modes are necessary to give good accuracy. At small squeeze number (low frequency), the viscous force dominates because the air has enough time to pass through the gap without being compressed. At large squeeze number (high frequency), the spring force increases because the plate is either too big or moving too fast so that the air is trapped in the gap, which gives rise to the compression of the air film.

For a rigid rectangular plate oscillating in tilting mode over a stationary plane, the analytical solution of the torque components due to pressure force [10] are
\[
\tau_d = \frac{32\sigma}{\gamma} \sum_{m,n\text{ odd}} \frac{m^2 + (n/\beta)^2}{\left((m^2 + (n/\beta)^2)^2 + \sigma^2 / \pi^4\right)}
\]

(2.3.2)\[
\tau_s = \frac{32\sigma^2}{\gamma} \sum_{m,n\text{ odd}} \frac{1}{\left((m^2 + (n/\beta)^2)^2 + \sigma^2 / \pi^4\right)}
\]

where \(\tau_d\) and \(\tau_s\) are non-dimensional damping and spring torque amplitudes of the total torque due to the pressure back-force, \(\gamma\) is the non-dimensional angular oscillation amplitude which has been normalized to the maximum angular displacement, and \(\beta\) is the rectangular plate’s length/width ratio. Note that \(\tau_d\) and \(\tau_s\) are normalized to \(P_c\beta L^2\).

Some approximate solutions of pressure forces for simple rectangular flexible structures also can be found in [10]. However, since these solutions are in fact the total spring/damping force components applied on the surface of flexible structures, they need to be transformed into modal coordinates \([11][12]\) in order to be used for dynamics analysis with corresponding modal parameters (structural spring constant and structural mass).

![Graph](image)

**Figure 2.3.1** Analytical results of the spring and damping components of pressure back-force due to squeeze film for a square rigid plate.
2.4 Slip Flow Consideration

If the gap thickness (i.e., characteristic length) is comparable to the mean free path of the fluid particles, the tangential velocity of the fluid at the boundary is no longer strictly zero, which is called the slip-flow effect. The slip velocity on the boundary gives rise to a decrease of the viscous forces in the squeezed air film. In general, flows can be classified into four regimes: the continuum regime, the transition regime, the slip-flow regime and the molecular regime [13][14][15]. The Knudsen number, whose definition is shown in Eqn. (2.4.1), determines in regimes of flows, as shown in Table 1.

\begin{equation}
Kn = \frac{L_m}{L_c}
\end{equation}

where \( L_m \) is the mean free path of a fluid particle and \( L_c \) is the characteristic length of region the flow passes through. In this study, \( L_c \) is the gap thickness between the parallel plates.

The mean free path of a gas is inversely proportional to pressure [16], and is given by

\begin{equation}
L_m = \frac{0.069}{P} \quad (\mu m)
\end{equation}

where \( P \) is the ambient pressure (atm).

In the continuum flow regime, the fluid is governed by the Navier Stokes equation; in the molecular flow regime, the fluid is governed by the Boltzmann transportation equation. In the slip flow and transition flow regimes, the Navier-Stokes equation and the Boltzmann equation start to degenerate and some special treatments are needed [15]. For the linearized Reynolds equation, several studies have demonstrated that the non-continuum effect (\( Kn > 0.001 \)) can be modeled by using an effective viscosity that is a function of the Knudsen number. Table 2 lists the effective viscosity models proposed by several authors. Some of them were obtained by curve-fitting experimental data, and some were derived from the Navier-Stokes equation or the Boltzmann equation by imposing appropriate velocity boundary conditions on wall-flow
interfaces. In the case that the Knudsen number is less than 0.1, the values of effective viscosities from most of the models are very close. However, in the case of larger Knudsen number ($Kn > 0.1$), they differ significantly. Most of the studies have experimental data to support their models, so it is not clear which model is the best one. We speculate that the discrepancy of those studies is because the effective viscosity is not only function of Knudsen number but also depends on the condition of flow/wall interface, and experimental configuration. In this study, we use Burgdorfer's model [17].

<table>
<thead>
<tr>
<th>FLOW REGIMES</th>
<th>Kn RANGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Continuum flow</td>
<td>$Kn &lt; 0.001$</td>
</tr>
<tr>
<td>Slip flow</td>
<td>$0.001 &lt; Kn &lt; 0.1$</td>
</tr>
<tr>
<td>Transition flow</td>
<td>$0.1 &lt; Kn &lt; 10$</td>
</tr>
<tr>
<td>Molecular flow</td>
<td>$Kn &gt; 10$</td>
</tr>
</tbody>
</table>

Table 1 Flow regimes and their Knudsen number ranges
<table>
<thead>
<tr>
<th>Author</th>
<th>Reference</th>
<th>Effective Viscosity ($\mu^*$)</th>
<th>Derived from</th>
</tr>
</thead>
<tbody>
<tr>
<td>Burgdorfer</td>
<td>1959 [17]</td>
<td>$\frac{\mu}{1+6K_n}$</td>
<td>Navier-Stokes equation</td>
</tr>
<tr>
<td>Hsia et al.</td>
<td>1983 [18]</td>
<td>$\frac{\mu}{1+6K_n+6K_n^2}$</td>
<td>Experimental data fitting</td>
</tr>
<tr>
<td>Fukui et al.</td>
<td>1988 [19]</td>
<td>$\frac{D}{6Q(D)}$ ; $D = \frac{\sqrt{\pi}}{2K_n}$</td>
<td>Boltzmann equation</td>
</tr>
<tr>
<td>Seidel et al.</td>
<td>1993 [6]</td>
<td>$\frac{0.7\mu}{K_n}$</td>
<td>Experimental data fitting</td>
</tr>
<tr>
<td>Mitsuya</td>
<td>1993 [20]</td>
<td>$\frac{\mu}{1+6\left(\frac{2-\alpha}{\alpha}\right)K_n + \frac{8}{3}K_n^2}$</td>
<td>Navier-Stokes equation</td>
</tr>
<tr>
<td>Veijola et al.</td>
<td>1995 [8]</td>
<td>$\frac{\mu}{1+9.638K_n^{1.159}}$</td>
<td>Approximation of Fukui’s model</td>
</tr>
<tr>
<td>C.-L Chen</td>
<td>1996 [21]</td>
<td>$\frac{\mu}{1+6\alpha K_n}$</td>
<td>Navier-Stokes equation</td>
</tr>
</tbody>
</table>

Table 2  
List of different effective viscosity models for the Linearized Reynolds equation. Note that $\alpha$ is the accommodation coefficient, which measures the rate of the momentum transfer from the gas particles to the wall due to collision. For most engineering surfaces, $\alpha$ is very close to unity [13][17]. $Q$ in the Fukui's model is the Poiseuille flow rate, whose definition can be found in the Appendix of [19].
CHAPTER 3

NUMERICAL APPROACH

This chapter presents several numerical methods for calculating squeeze-film damping based on the governing equations introduced in. Basically, there are two types of problems of squeeze-film damping: small amplitude oscillation and large amplitude motion. For the small amplitude case, we solve the Reynolds equation by using diffusion equation solvers found in commercial finite element fluidic packages. For the large amplitude case, we write our own codes to simulate the isothermal Reynolds equation. Furthermore, in order to reduce squeeze-film damping, or to release moving parts during fabrication process, perforated plates are widely used in MEMS devices. In the last part of this chapter, we propose a model to calculate the pressure leakage due to perforation, taking into account the flow resistance of the holes.

3.1 Numerical Scheme for Linear Model

For the small amplitude oscillation, a few analytical solutions are available for rectangular or circular rigid plates, which have been shown in CHAPTER 2. However, these solutions are not applicable to most of the devices. As a result, numerical approaches of solving the linearized Reynolds equation are necessary [22]. Since the linearized Reynolds equation has the same form
of the transient diffusion equation or, equivalently, the heat-flow equation, it can be solved numerically by using either the transient energy-equation solver found in some typical finite element fluids-modeling packages (e.g., FIDAP [23]) or by commercial heat-flow simulators. Eqns (3.1.1) and (3.1.2) show the analogy between the linearized Reynolds equation, which is an alternative form of Eqn (2.1.1), and the heat-flow equation.

\[
\frac{\sigma \partial \Theta}{\partial \tau} = \nabla^2 \Theta - \sigma V
\]

(3.1.1)

\[
\rho C_p \frac{\partial T}{\partial \tau} = \nabla^2 T + Q
\]

(3.1.2)

Table 3 shows the variable mapping for solving the linearized Reynolds equation by using typical heat-flow equation solvers. The temperature is analogous to the non-dimensional pressure in the Reynolds equation, and so on. In order to extract the small amplitude damping and spring components of the pressure back-force amplitude, we use a sinusoidal time function of spatially uniform \textit{velocity} as the input for the linearized Reynolds equation, then integrate it in time by using a heat-flow equation solver. After the resulting total pressure back-force reaches steady state, we examine the time phase shift between the input velocity and total pressure back-force. Since the linear damping force (spring force) is always in the same phase with velocity (displacement), we can easily resolve the total pressure back-force into damping and spring components:

\[
F_r = \int_A P(x, y) dx dy
\]

(3.1.3)

\[
f_d = F_r \cos \phi
\]

(3.1.4)

\[
f_z = F_r \sin \phi
\]

where \(P(x,y)\) is the pressure distribution over the plate, \(F_T\) is the non-dimensional total pressure back-force, \(A\) is the plate area and \(\phi\) is the phase shift between the input velocity and total pressure force. Note that since we use non-dimensional time in the simulation, the units of \(\phi\) is radians.
<table>
<thead>
<tr>
<th>HEAT-FLOW EQUATION</th>
<th>LINEARIZED REYNOLDS EQUATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$ (temperature)</td>
<td>$\Theta$ (non-dimensional pressure)</td>
</tr>
<tr>
<td>$\rho C_p$ (product of density and specific heat)</td>
<td>$\sigma$ (squeeze number)</td>
</tr>
<tr>
<td>$K$ (Thermal conductivity)</td>
<td>1</td>
</tr>
<tr>
<td>$Q$ (Heat source)</td>
<td>$-\sigma V$ (negative product of $\sigma$ and velocity)</td>
</tr>
<tr>
<td>$\tau$ (Non-dimensional time)</td>
<td>$\tau$ (Non-dimensional time)</td>
</tr>
</tbody>
</table>

Table 3 Analogy of variables and parameters for the heat-flow equation and the linearized Reynolds equation.

Because the pressure force components are dependent on squeeze number, as indicated in Section 2.3, it is necessary to solve the linearized Reynolds equation for a wide range of squeeze number to obtain this dependence. The procedure is illustrated in Figure 3.1.3.

For a square plate with edges exposed to ambient pressure, the simulation results agree with the analytical solutions (Eqn. (2.3.1)) very well (in this case, we used 20x20). The normalized transient responses of the total back-force for two different squeeze numbers are shown in Figure 3.1.1. For these two cases, the transient responses reach steady state in three oscillation periods. The forces have been normalized to their steady state maximum values. The force lags behind the sinusoidal plate velocity, and the phase lag increases with squeeze number because of the increase of air compression. An equivalent circuit model, which uses frequency-dependent resistors and inductors to model the squeeze film effect, can be found in [8]. Figure 3.1.2 is the contour plot of pressure for a perforated plate. Since the pressure contour is generated by the FIDAP’s heat-flow equation solver, the physical variable shown on the plot is temperature. Note that for this case we assume the plate thickness is much less than the size of each hole’s opening, so the pressure boundary condition on the perimeter of each hole is ambient pressure. A detailed discussion of squeeze-film damping for perforated thick plates will be presented in Section 3.3.
Figure 3.1.1 Transient responses of the total back-force for two different squeeze numbers (20 and 60).

Figure 3.1.2 A contour plot of pressure distribution for a perforated plate calculated by FIDAP
Figure 3.1.3 Procedure of using the diffusion equation solver to simulate the linearized Reynolds equation and obtain the spring and damping components of total pressure back-force.
3.2 Solving the Reynolds Equation

Operating in small amplitude oscillation has advantages in actuation and signal sensing. However, many devices have to be operated in large amplitude motion. For example, the deformable mirror devices (DMD) pull the edges of mirrors all the way down to substrate torsionally [24]; pull-in pressure sensors pull in a fixed-fixed beam to estimate pressure [26][27]; micromachined variable capacitor devices change their capacitance values by altering the gap thickness significantly [28]. For those devices, the small amplitude oscillation model is not valid. Therefore, we use finite difference methods to solve the isothermal Reynolds equation. The numerical schemes are: (1) use finite difference method the discretize the plate surface for the isothermal Reynolds equation, (2) integrate it in time by using Runge-Kutta method.

Figure 3.2.1 shows four cases of squeeze-film damping problems with either large amplitude motions or non-parallel relative normal motions. The 3D surface and contour plots of pressure distribution for one quarter of the plate in case (A) are shown in Figure 3.2.2. These plots are the results at the time when the pressure reaches its maximum value in a vibration cycle. Figure 3.2.3 is the dimensionless time response of the total pressure back force for case (A), obtained by integrating the pressure distribution over the plate area. The oscillation amplitude in this case is equal to 30% of the gap thickness. Because of the nonlinearity of the Reynolds equation, the back force curve, which is normalized to the ambient pressure force (i.e., ambient pressure times the plate area), is not symmetric about the ambient pressure force (P=1 in the figure). When the plate moves down, the maximum back-force is about 1.2 times of the ambient pressure force; when plate moves up, the minimum back-force is about 0.9 of the ambient pressure force. In other words, the plate undergoes a larger pressure resistant force when the plate moves down. It is because the absolute pressure in the gap increases rapidly when the gap thickness decreases. However, for the linearized model, the pressure variation is linearly proportional to the oscillation amplitude, which must be very small compared to the nominal gap thickness in order to satisfy the linearity assumption. Note that the velocity curve in Figure 3.2.3 has also been normalized. The figure indicates that the transfer function between force and velocity is clearly nonlinear.
Table 4 summarizes the maximum and minimum pressures for the four cases in Figure 3.2.1 when the plates are sinusoidally oscillating in normal relative motions or torsional motions. Note that the pressure caused by the torsional motion, which is also a potential second mode of motion for the accelerometer shown in Figure 1.1.1, is much less than that caused by normal relative motion (case (A) in Figure 3.2.1) under ambient pressure.

Figure 3.2.1 Four cases of squeeze-film damping problems with large amplitude motions or non-parallel motions
Figure 3.2.2 The 3D surface and contour plots of pressure distributions for case (A) in Figure 3.2.1.
Figure 3.2.3 Nonlinear time response of the total pressure back force for a square plate with a large amplitude oscillation.

<table>
<thead>
<tr>
<th>CASE</th>
<th>Max. d/h₀</th>
<th>Max. P/Pₐ</th>
<th>Min. P/Pₐ</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) Accel.</td>
<td>0.3</td>
<td>1.4484</td>
<td>0.7688</td>
</tr>
<tr>
<td>(B) Perforated Accel.</td>
<td>0.3</td>
<td>1.1007</td>
<td>0.9079</td>
</tr>
<tr>
<td>(C) Gyro</td>
<td>0.3</td>
<td>1.0902</td>
<td>0.9246</td>
</tr>
<tr>
<td>(D) DMD</td>
<td>0.3</td>
<td>1.0635</td>
<td>0.9446</td>
</tr>
</tbody>
</table>

Table 4 Summary of maximum and minimum pressure on the plates for the four cases in Figure 3.2.1. Note that d is the plate oscillation amplitude, h₀ is the initial gap thickness, P is the air film pressure on the plate, and Pₐ is the ambient pressure. The oscillation amplitude for case (C) and (D) is the maximum displacement of the plate tip.
3.3 Perforation Model

Perforations are often used in MEMS devices either to reduce the release-etch time or to reduce squeeze-film damping. However, numerous holes in a structure makes modeling much more complicated. Previous studies of squeeze-film damping for perforated plates neglect the flow resistance of the holes, and assume that the pressure on the perimeter of each hole is the same as the ambient pressure [9][24]. With this assumption, which is valid only for thin plates, the damping effect can be easily simulated by using a standard 2-D diffusion equation solver, as explained in Section 3.1 [8][22]. Figure 3.1.1 shows two perforated plates with identical hole sizes but different plate thicknesses. In Case (A), it is legitimate to assume that the pressure P is equal to ambient pressure $P_a$. For Case (B), the holes behave like pipes because of thick plate thickness, and the pipe-flow resistance gives rise to the pressure difference between P and $P_a$, which in turn results in higher damping.

![Diagram of perforated plates](image)

**Figure 3.3.1** Two examples of perforated plates.

Full simulation of the effects of the perforations requires 3D simulation. To avoid the complexity of full 3D, we use the enhanced 2D damping model shown in Figure 3.3.2. The gap regions between the holes are governed by the 2D (linearized) Reynolds equation, which is
solved with finite-difference methods [22]. The holes are represented by lumped resistance elements to ambient added to the finite-difference solver. The resistance value is estimated from the pipe-flow equation [30]. Finally, since the mass flow rate must be conserved, the continuity equation is used to match flow rates between regions. The pressures on the nodes which connect the gap and hole regions are self-consistently evaluated and serve as pressure boundary conditions at each time integration step.

![Diagram of Thick moving plate with holes](image)

- Regions governed by the Reynold's equation
- Regions governed by the pipe flow equation
- Nodes governed by the continuity equation

Figure 3.3.2 Cross-section view of a perforated plate with indication of regions governed by different governing equations.

The flowrate of a circular pipe flow can be written as [30]:

\[
Q = \frac{\pi D^4}{128 \mu} \left( -\frac{\partial P}{\partial z} \right)
\]

where \( Q \) is the flow rate along pipe (x-direction), \( D \) is the diameter of the circular pipe, \( \mu \) is viscosity and \( \frac{\partial P}{\partial z} \) is pressure gradient in z-direction.
In order to simplify the pipe flow equation, we assume the fluid flowing through the holes is fully developed and incompressible. Under these assumptions, the pressure gradient is constant and we can rewrite Eqn. (3.3.2) as

\[
Q = \frac{\pi D_c^4}{128 \mu} \left( \frac{P - P_a}{L} \right) = \frac{P - P_a}{R}
\]

(3.3.2)

\[
R = \frac{128 \mu L}{\pi D_c^4}
\]

where \( P \) is pressure at the inlet of the pipe, \( P_a \) is pressure at the outlet (ambient pressure), \( L \) is the length of the pipe, \( D_c \) is the characteristic diameter, and \( R \) is the equivalent flow resistance.

As mentioned in the previous section, the Reynolds equation is derived from the Navior-Stokes equation. However, the Reynolds equation itself does not provide any information about the flow velocity or flow rate for the flow inside the gap. Fortunately, the procedure of deriving the Reynolds equation [3] reveals enough information of the velocity profiles of the flow inside the gap, as shown in Eqn. (3.3.3).

\[
u = \frac{h z \left( -\frac{\partial p}{\partial x} \left( 1 - \frac{z}{h} \right) \right)}{2 \mu}
\]

(3.3.3)

\[
u = \frac{h z \left( -\frac{\partial p}{\partial y} \left( 1 - \frac{z}{h} \right) \right)}{2 \mu}
\]

where \( u \) and \( v \) are the velocity profiles in \( x \) and \( y \) directions.

The volume flow rates per unit length are derived by integrating the velocity profiles in \( z \) direction.
\[ q'_x = \frac{h^3}{12\mu} \left( -\frac{\partial p}{\partial x} \right) \]

(3.3.4)

\[ q'_y = \frac{h^3}{12\mu} \left( -\frac{\partial p}{\partial y} \right) \]

Since the compression of the air film in each hole is much less than that underneath the solid plate, we assume the flow is incompressible in each hole. In order to satisfy the mass conservation condition for both the Reynolds equation and pipe-flow equation, we apply the continuity equation to match the flow rate at the node right below each hole, as shown in Eqn. (3.3.5). The finite difference form of the continuity equation is shown in Eqn. (3.3.6), and the 1D schematic of the discretized pressure variables is described in Figure 3.3.3

\[ Q = q'_{M-1,x} \cdot \Delta y + q'_{M+1,x} \cdot \Delta y + q'_{N-1,y} \cdot \Delta x + q'_{N+1,y} \cdot \Delta x \]

(3.3.5)

\[ \frac{P_{M,N} - P_a}{R} = \frac{h^3}{12\mu} \left( \frac{P_{M-1,N} - P_{M,N}}{\Delta x} \right) \Delta y + \frac{h^3}{12\mu} \left( \frac{P_{M+1,N} - P_{M,N}}{\Delta x} \right) \Delta y + \frac{h^3}{12\mu} \left( \frac{P_{M,N-1} - P_{M,N}}{\Delta y} \right) \Delta x + \frac{h^3}{12\mu} \left( \frac{P_{M,N+1} - P_{M,N}}{\Delta y} \right) \Delta x \]

(3.3.6)

![1D Schematic view of discretized pressure variables for finite difference formulation.](image)

Figure 3.3.3 1D Schematic view of discretized pressure variables for finite difference formulation.
The time integration scheme is very similar to that discussed in Section 3.2. In each time integration step for the Reynolds equation, \( P_{M-1,N} \), \( P_{M+1,N} \), \( P_{M,N-1} \), and \( P_{M,N+1} \) in Eqn. (3.3.6) are actually the initial pressure values (i.e., they are given in the time step), so we can evaluate \( P_{M,N} \) by rearranging Eqn. (3.3.6) right before the integration process. The evaluated \( P_{M,N} \), as shown in Eqn. (3.3.7), will serve as a pressure boundary condition for that time integration step.

\[
P_{M,N} = \frac{P_g}{R} + \frac{h^3}{12 \mu} \left( \frac{\Delta y}{\Delta x} \left( P_{M-1,N} + P_{M+1,N} \right) + \frac{\Delta x}{\Delta y} \left( P_{M,N-1} + P_{M,N+1} \right) \right)
\]

(3.3.7)

Note that several factors affect the value of the flow resistance \( R \). For example, the flow in holes might not be fully developed flow. Also, there must be a slip-flow effect if the diameter of holes is comparable to the mean free path of the air. Moreover, the model does not count the flow resistance effect due to deflection of air flow flowing from gap to holes. In order to use this model more carefully, we suggest to redefine flow resistance \( R \) as Eqn. (3.3.8), and to determine the \( K_R \) experimentally.

\[
R = K_R \cdot \frac{128 \mu L}{\pi D_c^4}
\]

(3.3.8)

The benefits of this modification are two-fold. First of all, we preserve the dimensional dependence for the flow resistance \( R \), which is one of the important properties of a good macromodel. Secondly, in the early stage of design, we may simply assume \( K_R \) equals to 1 and estimate a reasonable flow resistance value based on the characteristic diameter and length of pipes (holes).

In our simulation, four resistance elements are actually used for each perforation, and each element is located at the edge of each hole. Figure 3.3.4 shows the 1-D sketch of the comparison of the original and modified models. Note that the resistance value in the modified model is four times as that in the original model so that the effective total resistance is the same. The main purpose of this modification is to capture the correct pressure profile as much as possible. For
example, if the hole size is much less than the distance between two adjacent holes (pitch distance), this modification is not necessary because the hole is like a point sink of pressure. However, if the hole size is bigger than one twentieth of the pitch distance, the pressure profile become sensitive to the location of perimeter of each hole. As a result, we need to use the modified model in order to accurately calculate the pressure on the perimeter of each hole, which is necessary to obtain accurate pressure profile underneath the plate.

![](image)

- Node for pressure computation

~\~\~\~\~  Pipe flow resistance element

Figure 3.3.4  (a) Pipe flow model of perforations for the case that the hole size is much smaller than the pitch distance. (b) Pipe-flow model for the case that the hole size is bigger than 1/20 of the pitch distance. Note that the resistance value in (b) is four times as that in (a) so that the effective resistance value of (b) is the same as that in (a).
Simulations of coupled-energy-domain systems with squeeze-film damping will be discussed in this chapter. A macromodel for an accelerometer under small amplitude motion is formulated and simulated. The frequency response of this device shows that both viscous damping and spring effects from squeezed film alter the device's dynamics significantly. Simulation results of the new perforation model, which has been described in Section 3.3, are also presented. The theory and method of calculating damping effect for flexible structures are derived and discussed. Simulation results of pull-in dynamics and small amplitude vibration for a fixed-fixed beam microrelay are also demonstrated. Experimental data show good agreement with simulation results for various types of devices.
4.1 Macromodels for Linearized Squeeze-film Damping - Uniform Displacement

In order to demonstrate squeeze-film damping effects for small amplitude vibrating devices, we calculate the damping and spring forces for a micro-accelerometer, which is fabricated by the MIT MEMS group [30], and formulate a transfer function of the device for frequency response analysis. Figure 4.1.1 shows the schematic of the accelerometer. The proof mass is a 500 µm by 500 µm square plate supported 1 µm above the substrate by flexible tethers. The damping and spring force components were calculated by the numerical method presented in CHAPTER 3. The results then are transferred into damping ratio and spring constant, as shown in Eqn. (4.1.1).

\[
c_a = \frac{f_d P_a \beta L^2}{\alpha h_0}
\]

(4.1.1)

\[
k_a = \frac{f_s P_a \beta L^2}{h_0}
\]

where \(c_a\) is the damping coefficient and \(k_a\) is the spring constant due to squeeze-film damping. Note that both \(c_a\) and \(k_a\) frequency dependent, as both \(f_d\) and \(f_s\) depend on the squeeze number.

Figure 4.1.2 is the 1-D mass-spring-damper macromodel of the accelerometer with squeeze-film damping. Note that the air spring constant and air damping coefficient are calculated by Eqn. (4.1.1), and hence are frequency dependent. As a result, it is not rigorously correct to write a “transfer function” in the Laplace transform variable \(s\). Nevertheless, as a compact way to present all of the results, and provided that we do not actually invest the transform into the time domain, we can write a frequency domain transfer function of the macromodel for the fundamental mode as:

\[
T(s) = \frac{Z(s)}{F_x(s)} = \frac{1}{s^2m + sc_a + (k + k_a)}
\]

(4.1.2)
where $Z$ is the displacement of the moving plate normal to the substrate, $F_E$ is the external excitation force, $m$ is the proof mass, and $k$ is the structural spring.

The simulated small-amplitude dynamical responses of the accelerometer are illustrated in Figure 4.1.3. Curve 1 is a typical response of a second order dynamical system without any air damping. Curve 2, which drops very fast, is the response with the air damping. Curve 3 of Figure 4.1.3 is the response of the device operating in low pressure. Curves 2 and 3 show that the spring effect of the gas film results in a resonance shift, which increases with pressure. At low frequencies (low squeeze number), the system is overdamped because of highly viscous damping. At higher frequencies, the viscous damping decreases while the spring force increases. The resonance occurs at a high enough frequency to make the resonance itself be underdamped. These effects have been reported by a few studies [6][7][8].

Figure 4.1.1 Schematic of an accelerometer designed and fabricated by MIT MEMS group [30]. (a) is the top view, and (b) is the cross-section view.
Figure 4.1.2 1-D mass-spring-damper macromodel of the accelerometer, as shown in Figure 4.1.1, with squeeze-film damping. For flexible structures represented with a modal coordinate, the $k$, $k_a$, $c_a$ and $Z$ are replaced with $k_G$, $k_{aG}$, $c_{aG}$ and $Q$, respectively. (see Eqn. (4.1.2) and (4.2.7))

Figure 4.1.3 Simulated dynamical responses of an accelerometer [30] with and without air film damping effects.
4.2 Macromodels for Linearized Squeeze Film Damping - Non-uniform Displacement

For flexible MEMS structures under small amplitude oscillation, the moving-boundary forcing function imposed on the Reynolds equation has both spatial and time dependences. Also, the shape of a oscillating structure is usually dominated by one mode, depending on the configuration of external excitation forces [8]. Therefore, it is possible to extract squeeze film damping effects by solving the Reynolds equation (as described in the Section 3.2) with a moving boundary determined by the dominant oscillating mode shape.

The modal parameters, such as oscillating mode shapes (eigenvectors), generalized spring constants, and generalized masses for each oscillation mode, can be obtained by commercial structural dynamics finite-element codes (in this study, we use Abaqus). The equation of motion can be constructed by using the modal parameters:

\[
M_G \ddot{q} + K_G q = -V' \{ F_P(V(q), \dot{V}(\dot{q})) \} + V' \{ F_E \}
\]

(4.2.1)

where \(V\) is the \(N \times N\) modal matrix (\(N\) is the number of nodes after discretizing the moving flexible plate) which contain the eigenvectors (mode shapes) for each oscillation mode, \(M_G\) and \(K_G\) are the generalized mass and spring matrices (\(N \times N\)) and are diagonal, \(F_P\) is the pressure force distribution (\(N \times 1\)) along the flexible moving plate and is a function of both the displacement and velocity on each node of the plate, \(F_E\) is a vector (\(N \times 1\)) representing the external excitation force, and \(\{q\}\) is a vector containing modal amplitude of each mode.

Since the dominant oscillation mode is of interest, Eqn (4.2.1), which is a system of \(N\) 2nd order differential equations, can be truncated into one 2nd order differential equation of the dominant oscillation mode, which, in the s-plane, becomes:
where $Q(s)$ is the modal variable of the dominant mode, $\{v\}$ is the corresponding eigenvector, $k_G$ and $m_G$ are the corresponding generalized spring constant and generalized mass respectively, and $F_{EG}$ is the generalized external force which excites the structure in the dominant mode.

Due to the assumption of small amplitude oscillation, the spring component ($F_{pc}$) and damping component ($F_{pd}$) of the pressure reactant force on each node of the moving plate can be decoupled [4][22]. The equation of motion becomes:

\begin{equation}
(4.2.3) \quad s^2 m_G Q(s) + k_G Q(s) = -\{v\}' \{F_{pc}(\{v\} Q(s)) - \{sQ(s)\}\} + F_{EG}(s)
\end{equation}

In order to obtain the effective spring constant and the damping coefficient numerically, we use the dominant oscillation mode shape as the sinusoidally moving boundary for the linearized Reynolds equation with a specific squeeze number, then integrate it in time until reaching steady state. The spring and damping forces on each node can be determined from the total node reaction force using the phase shift between the steady state local pressure force and the velocity on that node, as shown in Figure 4.2.1. The effective spring constant and damping coefficient on each node can be obtained from the decoupled node forces by dimensional analysis, as described in Section 4.1. With the effective parameters and eigenvector elements written explicitly, Eqn. (4.2.3) can be rewritten as:

\begin{equation}
(4.2.4) \quad s^2 m_G Q(s) + k_G Q(s) = -\{v_1 \ldots v_N\} \begin{bmatrix} k_{a1} v_1 \\ \vdots \\ k_{aN} v_N \end{bmatrix} Q(s) - \{v_1 \ldots v_N\} \begin{bmatrix} c_{a1} v_1 \\ \vdots \\ c_{aN} v_N \end{bmatrix} sQ(s) + F_{EG}(s)
\end{equation}

where $v_n$'s are the elements of the eigenvector, and $k_{an}$'s and $c_{an}$'s are the effective spring constants and damping coefficients on each node.
These effective spring and damping parameters are then projected back onto the mode shape to determine an effective spring and damping components for the dominant mode. Thus the generalized spring constant and damping coefficient for this oscillation mode are:

\[
(4.2.5) \quad k_{aG} = \sum_{n}^{N} k_{an} v_{n}^2; \quad c_{aG} = \sum_{n}^{N} c_{an} v_{n}^2
\]

and Eqn. (4.2.4) can be simplified as

\[
(4.2.6) \quad s^2 m_{G} Q(s) + k_{G} Q(s) = -k_{aG} Q(s) - s c_{aG} Q(s) + F_{EG}(s)
\]

Rearranging Eqn. (4.2.6), we obtain the transfer function of this oscillation mode with the effective generalized parameters, as shown in Eqn. (4.2.7).

\[
(4.2.7) \quad T(s) = \frac{Q(s)}{F_{EG}(s)} = \frac{1}{s^2 m_{G} + s c_{aG} + (k_{G} + k_{aG})}
\]

Note that Eqn. (4.2.7) has the same form as Eqn. (4.2.1), and the mass-spring-damper model shown in Figure 4.1.2 is applicable to this new transfer function of the dominant oscillation mode. The main difference between Eqn. (4.2.1) and Eqn. (4.2.7) is that the latter represents the dynamics in the modal coordinate \( Q(s) \) which is the variable describing the motion with dominant mode shape. Similar to the rigid-plate case, the frequency response for this dominant oscillation mode can be calculated by Eqn. (4.2.7).

After calculating the generalized parameters repeatedly for a wide range of different squeeze numbers, we obtain the squeeze number dependence for the pressure force components. Figure 4.2.2 shows the spring and damping components vs. squeeze number on different nodes of a microbridge with aspect ratio 10 (length/width) operating in its fundamental oscillation mode. In this calculation, the mesh for solving the Reynolds equation is based on the two-dimensional structural mesh of the fixed-fixed beam from ABAQUS. We used a symmetric finite difference formulation (zero-flux boundary) for the incremental pressure on the nodes located on the fixed ends of the beam, and the incremental pressure on the other two edges of the beam is zero. Figure 4.2.3 shows the squeeze number dependence of the resulting generalized spring and
damping forces for fixed-fixed beam structures with aspect ratio 10. Since Figure 4.2.3 is non-dimensional, this plot is valid for any thin fixed-fixed beam with aspect ratio 10. Based on this idea, we may build a macromodel database of squeeze-film damping by doing a series of generic calculations for different aspect ratios. This generic calculation is also applicable to other typical MEMS flexible structures, such as cantilevers and membranes.

![Diagram](image)

\[
P_{i,damping} = P_i \cdot \cos \phi_i \quad P_{i,spring} = P_i \cdot \sin \phi_i
\]

Figure 4.2.1 Schematic view of a flexible MEMS structure (a fixed-fixed beam) with squeeze-film damping. The reaction pressure force on each node can be decoupled into spring and damping components. \( P_i \) is the total local reaction pressure on the i-th node and \( \phi_i \) is the phase difference between the sinusoidal input velocity and \( P_i \) on the ith node.
Figure 4.2.2 Nondimensional spring and damping forces on selected nodes of a fixed-fixed beam (aspect ratio 10) in the fundamental oscillation mode. Note that the pressure force components on node A are smaller than those on node B because the relative displacement of node A is smaller than node B.

Figure 4.2.3 Nondimensional generalized spring and damping forces of the fundamental oscillation mode for a fixed-fixed beam with aspect ratio 10.
Figure 4.2.4 shows a schematic cross section of the fixed-fixed-beam microrelay which we simulated and tested. The actuation circuit is formed between the polysilicon layer of the microbridge (3\textsuperscript{rd} polysilicon electrode) and the n-doped region in the substrate (counter electrode). The working circuit consists of two polysilicon layers: the first one is on the substrate (1\textsuperscript{st} polysilicon) and the second one is under the beam (2\textsuperscript{nd} polysilicon). The detailed description of this device can be found in [32][33]. A static 7-volt bias in addition to a small ac excitation has been applied to the beam in order to drive the resonance. Because of the static bias, it was necessary to include the well-known spring-softening effect, and this was done in our simulation by adding to the structural spring constant the effective spring constant due to electrostatic force. Figure 4.2.5 shows the spring softening effect for a 300 \textmu m long microrelay. A change of about 2\% has been observed for 15V applied to the beam. Simulation results of resonance shift and quality factor are consistent with experimental results, as shown in Figure 4.2.6 and Figure 4.2.7. The quality factor Q (in both figures) is relatively independent of the beam's constitutive properties, but is a strong function of ambient pressure due to the damping force component on the fundamental mode. Agreement with experiment is excellent down to 1 mbar, at which point the intrinsic losses in the beam resonance limit the Q, while the calculation predicts a continuing increase in Q. The resonant frequency (in both figures) depends directly on the beam's constitutive properties. Here, the agreement between the calculated and measured resonant frequency was to within about 5\%. In order to demonstrate the perturbation of the resonance by the air damping spring force, the modal stiffness in the simulation was arbitrarily reduced slightly so that the low-pressure resonant frequency agreed with the experiment value. The shift to higher frequencies produced by the air spring force was calculated from pressures up to 20 mbar.
Figure 4.2.4 Schematic of a fixed-fixed-beam microrelay [32]

Figure 4.2.5 Spring softening effect: measurements and simulation for the 300 μm long microrelay
Figure 4.2.6 Quality factor and resonance shift vs. pressure for a 300 μm long microrelay beam in its fundamental oscillation mode.

Figure 4.2.7 Quality factor and resonance shift vs. pressure for a 350 μm long microrelay beam in its fundamental oscillation mode.
4.3 Pull-in Dynamics

In order to estimate the squeeze-film damping effects on contact-type MEMS devices, such as micro-relays and micro-switches, we model and simulate the pull-in dynamics of the fixed-fixed-beam microrelay discussed in Section 4.2 (see Figure 4.2.4). The pull-in voltage is measured with a reverse bias on the p-n junction, and a steady bias between the polysilicon beam and the p⁺ substrate (-33V). In the presence of this bias, the potential on the counter electrode (n-doped region) is increased until the device pulls in. The experimental result of the pull-in voltage is 48.7 V for the 300 µm long relay.

There are three energy domains involved in the dynamics of the system: structural dynamics (including both potential and kinetic energy), electrostatics and fluid dynamics (the squeeze-film damping). The simulation of the mechanical structure is based on the modal analysis [12], the squeeze-film damping simulation is based on the discussion in Section 3.2, and the electrostatics calculation is based on the assumption of perfectly conducting surfaces of the electrodes. Eqns. (4.3.1), (4.3.2) and (4.3.3) are the governing equation of the three energy domains.

\[
M_G \{q\} + K_G \{q\} = -V' \{F_p(z(x, y), \dot{z}(x, y))\} + V' \{F_E(z(x, y))\}
\]

(4.3.1)
\[
z(x, y) = V\{q\}
\]

(4.3.2)
\[
F_E(z(x, y)) = \frac{\varepsilon_0 V^2}{2(h_0(x, y) - z(x, y))^2} dA
\]

(4.3.3)
\[
F_p(z(x, y), \dot{z}(x, y)) = P(x, y) dA
\]

where \( V \) is the \( M \times N \) modal matrix (\( M \) is the mode number which is sufficient to calculate the mechanical behavior accurately, \( N \) is the number of nodes after discretizing the moving flexible plate) which contains \( M \) eigenvectors (mode shapes) for each oscillation mode, \( M_G \) and \( K_G \) are the generalized mass and spring matrices (\( M \times M \)) and are diagonal, \( F_E \) is a vector (\( N \times 1 \)) representing the electrostatic force, \( F_P \) is the pressure force distribution (\( N \times 1 \)) along the flexible moving plate and is a function of both the displacement and velocity on each node of the plate, \( \{q\} \) is a vector (\( M \times 1 \)) containing modal amplitudes (variables) of the \( M \) modes, \( P(x,y) \) is the
pressure distribution obtained by solving the Reynolds equation, and \( dA \) is the area of the discretized element. The generalized mass \( M_G \) and spring constants \( K_G \) of the microrelay were obtained by the MIT's MEMCAD system [34].

Since Eqns. (4.3.1) and (4.3.3) are time-dependent differential equations, the time response of the coupled system is solved by Runge-Kutta integration method. Figure 4.3.1 shows the experimental and simulation data of the device's pull-in time, which is the time needed for the beam to collapse onto substrate from its neutral position under a constant applied voltage. Because the stiffness matrix (structural spring constants) is based on the linear vibration analysis, it does not include the stress stiffening effect due to this large amplitude pull-in motion. Therefore, the calculation results underestimate the pull-in voltage and pull-in time. In order to match the pull-in voltage, we increased the spring constant of each mode by 8%. After the pull-in voltage is matched, the pull-in time agree with the experimental data very well. An application of pull-in time measurement is pressure measurement (vacuum detection), which is discussed in [27].

![Graph showing pull-in time vs. actuation voltage](image)

**Figure 4.3.1** Switching times versus voltages for a 300 \( \mu \)m long microrelay.
4.4 Effect of Perforation: Uniform Displacement

As discussed in Section 3.3, a small aspect ratio of hole diameter vs. hole length for a perforated plate makes the behavior of the perforations like that of pipes. In this Section, we calculate the damping ratio of a rigid perforated accelerometer by solving the Reynolds equation coupled with the pipe-flow model for each hole (see Section 3.3). The dimension of the proof mass is 515x515 μm² (as shown in Figure 4.1.1), and the pitch distance is 14 μm. There are a total of 36x36 holes distributed evenly over the plate. Figure 4.4.1 illustrates the configuration of the perforation of the proof mass. The damping ratio is calculated by imposing a small amplitude sinusoidal plate displacement with a constant oscillation frequency, which is 10 kHz in this case. Due to pressure leakage through the perforations, the compression of the air film is negligible and the resulting total pressure back-force is in phase with oscillation velocity. The damping coefficient is obtained by calculating the ratio of steady-state total pressure force amplitude and the oscillation velocity amplitude.

\[
B = \frac{\int P(x, y, t_\infty) dA}{V(t_\infty)} \tag{4.4.1}
\]

where \( t_\infty \) is the time when the total pressure force reaches steady state, \( V \) is the imposing plate velocity, \( A \) is the plate area, and \( B \) is the damping coefficient.

The damping ratio is:

\[
\zeta = \frac{B}{2\sqrt{MK}} \tag{4.4.2}
\]

where \( M \) is the mass of the proof mass and \( K \) is the structural spring constant.
Figure 4.4.1 Configuration of perforations in the proof mass of an accelerometer

The pressure distribution near on corner of the plate is shown in Figure 4.4.2. Note that the pressure at each hole does not drop to ambient pressure due to finite pipe-flow resistance. Furthermore, the global pressure profile is flat almost everywhere except the regions close to the edge, so it is possible to estimate the upper bound of the damping ratio just by calculating the pressure force on a small element near the center of the plate.

Figure 4.4.2 3-D surface plot of the pressure distribution for a perforated proof mass of an accelerometer
The simulated damping ratio as a function of the flow-resistance scaling factor $K_R$ (see Section 3.3) is shown in Figure 4.4.3. The correct scaling factor to use is estimated from a 2-D axially-symmetric steady incompressible fluid simulation of plane flow converging beneath a perforation, as shown in Figure 4.4.4 [23]. Note that the axis of symmetry in the figure is parallel to the Z-axis. It is found that the net effect of this convergent flow is to increase the effective pipe-flow resistance by 10%. With a unity scaling factor, the calculated damping ratio is 4.2, which is 16% less than the experimental result of 5 [30]. With the scaling factor of 1.1, the calculated damping ratio is 4.3, an improvement in the agreement with experiment. If the flow resistance is totally neglected, as in [9][24], the calculated damping ratio is 3.1, which is far from the experimental result.

![Graph showing damping ratio vs. scaling factor for pipe-flow resistance. The horizontal dashed line shows the experimental value.](image-url)
Figure 4.4.4 Velocity vector plot from Fidap [23], which is used to estimate the resistance effect due to deflection of air flow flowing from gap into holes. Note that the z-axis is horizontal in the plot.

4.5 **Effect of Perforation: Non-uniform Displacement**

For perforated flexible structures, the calculation strategy for damping ratio is in fact a combination of methods discussed in Section 3.3 and 4.2. In other words, we use the numerical model proposed in Section 3.3, and follow the calculation procedure described in Section 4.2 to obtain the damping ratio for a specific oscillation mode of a flexible perforated structure. The effects of perforation on the equivalent mechanical/material properties of the plate (Young’s modulus, Poison ratio, and so on) are discussed in [35].

Figure 4.5.1 shows a photograph of perforated flexible circular membranes that are made of aluminum. Note that the top left one is in pull-in condition so that we can visualize the actual shape of the membrane. We simulate the damping ratio for the 0.45-μm-thick diaphragm of
radius 103 μm with an array of 3μm holes on 8μm pitch, positioned 1.49 μm above a ground plane. The detailed mechanical/material properties can be found in [35]. Using these properties, we can obtain the modal parameters (oscillating mode shapes, generalized spring constants, and generalized masses) by Abaqua. Then we impose the moving boundary of a specific mode shape on the numerical model (squeeze-film damping/pipe-flow), and simulate its transient behavior with sinusoidal oscillation of the moving boundary with frequency of 50KHz. Finally, the damping ratio can be calculated from the simulated pressure distribution on the membrane.

![Image](image_url)

**Figure 4.5.1** Photograph of the perforated membranes. The upper left is in the pulled-in configuration

Because of symmetry, only one quarter of the membrane is considered into the numerical model in order to save computation. The 3D surface plot of pressure is shown in Figure 4.5.2. The simulated relation of damping ratios vs. ambient pressure is plotted in Figure 4.5.3. Note that since the size of each perforation is comparable to the pitch of the perforation array, the spring effect is negligible.
Figure 4.5.2 Simulated 3D surface plot of pressure for a flexible perforated aluminum membrane.

Figure 4.5.3 Damping ratio vs. ambient pressure for a flexible perforated aluminum membrane
CHAPTER 5

CONCLUSION

The theoretical background of squeeze-film damping and its governing equation, the Reynolds equation, were introduced. Under small amplitude vibrating conditions, the Reynolds equation can be linearized into a transient heat-flow (diffusion) equation, for which some useful analytical solutions exist. Slip-flow effects were also discussed. A few effective viscosity models from different studies were summarized.

For small amplitude vibrating motions, numerical simulations of the linearized Reynolds equation have been demonstrated. The linearized model can be simulated by the energy equation solver of the finite element package FIDAP for plate geometries too complex for analytical solution. The viscous damping and spring effects of squeeze-films can be decoupled and modeled as frequency-dependent parameters for the frequency response analysis. A macromodel of an accelerometer with squeeze-film damping is also demonstrated. When operated in ambient pressure, the viscous damping effect dominates at low frequencies. The spring effect of the squeezed air film becomes significant at high frequency, and shifts the resonance by a large amount. Theory and method of estimating damping effects for flexible devices are also presented. The theory is based on the structural dynamics modal analysis and the simulation of the linearized Reynolds equation. Experimental results show excellent agreement in quality factor and resonance shifting.

For large amplitude motion, a home-built Reynolds equation solver was implemented. Because the pressure inside the gap is a strong function of the gap thickness, significant non-
linear effects of pressure were observed for large amplitude motion. Four examples of different
types of motions are demonstrated. The maximum pressure back forces for each case under
specific motion amplitude were also summarized. A simulation of switching time for a fixed-
fixed beam microrelay is presented. Three energy domains are involved in this numerical model:
structural dynamics, fluid mechanics and electrostatics. With appropriate adjustment of stiffness
matrix by matching pull-in voltage of the device, the simulated results of pull-in time show good
agreement with the experimental results.

Perforated plates are frequently used for reducing/controlling squeeze-film damping. In
order to model the squeeze-film damping effect more carefully, a pipe-flow model for
perforations is proposed. A numerical scheme, which combines the Reynolds equation solver
with the pipe-flow model, was illustrated. The combined solver preserves the dimension
dependence of the perforations, and thus is more accurate to estimate damping effects without
significantly increasing computation cost. Numerical examples of rigid and flexible perforated
structures are demonstrated, and show that the pressure inside the hole does not drop to ambient
pressure due to finite flow resistance. Experimental results show that the proposed model gives
significant improvement in damping estimation compared with the models proposed in previous
studies.

Finally, this thesis is concluded by Table 5, which summarizes the comparison of simulation
and experiment results in this study. The table also shows the discrepancy (%) of experimental
and simulated results. The formula of discrepancy is defined as

\[
(5.1.1) \quad \left| \frac{E - S}{E} \right| = \text{DISCREPANCY} \, (\%) 
\]

where \( E \) is the experimental results, and \( S \) is the simulation results.

Apparently, the simulated results of the non-perforated devices have better discrepancy than
that of the perforated device because of complexity and uncertainty of damping modeling for
perforations.
<table>
<thead>
<tr>
<th>DISCREPANCY</th>
<th>Quality Factor (fixed-fixed beams)</th>
<th>Resonance Freq. (fixed-fixed beams)</th>
<th>Pull-in Time (fixed-fixed beam)</th>
<th>Damping Ratio (perforated accel.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E: experiment</td>
<td>300 μm long beam within 3%</td>
<td>300 μm long beam within 1%</td>
<td>within 3%</td>
<td>14%</td>
</tr>
<tr>
<td>S: simulation</td>
<td>350 μm long beam within 5%</td>
<td>350 μm long beam within 1%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>COMMENTS</td>
<td>1. Section 4.2 2. 30 μm wide 3. pressure range is from 0.5 mbar to 15 mbar.</td>
<td>1. Section 4.2 2. 30 μm wide 3. pressure range is from 0.5 mbar to 15 mbar. 4. resonance shift effect is small compared with resonance freq.</td>
<td>1. Section 4.3 2. 300x30 μm² 3. @ 1 atm 4. stiffness of the beam needs to be adjusted up to 8% 5. voltage range is from 48 to 53V.</td>
<td>1. Section 4.4 2. 515x515 μm² 3. @ 1 atm 4. 36x36 hole array 5. 14 μm pitch 6. resistance effect of converging flow is included</td>
</tr>
</tbody>
</table>

Table 5 Comparison of experimental and simulated results in this study.
REFERENCES


APPENDIX

In this section, the example codes of calculating squeeze-film damping of a oscillating fixed-fixed beam for a wide range of squeeze number are provided. There is one C code and three Matlab scripts:

d2000-20000.c

phase.m

phase2.m

phase3.m

The d200-20000.c solves the Reynolds equation by imposing a sinusoidally moving boundary of the first oscillation mode. The output is the pressure distribution on the beam for each squeeze number. The phase.m interprets the output data from d200-20000.c, and find out the phase shift between the input velocity and the pressure force on each node, then de-couples the pressure force into viscous damping component and air spring component on each node. According to the theory derived in Section 4.2, phase2.m transfers the de-coupled components (output from phase.m) into the viscous damping and spring components of the oscillation mode (generalized viscous damping ratio and generalized air spring constant). Phase3.m uses the generalized squeeze-film damping parameters and solve the transfer function of the oscillation mode. Quality factor and resonance can be extracted from the plots generated by phase3.m.
Furthermore, there are three input data files which are needed for the C code.

calc_para.dat

BCM_beam.dat

modes_beam_1d.dat

These three files define boundary conditions and calculation parameters. The detailed information will be explained.
#include <stdio.h>
#include <math.h>
#include "nrutil.h"

#define epsilon0 8.854e-12
#define rho 2231.0 /* density of material */
#define viscosity 1.82e-5 /* need to get viscosity */

/***********************************************************/
/* */
/* variables for finite-element model */
/* */
/**************************************************************/
int DOF;
int NODE;
int MODE;

/**************************************************************/
/* */
/* variables for finite-difference model */
/* */
/**************************************************************/
int M;
int N;
int PNODE;
int SKIP=4;

int nok,nbad;
double **S;
float *D0, *Qp;
double **H0, **NAREA;
double *GM, *GK;
int **NTYPE;
double L,W;
double dA;
double dX2;
double dY2;
double POS,VEL;
double **Q;
double h0,a0,w0;

double V;
double P0;
double step0;
double step;
double tf;
double eps;
double D;

int kmax=0,kount=0;
double *xp=0,**yp=0,dxsv=0,TotL;

do	
void odeint(double ystart[],int nvar, double x1, double x2,
double eps, double h1,
double hmin, int *nok, int *nbad,
void (*derivs)(double, double [], double []),
void (*rkqs)(double [], double [], int, double *
 double *, double *
 void (*) (double, double [], double []));

void rkqs(double y[], double dydx[], int n, double *x,
double htry, double eps,
double yscal[], double *hdid, double *hnex,
void (*derivs)(double, double [], double []));

void all_deriv(double t,double y_state[], double y_deriv[]);
void get_q_accel ( double *q_pos, double *ext_force,
double *q_accel );
void get_elec_force( double **z_pos, double **elec_force );
void get_pressure_deriv( double **H, double **dH,
double **pressure,
double **pressure_deriv );
void calc_ext_force( double **pressure,
double **elec_force, double *ext_force
 void q_2_z ( double *q, double *z );
void z_2_q ( double *z ,double *q );

void all_deriv(double t, double y_state[], double y_deriv[] )
{
 int i,j;
 double *q_pos,*q_vel,*q_accel,**pressure,**pressure_deriv;
 double **z_pos, **z_vel, *ext_force, **H, **elec_force, *z_tmp;

 q_pos =dvector(1,MODE);
 q_vel =dvector(1,MODE);
 q_accel =dvector(1,MODE);
 pressure =dmatrix(1,M,1,N);
 pressure_deriv =dmatrix(1,M,1,N);
 z_pos =dmatrix(1,M,1,N);
 z_vel =dmatrix(1,M,1,N);
elec_force =dmatrix(1,M,1,N);
ext_force =dvector(1,N);
H =dmatrix(1,M,1,N);
z_tmp =dvector(1,NODE*DOF);

for (i = 1; i <= M; i++)
   for (j = 1; j <= N; j++)
   {
      pressure[i][j]=y_state[(i-1)*N+j]; /*extract pressures */
   }

for (i=1; i<=N ; i++)
   for (j=1; j<=M ; j++)
   {
      z_pos[j][i]=S[(i-1)*SKIP*DOF+1][1]*a0*cos(t);
      z_vel[j][i]=S[(i-1)*SKIP*DOF+1][1]*(-a0)*sin(t);
   }

POS=a0*cos(t);
VEL=-a0*sin(t);

/* transfer z_pos to H */
for (i=1 ; i<=M ; i++)
   for (j=1 ; j<=N ; j++)
   {
      H[i][j]=z_pos[i][j]+H0[i][j];
   }

/* calculate pressure derivatives */
get_pressure_deriv(H,z_vel,pressure,pressure_deriv);

for (i = 1; i <= M; i++)
   for (j = 1; j <= N; j++)
      y_deriv[(i-1)*N+j]=pressure_deriv[i][j];

free_dvector(q_pos ,1,MODE);
free_dvector(q_vel ,1,MODE);
free_dvector(q_accel ,1,MODE);
free_dmatrix(pressure ,1,M,1,N);
free_dmatrix(pressure_deriv ,1,M,1,N);
free_dmatrix(z_pos ,1,M,1,N);
free_dmatrix(z_vel ,1,M,1,N);
free_dmatrix(elec_force ,1,M,1,N);
free_dvector(ext_force ,1,M);
free_dmatrix(H ,1,M,1,N);
free_dvector(z_tmp ,1,NODE*DOF);
void z_2_q ( double *z, double *q )
{
    int i,j;

    for ( i=1 ; i<=MODE ; i++ )
    {
        q[i]=0.0;
        for ( j=1 ; j<=NODE*DOF ; j++ )
            q[i]=S[j][i]*z[j]+q[i];
    }
}

void q_2_z ( double *q, double *z )
{
    int i,j;

    for ( i=1 ; i<=NODE*DOF ; i++ )
    {
        z[i]=0;
        for ( j=1 ; j<=MODE ; j++ )
            z[i]=S[i][j]*q[j]+z[i];
    }
}

void get_pressure_deriv( double **H, double **dH, double **P, double **dP )
{
    int i,j,m,n,mm,nn;

    for (m=1; m<=M; m++)
    {
        for (n=1; n<=N; n++)
        {

            if ( NTYPE[m][n]==0 ) /* Ambient Pressure */
            {
                dP[m][n] = 0;
            }

            else if ( NTYPE[m][n]==11 )
            {
                dP[m][n] = Q[m][n]*(H[m][n]*H[m][n]/D)*
                       ( (P[m][n])*( (P[m+1][n]+P[m-1][n]-
                           2*P[m][n])/dY2
                           +(P[m][n+1]+P[m][n-1]-
                           2*P[m][n])/dX2 ) );
            }
        }
    }
}
\[ + \frac{1}{(H[m][n]*D)*} \\
\quad (Q[m+1][n]*(H[m+1][n]*H[m+1][n]*H[m+1][n]) *P[m+1][n] \\
- Q[m][n]*(H[m][n]*H[m][n]*H[m][n]) *P[m][n] ) \\
\quad \cdot (P[m+1][n]-P[m][n])/(dY2) \\
+ (Q[m][n+1]*(H[m][n+1]*H[m][n+1]*H[m][n+1]) *P[m][n+1] \\
- Q[m][n]*(H[m][n]*H[m][n]*H[m][n]) *P[m][n] ) \\
\quad \cdot (P[m][n+1]-P[m][n])/(dX2)) \\
- P[m][n]*dH[m][n]/H[m][n] ; \\
\]

else if ( NTYPE[m][n]==15 ) /* Interior Nodes which are symmetric to y variable */
{
    if (m==1)
        mm=m+1;
    else
        mm=m-1;

    if (n==1)
        nn=n+1;
    else
        nn=n-1;

    dP[m][n] = Q[m][n]*(H[m][n]*H[m][n]) /D)* 
\quad (P[m][n]+(P[mm][n]+P[mm][n]- 
\quad 2*P[m][n])/(dY2) \\
\quad +(P[m][n+1]+P[m][nn]- 
\quad 2*P[m][n])/(dX2) ) \\
+ \frac{1}{(H[m][n]*D)*} \\
\quad (Q[mm][n]*(H[mm][n]*H[mm][n]*H[mm][n]) *P[mm][n] \\
- Q[m][n]*(H[m][n]*H[m][n]) *P[m][n] ) \\
\quad \cdot (P[mm][n]-P[m][n])/(dY2) \\
+ (Q[m][n+1]*(H[m][n+1]*H[m][n+1]) *P[m][n+1] \\
- Q[m][n]*(H[m][n]*H[m][n]) *P[m][n] ) \\
\quad \cdot (P[m][n+1]-P[m][n])/(dX2)) \\
- P[m][n]*dH[m][n]/H[m][n] ; \\
\}

else if ( NTYPE[m][n]==51 ) /* Interior Nodes which are symmetric to x variable */
{
    if (m==1)
        mm=m+1;
else
    mm=m-1;

if (n==1)
    nn=n+1;
else
    nn=n-1;

\[ dP[m][n] = Q[m][n]*(H[m][n]*H[m][n]/D)* \]
\[ (P[m][n]*((P[m+1][n]+P[mm][n]-2*P[m][n])/dY2
\]
\[ +(P[m][nn]+P[m][nn]-2*P[m][n])/dX2 ) ) + 1/(H[m][n]*D)* \]
\[ (Q[m+1][n]*(H[m+1][n]*H[m+1][n]*H[m+1][n])*P[m+1][n]
\]
\[ - Q[m][n]*(H[m][n]*H[m][n]*H[m][n])*P[m][n] \]
\[ *(P[m+1][n]-P[m][n])/(dY2) \]
\[ + (Q[m][nn]*(H[m][nn]*H[m][nn]*H[m][nn])*P[m][nn]
\]
\[ - Q[m][n]*(H[m][n]*H[m][n]*H[m][n])*P[m][n] \]
\[ *(P[m][nn]-P[m][n])/(dX2) \]
\[ - P[m][n]*dH[m][n]/H[m][n] ; \]

else if (NTYPE[m][n]==55) /* Interior Nodes which are symmetric to x and y variables */
{
    if (m==1)
        mm=m+1;
    else
        mm=m-1;

    if (n==1)
        nn=n+1;
    else
        nn=n-1;

    \[ dP[m][n] = Q[m][n]*(H[m][n]*H[m][n]/D)* \]
\[ (P[m][n]*((P[mm][n]+P[mm][n]-2*P[m][n])/dY2
\]
\[ +(P[m][nn]+P[m][nn]-2*P[m][n])/dX2 ) ) + 1/(H[m][n]*D)* \]
\[ (Q[mm][n]*(H[mm][n]*H[mm][n]*H[mm][n])*P[mm][n] \]
\[ - Q[m][n]*(H[m][n]*H[m][n]*H[m][n])*P[m][n] \]
\[ *(P[m][nn]-P[m][n])/(dX2) \]
\[ - P[m][n]*dH[m][n]/H[m][n] ; \]
- \( Q[m][n] * (H[m][n] * H[m][n] * H[m][n]) * P[m][n] \)
  \( * (P[mm][n] - P[m][n]) / (dY2) \)
  + \( (Q[m][nn] * (H[m][nn] * H[m][nn] * H[m][nn]) * P[m][nn] \)
  \( - Q[m][n] * (H[m][n] * H[m][n] * H[m][nn]) * P[m][nn] \)
  \( * (P[m][nn] - P[m][nn]) / (dX2) \)
  - \( P[m][n] * dH[m][n] / H[m][n] \)

void read_calc_para()
{
    FILE *fp;
    int i;
    float t1;
    int t2;

    if ((fp=fopen("calc_para.dat","r")) == NULL)
    {
        printf("reading error \n"); exit(0);
    }

    fscanf(fp, "%d\n", &t2);
    MODE=t2;
    fscanf(fp, "%d\n", &t2);
    DOF=t2;
    fscanf(fp, "%d\n", &t2);
    NODE=t2;

    fscanf(fp, "%f\n", &t1);
    V=t1;
    fscanf(fp, "%f\n", &t1);
    P0=t1;
    fscanf(fp, "%f\n", &t1);
    step0=t1;
    fscanf(fp, "%f\n", &t1);
    step=t1;
    fscanf(fp, "%f\n", &t1);
    tf=t1;
    fscanf(fp, "%f\n", &t1);
    eps=t1;

    fclose(fp);
}
void read_mode_shapes()
{
    FILE *fp;
    int i,j;
    float t1;

    if ((fp=fopen("modes_beam_1d.dat","r"))==$NULL) {
        printf("reading error \n"); exit(0);
    }

    for (i=1; i<=$MODE; i++)
    {
        for (j=1; j<=$NODE*DOF; j++)
        {
            fscanf(fp, "%f", &t1 );
            S[j][i]=t1;
        }
    }
    fclose(fp);
}

void read_BC()
{
    FILE *fp;
    int i,j;
    float t1;
    int t2;

    if ((fp=fopen("BCM_beam.dat","r"))==$NULL) {
        printf("reading error \n"); exit(0);
    }

    fscanf(fp, "%d", &t2 );
    M=t2;
    fscanf(fp, "%d", &t2 );
    N=t2;

    fscanf(fp, "%f", &t1 );
    W=t1;
    fscanf(fp, "%f", &t1 );
    L=t1;

    fscanf(fp, "%f", &t1 );
    h0=t1;

    fscanf(fp, "%f", &t1 );
    a0=t1;

    S =dmatrix(1, $NODE*DOF, 1, $MODE);
    D0 =vector(1,200);
Qp =vector(1,200);
GM =dvector(1,MODE);
GK =dvector(1,MODE);

NTYPE =imatrix(1,M,1,N);
H0 =dmatrix(1,M,1,N);
NAREA =dmatrix(1,M,1,N);

for (i=1; i<=M; i++)
{
    for (j=1; j<=N; j++)
    {
        fscanf(fp, "%d", &t2);
        NTYPE[M-i+1][j]=t2;
    }
}

for (i=1; i<=M; i++)
{
    for (j=1; j<=N; j++)
    {
        fscanf(fp, "%f", &t1);
        H0[M-i+1][j]=1;
    }
}

for (i=1; i<=M; i++)
{
    for (j=1; j<=N; j++)
    {
        fscanf(fp, "%f", &t1);
        NAREA[M-i+1][j]=t1;
    }
}

close(fp);

void main()
{

doall time=0,*y_state,*z_state,*p_state,*q_state,**PP,Pt;
    int i,j,k,middle;
    FILE *fp;

    float Kn, D1, Qp1, error;
float m_freepath = 0.069e-6;
    /* mean free path of air @ 1 atm and 25C */

float sqrt_pi = 1.7725;

read_calc_para();
read_BC();
read_mode_shapes();

PNODE=M*N;

dA=(W/L)/((M-1)*(N-1));
dY2=( (W/L)/(M-1) * (W/L)/(M-1) );
dX2=1.0/((N-1)*(N-1));

PP =dmatrix(1,M,1,N);
y_state=dvector(1,PNODE);
z_state=dvector(1,NODE*DOF);
q_state=dvector(1,MODE*2);
Q=dmatrix(1,M,1,N);

printf("%g %g %g \n",dA, dX2, dY2);
printf("%g %g %d %d \n",L, W, N, M);

/*
 //
 //
 // Set initial conditions
 //
 */

for (i=1 ; i<=PNODE ; i++) y_state[i]=1;

if ((fp=fopen("d100-20000.out","w"))==NULL)
    { printf("reading error \n"); exit(0); } ;
a0=0.01;

for (D=100; D<=20000 ; D+=100)
{
time=0;

while ( time < tf )
{
    odeint( y_state,PNODE,time,time+step,eps,step0,0.0,
          &nok,&nbad,all_deriv,rkgs);
time=time+step;

for (i=1 ; i<=M; i++)
    for (j=1 ; j<=N; j++)
    {
        PP[i][j]=y_state[(i-1)*N+j];
    }
fprintf(fp,"%10.6g  %12.9g\n",time,cos(time));

for (j=1 ; j<=N; j++)
{
    Pt=0;
    for (i=1 ; i<=M; i++)
        Pt=Pt+PP[i][j];
    Pt=Pt/M;
    fprintf(fp,"  %10.5g",Pt);
    if ( ((j/5)*5)==j ) fprintf(fp,"\n");
}
fprintf(fp,"\n");

} printf("Finished D=%g\n",D);
A1.1 calc_para.dat

This file contains the information regarding to configuration of the numerical solver. There are totally 9 entries, and the explanation of each entry is listed below.

integer                  number of modes (MODE)
integer                  degree of freedom
integer                  number of points of the mode shape data (NODE)
float                    initial voltage (V)
float                    ambient pressure (N/m²)
float                    time step (s)
float                    initial time step (s)
float                    final time (s)
float                    epsilon for Runge-Kutta integration

A1.2 BCM_beam.dat

This file stores the boundary data for the device of interest. The size of this file depends on the node numbers defined by M and N.

integer                  number of nodes in x direction (M)
integer                  number of nodes in y direction (N)
float                    dimension in x direction (m)
float                    dimension in y direction (m)
float                    gap thickness (m)
float                    oscillation amplitude (m)
M x N array of integers   11: internal node
                            00: zero pressure
                            15: symmetric to y
                            51: symmetric to x
                            55: symmetric to x and y
                            22: pipe-flow resistance element
M x N array of floats     ratio of thickness to initial thickness (usually 1)
M x N array of integers   1: internal nodes
                            2: nodes on sides
                            4: nodes on corners
A1.3 modes_beam_1d.dat

The mode shape information of the device of interest is stored in this file. The variables NODE and MODE are defined in A1.1. The mode shapes are actually the eigenvectors obtained from structural dynamics analysis of the device by using ABAQUS.

NODE x MODE array of floats eigenvectors of modes
A2  phase.m

%  This program read the output file from the program which
%  generates the pressure of small amplitude oscillation
%  , and then calculate the phase shift between spring force
%  and damping force
%  
%  Yang, Yao-Joe    July 1996
%  
clear all
fid=fopen('d100-20000.out')
sqi=1;
for SQ=100:100:20000
  for SQ=31000:200:37000
    clear T Vr P
    i=1;
    
    for time=0.05:0.05:18.90
      [t1,count]=fscanf(fid,'%f',1);
      [t2,count]=fscanf(fid,'%f',1);
      [t3,count]=fscanf(fid,'%f',26);
      T(i)=t1;
      Vr(i)=sin(t1);
      P(i,:)=t3';
      i=i+1;
    end
    
    SQ
    [tt1,tt2]=size(T);
    range=round((126)*(3-0.7));
    range_end=round(range+126*0.6);
    for node=1:1:26
      for iii=range:range_end
        if (P(iii,node)-1)>0 & (P(iii+1,node)-1)<0
          rangel=iii-1;
          range_end1=iii+1;
          iii=range_end;
        end
      end
      range2=round((126)*2.4);
      range_end2=round(126*2.6);
Phi_P(sqi,node)=interp1(P(range1:range_end1,node)-1,T(range1:range_end1),0,'spline');
Phi_Vr(sqi,node)=interp1(Vr(range2:range_end2),T(range2:range_end2),0,'spline');

Phi_P(sqi,node)=Phi_P(sqi,node)/pi*180;
Phi_Vr(sqi,node)=Phi_Vr(sqi,node)/pi*180;
dPhi(sqi,node)=Phi_P(sqi,node)-Phi_Vr(sqi,node);

Pmax(sqi,node)=max(P(265:365,node)-1);

end

SQN(sqi)=SQ;
sqi=sqi+1;
end

fclose(fid)

save d100-20000
A3  phase2.m

%%%%
%%%%  This program should be executed after phase.m
%%%%  It calculates the pressure force (spring and damping components)
%%%%  on a fixed-fixed beam which is under small amplitude oscillation
%%%%  of first mode
%%%%
%%%%
%%%%  Joseph Yang  July 1996
%%%%
%%%%
L=300e-6;
W=30e-6;
dx=L/25;
dy=W;
da=dx*dy;
h0=1.6e-6;
a0=0.01;

% SKIP=2  % for cantilever
SKIP=4  % for FF beam

% LASTNODE=0  % for cantilever
LASTNODE=1  % for FF beam

% File Name
%
load d100-20000

% Read mode shape
%fid=fopen('modes_beam_1d.dat','r');
% fid=fopen('modes_cant_1d.dat','r');
for i=1:1:25
    [temp, count]=fscanf(fid,'%f',SKIP);
    S(i)=temp(1);
end
[temp, count]=fscanf(fid,'%f',1);
S(26)=temp(1);

[tt1,tt2]=size(dPhi);
for node=2:tt2-LASTNODE
    node
    for sqn=1:tt1

    Pb(sqn,node)=cos(dPhi(sqn,node)/180*pi)*Pmax(sqn,node)/(S(node)*a
0);

    Ps(sqn,node)=sin(dPhi(sqn,node)/180*pi)*Pmax(sqn,node)/(S(node)*a
0);
    end
    plot (SQN,Pb(:,node),'+',SQN,Ps(:,node),'*');
    pause
end

for sqn=1:tt1
    GFdn(sqn)=0;
    GFsn(sqn)=0;
    GFd(sqn)=0;
    GFs(sqn)=0;
    for node=2:tt2-1
        % GFd(sqn)=GFd(sqn)+Pb(sqn,node)*S(node)*S(node)*P0*dA/h0;
        % GFs(sqn)=GFs(sqn)+Ps(sqn,node)*S(node)*S(node)*P0*dA/h0;

        GFdn(sqn)=GFdn(sqn)+Pb(sqn,node)*S(node)*S(node)*(1/1)*(1/1)*10/9
    ; % normalized generalized-damping-force

        GFsn(sqn)=GFsn(sqn)+Ps(sqn,node)*S(node)*S(node)*(1/1)*(1/1)*10/9
    ; % normalized generalized-spring-force
        % GFd(sqn)=GFd(sqn)+Pb(sqn,node)*S(node)*S(node)/(norm(S,2)^2);
        % GFs(sqn)=GFs(sqn)+Ps(sqn,node)*S(node)*S(node)/(norm(S,2)^2);
    end
end

%plot(SQN, GFd, SQN, GFs);

plot(SQN, GFdn,'--', SQN, GFsn);
% axis([0,2000,0,1])

save G_SQFD_force_d200-20000
A4 phase3.m

load G_SQFD_force_d100-20000

% % The dynamics response can be obtained by this code for all
% fixed-fixed beam.
% % Please change the value of f_Kn to specify the slip-flow
% model we want to use.
%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Burg's model 1+6*Kn
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%f_Kn=17.98; % 15mbar
%P0=1500;

%f_Kn=26.44; % 10mbar
%P0=1000;

%f_Kn=51.88; % 5 mbar
%P0=500;

%f_Kn=255.5; % 1 mbar
%P0=100;

%f_Kn=2546.2; % 0.1 mbar
%P0=10;

f_Kn=5089; % 0.05 mbar
P0=1;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Seidel's model 1+Kn/0.7
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%f_Kn=5.043; % 15mbar
%P0=1500;

%f_Kn=7.057; % 10mbar
%P0=1000;

%f_Kn=13.114; % 5 mbar
%P0=500;
%f_Kn=61.6;  % 1 mbar
%P0=100;

%f_Kn=287.1;  % 0.8 mbar
%P0=80;

%f_Kn=382.73;  % 0.6 mbar
%P0=60;

%HH's model  1+6*a*Kn  a=0.9

%f_Kn=16.28;  % 15mbar
%P0=1500;

%f_Kn=23.90;  % 10mbar
%P0=1000;

%f_Kn=46.79;  % 5 mbar
%P0=500;

%f_Kn=230;  % 1 mbar
%P0=100;

%f_Kn=287.1;  % 0.8 mbar
%P0=80;

%f_Kn=382.7;  % 0.6 mbar
%P0=60;

%HH's model  1+6*a*Kn  a=0.8

%f_Kn=14.58;  % 15mbar
%P0=1500;

%f_Kn=21.53;  % 10mbar
%P0=1000;

%f_Kn=41.70;  % 5 mbar
%P0=500;

%f_Kn=204.6;  % 1 mbar
%P0=100;

%HH's model  1+6*a*Kn  a=0.7
\% f_Kn=12.89; \% 15mbar
\% P0=1500;

\% f_Kn=18.81; \% 10mbar
\% P0=1000;

\% f_Kn=36.62; \% 5 mbar
\% P0=500;

\% f_Kn=179.2; \% 1 mbar
\% P0=100;

\% HH's model \hspace{1cm} 1+6*a*Kn \hspace{1cm} a=0.6

\% f_Kn=11.19; \% 15mbar
\% P0=1500;

\% f_Kn=16.26; \% 10mbar
\% P0=1000;

\% f_Kn=31.53; \% 5 mbar
\% P0=500;

\% f_Kn=153.7; \% 1 mbar
\% P0=100;

\% Kunsen's model \hspace{1cm} f_Kn=1+Z*Kn/0.1474
\% \hspace{1cm} Z=(Kn+2.507)/(Kn+3.095)

\% f_Kn=18.294; \% 15mbar
\% P0=1500;

\% f_Kn=27.46; \% 10mbar
\% P0=1000;

\% f_Kn=55.61; \% 5 mbar
\% P0=500;

\% f_Kn=285.1; \% 1 mbar
\% P0=100;

\% f_Kn=356.7; \% 0.8 mbar
\% P0=80;

\% f_Kn=476.8; \% 0.6 mbar
\% P0=60;
\%W=30e-6;
\%L=350e-6;
\%h0=1.6e-6;
\%M=1.09831E-11;
\%K=43.4609*0.998;

\%
\% Parameters for 300*30 beam
\%
W=30e-6;
L=300e-6;
h0=1.6e-6;
K=4.7161918e+01*1.01045;
M=9.3142500e-12;

visc_eff=1.77e-5/f_Kn;

www=sqrt(K/M);

sqn_ref=round(12*visc_eff*L^2*www/(P0*h0^2));

sqn=sqn_ref-100:0.25:sqn_ref+100;

for f=1:1:800
  w(f)=sqn(f)/(12*visc_eff*L^2/(P0*h0^2));
  s=j*w(f);
  ww(f)=w(f)/(2*pi);
  
  \%
  \% Get damping force (ratio) and spring constant of the air film
  \%
  for i=1:100
    if ( sqn(f)>=SQN(i) ) \& ( sqn(f)<SQN(i+1) )
      break;
    end
  \% [i SQN(i) sqn(f)]
  end
  \% i
  SQNt=SQN(i:i+1);
  GFdt=GFdn(i:i+1);
  GFst=GFsn(i:i+1);
  % Ca=interp1(SQNt,GFdt,sqn(f))*P0*W*(9/10)*L/25/h0/w(f);
  % Ka=interp1(SQNt,GFst,sqn(f))*P0*W*(9/10)*L/25/h0;
  Ca=interp1(SQNt,GFdt,sqn(f))*P0*W*L/25/h0/w(f);
\[ Ka = \text{interp1}(\text{SQNt}, \text{GFst}, \text{sqn(f)}) \times 0.05W/L/25/h0; \]

\[ TF = \frac{K}{(M's's + Ca's + (K + Ka))}; \]
\[ TF_p = \frac{K}{(M's's + Ca's + K)}; \]

\% Mag(f) = 20*log(abs(TF));
\[ Mag(f) = \text{abs(TF)}; \]
\[ \text{Phase}(f) = \text{angle(TF)}; \]

\% Mag_p(f) = 20*log(abs(TF_p));
\[ Mag_p(f) = \text{abs(TF_p)}; \]
\[ \text{Phase}_p(f) = -\text{angle(TF_p)}; \]

end

[Mag_max iw] = max(Mag);

Mag_hp = Mag_max/sqrt(2);

for i = 1:1:800
    if (Mag(i) < Mag_hp) & (Mag(i+1) > Mag_hp)
        break
    end
end

Mag_max;
ww = w*w(iw);

wwi = w*w(i-2:i+2);
Magi = Mag(i-2:i+2);

BW_h = w*r - interp1(Magi, wwi, Mag_hp);
BW = BW_h*2;
QF = w*r/BW;

wr
BW
QF

\% semilogx(ww, Mag, '.' , w, Mag_p, ' - ');
plot(ww, Mag, '.' , w, Mag_p, ' - ');
\%axis([ 3.5e5 3.7e5 0 300]);
xlabel('Frequency (Hz)');
ylabel('Magnitude');

\% legend('Nominal (low pressure)', 'Proof mass with holes (1 atm)', 'Proof mass without holes (1 atm)')

\% Phase = Phase\*180/(2*pi);
Phase_p = Phase_p * 180 / (2 * pi);
Phase2 = Phase2 * 180 / (2 * pi);

plot (sq_no, Fb, sq_no, Fs)
xlabel('Squeeze Number')
ylabel('Non-dimensional Force')
text(25, 0.2, 'Damping Force')
text(25, 0.63, 'Spring Force')
A5. List of codes from the Numerical Recipe in C

The following codes are subroutines from the *Numerical Recipes in C* [36]. These codes should be complied and linked with the C code listed in A1.

1. nrutil.c
2. rkck.c
3. rkqc.c
4. odeint.c
5. rkqs.c
6. polint.c
7. nrutil.h