

**Essays on Aggregate and Individual Consumption
Fluctuations**

by

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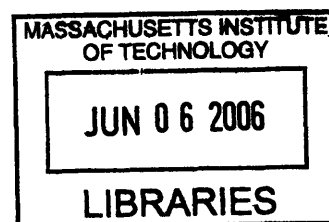
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Abstract

This thesis consists of three essays on aggregate and individual consumption fluctuations.

Chapter 1 develops a quantitative model to explore aggregate and individual consumption dynamics when the income process exhibits regime-switching features, and compares its performance with the conventional linear model. For this purpose, I consider an economy populated by a large number of consumers whose incomes are subject to both aggregate and idiosyncratic shocks. The notable element of the model is that a latent regime-switching stochastic variable governs both the trend growth of the aggregate component and the counter-cyclical variances of the idiosyncratic components in individual earnings. I demonstrate that the model can provide a reasonable description of the cyclical behavior of actual consumption fluctuations, and can successfully replicate some key empirical properties of aggregate consumption growth, such as smaller volatility than income growth, greater volatility in recessions than in expansions, and a negatively skewed and leptokurtic distribution, while the typical linear model fails to do so. The model highlights that the interaction between aggregate and idiosyncratic shocks, shifts in the cross-sectional distribution of individual consumers' optimal consumption decisions, and learning about the underlying business-cycle regime play a critical role in explaining aggregate consumption dynamics. In addition, comparison between individual and aggregate consumption and aggregation issues are addressed. Finally, I show that the regime-switching features, combined with heteroskedastic income risk, can account for more than 50% of aggregate consumption fluctuations, but less than 4% of individual consumption fluctuations.

It is widely believed that utility maximization implies that expected consumption growth should be higher in recessions which are associated with higher income uncertainty because consumers with precautionary saving motives save more to tilt up their consumption path. Evidence in the literature, however, does not seem to support this prediction. Chapter 2 tries to reconcile these seemingly contradictory observations. First, noting that recessions are times of both higher income uncertainty and lower income growth, I perform comparative experiments to see each effect on expected consumption growth. Higher income uncertainty indeed increases expected consumption growth, while lower income growth does the opposite. Next, in a calibrated switching regime income process example, where recessions are associated with lower income growth and higher uncertainty, I show the net effect may well decrease, rather than increase, expected consumption growth. I then compare my results to the usual argument in the literature, based on approximations to the Euler equation.

Chapter 3 develops econometric methods to estimate consumers' risk aversion and time

discount rate parameters in a dynamic stochastic general equilibrium (DSGE) model set-up, using the simulated method of moments (SMM) technique. This approach, based on a numerical solution algorithm, offers several advantages over traditional methods, which directly estimate a (linearized) Euler equation. In particular, the model allows us to incorporate a possibly non-linear underlying income process and the selection of moment conditions into the estimation procedure. I also consider two extensions by (1) allowing for the parameters of the model to be state-dependent and (2) incorporating the agent's learning about the latent aggregate state.

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Chapter 1

Regime-Switching Fluctuations, Aggregate and Individual Consumption Fluctuations

1.1 Introduction

The dynamics of recessions are qualitatively different from those of expansions in many aspects. For instance, recessions are relatively shorter than expansions, and the decline in economic activity in recessions is generally more rapid and disruptive than the recovery, which is typically a slow and gradual process. These intriguing non-linear features of business cycles have long been of great interest to researchers in dynamic macroeconomics and macroeconometrics. Indeed, various time-series econometric modeling techniques have generated a great deal of evidence that suggests the non-linear and asymmetric nature of business cycle dynamics.¹

Among these, the Markov regime-switching model (Hamilton [1989]) is one of the most popular non-linear time series models, and has been quite successful in characterizing time series behavior of major macro variables over business cycles and a variety of financial market

¹See Rothman [1999], Franse and van Dijk [2000], and Milas and Rothman, and van Dijk [2005] for survey on this.

phenomena.² In Hamilton's regime-switching setup, contractions and expansions are modeled as switching regimes of the stochastic process generating the growth rate of GDP, and the model allows the dynamics of recessions to be qualitatively distinct from those of expansions.

At the same time, another front of macroeconomic research has focused on the implication of idiosyncratic risk under incomplete markets and its magnitude in various contexts, such as aggregate saving and wealth distribution (Aiyagari [1994], and Krusell and Smith [1998]); asset pricing (Lucas [1994], Heaton and Lucas [1996], and Constantinides and Duffie [1996]); physical and human capital accumulation and growth (Krebs [2003a], and Angeletos and Calvet [2005]); and the welfare cost of business cycles (İmrohoroğlu [1989], Atkeson and Phelan [1994], and Krusell and Smith [1999]).

In particular, recent evidence suggests that the periods of recession are regarded as phases of reallocation, and are accordingly considered to involve more risks at micro level. For example, Davis and Haltiwanger [1992] conclude from data on the US manufacturing sector that job reallocation, the sum of job creation and job destruction, exhibits significant counter-cyclical time variations. As the changes in employment status are likely to be strongly correlated with individual earnings over business cycle fluctuations, recessions tend to be associated with more idiosyncratic income risk.

More direct evidence of the counter-cyclical income risk is provided by Storesletten, Telmer and Yaron [2004a]. They observe the fact that the cross-sectional variance of the residuals from an earning regression contains the entire history of macroeconomic shocks, which allows them to identify the cross-sectional effects of aggregate shocks with the relatively short time series. Based on the GMM estimation on PSID data, they argue that individual labor income is 80% more risky in recessions. Hence, the size of idiosyncratic income risk over cycles and its interaction with aggregate risk is an important question in macroeconomics.³

All the research projects mentioned above suggest that recessions are qualitatively distinctive phases, and that business cycles can be better characterized by non-linear dynamics (pre-

²For some examples of applications and extensions of the regime-switching model, see Kim and Nelson [1999] and Hamilton and Raj [2002].

³Meghir and Pistaferri [2004] also question the conventional linearity/i.i.d. assumption in the individual earning process, and find the evidence of ARCH type heteroskedastic risk.

sumably the regime-switching type) at both macro and micro levels. If the regime-switching model is indeed the true data-generating process for the economy, several interesting questions naturally arise with respect to agents' expectations and optimal consumption behavior in response to stochastic switching between two regimes over business cycles.

This paper is motivated by these observation and aims to explore individual agents' consumption behavior and the resulting aggregate consumption dynamics when individual agents experience non-linear/regime-switching type fluctuations in their income variations. The regime-switching model, supported by wide and successful applications in characterizing business fluctuations, indeed provides a natural framework to explore the effects on the consumption behavior of counter-cyclical income risk over cycles in this environment.

The notable element of the model is that a latent regime-switching stochastic variable governs both the trend growth of the aggregate component and the counter-cyclical variances of the idiosyncratic components in individual earnings, an assumption that is supported by recent empirical evidence.

This specification provides us with a rich framework to study the interactions between aggregate and idiosyncratic shocks, as well as the cyclical evolution of cross-sectional consumption distributions. The model highlights the following as key ingredients in understanding consumption dynamics in this regime-switching environment.

First, the model points out the different natures of two aggregate shocks: while the frequent stationary shocks to aggregate growth have relative small effect on both aggregate and individual consumption, the regime-switching shocks, which affect both the growth rate and variances of the individual income process, lead to changes in the location and shape of the distribution of individual consumption. Combined with counter-cyclical idiosyncratic income risks, these regime-switching features generate more volatile responses in aggregate consumption growth in recessions and/or at business cycle turning points.⁴

Second, the shift in the distribution of individual consumption is asymmetric across two regimes: in recessions, the entire distribution not only shifts down but also fans out. In particular, the lower tail of the distribution seems to be hit by a recession more severely due to the concavity of optimal consumption policy and/or the presence of liquidity con-

⁴Comparison between individual and aggregate consumption and aggregation issues are also addressed in this context.

straint, combined with the counter-cyclical risk, and this creates further drops in aggregate consumption growth in recessions.

Third, the model also emphasizes that the individual learning process concerning the underlying business-cycle regime plays a critical role in explaining aggregate consumption dynamics. Individual consumers' learning about the unobserved aggregate regime, often associated with delay and misperception about the true state, makes consumption fail to immediately respond at the true business-cycle turning points, and thus generates smoothing effects.⁵

Building on these elements, I demonstrate, through a series of simulations, that the model can provide a reasonable description of the cyclical behavior of actual consumption fluctuations, and can successfully replicate some key empirical properties of aggregate consumption growth observed in data: smaller volatility than income growth, greater volatility in recessions than in expansions, and a negatively skewed and leptokurtic distribution. The alternative linear model without the non-linear/regime-switching features however fails to do so. In addition, comparison between individual and aggregate consumption and aggregation issues are addressed in this context.

In fact, despite the wide application and notable success of the Markov regime-switching model in the areas of macroeconometrics and empirical finance, there has been limited work in the aggregate consumption literature incorporating this feature; research in the aggregate consumption and precautionary saving literature has been mainly conducted with a linear framework, which usually specifies fluctuations in incomes as some type of ARIMA process (for example, Blinder and Deaton [1985], Deaton [1991, 1992], Carroll [1992, 1997, and 2001], and Ludvigson and Michaelides [2001]). The performance of the linear models, however, has not been very successful; it seems that they cannot replicate some key features of aggregate consumption fluctuations.⁶

⁵In this sense, this paper shares another line of research that emphasizes individual learning about the underlying unobservable states; A few examples include Veronesi [1999], Van Nieuwerburgh and Veldkamp [2004], Wang [2004], and Guvenen [2005].

⁶For example, Deaton [1991] develops an aggregate version of buffer stock model in which individual income is subject to both aggregate and idiosyncratic shocks along with liquidity constraints, and individual agents cannot distinguish two types of shocks, which is emphasized as one of key features in consumers' behavior by Pischke [1995]. He shows that such a model can produce some smoothing at aggregate level, but the smoothing effect is very small. In an extension of Deaton's work, Ludvigson and Michaelides [2001] show that the standard buffer-stock model, which incorporates borrowing restrictions, impatience, and precautionary

Although some recent papers argue that counter-cyclical idiosyncratic risk has significant implications in macro research,⁷ it seems that several specific aspects of consumption dynamics are not studied in detail, e.g., the cyclical evolution of cross-sectional distribution of individual variables, individual consumers' learning about the aggregate state (under the time-varying uncertainty), and conditional dynamics which exhibit asymmetric features depending on aggregate states; all of which may have potentially significant implications for understanding consumption dynamics over business cycles, especially when some non-linear patterns are present in them.

This paper addresses these issues in detail and shows that they can provide several important insights in understanding consumption fluctuations. I also look at the *conditional* aspect of consumption dynamics as well as the *unconditional* picture with which most research on consumption over cycles is concerned.

Finally, I investigate the quantitative implications of regime-switching features combined with heteroskedastic income risk for consumption fluctuations through a series of simulations with the nature of these shocks modified. The results show that these non-linear/ regime-switching features can account for a large fraction of aggregate consumption fluctuations (more than 50%), but a very small part of individual consumption fluctuations (less than 4%).

The remainder of this paper is organized as follows. Section 2 documents some relevant empirical regularities of aggregate consumption which support the non-linear/ regime-switching view. Section 3 presents the baseline model, followed by a discussion of the calibration/numerical solution method in Section 4. Section 5 then presents the basic key results of the model, and Section 6 reports a set of main results from simulation exercises. The last section concludes.

motives, does not generate robust "excess sensitivity" (consumption growth is positively correlated with lagged income growth) and "excess smoothness" (consumption growth is less volatile than income growth) of aggregate consumption growth. They argue that these implications arise only in the alternative framework which presumes that individuals do not observe the economy-wide component of their earnings. However, even with this incomplete information assumption, they find the aggregate consumption is still too volatile, and its changes are too excessively correlated with lagged income growth.

⁷Some examples include aggregate fluctuations (Gourinchas [2000]), the equity premium (Storesletten, Telmer and Yaron [1999]), and the welfare cost of business cycles (Storesletten, Telmer and Yaron [2001] and Krebs [2003b]).

1.2 A Few Empirical Findings from U.S. Aggregate Consumption Data

Before presenting the baseline model and its results, this section provides documentation of some empirical evidence about the U.S. aggregate consumption data to motivate the non-linear/regime-switching framework. Figure 1.A shows the growth rates of quarterly per-capita real disposable income and (non-durable) consumption of the U.S. (along with the 9-quarter moving averages) for 1953:I - 2003:III. Aggregate consumption growth seems to be less volatile than income growth, and a closer look reveals that the movement of consumption growth seems to be relatively stable within business-cycle regimes - booms and recessions, while it exhibits large fluctuations around recession times in several business cycle episodes. These features are further confirmed in Figure 1.B. This figure plots the volatility of (non-durables and services, and non-durables) consumption growth, which I calculate as the squares of deviations from the 9-quarter moving averages. Although consumption growth is fairly stable for most of the time, it seems to be a lot more volatile around the times of recessions.

I also look at these intriguing asymmetric features from the distributional point of view, and Figure 1.C shows the kernel density estimation of aggregate consumption growths.⁸ To make the point clear, I also plot two normal densities with the same means and standard deviations of the data. A few points are worth mentioning. First, the distributions do not seem to be symmetric, rather negatively skewed. Second, having the slight humps on the left, the densities also imply the underlying distribution for aggregate consumption growth may be a mixture of two distributions (with the main one centered around 0.5% and the other small one around -1%).

Table 1 summarizes the above discussion and reports relevant statistics for the U.S. aggregate consumption (non-durables and services, non-durables, services, and durable good expenditure) growth data. It presents the following four statistics: relative volatility (ratio of the standard deviation of consumption growth to that of income growth), cross-regime volatility (ratio of the average squares of deviations from the whole sample mean in reces-

⁸The kernel density estimation uses the Epanechnikov kernel and 200 equidistant points; the bandwidth is set to 0.002.

sions to that of the rest of the sample),⁹ skewness, kurtosis, and Jacque-Bera statistics for the normality test.¹⁰

Except for durable goods whose growth is significantly more volatile than income growth, the growth rates of all the other measures of consumption are about half to two thirds times as volatile as income growth (relative volatility being 0.510 to 0.789, second column). However, as suggested in Figure 1.B, the consumption growth is about two times more volatile in times of recessions than in expansions (third column).¹¹ The table also shows that the distributions are negatively skewed and leptokurtic - peaked at center and/or fat-tailed, (fourth and fifth column) so that they have some type of non-normal distribution. This is statistically confirmed by a Jarque-Bera test (sixth column); in all cases the null hypotheses of normality are rejected.

Overall empirical characteristics of consumption growth documented above are at odds with the standard linear and/or normal view of aggregate consumption fluctuations over business cycles in dynamic macro research. In particular, the linear model could not provide a convincing argument as to the disruptive changes in consumption volatility associated with recessions.

1.3 A Baseline Model

Motivated by the evidence in Section 2, this section presents a baseline model economy to explore aggregate and individual consumption fluctuations under the assumption that the individual income process has the regime-switching features.

⁹In my calculation of the cross-regime volatility, the recession periods include the NBER recession dates, and I also add 1 (left column) or 2 (right column) pre-/post- quarters to each recession episode. This is because in some cases NBER business cycle dates, which are based on monthly frequency, do not provide clear lines to cut the sample. For instance, when the month of February is a business cycle trough, it is not obvious whether to count the first quarter in a recession, or expansion regime. Moreover, given that there is no clear evidence that the NBER business cycle dating methodology is necessarily based on the non-linear/regime-switching view as in this paper, this method may give a conservative measure about the cross-regime volatility of consumption growth.

¹⁰Skewness (S) and kurtosis (K) are calculated as follows: $S = \sum ((y_n - \bar{y}) / \hat{\sigma})^3 / N$ and $K = \sum ((y_n - \bar{y}) / \hat{\sigma})^4 / N$, where N is the sample size, y are data, and $\hat{\sigma}$ is based on the biased estimator for the variance (Bickel and Doksum [1977]).

¹¹A simple F-test rejects the null hypothesis that variances are the same across two regimes in most cases.

1.3.1 Setup

A. Consumer's Problem

Consider an individual consumer's problem in which he wishes to maximize the discounted sum of expected lifetime utility with the isoelastic preference:¹²

$$\max E_0 \left[\sum_{t=0}^{\infty} \beta^t U(C_{it}) \right] \quad (1.1)$$

with

$$U(C_{it}) = \begin{cases} \frac{C_{it}^{1-\gamma}-1}{1-\gamma}, & \gamma \neq 1 \\ \ln C_{it}, & \gamma = 1 \end{cases}$$

subject to the constraints:

$$W_{it+1} = (1+r)(W_{it} - C_{it}) + Y_{it+1}, \quad (1.2)$$

$$A_{it} \geq -B, \quad (1.3)$$

where Y_{it} denotes (labor) income, A_{it} is (liquid) asset, $W_{it} = A_{it} + Y_{it}$ is wealth, or cash-on-hand as in the precautionary/buffer stock saving literature, and $(1+r)$ is the (constant) gross interest factor. The first equation, (2), is the usual intertemporal budget constraint, and the second inequality, (3), states that the available credit is limited by a certain amount, B .

B. Individual Income Process: Specification

For the specification of an individual consumer's income process, I deviate from the convention in the precautionary saving and aggregate consumption literature, which is mainly formulated with the linear specification for income process, and incorporate the non-linear/regime-switching features in the trend growth and variances into income fluctuations. More specifically, I assume that the evolution of income dynamics (in terms of log) is given by the

¹²Throughout the paper, I distinguish individual specific variables from aggregate variables by putting subscript i on them.

following stochastic processes:

$$\begin{aligned}
\ln Y_{it} &= \ln P_{it} + \ln U_{it}, \\
\ln P_{it} &= \ln P_{it-1} + \ln G_t + \ln N_{it}, \\
\ln G_t &= \alpha_0 + \alpha_1 S_t + \varepsilon_t,
\end{aligned} \tag{1.4}$$

where

$$\begin{aligned}
\varepsilon_t &\sim N(0, \sigma_\varepsilon^2); \\
n_{it} \equiv \ln N_{it} &\sim (0, \sigma_{n|S}^2), \quad \sigma_{n|S}^2 = S_t \sigma_{n|E}^2 + (1 - S_t) \sigma_{n|R}^2, \quad \sigma_{n|R}^2 > \sigma_{n|E}^2; \\
u_{it} \equiv \ln U_{it} &\sim (0, \sigma_{u|S}^2), \quad \sigma_{u|S}^2 = S_t \sigma_{u|E}^2 + (1 - S_t) \sigma_{u|R}^2, \quad \sigma_{u|R}^2 > \sigma_{u|E}^2;
\end{aligned}$$

and $S_t \in \{0, 1\}$, whose stochastic process is a first order Markov process to be specified below.

The formulation in (4) says that individual consumer's income, Y_{it} , consists of permanent component, P_{it} , and transitory component, U_{it} ; the growth of the permanent income, $\Delta \ln P_{it}$, can be further decomposed into aggregate component, $\ln G_t$, and idiosyncratic component, $\ln N_{it}$. This type of income decomposition is widely used in the consumption literature and seems to fit the data quite well.¹³

The distinctive feature of the model is that the trend growth of the aggregate component and the variances of idiosyncratic-permanent/transitory components in individual income are assumed to follow a regime-switching process. That is, the conditional mean growth rate of aggregate income, $\ln G_t = \alpha_0 + \alpha_1 S_t$, and the counter-cyclical variances of two idiosyncratic shocks, $\sigma_{n|S}^2$ and $\sigma_{u|S}^2$, are determined by the stochastic latent variable S_t , which switches between two unobserved states: $S_t = 0$ (recession, R) or 1 (expansion, E). And the transition

¹³For example, Carroll [1992, 1997], Hubbard, Skinner and Zeldes [1995], and Gourinchas and Parker [2002]; see MaCurdy [1982] and Abowd and Card [1984] for more microeconomic foundations on this.

between two states is governed by the following probabilities:¹⁴

$$\Pr[S_t = 1|S_{t-1} = 1] = p, \quad \Pr[S_t = 0|S_{t-1} = 0] = q. \quad (1.6)$$

Given the above setup, the growth rate of an individual consumer's income then can be compactly written as follows:

$$\begin{aligned} \Delta \ln Y_{it} &= \Delta \ln P_{it} + \Delta \ln U_{it} \\ &= \ln G_t + \ln N_{it} + \Delta \ln U_{it} \\ &= \underbrace{\underbrace{\alpha_0 + \alpha_1 S_t}_{\text{regime-switching}} + \underbrace{\varepsilon_t}_{\text{stationary}}}_{\text{aggregate}} + \underbrace{\underbrace{n_{it}}_{\text{permanent}} + \underbrace{u_{it} - u_{it-1}}_{\text{transitory}}}_{\text{idiosyncratic}} \end{aligned} \quad (1.7)$$

C. Individual Income Process: Discussion

The above specification of the income process has several notable features, and some major implications of the model are closely related to them. First, each period an individual consumer's income is subject to four types of shocks. Among the two aggregate components, one is a small/stationary and frequent shock to the cyclical component (ε_t), and the other is an occasional, but trend-reverting innovation (V_t). Hence all the consumers are expected to experience occasional revisions in the trend in their income growth along with some stationary variation around it since the aggregate components are common to them. In addition to the aggregate shocks, they also face two types of individual specific shocks: permanent (n_{it}) and

¹⁴These two parameters fully characterize the stochastic process for S_t , and it can be represented by the following AR(1) process:

$$S_t = (1 - q) + (p + q - 1)S_{t-1} + V_t, \quad (1.5)$$

where, conditional on $S_{t-1} = 1$,

$$V_t = \begin{cases} (1 - p) & \text{with probability } p \\ -p & \text{with probability } 1 - p, \end{cases}$$

conditional on $S_{t-1} = 0$,

$$V_t = \begin{cases} -(1 - q) & \text{with probability } q \\ q & \text{with probability } 1 - q. \end{cases}$$

transitory (u_{it}).

Second, the state variable S_t governing the current business-cycle state as well as the variances of idiosyncratic shocks may not be directly observable even after its realization. Instead, each period consumers need to make an inference about the current state in order to make their optimal consumption decision. Later discussion will demonstrate that this inference problem due to the incomplete information plays an important role in matching the model's simulated data with actual data.

Third, if the expansion regime is likely to persist longer than the recession regime, that is, $p > q > 1/2$,¹⁵ the conditional variance of V_{t+1} is greater when the current regime is a recession: $Var_t[V_{t+1}|S_t = 0] = q(1 - q) > Var_t[V_{t+1}|S_t = 1] = p(1 - p)$. In addition, the model assumes that the idiosyncratic-permanent/transitory component has greater variability in recession. These patterns of heteroskedasticity in the income process thus imply that individual consumers' income growths tend to be not only lower but also more volatile in recessions (both in aggregate and idiosyncratic components). Analytically, it is straightforward to show:

$$\frac{Var_t[\Delta \ln Y_{it+1}|S_t = 0 \text{ (recession)}]}{Var_t[\Delta \ln Y_{it+1}|S_t = 1 \text{ (expansion)}]} = \frac{\alpha_1^2 q(1 - q) + \sigma_\varepsilon^2 + \sigma_{n|R}^2 + \sigma_{u|R}^2}{\alpha_1^2 p(1 - p) + \sigma_\varepsilon^2 + \sigma_{n|E}^2 + \sigma_{u|E}^2}. \quad (1.8)$$

As $q(1 - q) > p(1 - p)$, $\sigma_{n|R}^2 > \sigma_{n|E}^2$, and $\sigma_{u|R}^2 > \sigma_{u|E}^2$, this ratio is bigger than one.¹⁶

The income dynamics assumed in (4) - (7) are (at least qualitatively) consistent with the actual features of earning data both in micro and macro dimensions. Individual incomes are much more volatile than aggregate per-capita income due to large idiosyncratic shocks, and are often believed to contain a moving average component – typically MA(1).¹⁷ These are well reflected in the actual individual income process in the model, and the regime-switching specification of aggregate (per-capita) income also seems reasonable given the model's wide and successful application in most macro time series, including aggregate income.

Finally, through (3), I assume that consumers can borrow up to some amount. Specifically, individual consumers' borrowing limit is assumed to be stochastic, and is set equal to a fixed

¹⁵This is an empirically valid assumption; see the calibration section below.

¹⁶Using the benchmark calibrated values (to be specified below), this ratio is as large as 2.34. (and 3.19 when I consider permanent components only)

¹⁷See MaCurdy(1982) for example.

fraction of the current permanent income, $B(P_{it}) = bP_{it}$. This is consistent with the view that actual lenders try to limit the ratio of the borrower's debt to income (see Ludvigson (1999) and Carroll (2001)).

I now look at individual consumers' optimal behavior under this environment, and consider two versions of the regime-switching model depending on the information assumption. Subsection 3.2 considers the complete information case in which consumers are assumed to know the true current state, S_t , as a baseline case, and Subsection 3.3 discusses more realistic version, the incomplete information case where they are supposed to make an inference about the current aggregate state which is typically unobserved.

1.3.2 The Complete Information Case

In the complete information case, a consumer's optimal consumption decision at time t depends on three state variables: wealth (W_{it}), permanent income (P_{it}), and aggregate state (S_t), and the Euler equation for his problem is:

$$C_{it}^{-\gamma} = \max\{(W_{it} + bP_{it})^{-\gamma}, \beta(1+r)E_t[C_{it+1}^{-\gamma}]\}. \quad (1.9)$$

Using the homogeneity of the optimal consumption, the Euler equation (9) can be expressed in terms of normalized variables (ratio of the variables to the permanent income; $c_{it} = C_{it}/P_{it}$, and $w_{it} = W_{it}/P_{it}$) in each regime.¹⁸ For the expansion regime, we have

$$\begin{aligned} & c(w_{it}, E)^{-\gamma} & (1.10) \\ = & \max \left\{ (w_{it} + b)^{-\gamma}, \beta(1+r) \left\{ pE_t \left[(e^{g_{it+1|E}} c(w_{it+1|EE}, E))^{-\gamma} \right] \right. \right. \\ & \left. \left. + (1-p)E_t \left[(e^{g_{it+1|R}} c(w_{it+1|ER}, R))^{-\gamma} \right] \right\} \right\}, \end{aligned}$$

¹⁸See Appendix for the detailed derivation.

Likewise, for the recession regime,

$$\begin{aligned}
& c(w_{it}, R)^{-\gamma} \\
= & \max \left\{ (w_{it} + b)^{-\gamma}, \beta(1 + r) \left\{ (1 - q) E_t \left[(e^{g_{it+1|E}} c(w_{it+1|RE}, E))^{-\gamma} \right] \right. \right. \\
& \left. \left. + q E_t \left[(e^{g_{it+1|R}} c(w_{it+1|RR}, R))^{-\gamma} \right] \right\} \right\},
\end{aligned} \tag{1.11}$$

where

$$\begin{aligned}
g_{it+1|S_{t+1}} &= \alpha_0 + \alpha_1 S_{t+1} + \varepsilon_{t+1} + n_{it+1|S_{t+1}}, \\
w_{it+1|S_t S_{t+1}} &= e^{-g_{it+1|S_{t+1}}} (1 + r) (w_{it} - c(w_{it}, S_t)) + e^{u_{it+1|S_{t+1}}}.
\end{aligned}$$

The above system of two simultaneous equations implicitly defines the optimal consumption to income ratio (or normalized consumption) as a function of wealth to income ratio (or normalized wealth) for each regime, $c_{it} = c(w_{it}, S_t)$. And under the following ‘‘impatience’’ condition which I assume to be met below, there exists a unique solution to the problem:¹⁹

$$\beta(1 + r) E_t \left[(P_{it+1}/P_{it})^{-\gamma} \mid S_t \right] = \beta(1 + r) E_t \left[(e^{g_{it+1|S_{t+1}}})^{-\gamma} \mid S_t \right] < 1. \tag{1.12}$$

1.3.3 The Incomplete Information Case

The previous section relies on the strong information assumption that consumers observe the true business cycle regimes every period. This section discusses the incomplete information case where consumers do not know the current regime, which is presumably a more realistic description. When consumers do not know in which business cycle regime they are, they infer it to help their optimal consumption decision. Hence, in addition to the standard dynamic optimization problem, consumers also face a filtering, or signal-extraction, problem, in which they must infer the current state utilizing all the information they have, and update their belief in Bayesian fashion.

It is useful to introduce some additional notations to describe the consumer’s optimal behavior in this incomplete information case:

¹⁹See Deaton and Laroque [1992] for proof. This condition is a necessary condition for the consumer’s problem to have a unique solution, insures that consumers want to run down consumption over time.

- ψ_{it} : consumer i 's information set at time t ,
- μ_{it} : consumer i 's belief about the current state given his information set;
 $\mu_{it} = E_{it}S_t = \Pr[S_t = 1|\psi_{it}]$,
- $\tilde{\mu}_{it+1}$: consumer i 's interim probability about the aggregate regime at $t + 1$
before observing time $t + 1$ data;
 $\tilde{\mu}_{it+1} = \Pr[S_{t+1} = 1|\psi_{it}] = p\mu_{it} + (1 - q)(1 - \mu_{it})$.

As in the complete information case, we then can write the Euler equation in this case in terms of normalized variables

$$c(w_{it}, \mu_{it})^{-\gamma} = \max \left\{ (w_{it} + b)^{-\gamma}, \beta(1 + r) \left\{ \tilde{\mu}_{it+1} E_t \left[\left(e^{g_{it+1|E}} c(w_{it+1}, \mu_{it+1|E}) \right)^{-\gamma} \right] \right. \right. \\ \left. \left. + (1 - \tilde{\mu}_{it+1}) E_t \left[\left(e^{g_{it+1|R}} c(w_{it+1}, \mu_{it+1|R}) \right)^{-\gamma} \right] \right\} \right\}. \quad (1.13)$$

where $\mu_{it+1|E}$ and $\mu_{it+1|R}$ are the (conditional) beliefs that aggregate regime at $t + 1$ is expansion and recession, given the current information, respectively.

While more attractive and realistic, the incomplete information case creates a few issues which do not arise in the complete information environment. In order to make the (numerical) analysis simple, I make a few assumptions. First, I assume that consumers know the true stochastic structure of the income process and all the related parameters. So the only unobserved variable involved in a consumer's problem is the latent business-cycle regime variable, S_t .

The second issue is what kind of (income) data, ψ_{it} , are available to consumers in this inference problem. For the choice of ψ_{it} , I consider two cases. In the first case, it is assumed that consumers use aggregate income growth data only and ignore (noisier) individual information (denoted AGG), and the second case assumes they use all the available data, both aggregate and individual income growth data with appropriate weights (denoted AGG+IND).^{20 21}

²⁰Note that although a lot noisier, individual data also contain some information about the current regime via their second moments, so consumers may want to use them (with a smaller weight).

²¹One may also consider the remaining IND case where consumers use their income data only for their inference. The earlier version of this paper considers this case as well, and in turns out that aggregate consumption growth is too smooth compared to the income growth, and this assumption is not very convincing as the empirical findings from the consumer sentiments indicate that consumers' expectation, or sentiment are, on average, closely correlated with aggregate variables (Carroll [1992], and Carroll, Fuhrer and Wilcox [1994], although at the same time there is a substantial heterogeneity among agents (Souleles [2004]).

Finally, I make the following two behavioral, or bounded rationality, assumptions to make numerical solution algorithm and simulation procedure simple: (i) Consumers in this economy are assumed to be only forward-looking, but not backward-looking. Recall that individual agents update their beliefs ceaselessly in every period, and hence with more data accumulating, they revise beliefs about the past regimes accordingly.²² This “error-correction” of the past belief will in turn induce the adjustment of past consumption expenditure under the assumption of backward-looking behavior. This behavior is ignored in my analysis. (ii) When evaluating the future marginal utility in the Euler equation, $E_t\left[\left(e^{g_{it+1}S_{t+1}}c(w_{it+1}, \mu_{it+1|S_{t+1}})\right)^{-\gamma}\right]$, I assume that once consumers form their belief about the current regime, μ_{it} , they do not consider future income data for their future belief formation. Hence, I assume that $\mu_{it+1|E} = 1$ and $\mu_{it+1|R} = 0$. Without this assumption, the evaluation of the future belief, $\mu_{it+1|S_{t+1}}$, involves the integration over all three lognormal shocks, in which case the computation time increases by multiple times.

Given the above assumptions, consumers’ behavior is as follows. First, in the beginning of each period they start with a prior belief about the current regime, $\tilde{\mu}_{it}$. Once their income data are realized, they calculate the conditional likelihood of each regime using the data, $f(\psi_{it}|S_t)$, and update their belief about the current aggregate state using Bayes rule:²³

$$\mu_{it} = \frac{f(\psi_{it}|S_t = 1)\tilde{\mu}_{it}}{f(\psi_{it}|S_t = 1)\tilde{\mu}_{it} + f(\psi_{it}|S_t = 0)(1 - \tilde{\mu}_{it})}. \quad (1.14)$$

Based on this belief and realized (normalized) wealth, they then decide optimal consumption.

Using a method similar to that in the complete information case, one can also solve (13) to obtain the optimal normalized consumption as a function of the normalized wealth, w_{it} , and the inferred business-cycle regime, $\mu_{it} : c_{it} = c(w_{it}, \mu_{it})$.²⁴

²²That is, it can be that $\Pr[S_t = 1|\psi_{it+s}] \neq \Pr[S_t = 1|\psi_{it}]$ for $s > 0$; although the true underlying process is Markovian, this error correction depends on the entire history of beliefs due to the random noises.

²³This filtering problem here is essentially the same method used in Hamilton [1989]. See the Appendix for the derivation.

²⁴See Appendix for the detailed numerical solution procedure.

1.4 Calibration and Numerical Solution Procedures

In the case of stochastic labor income with the isoelastic utility, there is no known analytical solution to an individual consumer's problem. Hence, I instead resort to a numerical solution method. This section briefly describes the calibration procedure and the numerical solution method used to obtain the optimal consumption policy of the model.

I first address the issues concerning calibration of parameters used in the model. The time unit of the model is quarter. For the post-Korean War quarterly U.S. aggregate data (1953:I - 2003:III), the average durations of expansion and recession are 20.25 and 3.56 (both in quarters), respectively. These numbers help us pin down the transition probabilities between the two regimes. Using the expected duration formula for each regime, $1/(1-p) = 20.25$ and $1/(1-q) = 3.56$, I set $p = 0.95$ and $q = 0.72$ for each regime's retention probabilities. For the average income growth rates in each regime and the standard deviation of the within-regime innovation, I follow the values from aggregate data to set $\alpha_0 = -0.00585$, $\alpha_1 = 0.0115$, and $\sigma_\varepsilon = 0.008$.²⁵

As to the risk aversion coefficient, I assign the typical value in the literature, $\gamma = 1$. And the interest rate is set to $r = 0.005$ (0.5%); the discount rate is parameterized as $\delta = 0.01$, which amounts to the discount factor of $\beta = 1/(1 + \delta) = 0.9901$. The borrowing constraint parameter is set $b = 0.0$.

To the best of my knowledge, there is no empirical work that estimates an individual consumer's (log linear) income process at quarterly frequency. Hence, with reference to the estimates using annual data (MaCurdy [1982], Abowd and Card [1989], Carroll [1992], Carroll and Samwick [1997], and Gourinchas and Parker [2002]), I need to convert the estimates at annual frequency to the parameter values at quarterly frequency for the model.

A few challenges arise when I try to do this conversion. A natural approach would be to simply model annual income as a sum of four quarterly incomes. This method, although right by definition, does not seem to give us any analytically tractable expression to obtain the standard deviations for two idiosyncratic shocks, and as pointed out by Ludvigson and Michaelides (2001), time aggregation of permanent shocks also significantly reduces the

²⁵Note that *per-capita* income growth is: $\ln \left(\sum_{i=1}^I \ln Y_{i,t}/I \right) - \ln \left(\sum_{i=1}^I \ln Y_{i,t-1}/I \right)$. Hence, the conditional mean income growth rate in each regime is $\alpha_0 + \alpha_1 S_t + 0.5\sigma_{n|S_t}^2$, rather than $\alpha_0 + \alpha_1 S_t$.

permanent component of quarterly income in order to match the observed negative autocorrelation in annual income growth at individual level. Alternatively, one could assume that the annual income growth is the sum of the four quarterly log differences during the year. While this method yields a neat conversion formula, it relies on quite strong an assumption that individual consumers receive exactly a quarter of their yearly income each quarter.²⁶

Rather than using either procedure above, I employ a less problematic approach, which is described as follows. Notice that the annual income growth rate can be well-approximated by the average of four 4-quarter log-differences of quarterly income. That is, ignoring the common/aggregate component, we can write annual growth rate as:

$$\begin{aligned}
& \ln Y_{it|A} - \ln Y_{it-1|A} & (1.15) \\
& \approx \frac{1}{4} \left(\sum_{s=0}^3 \ln Y_{it-s|Q} - \ln Y_{it-s-4|Q} \right) \\
& = \frac{1}{4} \left[\left(\sum_{j=0}^3 \sum_{i=0}^3 n_{it-j-i|Q} \right) + \left(\sum_{i=0}^3 u_{t-i|Q} - \sum_{i=0}^3 u_{it-4-i|Q} \right) \right]
\end{aligned}$$

where subscripts $|A$ and $|Q$ denote annual and quarterly frequencies of data. Working on (15) allows us to obtain the equation that links the variances at both frequencies: $\sigma_{n|A}^2 + (1^2 + (-0.5)^2) \sigma_{u|A}^2 = \frac{1}{4}(11\sigma_{n|Q}^2 + 2\sigma_{u|Q}^2)$. So this conversion method implies that the standard deviation of permanent shock at quarterly frequency is about $\sqrt{4/11} \approx 0.6$ of its annual counterpart, while the standard deviation of transitory shock at quarterly frequency is about $\sqrt{2 \times 1.25} \approx 1.6$ of its annual counterpart. Note that the variance of the permanent component has decreased while that of the transitory component has increased in this conversion because transitory shocks only partially cancel over time.²⁷

The estimates of standard deviation of permanent shocks in the individual earning process from annual panel data range about 0.1 to 0.15. So based on the above method and the empirical evidence on counter-cyclical idiosyncratic risk (Storesletten, Telmer and Yaron

²⁶This method indeed is used in Ludvigson and Michaelides [2001].

²⁷To get around these conversion issues, one may favor an annual model to analyze consumption behavior, rather than a quarterly model. Despite this advantage of annual frequency model, this paper favors a quarter as an individual agent's decision time period for the following reasons: first, most of the precautionary saving and empirical business cycle research have been performed at quarterly frequencies as a convention, and the use of annual model may hide the significant autocorrelation of income data at quarterly frequency. Second, as the average duration of a recession is less than a year, an annual model does not allow for a plausible transition probability.

[2004a]), I set $\sigma_{n|E} = 0.05$, and $\sigma_{n|R} = 0.09$. For the standard deviation of the transitory component, the estimates in the literature are about 0.17 to 0.25, depending on the sample and its selection criteria. As it is likely that a significant (but unknown) part of this value is inflated due to measurement error, and with lack of sufficient empirical evidence on the degree of counter-cyclicality of the transitory component, I decide to be on conservative side; I set $\sigma_{u|E} = 0.16$ and $\sigma_{u|R} = 0.24$, so that transitory income is 50% more volatile in recession.

Throughout the numerical exercise and simulation exercise below, I use this set of parameter values as a benchmark case, and consider cases with $b = 0.3$ and/or $\gamma = 2$ as a part of robustness check.

Note that, given the assigned values of the variances of each income component, the aggregate component accounts for a very small fraction of overall variations in individual income changes.²⁸

In the numerical solution procedure, the system of simultaneous equations, (10) and (11) in the complete information case, and (13) in the incomplete information case, respectively, is solved by the policy function iteration algorithm. The details of these procedures are given in Appendix.

1.5 The Dynamics of Consumption under the Regime-Switching Income Process

The optimal consumption policy in the complete information case obtained from the numerical method is shown in Figure 2.A. A few facts are worth mentioning. First, as well known, the optimal consumption function is (strictly) concave, and normalized consumption is higher in an expansion for the unconstrained part of optimal policy. Second, consumers start engaging in additional precautionary saving due to the liquidity constraint and decrease consumption well below the point at which they would be exogenously constrained. That is, even when consumers are not credit-constrained *per se*, they would want to start saving by taking into account the chance in which they are constrained. This precautionary behavior is more pronounced for the recession consumption policy. Figure 2.B shows the optimal con-

²⁸Table 2 summarizes the calibrated values for parameters, and Table 3 calculates the relative share of each component to the total variations in the individual income process.

sumption policy in the incomplete information case, as a function of normalized wealth and belief. As in the complete information case, one can observe similar features, namely that the (normalized) consumption is an increasing and concave function of wealth and belief.

With the optimal consumption policy at hand, we are in a position to investigate the key characteristics of the dynamics of aggregate and individual consumption. Many ideas and implications of the model are easiest to see for the complete information case, so I focus the main arguments on this case, and the discussion for the incomplete case is provided later. Below, I present a set of key results from a series of simulations to help understand the consumption dynamics both at aggregate and individual levels.

1.5.1 The Cyclical Dynamics of (Normalized) Consumption and Wealth

First, Figure 3.A shows a time-series simulation of normalized consumption and wealth of one randomly selected consumer over business-cycle fluctuations (along with the regime indicator). One can observe that there are frequent and large swings in wealth, but consumption is relatively stable even at the times of extreme variations in wealth, and that there seems no noticeable correlation between individual consumption behavior and the aggregate state. This result is essentially because the aggregate component is responsible for a very small fraction of individual income fluctuations.

A clearer picture of cyclical properties of consumption emerges when we look at the average consumer's behavior, and normalized consumption and wealth exhibit interesting dynamics, showing dramatic swings over the regime-switches. Figure 3.B displays a simulation result of the same variables for the (cross-sectional) average consumer.²⁹ One can easily see the following consumers' behavior over a typical business cycle: on the onset of a recession, consumers on average start to accumulate assets by decreasing consumption sharply; wealth keeps increasing as the recession continues. Then, as the recession ends and the economy switches back to an expansion, they use most of accumulated assets to finance their consumption boost. Hence, as soon as a boom is announced, consumption virtually jumps up, and wealth starts to decline. This pattern repeats over cycles maintaining the same qualitative

²⁹In this section, the average and aggregate variables are calculated using 2000 individual consumers' data (see the simulation section more on this). To make comparison clear and consistent, Figures 3.A – F use the same realization of the regime-switching variable.

features.

It is also interesting to notice the asymmetric pattern in wealth dynamics across two regimes, which indicates that consumers seem to be engaged in an S-s band-like behavior with conditional targets for each regime. For instance, as an expansion regime continues, the decline in wealth diminishes, and after reaching a certain level, there is no further decrease. Similarly, for a sufficiently long recession (not shown in the figure), once a good amount of wealth accumulates, consumers do not want to hold more assets. Hence, the typical wealth accumulation/decumulation patterns in an expansion/recession are concave/convex in time. This behavior is directly related to the asymmetric nature of business cycles, impatience, and prudence in consumer's behavior. Recall that consumers in this economy are impatient as well as prudent, and these two forces are at a war all the time. The relative strength of each depends on the aggregate regime and amount of wealth. For example, when the economy enters a recession, consumers' precautionary motives dominate impatience so that they will accumulate asset until they have a sufficient amount of wealth. However, as the recession is relatively short and is expected to end soon, holding too much asset is not optimal for consumers and is checked by their impatience. Their behavior in expansion is a mirror image of this, and a similar argument applies.

1.5.2 The Cyclical Evolution of the Distribution of (Normalized) Consumption

It is also important for understanding the dynamics of consumption to look at the changes in the distribution of individual consumption over business cycles. Figure 3.C displays the cyclical evolution of the cross-sectional distribution of individual consumption by plotting some selected percentiles of the distribution. Several points are worth noting. First, the distribution moves gradually within the regimes, but at the business cycle turning points, the entire distribution shifts in a discrete way from one consumption rule to the other. Note also that the median consumer's behavior is quite similar to the average consumer as shown in Figure 3.B. The second point to notice is that the distribution not only shifts down but also fans out in recessions, but the dispersion is asymmetric. The lower tail of distribution, representing poor consumers, is hit more severely by recessions while for the upper tail, rep-

representing wealthy consumers, the changes in normalized consumption are relatively smaller.³⁰ Figure 3.D also shows the cross-sectional evolution of the entire distribution over one typical business cycle, and one can see essentially the same features for the changing distribution.³¹ This asymmetric effect of recessions is basically due to the counter-cyclical idiosyncratic risks and the concavity of optimal consumption and/or the presence of liquidity constraint. When faced with increased income risks in recessions, consumers with low wealth are either liquidity-constrained or are on the high curvature part of the consumption policy, so they are forced to cut down consumption more than other consumers who stay on the relatively flat part of the consumption policy. These discrete and asymmetric shifts of the distribution will prove to have an important implication for aggregate consumption fluctuations.

1.5.3 The Cyclical Dynamics of Consumption Growth

I next move on to analyze the growth rates of individual and aggregate consumption. Before looking at the simulation results, it is useful to notice that the growth rates of individual and (*per capita*) aggregate consumption can be expressed as follows.³²

- individual consumption growth:

$$\Delta \ln C_{it} = \Delta \ln c_{it} + (\alpha_0 + \alpha_1 S_t + \varepsilon_t + n_{it}), \quad (1.16)$$

- aggregate consumption growth:

$$\begin{aligned} \Delta \ln C_t &= \ln \left(\sum_i c_{it}(w_{it}, \mu_{it}) P_{it} \right) - \ln \left(\sum_i c_{it-1}(w_{it-1}, \mu_{it-1}) P_{it-1} \right) \\ &= (\alpha_0 + \alpha_1 S_t + \varepsilon_t) + \chi_t, \end{aligned} \quad (1.17)$$

where $\chi_t = \ln \left(\sum_i c_{it}(w_{it}, \mu_{it}) P_{it-1} N_{it} \right) - \ln \left(\sum_i c_{it-1}(w_{it-1}, \mu_{it-1}) P_{it-1} \right)$.

Expression (16) states that the individual consumption growth can be broken into two parts, the growth rate of normalized consumption and the growth rate of permanent com-

³⁰Note that they do not necessarily correspond to rich or poor consumers (in level), as variables in question are defined as normalized ones.

³¹The densities are calculated using 50 sets of simulated individual consumption data.

³²Note that, as there is no population growth in this model, *per capita* aggregate consumption growth is the same as aggregate consumption growth.

ponents of individual income. As the income process is exogenous, individual consumers' smoothing behavior is captured in the first term.

Aggregate consumption growth, shown in (17), shares the stochastic trend of aggregate income growth, $\ln G_t = \alpha_0 + \alpha_1 S_t + \varepsilon_t$, but there is additional term, χ_t , that reflects the changes in distribution of individual consumers' optimal consumption decisions resulting from their idiosyncratic income realizations. Therefore the crucial step to understand the dynamics of aggregate consumption is to look at the dynamic behavior of this term and its interaction with shocks over business cycles.

Figure 3.E shows the cyclical changes in the cross-sectional distribution of individual consumption growth. The distribution also disperses in recessions; the dispersion seems to be more disruptive for lower tail, essentially due to the same reason for the asymmetric response of cross-sectional normalized consumption. The ratio of standard deviations across the two regimes is about 1.8. This cyclical pattern of cross-sectional consumption growth across two regimes are largely consistent with empirical findings in the literature, at least qualitatively. Attanasio and Davis [1996] report that individual consumption growth rates not only tend to be low on average but also diverge in recessions, and consumers with less education, who are also likely to be on low wealth position, suffer more from recessions than more educated group.³³

In addition to the estimations on counter-cyclical income risks, Storesletten, Telmer, and Yaron [2004a] also show that the cross-sectional variations of individual (food) consumption exhibit a similar pattern - counter-cyclical volatility, but the difference between contraction and expansion is somewhat mitigated, the ratio being around 1.4.³⁴ This is a bit smaller than the model's prediction. Notice however that the consumers in this model are allowed to hedge risks due to the incomplete market through self insurance only, and there is no risk sharing, either public (such as social security, or unemployment insurance) or private (among agents,

³³Based on CEX data for the years of 1980-1990, they show that in the first half of this period (which include the two NBER recession episodes), the difference in the average consumption growth by educational attainment is as large as 0.22: from -0.15 (less than 12 years) to 0.07 (college), while in the second half of the sample (where there is no recession), the difference is only 0.09: from 0.01 to 0.10.

³⁴PSID data contain only food expenditure for consumption, whose income elasticity is significantly lower than other expenditures. Hence, the volatility of overall consumption expenditure will be presumably more greater in recession. The non-separability in preferences between food and other non-durable consumption also raises additional issues regarding this; see Attanasio and Weber [1995].

e.g., relatives or friends). Hence, one way to interpret the difference between the estimates from data and the model's prediction is that it can be accounted for by some type of risk sharing among agents.³⁵

Figure 3.F shows a sample time series simulation for growth rates of aggregate consumption and income for the complete information case. As expected from the previous discussion, within a given regime consumption growth rates are fairly stable even in times of large fluctuation in incomes. However, at the times of regime-switches, the consumption growth rate exhibits large and sharp swings, by more than the change in income growth.

To better understand this result, Expression (17) is useful. First, notice that, conditional on the regime, the correlation between the aggregate stationary shock, ε_t and χ_t is negative, i.e., $Corr(\chi_t, \varepsilon_t | S_t) < 0$; a positive realization of ε_t decreases the normalized wealth on average. This dampens the aggregate consumption fluctuations and make them less volatile than income. At the times of regime-switches, however the entire consumption rules in the economy shift in a discrete way, which is reflected in χ_t , and this creates further excessive movement in addition to the change in aggregate income.

In the incomplete information case (AGG), overall pictures are largely similar, but consumption seems to be smoother both than aggregate income growth and than in the complete information case (see Figures 4.A and 4.B). Notice that however, the disruptive changes in consumption fluctuations around business-cycle turning points are still present in a moderate way (Figure 4.A). The reason for this smoothing effect is that individual consumers' beliefs are rarely completely accurate (beliefs are strictly between 0 and 1 most of the time), and often exhibit significant delay to reflect the switches of the regimes in time (Figure 4.B), thus aggregate consumption fails to respond immediately to the true changes in the aggregate regime. The overall results are largely similar for the AGG+IND incomplete information case, but there seems to be further smoothing in aggregate consumption growth as there are heterogeneity in individual consumers' belief (see Figures 4.C and 4.D).

³⁵In fact, the literature on the risk sharing shows that although there is some evidence for risk-sharing, it is far from being complete. (see Cochrane [1991], for example).

1.6 Further Investigation of Aggregate and Individual Consumption

1.6.1 Matching the (Conditional) Moments

One of the key exercises in this paper is to see whether the regime-switching model can successfully replicate some key empirical regularities of actual data in comparison with the conventional linear model. In particular, I am mainly interested in matching the model with the four (conditional) moments of aggregate consumption growth data presented in Table 1. This subsection describes the simulation procedure for this exercise and presents the results.

The simulation exercises are done in the following way. First, for a large number of consumers, I use Monte Carlo simulations to generate idiosyncratic shocks to their income for long periods. All the consumers in each period also receive two aggregate shocks which are generated by the regime-switching process in Section 3. Second, given an initial value of normalized wealth, w_{it} , and the regime indicator/belief, or S_t/μ_{it} , for each consumer, I use the optimal consumption policy obtained in Section 4 to determine the optimal normalized consumption. Aggregate consumption and income data are then constructed by summing over individual data, after individual agents' level data are recovered from the normalized variables.

In each simulation, I generate 2000 individual agents' income and consumption data to obtain aggregate data for 250 periods, then discard the first 50 observations since initial wealth distributions are arbitrarily set; the resulting 200 periods are roughly the same length as the actual data in Table 1.³⁶

In order to contrast the regime-switching model's performance with a representative linear model, I also construct and solve an alternative model in which all the non-linear/regime switching features in the individual consumer's income process are replaced by a homoskedastic linear AR(1) version. Specifically, I assume the linear AR(1) model has the following

³⁶I have tried several experiments with different numbers of consumers, and it seems that 2000 consumers is sufficient, and artificial aggregate income data created from simulation, once aggregated, closely match with the analytical values of the model (both in the first and second moments); The initial distribution of (normalized) wealth converges to a stationary distribution rather quickly, usually within 20 quarters.

income process:

$$\begin{aligned}
 \ln Y_{it} &= \ln P_{it} + \ln U_{it} & (1.18) \\
 \ln P_{it} &= \ln P_{it-1} + \ln G_t + \ln N_{it} \\
 (\ln G_t - \alpha) &= \phi (\ln G_{t-1} - \alpha) + \varepsilon_t,
 \end{aligned}$$

where

$$\varepsilon_t \sim N(0, \tilde{\sigma}_\varepsilon^2), \quad n_{it} \equiv \ln N_{it} \sim N(0, \tilde{\sigma}_n^2), \quad \text{and } u_{it} \equiv \ln U_{it} \sim N(0, \tilde{\sigma}_u^2),$$

and the first two moments of income process are set equal to the unconditional values of those in the regime-switching model. As far as the components and the unconditional first two moments in the individual income process are concerned, both the regime-switching and the linear models thus are the same.

Regarding the numerical solution algorithm, to avoid the curse of dimensionality due to additional state variables, I use an approximate AR(1) process suggested by Tauchen (1986), which greatly simplifies the computational problem. In this case, the optimal consumption policy takes the form, $c_{it} = c(w_{it}, z_t)$, where z_t is aggregate state.³⁷

Tables 4 summarizes the results from the simulation exercises.³⁸ In Panel A, I also reproduce the statistics from the actual U.S. consumption growth data to make comparison easier. For the linear AR(1) case shown in Panel B, it seems clear that the model does not do a good job. Although aggregate consumption growth is less volatile than aggregate income growth, it is still too volatile as opposed to the data, the relative volatility being around 0.9. And the distribution of aggregate consumption growth looks essentially symmetric and that of the normal, with skewness and kurtosis coefficients of around 0 and 3, respectively. To see this result, note that unlike the regime-switching model the shocks to aggregate growth, ε_t , in (18), are expected to be persistent in this linear AR(1) model. In addition, all the consumers' decision rules are shifting in every period along with the realizations of aggregate shocks so

³⁷The detailed discussion of the calibration and numerical solution method in the linear AR(1) model is contained in Appendix.

³⁸Each cell in the table is the average of each statistics from 50 simulation runs, and the standard error accross simulations are in parentheses.

the potential smoothing effect due to aggregation over heterogeneous agents becomes relatively small. As the underlying income process is symmetric regardless of aggregate states (and so is the shift in the distribution), it seems that the model also fails to capture the asymmetric/non-normal features of the actual aggregate consumption data, although the liquidity constraint and/or the concavity of consumption policy might be responsible for them in part.³⁹

How does the regime-switching model fare in this exercise? Panels C - E in Table 4 present the results from the regime-switching model. For the complete information case, although it can produce reasonable skewness coefficients, other moments are hard to match with the data. In particular, aggregate consumption growth is more volatile than income growth, and is excessively volatile in recession times. This is mainly because all the consumers' decision rules simultaneously shift in a discrete way at business cycle turning points, due to the complete information assumption, which creates large amplification in aggregate consumption fluctuation. This seems to dominate the within-regime smoothing effects. As we move to the incomplete information cases, the model's performance improves a lot. Both in the AGG and the AGG+IND cases, aggregate consumption growth is significantly less volatile than income growth, having a relative volatility around 0.7. Moreover, the higher moments of consumption growth data generated by the model seem quite similar to the data - negatively skewed and leptokurtic with quantitatively reasonable values. For the cross-regime volatility, the model also seems able to replicate reasonable statistics, although they are a little bit bigger than actual data. Overall, the results are slightly better with the AGG+IND case; the heterogeneity in agents' belief generates further smoothing effects in this case.

1.6.2 Individual and Aggregate Consumption, and Aggregation Issues

One of the traditional approaches in dynamic macro research has been to study a single representative agent's behavior, assuming the existence of the complete market. While this representative agent approach simplifies the formulation and analysis of the problem in several ways, the approach, by assuming a single agent in the entire economy, also typically ignores several important aspects of aggregate fluctuations and/or does not allow us to investigate

³⁹This suggests that, contrary to the common belief, liquidity constraint alone cannot explain the sharp drop in consumption in recessions.

them.⁴⁰

First, the dynamics resulting from the interactions between aggregate and idiosyncratic shocks are neglected. As the previous discussion emphasizes, these two types of shocks are different in their nature and size, and have sharply different implications both for aggregate and individual fluctuations. Hence, by collapsing the multi-dimensional shocks with different natures to one or two, we may lose important features in the dynamics of variables of interest.

Second, due to the single agent assumption, the representative agent framework fails to incorporate the heterogeneity of individuals in their characteristics induced by the incomplete market. This not only ignores the fact that aggregate variables are the result of a large number of heterogeneous agents' decisions, but also eliminates the potential role of the dynamics of the cross-sectional distribution of individual agents' variables. As discussed above, this "distributional effects" can be one of the key elements in understanding aggregate consumption dynamics.

It is then of natural interest to compare the implications from a representative or single agent version (both in the regime switching and in the linear models) with those from the aggregated version. Table 5 reports the results from the same experiment as in Table 4, assuming that there is only one (representative) consumer in the economy.

The results illustrate several important differences between a single individual consumption and the aggregated consumption. First, there is a substantial consumption smoothing at the individual level. For both the linear and the regime-switching income models, the relative volatilities of consumption growth (to income growth) are about 0.3, which is less than half of the values in the aggregated version. This is because, while the effects of the transitory shocks are largely cancelled out through aggregation, this kind of shock is easily insurable with a small amount of assets and the changes in individual consumption are significantly smaller compared to the income fluctuations. This suggests that the smoothing mechanism of consumption at the individual level is quantitatively and qualitatively different from that at the aggregate level, at which learning about the aggregate states and distributional effects

⁴⁰The models with uninsurable idiosyncratic risk only (Aiyagari [1992], Huggett [1994]), or with both idiosyncratic and aggregate risks (Krusell and Smith [1998]) show that the result from these aggregated versions is not much different from that of the representative agent model. However, these insignificant 'improvement' is probably due to the fact that the heterogeneity in these models comes from largely from idiosyncratic *transitory* shocks to individual income; see Heaton and Lucas [1996], and Gourinchas [2000].

are likely to play a main role.

Second, there are also additional interesting features regarding the distributions of consumption growth in the linear versus the regime-switching models, and/or at individual versus aggregate levels. On the one hand, it seems that the unconditional consumption behaviors are remarkably similar both in the linear and the regime switching models at the individual level. As the first two unconditional moments of income process are the same for both models, the difference in consumption behavior is essentially attributed to higher moments in the individual income process. Given the assigned parameter values of the model, it seems that negative skewness and leptokurtosis in the regime-switching model seem to be not important to the unconditional behavior of consumption at individual level.⁴¹

On the other hand, contrary to the findings in the aggregate data, the distributions of individual consumption growth are essentially symmetric, or slightly positively skewed, even in the regime-switching model. This confirms that the negative skewness observed in the aggregate consumption growth data is mainly due to the asymmetric shifting effects in the cross-sectional distribution, while the dynamics of consumption in a representative agent model lack such a mechanism.

The discussion in this and previous sections suggests that shifts in the cross-sectional distribution of individual consumption rule is a critical element in aggregate consumption dynamics. Noting also that the changes in consumption growth come from two sources: exogenous stochastic income changes and endogenous optimal consumption response to them (see (16) and (17)), it is then useful to decompose these two effects to get a quantitative sense of the shifting effect.

To do this, I perform a simple experiment in the following way: the consumption rule is set to a constant fictitious steady-state rule, regardless of the aggregate states, i.e., $c_{it} = c(w_{it}, \mu_{it} = 0.845)$, so that consumers are completely inattentive about the aggregate states and there is no shifting in the cross-sectional consumption rule over business cycles. I then calculate the same statistics in Table 4. The result is as follows: 0.489 (relative volatility),

⁴¹This is consistent with the welfare cost calculation of the counter-cyclical idiosyncratic risk in Storesletten, Telmer, and Yaron [2001]. They analytically show that, in a simple autarkic economy, for small risk aversion parameter value ($\gamma \leq 2$), the welfare gain from eliminating aggregate productivity shock and counter-cyclicity of idiosyncratic shocks is negligible, or can be negative, although the welfare gain non-linearly increases in the risk aversion coefficient.

11.495 (cross-regime volatility, ± 1), 4.343 (cross-regime volatility, ± 2), -1.086 (skewness), and 5.838 (kurtosis).⁴² One can see that the distributional effect plays a quantitatively important role in aggregate consumption dynamics, especially for the volatility.

The evidence from these simulations suggests that the analysis of individual consumption behavior based on the aggregate consumption dynamics, or vice versa may deliver quite misleading implications for understanding consumption dynamics at different aggregation levels.

A related, but separate issue is the aggregation problem. Several recent papers have shown that the dynamics of aggregate consumption implied by the Euler equation for individual consumption cannot be described simply by the cross-sectional average of the distribution of consumption (see Attanasio [1999] for survey). In particular, Attanasio and Weber (1993) show that omitted higher moments may play an important role to explain the bias in this kind of estimation. The idea of the bias from using the logs of the arithmetic means rather than the logs of the geometric means can be seen in the following expression:

$$\begin{aligned} Bias_t &= \Delta \ln \left(\frac{1}{I} \sum_i C_{it} \right) - \Delta \left(\frac{1}{I} \sum_i \ln C_{it} \right) \\ &\approx \Delta \ln \left(1 + \frac{1}{2!} M_{2,t} + \frac{1}{3!} M_{3,t} + \frac{1}{4!} M_{4,t} \right), \end{aligned} \quad (1.19)$$

where the second line is obtained by a Taylor-series expansion of $\exp\left(\frac{1}{I} \sum_i \ln C_{it}\right)$ around $\ln\left(\frac{1}{I} \sum_i C_{it}\right)$ up to fourth order and taking the first differences, and $M_{k,t} = \frac{1}{I} \sum_i (\ln C_{it} - \overline{\ln C_{it}})^k$ is the k 's central moment of cross-sectional log consumption distribution.

Assuming that errors resulting from (i) the approximation, for small x , $\ln(1+x) \approx x$, and (ii) truncating the moments higher than fourth order are quantitatively negligible, the sources of the aggregation bias come from the omitted three moments in the cross-sectional distribution.

In light of the regime-switching income model, this aggregation bias can be more serious, as the simulation results in the previous section imply that the cross-sectional distribution tends to exhibit rich dynamics over cycles with time-varying higher moments as well as the

⁴²This result comes from the AGG+IND incomplete information case with $\gamma = 1$ and $b = 0.0$; the results are largely similar for other parameter values.

mean. This point can be easily visible in Figure 5, which shows a time series pattern of the bias and the first difference of the (adjusted) higher-moments (divided by $2!$, $3!$, and $4!$, respectively) from a simulation. The magnitude of the bias is non-negligible and tends to be larger in recessions, and it seems that all three higher moments are highly correlated with the bias. A simple regressions below confirms this:

$$Bias_t = \underset{(57.401)}{0.997} \Delta \left(\frac{1}{2!} M_{2,t} \right) + \underset{(19.057)}{0.309} \Delta \left(\frac{1}{3!} M_{3,t} \right) + \underset{(8.254)}{0.226} \Delta \left(\frac{1}{4!} M_{4,t} \right), \quad (1.20)$$

$$R^2 = 0.955.$$

where the result in (20) reports average estimates, t -values (in parentheses), and R^2 in the regression of bias on three (adjusted) higher moments using simulated data.⁴³

1.6.3 Volatility Accounting

Recall that in the baseline model presented in Section 3, the individual incomes are subject to four types of shocks each period: two aggregate shocks (regime-switching, V_t ; stationary, ε_t) and two idiosyncratic shocks (permanent, n_{it} ; transitory, u_{it}). Another important question of interest then is: what are the relative effects of each shock on individual and aggregate consumption fluctuations? In particular, how much are the regime-switching features in the individual income variations, combined with the counter-cyclicality of idiosyncratic risks, responsible for consumption fluctuations at both macro and micro levels? This section attempts to provide answers to this question.

Again due to the lack of an analytical solution to the dynamic consumption problem, I mainly rely on Monte Carlo simulation for this exercise, and solve new consumers' problem for a series of alternative economies in which the shocks of interest are eliminated or modified.

More specifically, in addition to the benchmark model (Model A), I solve four additional models in which I remove or alter each component of the regime-switching features: first, I solve an alternative economy in which there is no regime switching shock in the aggregate component of income, i.e., $V_t = 0$ for all t , and the mean growth rate is set to the uncondi-

⁴³The model used is the incomplete information (AGG+IND) case with $\gamma = 1$ and $b = 0.0$. The regression also includes a constant term which is insignificant. I obtained a bit similar result for the linear models, but with lower R^2 .

tional value of the regime switching model (labeled Model C). In the second and third cases, I consider models in which, maintaining no regime-switching feature of aggregate growth in Model C, the heteroskedasticity of idiosyncratic permanent/transitory income shocks are removed by setting the unconditional values to variances in both regimes (Models C1/C2, respectively). Finally, I solve the case without the counter-cyclical variations in both types of idiosyncratic risks, still maintaining no regime-switching feature in aggregate growth (Model C3). Hence, the new economy in Model C3 is essentially a linear version of the original model, with all the non-linear/regime switching elements removed.

After solving each model, I use optimal consumption policies obtained from the new models to generate artificial individual and aggregate consumption growth data, and then compare their standard deviations with those of the benchmark economy.

The results from this exercise is shown in Tables 6.A and 6.B, which report the average standard deviations of aggregate and individual consumption growth in each case, and the percentage value compared to the benchmark case (in bracket). First, note that there are significant smoothing effects at the aggregate level; the main effect comes from the elimination of the regime-switching type shock in the aggregate component (transition from Model A to Model C, accounting for 40 ~ 50% in the decrease in volatility). And the elimination of the counter-cyclical of idiosyncratic shocks also accounts for about 3% ~ 14% of aggregate consumption fluctuations depending on the parameter values and information assumption. When all the regime-switching elements in the income process are removed, the decrease in volatility is as large as three quarters of the benchmark values (in Model C3).

At the individual level, however, consumption growth is virtually as equally variable as the benchmark case regardless of the types of removed shocks, and the decrease in volatility is less than 4%.

Why is the smoothing effect from the removal of regime-switching features so negligible for individual consumption fluctuations? On the contrary, why does it account for a significant part of aggregate consumption fluctuations?

As for the aggregate consumption, first note that although both aggregate shocks affect individuals' wealth and consumption decision every period, their natures are sharply different. Within a given regime, although stationary shocks partly determine agents' wealth positions,

a majority of their effects on aggregate consumption is mitigated due to large idiosyncratic shocks, while the regime-switching variations essentially lead to the shift in aggregate wealth distribution, and accordingly individual consumers' optimal consumption decisions. Hence, the regime-switching variations explain a large part of consumption changes at the aggregate level. Note that the magnitude of the smoothing effects is the largest in the AGG case as all the agents have the identical beliefs about aggregate regimes every period.

For the individual level, one straightforward explanation to the small smoothing effect is that, given the specification of the model and the assigned parameter values, aggregate fluctuations are responsible for a very small part of individual income variations, most of which is driven by idiosyncratic shocks. The effect of counter-cyclical idiosyncratic risk is also quite small.⁴⁴ In addition, it seems that consumers want to and actually do stay in the relatively flat part of the optimal consumption policy,⁴⁵ which may mitigate the effects of heteroskedastic income risks.

Another interesting account for the small smoothing effect on individual consumption comes from the incomplete information assumption. Notice that, in regard to individual agents' information processing and learning, the elimination of regime-switching features in the income process also implies that they lose part of their information about aggregate regimes in the new economies, and data available to them are noisier than the benchmark case. This poor quality of information creates additional sources of fluctuations through misperception in the course of learning, and tends to weaken the smoothing effects at individual level. Therefore this suggests that eliminating (the counter-cyclicity of) a shock might lead to greater variations in the individual consumptions. This feature can be found in the results from the AGG+IND incomplete information case, although the magnitude is quite small.

1.7 Concluding Remarks

This paper develops a quantitative model to explore aggregate and individual consumption dynamics when the income process exhibits regime-switching features, and compares its

⁴⁴The effect of eliminating counter-cyclical idiosyncratic risks is also quite small so it seems that the changes in higher moments in the individual income process do not matter much; recall the discussion in Subsection 6.2, where I compare the representative agent model both in the linear and the regime-switching model.

⁴⁵The fraction of liquidity-constrained consumers is less than 10% on average.

performance with the conventional linear model. For this purpose, I consider an economy populated by a large number of consumers whose incomes are subject to both aggregate and idiosyncratic shocks. The notable element of the model is that a latent regime-switching stochastic variable governs both the trend growth of the aggregate component and the counter-cyclical variances of the idiosyncratic components in individual earnings.

I demonstrate that the model can provide a reasonable description of the cyclical behavior of actual consumption fluctuations, and can successfully replicate some key empirical properties of aggregate consumption growth, such as smaller volatility than income growth, greater volatility in recessions than in expansions, and a negatively skewed and leptokurtic distribution, while the typical linear model fails to do so. The model highlights that the interaction between aggregate and idiosyncratic shocks, shifts in the cross-sectional distribution of individual consumers' optimal consumption decisions, and learning about the underlying business-cycle regime play a critical role in explaining aggregate consumption dynamics.

A few possible future extensions are worth pursuing. First, the introduction of durable goods will be an interesting exercise as the dynamics of durable goods seem to be more volatile than other consumption measures, and more sensitive to business cycle changes. In addition, as the cyclical evolution of cross-sectional distribution of durable stocks is critical to understanding the dynamic behavior of aggregate durable good expenditure, this regime-switching set-up may provide richer insights to this issue.

Another interesting point is to look at the effects of recessions on different groups of consumers. As the main text points out, the effects of recessions are not homogeneous across individual consumers; instead it seems that some consumers suffer more in recessions than others. This has a clear welfare implication for business cycle research in which the existing studies explore this issue in terms of the representative agent, or average individual consumers.

Probably the most ambitious future project would be the general equilibrium extension of the model. One shortcoming of the paper is that the model is developed with a partial equilibrium approach. Hence the cyclical variations of prices, such as interest rates, are largely ignored. Incorporating the production, capital accumulation and endogenous choice of labor supply in a general equilibrium model may enrich many aspects of the model, and

it should give us some new insights to understanding macroeconomic fluctuations.

1.8 Appendix

The appendix contains detailed discussion on some of the results/procedure such as the derivation of Euler equation, and numerical solution methods used in the main text.

1.8.1 The Complete Information Case

Using the notations in the main text, the evolution of normalized wealth can be written as:

$$\begin{aligned}
 w_{it+1} &= (P_{it+1}/P_{it})^{-1}(1+r)(w_{it} - c_{it}) + e^{u_{it+1}} \\
 &= (G_{t+1}N_{it+1})^{-1}(1+r)(w_{it} - c_{it}) + e^{u_{it+1}} \\
 &= e^{-(\alpha_0 + \alpha_1 S_{t+1} + \varepsilon_{t+1} + n_{it+1})}(1+r)(w_{it} - c_{it}) + e^{u_{it+1}}.
 \end{aligned} \tag{1.21}$$

As the optimal consumption is homogenous of degree 1 (and the value function is homogenous of degree $(1 - \gamma)$), dividing both sides of the Euler equation by $P_{it}^{-\gamma}$, we can rewrite it in terms of normalized variables:

$$\begin{aligned}
 &P_{it}^{-\gamma} c \left(\frac{W_{it}}{P_{it}}, 1, S_t \right)^{-\gamma} \\
 = &\max \left\{ P_{it}^{-\gamma} \left(\frac{W_{it}}{P_{it}} + b \right)^{-\gamma}, \beta(1+r)E \left[P_{it+1}^{-\gamma} c \left(\frac{W_{it+1}}{P_{it+1}}, 1, S_{t+1} \right)^{-\gamma} \right] \right\}
 \end{aligned} \tag{1.22}$$

or

$$\begin{aligned}
 &c(w_{it}, S_t)^{-\gamma} \\
 = &\max \left\{ (w_{it} + b)^{-\gamma}, \beta(1+r)E \left[\left(\frac{P_{it+1}}{P_{it}} \right)^{-\gamma} c(w_{it+1}, S_{t+1})^{-\gamma} \right] \right\}.
 \end{aligned} \tag{1.23}$$

For expansion regime, this becomes

$$\begin{aligned}
 &c(w_{it}, E)^{-\gamma} \\
 = &\max \left\{ (w_{it} + b)^{-\gamma}, \beta(1+r) \left[pE_t \left[(e^{g_{it+1}|E} c(w_{it+1}|EE, E))^{-\gamma} \right] \right. \right. \\
 &\left. \left. + (1-p)E_t \left[(e^{g_{it+1}|R} c(w_{it+1}|ER, R))^{-\gamma} \right] \right] \right\},
 \end{aligned} \tag{1.24}$$

Likewise, for recession regime

$$\begin{aligned}
& c(w_{it}, R)^{-\gamma} \\
&= \max \left\{ (w_{it} + b)^{-\gamma}, \beta(1+r) \left[(1-q) E_t \left[(e^{g_{it+1|E}} c(w_{it+1|RE}, E))^{-\gamma} \right] \right. \right. \\
&\quad \left. \left. + \beta(1+r) q E_t \left[(e^{g_{it+1|R}} c(w_{it+1|RR}, R))^{-\gamma} \right] \right] \right\}
\end{aligned} \tag{1.25}$$

where $g_{it+1|S_{t+1}} = \alpha_0 + \alpha_1 S_{t+1} + \varepsilon_{t+1} + n_{it+1|S_{t+1}}$, and $w_{it+1|S_t S_{t+1}} = e^{-g_{it+1|S_{t+1}}} (1+r)(w_{it} - c(w_{it}, S_t)) + e^{u_{it+1|S_{t+1}}}$; (24) and (25) are the equations (11) and (12) in the main text.

Now I turn to the numerical solution method, which employs the policy function iteration algorithm. I first discretize the space for normalized wealth with 100 grid points. In doing so, I allocate 80 points between 0 to 3 in order to capture the high curvature of consumption function for low normalized wealth, with the remaining 20 points for 3 to 40. Then given this grid space, starting from the initial guess, $c(w_{it}, S_t) = w_{it}$, the consumption policy satisfying the Euler equation is updated until the convergence occurs. The convergence criterion used is: $|c_{k+1}(\cdot) - c_k(\cdot)| < 1.0 \times 10^{-5}$, where $c_k(\cdot)$ is the updated consumption policy in k 's iteration.

Each iteration requires the calculation of expected marginal utility in solving the Euler equation. The calculation on the right hand side of (25), for example, involves the evaluation of triple integrals

$$\begin{aligned}
& E_t \left[(g_{it+1|R} c(w_{it+1|RR}, R))^{-\gamma} \right] \\
&= \int \int \int \left(e^{(\alpha_0 + \varepsilon_{t+1} + n_{it+1|R})} c \left[\frac{(1+r)(w_{it} - c(w_{it}, R))}{e^{(\alpha_0 + \varepsilon_{t+1} + n_{it+1|R})}} + e^{u_{it+1|R}}, R \right] \right)^{-\gamma} \\
&\quad \times d\Phi_\varepsilon d\Phi_{n(R)} d\Phi_{u(R)},
\end{aligned} \tag{1.26}$$

where $\Phi_{(\cdot)}$'s are normal cdfs associated with each shock. As all the shocks follow lognormal distribution conditional on the regime, it would be natural to use a three-dimensional Gauss-

Hermite quadrature for numerical integration in (26) (see Judd [1998]):

$$\begin{aligned}
& E_t \left[(g_{it+1|R} c(w_{it+1|RR}, R))^{-\gamma} \right] \\
&= \frac{1}{\pi^{3/2}} \sum_j \sum_k \sum_l \left(e^{(\alpha_0 + \sqrt{2}\sigma_\varepsilon \varepsilon_j + \sqrt{2}\sigma_{n|R} n_k)} c \left[\frac{(1+r)(w - c(w, R))}{e^{(\alpha_0 + \sqrt{2}\sigma_\varepsilon \varepsilon_j + \sqrt{2}\sigma_{n|R} n_k)} + e^{\sqrt{2}\sigma_{u|R} u_l}}, R \right] \right)^{-\gamma} \\
& \quad \times \omega_l \omega_k \omega_j,
\end{aligned} \tag{1.27}$$

where ε_j , n_k , and u_l are nodes used for each shock, and ω 's are the corresponding weights. In the actual implementation, I use a quadrature of order 7. The other terms in (24) and (25) are calculated in a similar way.

Whenever the policy function needs to be evaluated between the original grid points, the cubic spline interpolation scheme is used.

1.8.2 The Incomplete Information Case

Bayesian Updating

Following the notation in the main text, let ψ_{it} denote consumer's information set, where $\psi_{it} = \{\Delta \ln Y_t, \Delta \ln Y_{t-1}, \dots\}$, or $\psi_{it} = \{\Delta \ln Y_{it}, \Delta \ln Y_t, \Delta \ln Y_{it-1}, \Delta \ln Y_{t-1}, \dots\}$, depending on the information assumption; let μ_{it} be consumer i 's belief that the current aggregate state is expansion, conditional on all the information up to time t , i.e., $\mu_{it} = \Pr[S_t = 1 | \psi_{it}]$.

Given μ_{it-1} , we can calculate the interim/prior probability that time t is an expansion before observing time t data, $\tilde{\mu}_{it}$:

$$\tilde{\mu}_{it} = \Pr[S_t = 1 | \psi_{it-1}] = p\mu_{it-1} + (1-q)(1 - \mu_{it-1}). \tag{1.28}$$

Then, as the new (income growth rate) data at time t , x_{it} , are available, he updates his belief

using Bayes rule:

$$\begin{aligned}
\mu_{it} &= \Pr[S_t = 1 | \psi_{it}] \\
&= \Pr[S_t = 1 | \psi_{it-1}, x_{it}] \\
&= \frac{f(S_t = 1, x_{it} | \psi_{it-1})}{f(x_{it} | \psi_{it-1})} \\
&= \frac{f(x_{it} | S_t = 1) \tilde{\mu}_{it}}{f(x_{it} | S_t = 1) \tilde{\mu}_{it} + f(x_{it} | S_t = 0) (1 - \tilde{\mu}_{it})},
\end{aligned} \tag{1.29}$$

where $f(\cdot)$ is a conditional normal pdf such that, depending on the data used for inference:

- aggregate data only [AGG]: $x_{it} = \{\Delta \ln Y_t\}$,

$$f(x_{it} | S_t) = \frac{1}{(2\pi)^{1/2} \sigma_\varepsilon} \exp \left\{ -\frac{(\Delta \ln Y_t - \alpha_0 - S_t \alpha_1 - 0.5 \sigma_{n|S_t}^2)^2}{2\sigma_\varepsilon^2} \right\}, \tag{1.30}$$

- both individual and aggregate data [AGG+IND]: $x_{it} = \{\Delta \ln Y_{it}, \Delta \ln Y_t\}$,

$$\begin{aligned}
&f(x_{it} | S_t) \\
&= \frac{1}{(2\pi) \sigma_\varepsilon \sqrt{\sigma_\varepsilon^2 + \sigma_{n|S}^2 + \sigma_{u|S}^2}} \exp \left\{ -\frac{1}{2(1 - \rho_S^2)} \right. \\
&\quad \left[\frac{(\Delta \ln Y_{it} - \alpha_0 - \alpha_1 S_t + u_{it-1})^2}{\sigma_\varepsilon^2 + \sigma_{n|S}^2 + \sigma_{u|S}^2} + \frac{(\Delta \ln Y_t - \alpha_0 - \alpha_1 S_t + 0.5 \sigma_{n|S}^2)^2}{\sigma_\varepsilon^2} \right. \\
&\quad \left. \left. - 2\rho_S \frac{(\Delta \ln Y_{it} - \alpha_0 - \alpha_1 S_t + u_{it-1})(\Delta \ln Y_t - \alpha_0 - \alpha_1 S_t + 0.5 \sigma_{n|S}^2)}{\sigma_\varepsilon \sqrt{\sigma_\varepsilon^2 + \sigma_{n|S}^2 + \sigma_{u|S}^2}} \right] \right\}
\end{aligned} \tag{1.31}$$

where $\rho_S = \sigma_\varepsilon / \sqrt{\sigma_\varepsilon^2 + \sigma_{n|S}^2 + \sigma_{u|S}^2}$.

Euler Equation and Numerical Solution Procedure

Given the realization of normalized wealth and the belief about the aggregate regime for individual agents, the Euler equation (in terms of normalized variables) in the incomplete

case is

$$\begin{aligned}
& c(w_{it}, \mu_{it})^{-\gamma} \tag{1.32} \\
= & \max \left\{ (w_{it} + b)^{-\gamma}, \beta(1+r) \left[\tilde{\mu}_{it+1} E_{it} \left[\left(g_{it+1|S_{t+1}} c(w_{it+1}, \mu_{it+1|1}) \right)^{-\gamma} \right] \right. \right. \\
& \left. \left. + (1 - \tilde{\mu}_{it+1}) E_{it} \left[\left(g_{it+1|S_{t+1}} c(w_{it+1}, \mu_{it+1|0}) \right)^{-\gamma} \right] \right] \right\} \\
= & \max \left\{ (w_{it} + b)^{-\gamma}, \beta(1+r) \left[(p\mu_{it} + (1-q)\mu_{it}) E_{it} \left[(e^{g_{it+1|E}} c(w_{it+1}, 1))^{-\gamma} \right] \right. \right. \\
& \left. \left. + (1 - p\mu_{it} - (1-q)(1 - \mu_{it})) E_{it} \left[(e^{g_{it+1|R}} c(w_{it+1}, 0))^{-\gamma} \right] \right] \right\}.
\end{aligned}$$

In the numerical procedure, the space for individual's belief, μ_{it} , is discretized using 11 grid points, from 0.0 to 1.0 with the increment of 0.1. The optimal consumption policy is then the solution to a system of 11 simultaneous equations. Other details are essentially the same as in the complete information case.

1.8.3 The Linear AR(1) Income Process

For the linear AR(1) income process, the variances of two idiosyncratic shocks and the mean growth rate are set equal to the unconditional values of the regime-switching counterparts: $\tilde{\sigma}_n^2 = p^* \sigma_{n|E}^2 + q^* \sigma_{n|R}^2$; $\tilde{\sigma}_u^2 = p^* \sigma_{u|E}^2 + q^* \sigma_{u|R}^2$; and $\alpha/(1-\phi) + 0.5\tilde{\sigma}_n^2 = p^* (\alpha_0 + \alpha_1 + 0.5\sigma_{n|E}^2) + q^* (\alpha_0 + 0.5\sigma_{n|R}^2)$, where $p^* = (1-q)/(2-p-q)$ is the steady-state probability of an expansion, and $q^* = 1 - p^*$ is the steady-state probability of a recession.

As for the numerical solution procedure, I follow the finite state approximation technique proposed by Tauchen (1986). Since the aggregate component of the individual income follows a first order AR process, it follows that $\ln G_t = g_t \sim N(\alpha, \varphi^2)$, where $\varphi^2 = \tilde{\sigma}_e^2/(1-\phi^2)$. I approximate this with a discrete first-order Markov process in which the growth rate takes one of M discrete values, $\alpha + \varphi z_m$, where z_m is a discretized standard normal variable. The transition probability between the aggregate states m and n , $\omega_{m,n}$, is calculated as:

$$\begin{aligned}
\omega_{m,n} &= \Pr(\varphi z_n \geq g_t - \alpha \geq \varphi z_{n-1} | \varphi z_m \geq g_{t-1} - \alpha \geq \varphi z_{m-1}) \tag{1.33} \\
&= \frac{1}{\varphi \sqrt{2\pi}} \int_{\varphi z_{m-1}}^{\varphi z_m} e^{-\frac{x^2}{2\varphi^2}} \left\{ \Phi \left(\frac{\varphi z_n - \phi x}{\tilde{\sigma}} \right) - \Phi \left(\frac{\varphi z_{n-1} - \phi x}{\tilde{\sigma}} \right) \right\} dx
\end{aligned}$$

where Φ is a normal cdf.

The expected marginal utility in the Euler equation when the current aggregate state is m , for example, is also numerically approximated using a Gauss-Hermite quadrature:

$$\begin{aligned}
& E_t \left[\left(\frac{c(w_{it+1}, z_{n,t+1})}{e^{-(\alpha + \varphi z_{n,t+1} + n_{it+1})}} \right)^{-\gamma} \right] \tag{1.34} \\
&= \frac{1}{\pi^{3/2}} \sum_n \sum_k \sum_l \left(\frac{c[e^{-(\alpha + \sqrt{2}\varphi\tilde{\sigma}_\varepsilon\varepsilon_j + \sqrt{2}\tilde{\sigma}_n n_k)(1+r)}(w_{it} - c(w_{it}, z_{m,t})) + e^{\sqrt{2}\tilde{\sigma}_n u_l}]}{e^{-(\alpha + \sqrt{2}\varphi\tilde{\sigma}_\varepsilon\varepsilon_j + \sqrt{2}\tilde{\sigma}_n n_k)}} \right)^{-\gamma} \\
&\quad \times \omega_{m,n} \omega_l \omega_k.
\end{aligned}$$

In the actual implementation, I set $M = 13$; the other aspects of numerical procedure are essentially the same as the regime-switching model.

Bibliography

- [1] Abowd, John and David Card [1989], "On the Covariance Structure of Earnings and Hours Changes," *Econometrica*, pp. 411-445.
- [2] Aiyagari, Rao [1994], "Uninsured Idiosyncratic Risk and Aggregate Saving," *Quarterly Journal of Economics*, pp. 659-684.
- [3] Angeletos, George-Marios and Laurent Calvet [2005], "Idiosyncratic Production Risk, Growth and the Business Cycle," forthcoming in *Journal of Monetary Economics*.
- [4] Atkeson, Andrew and Christopher Phelan [1994], "Reconsidering the Costs of Business Cycles with Incomplete Markets," in Stanley Fischer and Julio Rotemberg (eds.), *NBER Macroeconomics Annual 1994*, pp. 187-207.
- [5] Attanasio, Orazio [1999], "Consumption," in John Taylor and Michael Woodford (eds.), *Handbook of Macroeconomics*, pp. 741-812.
- [6] Attanasio, Orazio and Steven Davis [1996], "Relative Wage Movements and the Distribution of Consumption," *Journal of Political Economy*, pp. 1227-1262.
- [7] Attanasio, Orazio and Guglielmo Weber [1993], "Consumption Growth, the Interest Rate and Aggregation," *Review of Economic Studies*, pp. 631-649.
- [8] Attanasio, Orazio and Guglielmo Weber [1995], "Is Consumption Growth Consistent with Intertemporal Optimization? Evidence from the Consumer Expenditure Survey," *Journal of Political Economy*, pp. 1121-1157.
- [9] Bickel, Peter and Kjell Doksum [1977], *Mathematical Statistics*, Holden-Day.

- [10] Blinder, Alan and Angus Deaton [1985], "The Time-Series Consumption Revisited," *Brookings Papers on Economic Activity*, pp. 465-521.
- [11] Campbell, John and Gregory Mankiw [1989], "Consumption, Income, and the Interest Rates: Reinterpreting the Time Series Evidence," *NBER Macroeconomics Annual*, pp. 185-215.
- [12] Carroll, Christopher [1992], "Buffer-Stock Theory of Saving: Some Macroeconomic Evidence," *Brookings Papers on Economic Activity*, pp. 61-135.
- [13] Carroll, Christopher [1997], "Buffer-Stock Saving and the Life Cycle/Permanent Income Hypothesis," *Quarterly Journal of Economics*, 1-55.
- [14] Carroll, Christopher [2001], "A Theory of the Consumption Function, With and Without Liquidity Constraints," NBER working paper 8387.
- [15] Carroll, Christopher, Jeffrey Fuhrer, and David Wilcox [1994], "Does Consumer Sentiment Forecast Household Spending? If So, Why?" *American Economic Review*, pp. 1397-1408.
- [16] Carroll, Christopher, and Miles Kimball [1996], "On The Concavity of The Consumption Function," *Econometrica*, pp. 981-992.
- [17] Carroll, Christopher and Andrew Samwick [1997], "The Nature of Precautionary Wealth," *Journal of Monetary Economics*, pp. 41-71.
- [18] Cochrane, John [1991], "A Simple Test of Consumption Insurance," *Journal of Political Economy*, pp. 957-976
- [19] Constantinides, George and Darrell Duffie [1996], "Asset Pricing with Heterogeneous Consumers," *Journal of Political Economy*, pp. 219-240.
- [20] Davis, Steven and John Haltiwanger [1992], "Gross Job Creation, Gross Job Destruction, and Employment Reallocation," *Quarterly Journal of Economics*, pp. 819-63.
- [21] Davis, Steven and John Haltiwanger and Scott Schuh [1996], *Job Creation and Destruction*, Cambridge, MA: MIT Press.

- [22] Deaton, Angus [1991], "Saving and Liquidity Constraints", *Econometrica*, pp. 1221-1248.
- [23] Deaton, Angus [1992], *Understanding Consumption*, Oxford: Oxford University Press.
- [24] Deaton, Angus and Guy Laroque [1992], "On the Behavior of Commodity Prices." *Review of Economic Studies*, pp. 1-23.
- [25] Franses, Philip Hans and Dick van Dijk [2000], *Nonlinear Time Series Models in Empirical Finance*, Cambridge: Cambridge University Press.
- [26] Gourinchas, Pierre-Olivier [2000], "Precautionary Savings, Life-Cycle and Macroeconomics," mimeo.
- [27] Gourinchas, Pierre-Olivier and Jonathan Parker, [2002], "Consumption Over the Life Cycle," *Econometrica*, pp. 47-89.
- [28] Guvenen, Fatih [2005], "Learning Your Earning: Are Labor Income Shocks Really Very Persistent?" mimeo.
- [29] Hamilton, James [1989], "A New Approach to the Economic Analysis of Nonstationary Time Series and The Business Cycle," *Econometrica*, pp. 357-384.
- [30] Hamilton, James and Baldev Raj [2002], (eds.) *Advances in Markov-Switching Models*, Physica-Verlag.
- [31] Heaton, John and Deborah Lucas [1996], "Evaluating the Effects of Incomplete Markets on Risk Sharing and Asset Pricing," *Journal of Political Economy*, pp. 443-487.
- [32] Hubbard, Glenn, Jonathan Skinner and Stephen Zeldes [1994], "The Importance of Precautionary Motives in Explaining Individual and Aggregate Saving," *Carnegie-Rochester Conference Series on Public Policy*, pp. 59-125.
- [33] Hubbard, Glenn, Jonathan Skinner and Stephen Zeldes [1995], "Precautionary Saving and Social Insurance," *Journal of Political Economy*, pp. 360-399.
- [34] İmrohoroğlu, Ayşe [1989], "Cost of Business Cycles with Indivisibilities and Liquidity Constraints," *Journal of Political Economy*, pp. 1364-1383.

- [35] Judd, Kenneth [1998], *Numerical Methods in Economics*, Cambridge, MA: MIT Press.
- [36] Kim, Chang-Jin and Charlse Nelson [1999], *State-Space Models and Regime Switching: Classical and Gibbs Sampling Approaches with Applications*, Cambridge, MA: MIT Press.
- [37] Krebs, Tom [2003a], "Human Capital Risk and Economic Growth," *Quarterly Journal of Economics*, pp. 709-744.
- [38] Krebs, Tom [2003b], "Growth and Welfare Effects of Business Cycles in Economy with Idiosyncratic Human Capital Risk," *Review of Economic Dynamics*, pp. 846-868.
- [39] Krusell, Per and Anthony Smith [1998], "Income and Wealth Heterogeneity in Macroeconomics," *Journal of Political Economy*, pp. 867-896.
- [40] Krusell, Per and Anthony Smith [1999], "The Welfare Effects of Eliminating Business Cycles," *Review of Economic Dynamics*, pp. 245-272.
- [41] Lucas, Deborah [1994], "Asset Pricing with Undiversifiable Income Risk and Short Sale Constraint: Deepening the Equity Premium Puzzle," *Journal of Monetary Economics*, pp. 325-341.
- [42] Ludvigson, Sydney [1999], "Consumption and Credit: A Model of Time-Varying Liquidity Constraints", *Review of Economics and Statistics*, pp. 434-447.
- [43] Ludvigson, Sydney and Alexander Michaelides [2001], "Does Buffer-Stock Saving Explain the Smoothness and Excess Sensitivity of Consumption?" *American Economic Review*, pp. 631-647.
- [44] MaCurdy, Thomas [1982], "The Use of Time Series Process to Model the Error Structure of Earnings in a Longitudinal Data Analysis," *Journal of Econometrics*, pp. 83-114.
- [45] Meghir, Costas and Luigi Pistaferri [2004], "Income Variance Dynamics and Heterogeneity," *Econometrica*, pp. 1-32.
- [46] Milas, Costas, Dick van Dijk, and Phillip Rothman [2005], (eds.) *Nonlinear Time Series Analysis of Business Cycles*, Amsterdam: Elsevier Science.

- [47] Pischeke, Jörn-Stephen [1995], "Individual Income, Incomplete Information, and Aggregate Consumption," *Econometrica*, pp. 805-840.
- [48] Rothman, Philip [1999], (ed.) *Nonlinear Time Series Analysis of Economic and Financial Data*, Kluwer Academic Press.
- [49] Souleles, Nicholas [2004], "Expectations, Heterogeneous Forecast Errors, and Consumption: Micro Evidence from the Michigan Consumer Sentiment Surveys," *Journal of Money, Credit, and Banking*, pp. 39-72.
- [50] Storesletten, Kjetil, Chris Telmer, and Amir Yaron [1999], "Asset Pricing with Idiosyncratic Risk and Overlapping Generations," mimeo.
- [51] Storesletten, Kjetil, Chris Telmer, and Amir Yaron [2001], "The Welfare Cost of Business Cycles Revisited: Finite Lives and Cyclical Variation in Idiosyncratic Risk," *European Economic Review*, pp. 1311-1339
- [52] Storesletten, Kjetil, Chris Telmer, and Amir Yaron [2004a], "Cyclical Dynamics in Idiosyncratic Labor-Market Risk," *Journal of Political Economy*, pp. 695-717.
- [53] Storesletten, Kjetil, Chris Telmer, and Amir Yaron [2004b], "Consumption and Risk Sharing over the Life Cycle," *Journal of Monetary Economics*, pp. 609-633.
- [54] Tauchen, George [1986], "Finite State Markov Chain Approximations to Univariate and Vector Autoregression", *Economics Letters*, pp. 177-181.
- [55] Van Nieuwerburgh, Stijn and Laura Veldkamp [2006], "Learning Asymmetries in Real Business Cycles," *Journal of Monetary Economics*, forthcoming.
- [56] Veronesi, Pietro [1999], "Stock Market Overreaction to Bad News in Good Times: A Rational Expectation Equilibrium Model," *Review of Financial Studies*, pp. 975-1007.
- [57] Wang, Neng [2004], "Precautionary Saving and Partially Observed Income," *Journal of Monetary Economics*, pp. 1645-1681.

Table 1 Quarterly Growth Rates of per-capita Consumption

| | relative volatility | cross-regime volatility | | skewness | kurtosis | Jarque-Bera statistics (<i>p</i> -value) |
|---------------------------|---------------------|-------------------------|--------------------|----------|----------|--|
| | | ± 1 | ± 2 | | | |
| non-durables and services | 0.510 | 2.580 ^a | 1.776 ^a | -0.562 | 4.270 | 27.077 (0.000) |
| non-durables | 0.789 | 1.437 ^b | 1.113 | -0.296 | 4.205 | 16.970 (0.000) |
| services | 0.516 | 2.539 ^a | 1.860 ^a | -0.344 | 4.225 | 18.577 (0.000) |
| durables | 3.701 | 1.606 ^b | 1.773 ^a | -0.458 | 4.205 | 19.382 (0.000) |

Notes: The data are from the national income and product account (NIPA) and cover 1953:I - 2003:III. All series are seasonally adjusted at annual rates. The relative volatility is the ratio of standard deviation of the variable's growth to that of disposable income growth, and the cross regime volatility is defined as the ratio of the average of squares of deviation from the whole sample mean in recessions to that in expansions, where the recession periods include the NBER recessions dates. I also add one (left column, ± 1), or two (right column, ± 2) pre-/post- quarters for each recession episodes. See the main text for details. The superscripts, *a*, *b*, and *c* indicate the the null hypothesis that the variances of growth rates are the same is rejected at 1%, 5%, and 10% significance levels, respectively.

Table 2 Parameter Values of the Model

| symbol | numerical value | description |
|--|-----------------|--|
| <i>common (preference) parameters</i> | | |
| β | 0.9901 | discount factor; δ (discount rate) = $1/\beta - 1 = 0.01$ |
| r | 0.005 | risk free real interest rate |
| b | 0, 0.3 | borrowing limit (fraction of permanent income) |
| γ | 1, 2 | (constant) risk aversion coefficient |
| <i>regime-switching income process</i> | | |
| p | 0.95 | retention probability of expansion regime |
| q | 0.72 | retention probability of recession regime |
| α_0 | -0.00585 | aggregate income growth rate (recession) |
| α_1 | 0.0115 | $\alpha_0 + \alpha_1 =$ aggregate income growth rate (expansion) |
| σ_ε | 0.008 | std. dev. of aggregate stationary shock |
| $\sigma_{n E}$ | 0.05 | std. dev. of idiosyncratic permanent shock (expansion) |
| $\sigma_{n R}$ | 0.09 | std. dev. of idiosyncratic permanent shock (recession) |
| $\sigma_{u E}$ | 0.16 | std. dev. of idiosyncratic transitory shock (expansion) |
| $\sigma_{u R}$ | 0.24 | std. dev. of idiosyncratic transitory shock (recession) |
| <i>AR(1) income process</i> | | |
| α | 0.0032 | unconditional aggregate income growth rate |
| ϕ | 0.20 | AR(1) coefficient of aggregate growth rate |
| $\tilde{\sigma}_\varepsilon$ | 0.0084 | std. dev. of aggregate stationary shock |
| $\tilde{\sigma}_n$ | 0.056 | std. dev. of idiosyncratic permanent shock |
| $\tilde{\sigma}_u$ | 0.172 | std. dev. of idiosyncratic transitory shock |

Notes: per-capita income growth is $\ln \left(\sum_{i=1}^H \frac{1}{H} \ln Y_{i,t} \right) - \ln \left(\sum_{i=1}^H \frac{1}{H} \ln Y_{i,t-1} \right) = \alpha_0 + \alpha_1 S_t + 0.5\sigma_{n|S}^2 + \varepsilon_t$, so that in the regime switching case, the (ex-post) recession growth rate is $\alpha_0 + 0.5\sigma_{n|R}^2 + \varepsilon_t$, and expansion growth rate is $\alpha_0 + \alpha_1 + 0.5\sigma_{n|E}^2 + \varepsilon_t$. In the linear AR(1) process, the first two moments are set equal to the unconditional values of the regime switching model; see the appendix for the details. The preference parameters are set to the same values both in the regime-switching

model and in the linear models.

Table 3 Variance Decomposition for Income Process

| | aggregate | | idiosyncratic | |
|---------------------------------------|-------------------------------|-----------------------------------|---------------------------|----------------------------|
| | regime-switching (V_t) | stationary (ε_t) | permanent (n_{it}) | transitory (u_{it}) |
| Expansion | 0.25% | | 99.75% | |
| | 0.02% | 0.23% | 8.87% | 90.88% |
| Recession | 0.14% | | 99.86% | |
| | 0.04% | 0.10% | 12.31% | 87.55% |
| Unconditional | 0.22% | | 99.78% | |
| | 0.03% | 0.19% | 9.89% | 89.90% |
| $\frac{Var(\cdot R)}{Var(\cdot E)}$ | 4.24 | 1 | 3.24 | 2.25 |
| $\frac{S.D.(\cdot R)}{S.D.(\cdot E)}$ | 2.06 | 1 | 1.8 | 1.5 |

Notes: The second and third rows show the relative share of each shock to the total variance of the individual income conditional on each regime. The fourth row reports the unconditional counterparts using the steady-state probabilities. The fifth/sixth rows are the relative values of each component across two regimes in terms of the variance/standard deviation.

Table 4 (Conditional) Moments of Aggregate Consumption Growth

| | relative volatility | cross-regime volatility | | skewness | kurtosis |
|--|------------------------|----------------------------|-------------------|-------------------|-------------------|
| | | ± 1 | ± 2 | | |
| A. U.S. data | | | | | |
| non-durables | 0.789 | 2.021 | 1.480 | -0.296 | 4.205 |
| non-durables and service | 0.510 | 2.655 | 2.167 | -0.562 | 4.270 |
| service good | 0.516 | 2.093 | 2.071 | -0.344 | 4.225 |
| B. Linear AR(1) Model | | | | | |
| $\gamma = 1, b = 0.0$ | 0.867 (0.035) | - | - | -0.043 (0.162) | 3.033 (0.412) |
| $\gamma = 1, b = 0.3$ | 0.935 (0.039) | - | - | -0.016 (0.194) | 3.066 (0.300) |
| $\gamma = 2, b = 0.0$ | 0.881 (0.034) | - | - | -0.031 (0.136) | 2.926 (0.308) |
| $\gamma = 2, b = 0.3$ | 0.919 (0.044) | - | - | 0.037 (0.177) | 2.894 (0.383) |
| C. Regime-Switching Model: Complete Information | | | | | |
| $\gamma = 1, b = 0.0$ | 1.125 (0.123) | 63.463 (26.462) | 6.169 (6.095) | -0.581 (0.236) | 11.942 (3.847) |
| $\gamma = 1, b = 0.3$ | 1.186 (0.120) | 64.381 (25.656) | 8.778 (8.355) | -0.614 (0.269) | 12.020 (2.886) |
| $\gamma = 2, b = 0.0$ | 1.232 (0.132) | 73.668 (38.837) | 9.530 (13.167) | -0.575 (0.267) | 11.843 (3.530) |
| $\gamma = 2, b = 0.3$ | 1.271 (0.136) | 75.198 (39.985) | 9.660 (13.483) | -0.581 (0.264) | 11.887 (3.547) |

Notes: The numbers in each cell are the average of 50 simulation runs, and standard deviations across simulations are in parentheses. See the notes to Table 1 for the definitions of statistics. In the

first column, b is the borrowing limit (as a fraction of permanent income), and γ is the risk aversion coefficient.

Table 4 (Conditional) Moments of Aggregate Consumption Growth
(continued)

| | relative volatility | cross-regime volatility | | skewness | kurtosis |
|--|------------------------|----------------------------|------------------|-------------------|------------------|
| | | ± 1 | ± 2 | | |
| D. Regime-Switching Model: Incomplete Information (AGG) | | | | | |
| $\gamma = 1, b = 0.0$ | 0.723 (0.057) | 14.219 (4.840) | 4.564 (2.540) | -0.847 (0.270) | 6.605 (1.465) |
| $\gamma = 1, b = 0.3$ | 0.759 (0.062) | 16.385 (4.880) | 4.746 (2.377) | -0.982 (0.375) | 7.554 (2.149) |
| $\gamma = 2, b = 0.0$ | 0.791 (0.054) | 10.825 (3.736) | 4.248 (2.094) | -1.028 (0.378) | 6.525 (1.742) |
| $\gamma = 2, b = 0.3$ | 0.810 (0.065) | 11.278 (3.370) | 4.854 (1.901) | -1.101 (0.399) | 7.226 (2.238) |
| E. Regime-Switching Model: Incomplete Information (AGG+IND) | | | | | |
| $\gamma = 1, b = 0.0$ | 0.703 (0.050) | 4.869 (3.667) | 2.808 (2.061) | -0.653 (0.349) | 4.822 (1.395) |
| $\gamma = 1, b = 0.3$ | 0.721 (0.051) | 5.174 (4.224) | 2.189 (0.721) | -0.720 (0.382) | 5.186 (1.361) |
| $\gamma = 2, b = 0.0$ | 0.748 (0.032) | 3.365 (1.861) | 2.098 (0.950) | -0.699 (0.368) | 4.695 (1.258) |
| $\gamma = 2, b = 0.3$ | 0.774 (0.037) | 3.604 (2.695) | 2.412 (1.102) | -0.657 (0.246) | 4.539 (0.989) |

Notes: The numbers in each cell is the average of 50 simulation runs, and standard deviations across simulations are in parentheses. See the notes to Table 1 for the definitions of statistics. In the first column, b is the borrowing limit (as a fraction of permanent income), and γ is the risk aversion coefficient. AGG refers to the case in which consumers use aggregate income data only for their inference, while AGG+IND refers to the case in which they use both aggregate and individual income data.

Table 5 Comparison with Representative Agent Model

| | relative volatility | cross-regime volatility | | skewness | kurtosis |
|--|------------------------|----------------------------|------------------|-------------------|------------------|
| | | ± 1 | ± 2 | | |
| A. Linear AR(1) | | | | | |
| $\gamma = 1, b = 0.0$ | 0.319 (0.037) | – | – | 0.177 (0.311) | 5.280 (2.190) |
| $\gamma = 1, b = 0.3$ | 0.325 (0.041) | – | – | 0.201 (0.415) | 5.488 (2.109) |
| $\gamma = 2, b = 0.0$ | 0.262 (0.030) | – | – | 0.071 (0.342) | 4.648 (2.189) |
| $\gamma = 2, b = 0.3$ | 0.269 (0.033) | – | – | 0.095 (0.309) | 4.450 (1.972) |
| B. Regime-Switching Model: Complete Information | | | | | |
| $\gamma = 1, b = 0.0$ | 0.319 (0.040) | 2.315 (1.165) | 1.562 (0.733) | 0.184 (0.463) | 6.605 (3.481) |
| $\gamma = 1, b = 0.3$ | 0.328 (0.042) | 2.325 (1.279) | 1.630 (0.757) | 0.059 (0.420) | 6.238 (2.918) |
| $\gamma = 2, b = 0.0$ | 0.265 (0.034) | 2.322 (0.987) | 1.634 (0.673) | 0.027 (0.431) | 5.563 (3.053) |
| $\gamma = 2, b = 0.3$ | 0.271 (0.035) | 2.535 (1.172) | 1.725 (0.662) | –0.061 (0.483) | 5.743 (3.242) |

Notes: The table reports results for a representative case; see the main text.

Table 5 Comparison with Representative Agent Model (continued)

| | relative volatility | cross-regime volatility | | skewness | kurtosis |
|--|------------------------|----------------------------|------------------|------------------|-------------------|
| | | ± 1 | ± 2 | | |
| C. Regime-Switching Model: Incomplete Information | | | | | |
| $\gamma = 1, b = 0.0$ | 0.298 (0.041) | 2.351 (1.087) | 1.765 (0.863) | 0.179 (0.449) | 6.210 (3.457) |
| $\gamma = 1, b = 0.3$ | 0.302 (0.037) | 2.482 (1.328) | 1.726 (0.791) | 0.111 (0.455) | 5.9530 (3.209) |
| $\gamma = 2, b = 0.0$ | 0.250 (0.029) | 2.691 (1.190) | 1.740 (0.676) | 0.118 (0.402) | 5.247 (3.432) |
| $\gamma = 2, b = 0.3$ | 0.255 (0.032) | 2.505 (1.174) | 1.829 (0.861) | 0.017 (0.371) | 4.924 (2.976) |

Notes: The table reports results for a representative case; see the main text.

Table 6.A Volatility Accounting: Regime-switching Model (AGG)

| Model | aggregate consumption | individual consumption | aggregate consumption | individual consumption |
|--|--------------------------|---------------------------|--------------------------|---------------------------|
| | $\gamma = 1, b = 0.0$ | | $\gamma = 1, b = 0.3$ | |
| A (benchmark) | 0.635% | 8.113% | 0.647% | 8.284% |
| C (\bar{V}_t) | 0.313% [49.31%] | 7.838% [96.61%] | 0.322% [49.73%] | 8.142% [98.28%] |
| C1 $(\bar{V}_t + \bar{\sigma}_n)$ | 0.286% [45.08%] | 7.883% [97.16%] | 0.280% [43.22%] | 8.113% [97.94%] |
| C2 $(\bar{V}_t + \bar{\sigma}_u)$ | 0.249% [39.17%] | 7.944% [97.92%] | 0.261% [40.28%] | 8.255% [99.65%] |
| C3 $(\bar{V}_t + \bar{\sigma}_n + \bar{\sigma}_u)$ | 0.215% [33.92%] | 7.830% [96.51%] | 0.217% [33.48%] | 8.087% [97.62%] |
| | $\gamma = 2, b = 0.0$ | | $\gamma = 2, b = 0.3$ | |
| A (benchmark) | 0.688% | 6.743% | 0.695% | 6.815% |
| C (\bar{V}_t) | 0.268% [38.89%] | 6.655% [98.70%] | 0.278% [40.07%] | 6.801% [99.79%] |
| C1 $(\bar{V}_t + \bar{\sigma}_n)$ | 0.214% [31.12%] | 6.595% [97.81%] | 0.212% [30.45%] | 6.707% [98.41%] |
| C2 $(\bar{V}_t + \bar{\sigma}_u)$ | 0.235% [34.18%] | 6.613% [98.08%] | 0.236% [33.95%] | 6.790% [99.64%] |
| C3 $(\bar{V}_t + \bar{\sigma}_n + \bar{\sigma}_u)$ | 0.183% [26.64%] | 6.557% [97.25%] | 0.185% [26.64%] | 6.705% [98.39%] |

Notes: The table reports the average standard deviation of aggregate and individual consumption growth rates from 50 simulation runs for the five models: [A] the benchmark model; [C] constant mean growth rate of aggregate component; [C1] C + homoskedasticity of idiosyncratic permanent shock; [C2] C + homoskedasticity of idiosyncratic transitory shock; [C3] C + homoskedasticity of

idiosyncratic permanent/transitory shock; see the main text for details. The numbers in bracket are the percentage value compared to the benchmark case.

Table 6.B Volatility Accounting: Regime-switching Model (AGG+IND)

| Model | aggregate consumption | individual consumption | aggregate consumption | individual consumption |
|--|--------------------------|---------------------------|--------------------------|---------------------------|
| | $\gamma = 1, b = 0.0$ | | $\gamma = 1, b = 0.3$ | |
| A (benchmark) | 0.601% | 7.811% | 0.597% | 7.919% |
| C (\bar{V}_t) | 0.317% [52.69%] | 7.888% [100.99%] | 0.315% [52.79%] | 8.122% [102.56%] |
| C1 $(\bar{V}_t + \bar{\sigma}_n)$ | 0.283% [47.17%] | 7.756% [99.30%] | 0.280% [46.90%] | 8.046% [101.60%] |
| C2 $(\bar{V}_t + \bar{\sigma}_u)$ | 0.241% [40.08%] | 7.919% [101.39%] | 0.247% [41.47%] | 8.204% [103.60%] |
| C3 $(\bar{V}_t + \bar{\sigma}_n + \bar{\sigma}_u)$ | 0.212% [35.30%] | 7.836% [100.32%] | 0.218% [36.58%] | 8.094% [102.22%] |
| | $\gamma = 2, b = 0.0$ | | $\gamma = 2, b = 0.3$ | |
| A (benchmark) | 0.655% | 6.513% | 0.661% | 6.605% |
| C (\bar{V}_t) | 0.243% [37.20%] | 6.528% [100.24%] | 0.254% [38.48%] | 6.694% [101.35%] |
| C1 $(\bar{V}_t + \bar{\sigma}_n)$ | 0.214% [32.65%] | 6.502% [99.84%] | 0.210% [31.82%] | 6.665% [100.90%] |
| C2 $(\bar{V}_t + \bar{\sigma}_u)$ | 0.215% [32.90%] | 6.618% [101.62%] | 0.212% [32.07%] | 6.694% [101.35%] |
| C3 $(\bar{V}_t + \bar{\sigma}_n + \bar{\sigma}_u)$ | 0.182% [27.87%] | 6.561% [100.74%] | 0.186% [28.14%] | 6.702% [101.46%] |

Notes: The table reports the average standard deviation of aggregate and individual consumption growth rates from 50 simulation runs for the five models: [A] the benchmark model; [C] constant mean growth rate of aggregate component; [C1] C + homoskedasticity of idiosyncratic permanent shock; [C2] C + homoskedasticity of idiosyncratic transitory shock; [C3] C + homoskedasticity of

idiosyncratic permanent/transitory shock; see the main text for details. The numbers in bracket are the percentage value compared to the benchmark case.

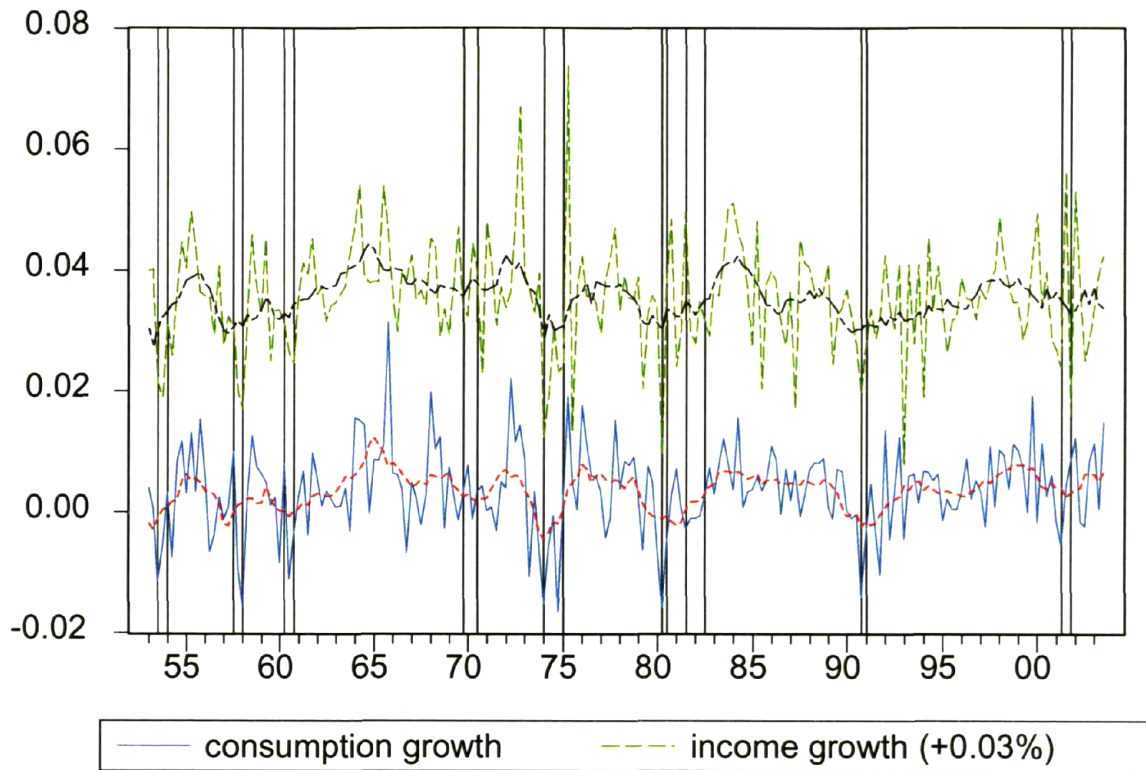


Figure 1.A Aggregate Consumption and Income Growth

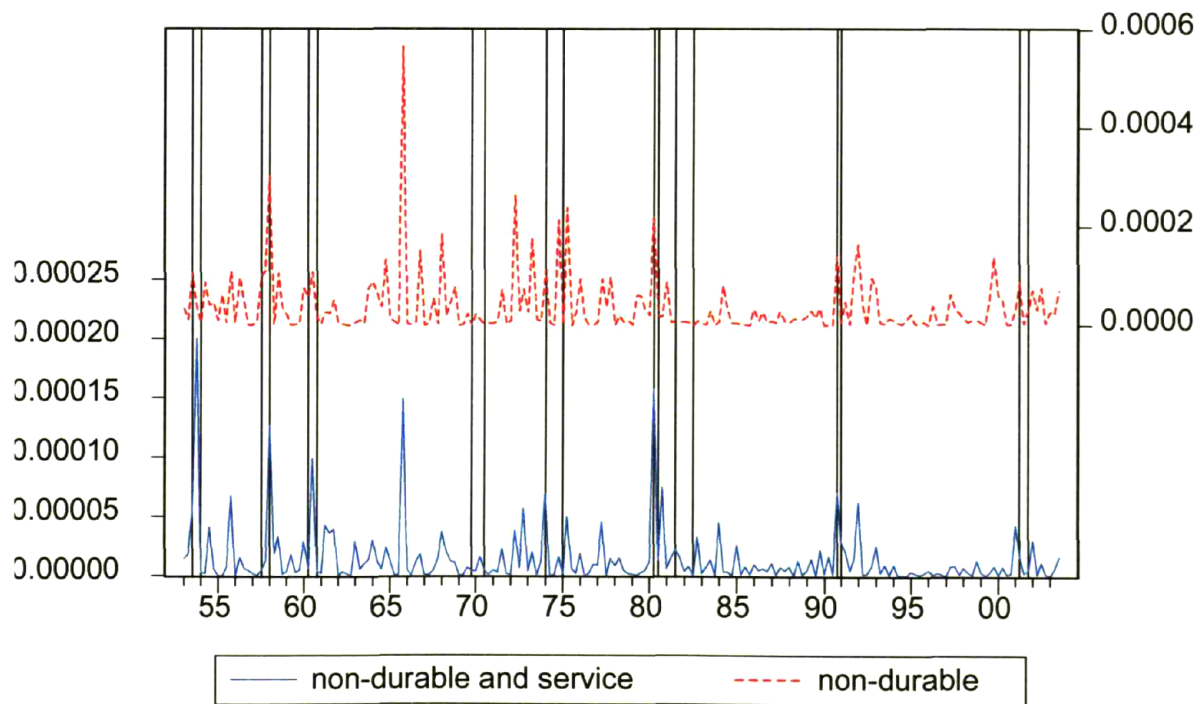


Figure 1.B Volatility of Consumption Growth

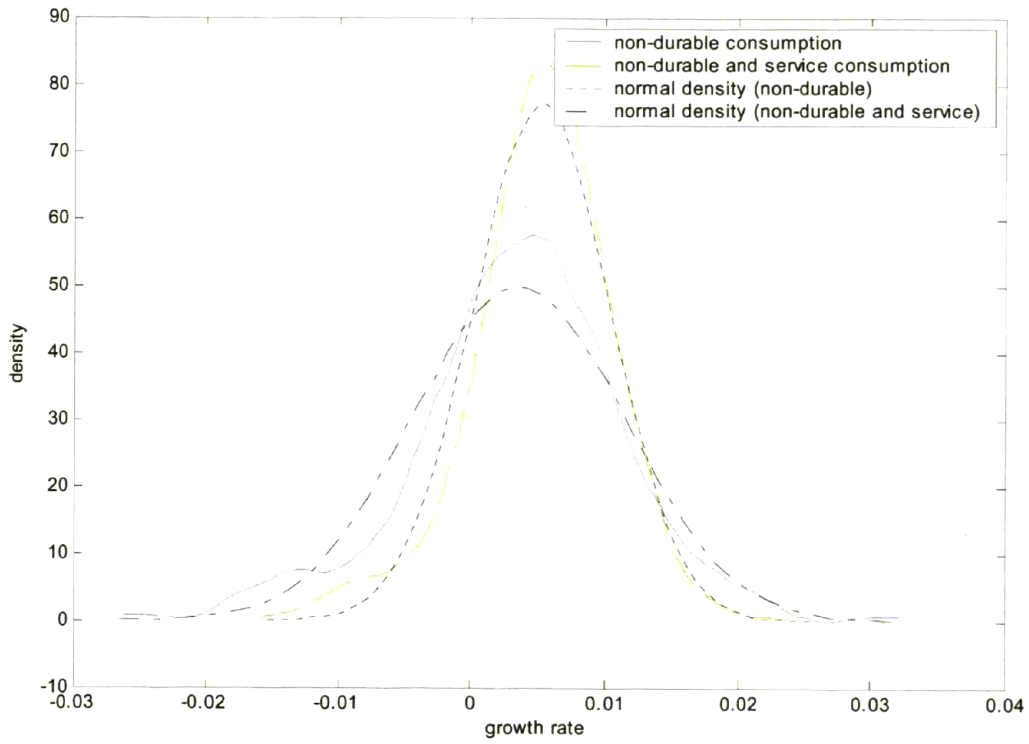


Figure 1.C Kernel Density of Aggregate Consumption Growth

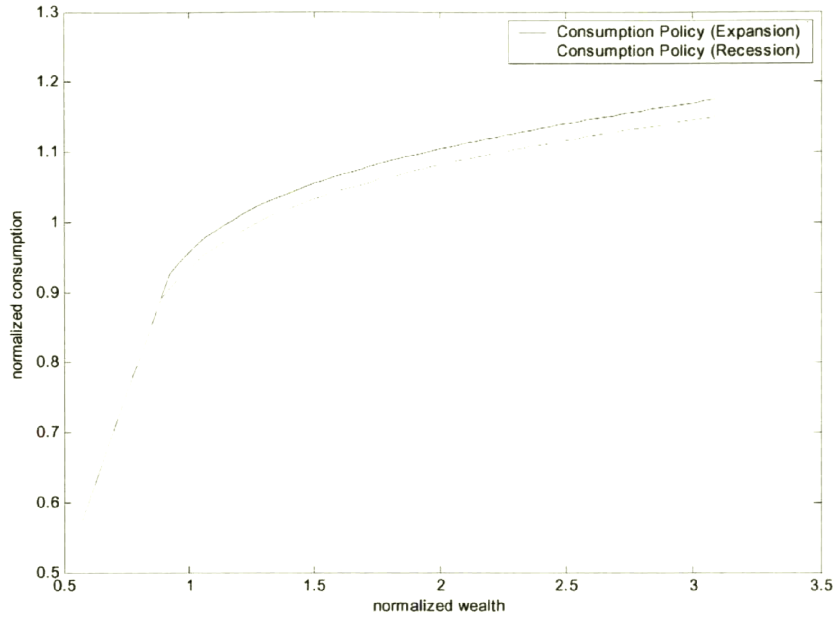


Figure 2.A Optimal Consumption Policy (Complete Information Case)

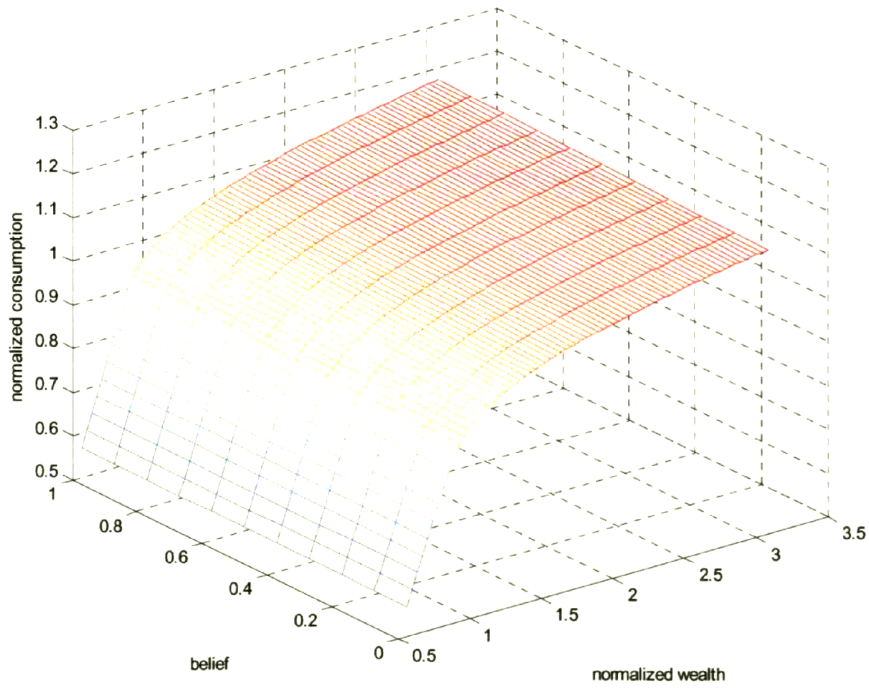


Figure 2.B Optimal Consumption Policy (Incomplete Information Case)

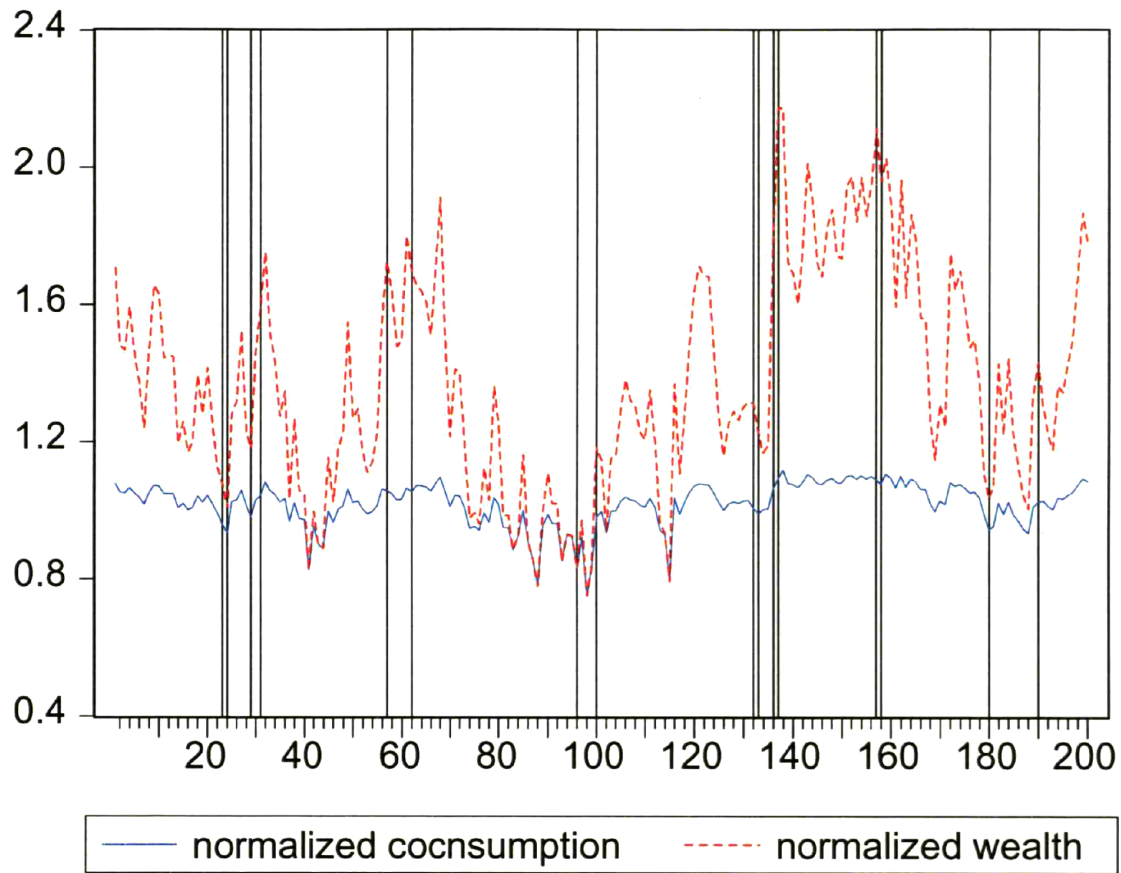


Figure 3.A Dynamics of Normalized Variables: A Random Consumer (Complete Information Case)

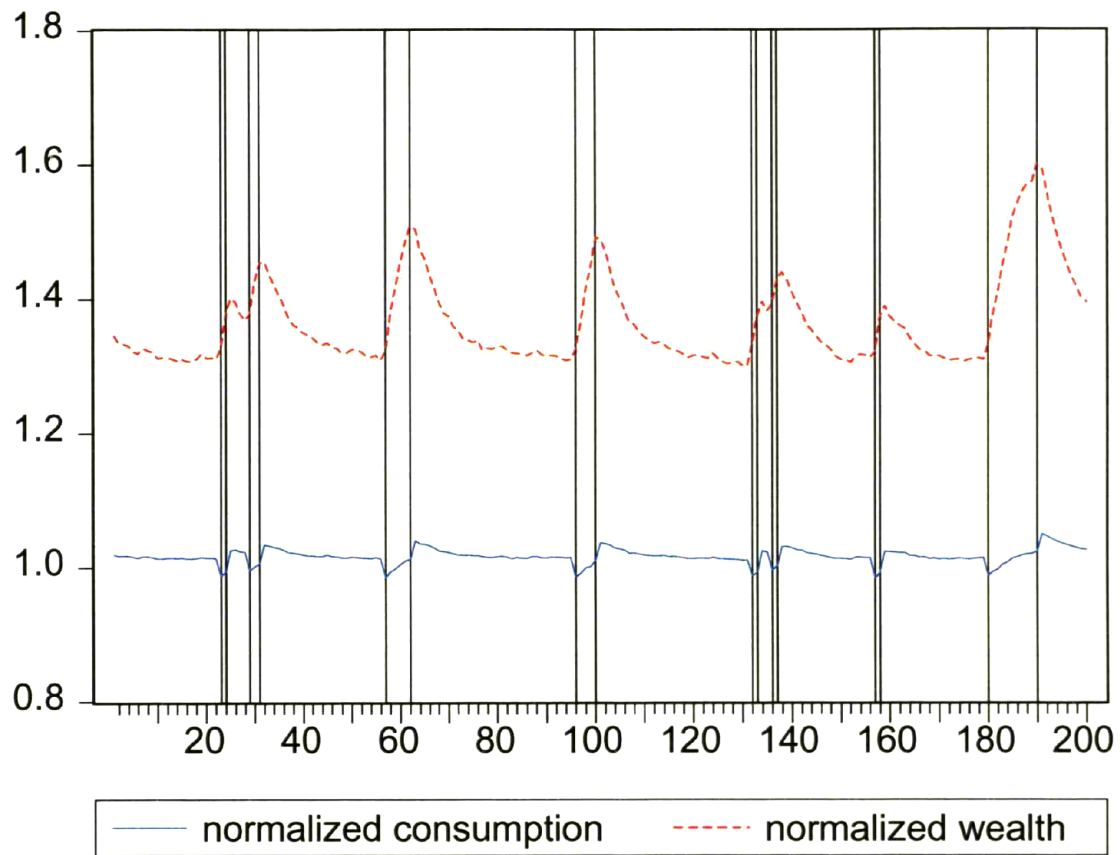


Figure 3.B Dynamics of Normalized Variables: Average Consumer (Complete Information Case)

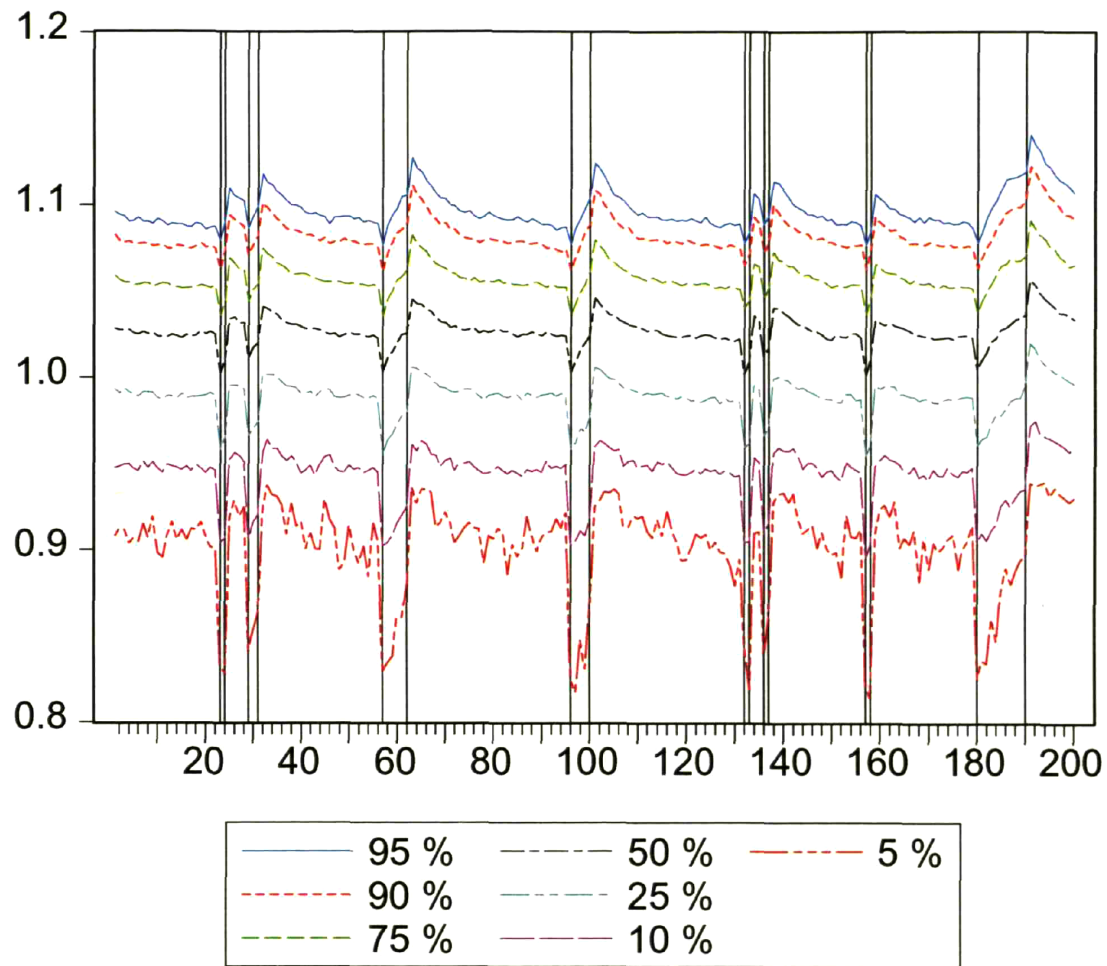


Figure 3.C Cyclical Evolution of Cross-sectional Distribution of Normalized Consumption (Complete Information case)

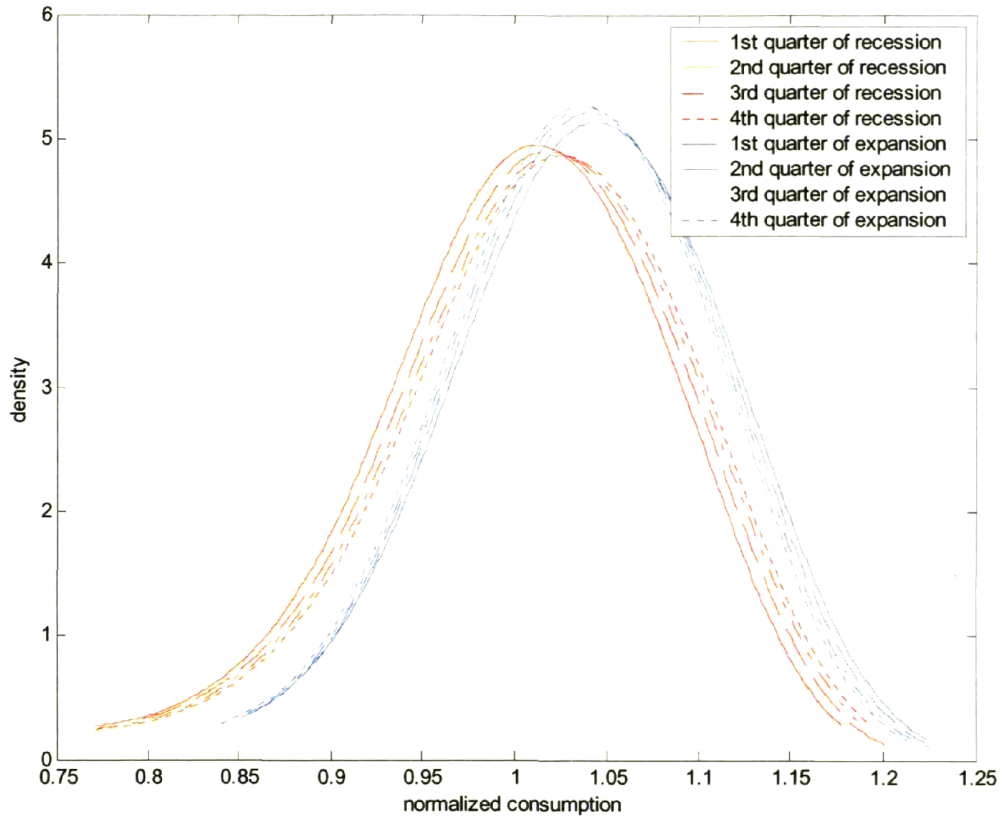


Figure 3.D Shift in Cross-Sectional Distribution of Normalized Consumption over a Business Cycle (Complete Information case)

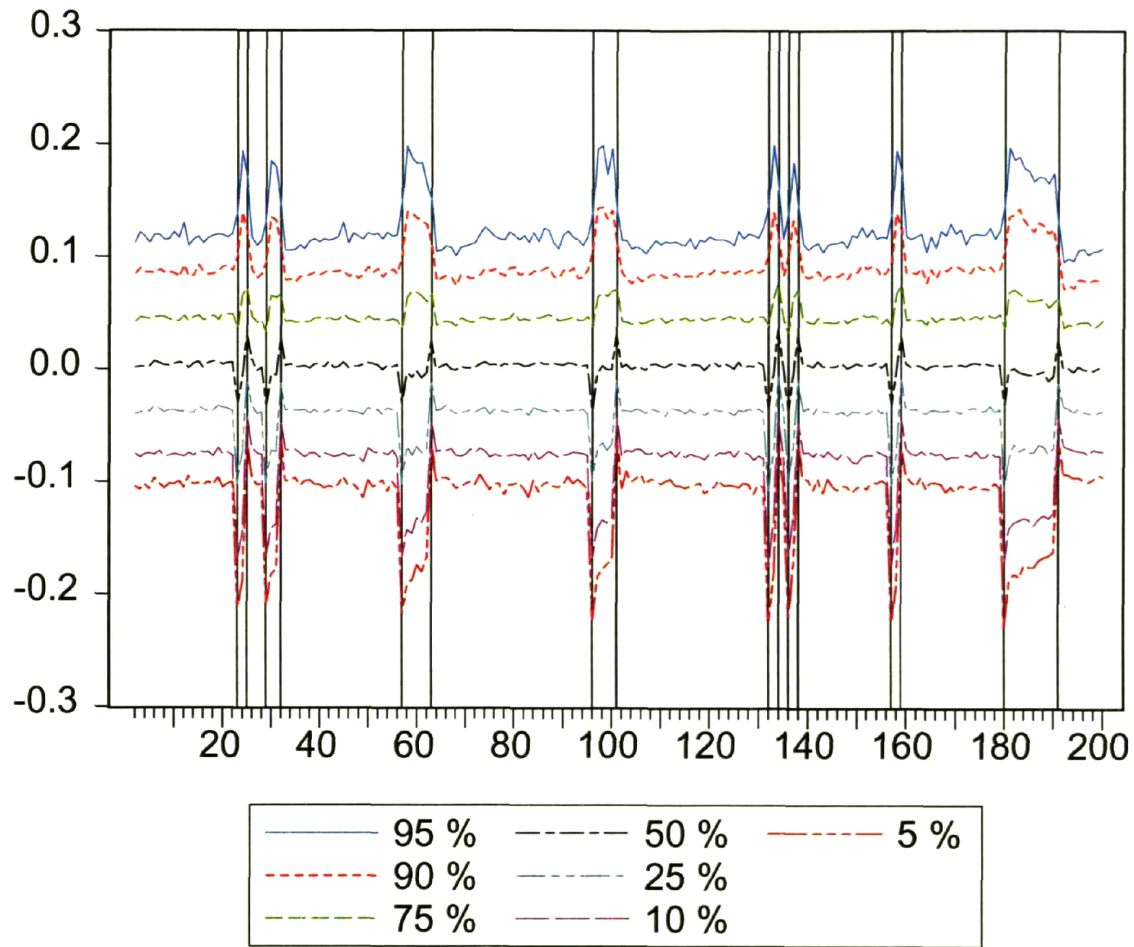


Figure 3.E Evolution of Cross-Sectional Distribution of Individual Consumption Growth (Complete Information case)

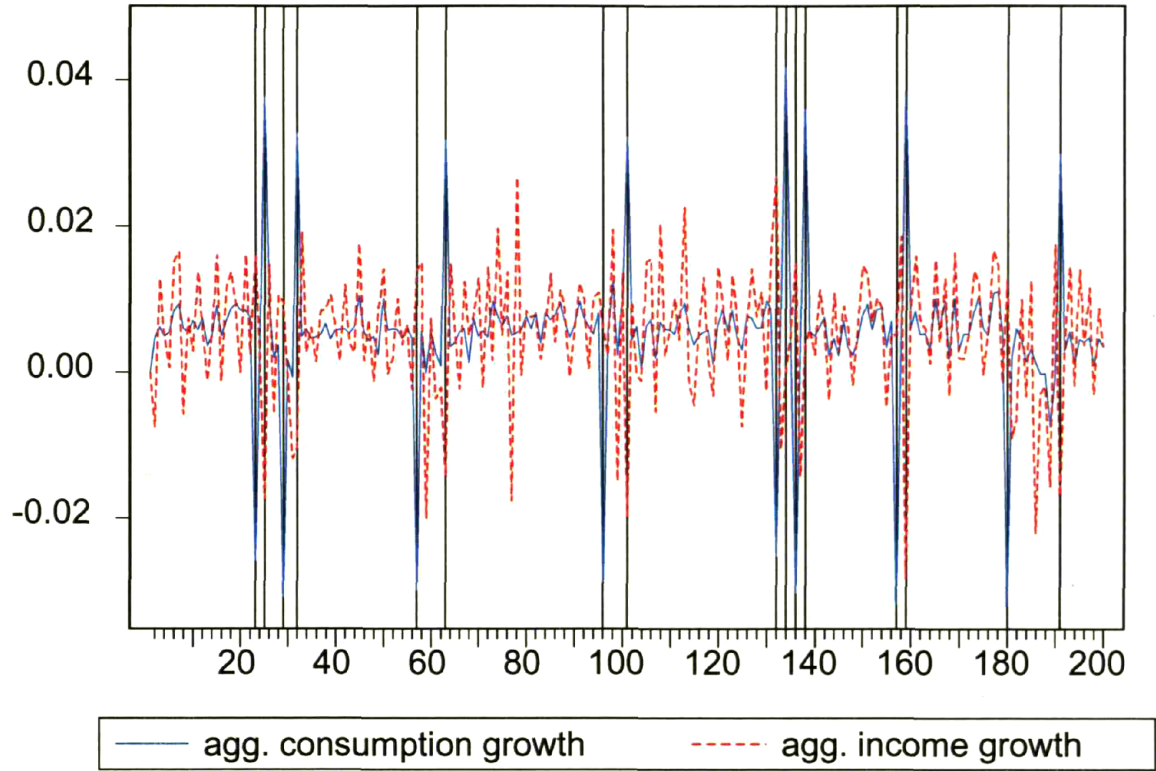


Figure 3.F Growth Rates of Aggregate Variables (Complete Information case)

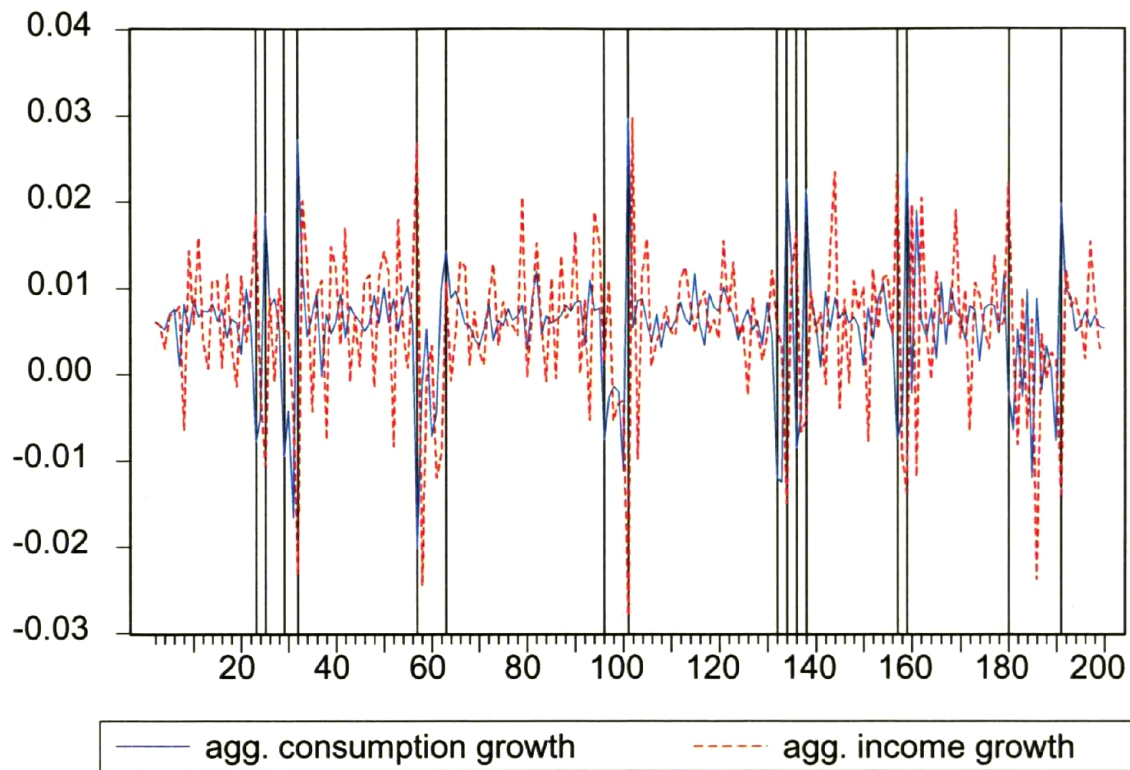


Figure 4.A Growth Rates of Aggregate Variables (Incomplete Information case: AGG)

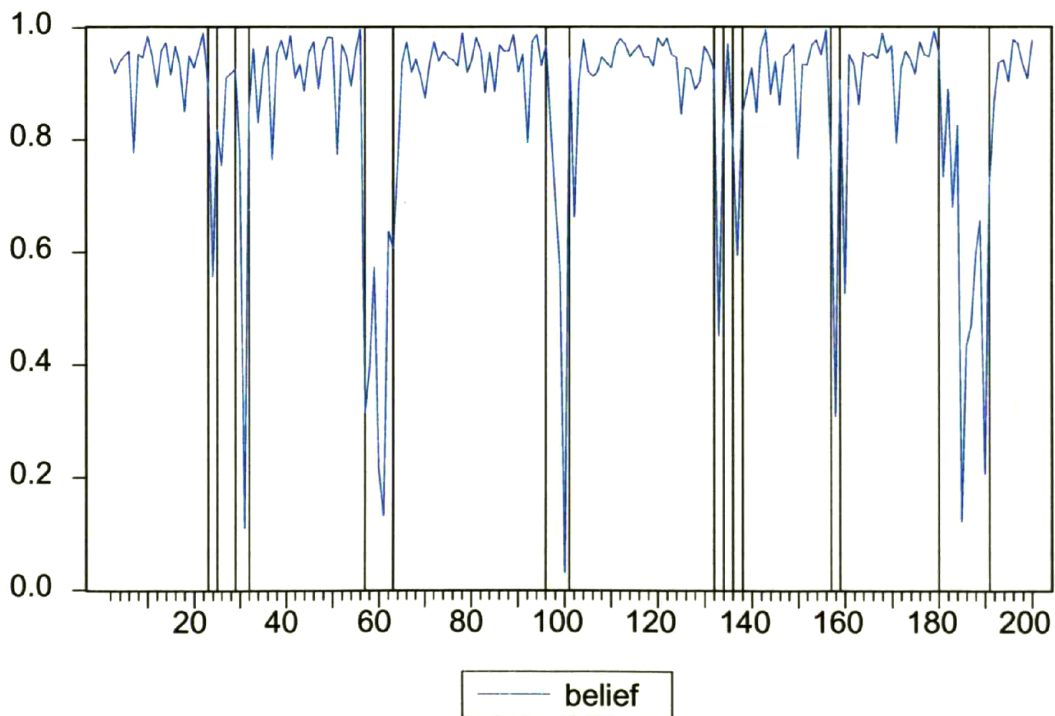


Figure 4.B Evolution of Belief (Incomplete Information Case: AGG)

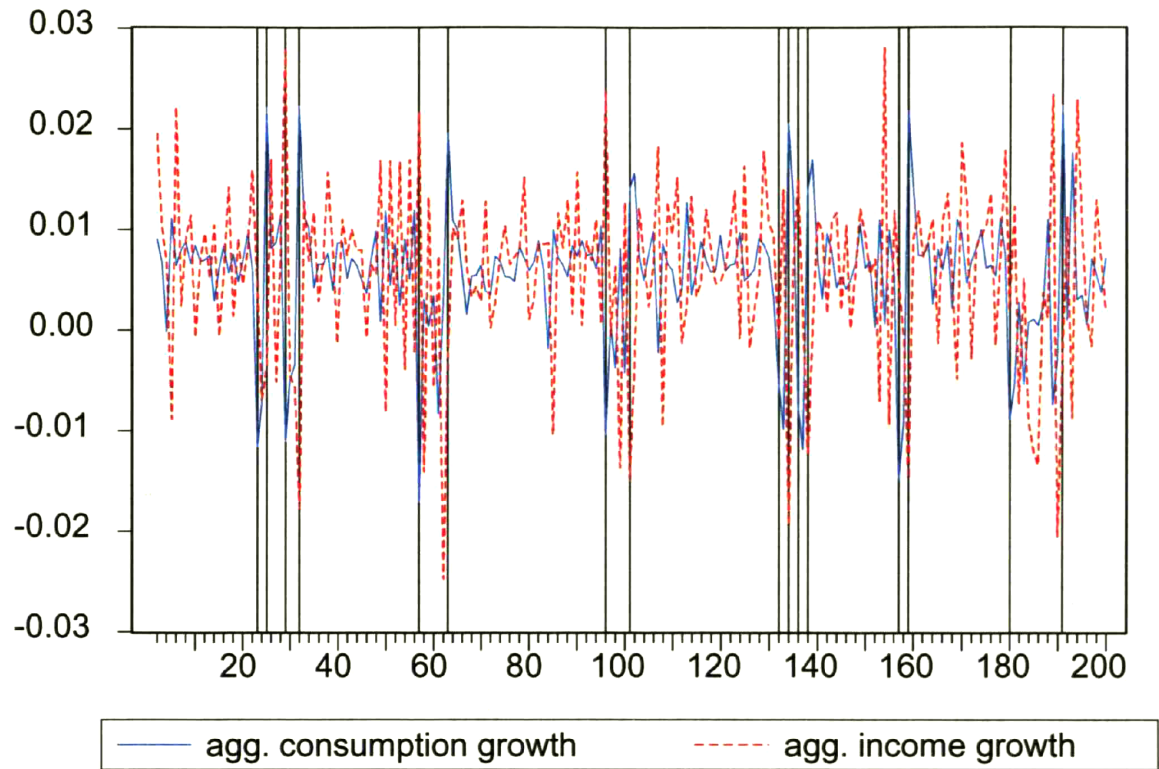


Figure 4.C Growth Rates of Aggregate Variables (Incomplete Information case: AGG+IND)

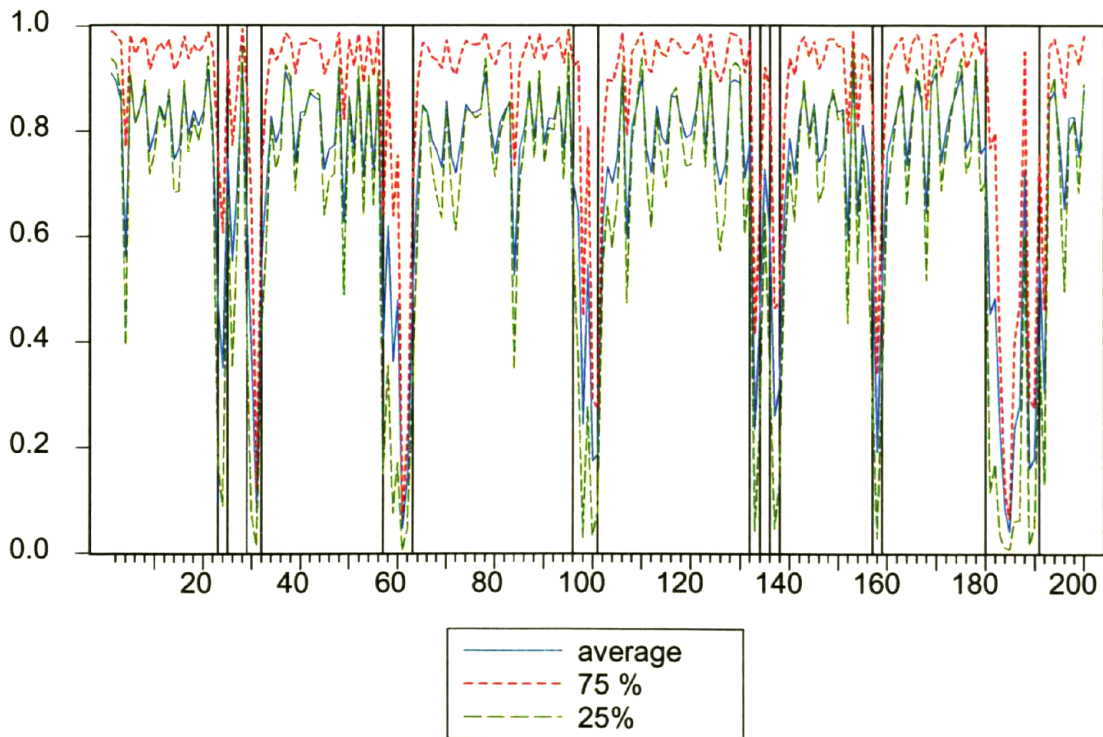


Figure 4.D Evolution of Belief (Incomplete Information Case: AGG+IND)

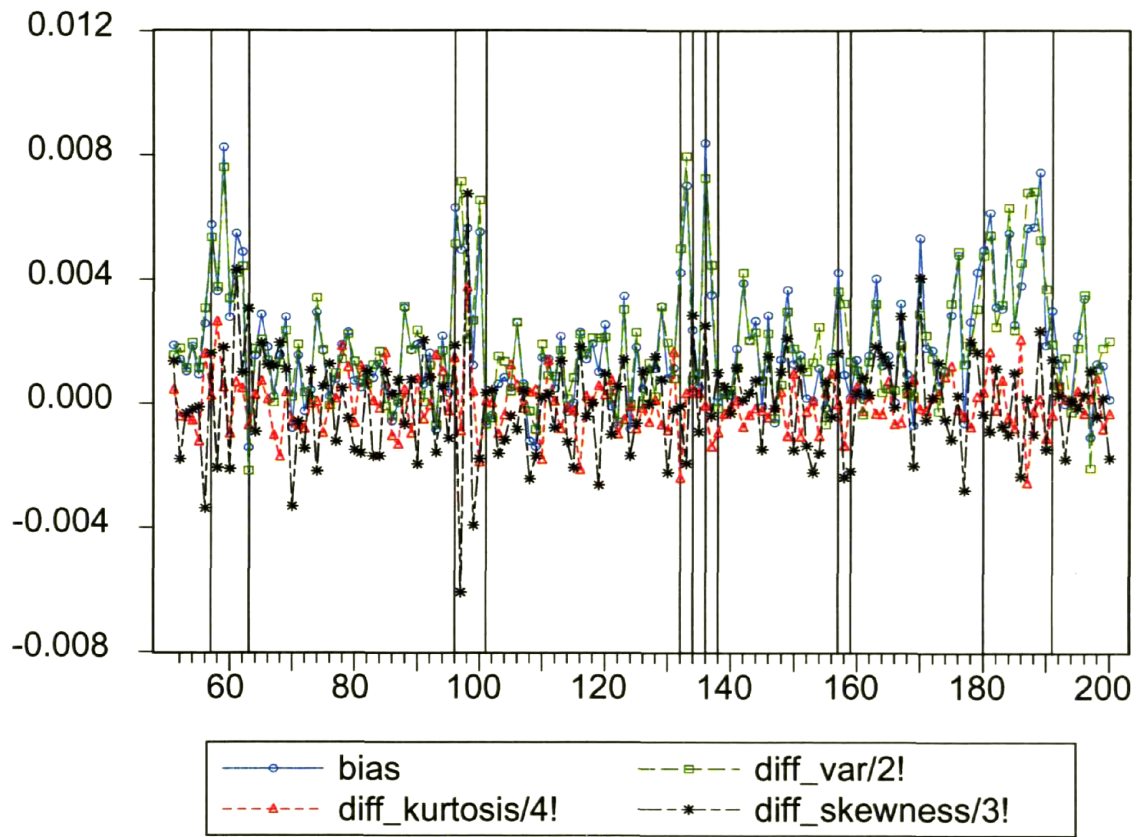


Figure 5 Aggregation Bias and Higher Moments

Chapter 2

Expected Consumption Growth In Recessions

2.1 Introduction

It is widely believed that recessions are times of greater uncertainty.¹ This implies that there will be greater precautionary saving motives in consumers' behavior during recessions; consumers will cut down consumption and save more hedge against greater uncertainty. The expected consumption growth then will increase in recessions.

However, evidence in the literature does not seem supportive of this prediction. In regressions of consumption growth on some measures of future income uncertainty, such as unemployment expectations or the variance across surveyed GDP forecasts, coefficients for uncertainty variables typically turn out to be either insignificant or even negative (for example, see Carroll [1992], and Hahm and Staigerwald [1999]). This suggests that lower income growth, another feature of recessions, may matter for consumption growth as well, especially for poor people with a relatively small amount of assets; having little buffer or liquidity constrained, they perhaps have to be more dependent on exogenous factors such as income to

¹Evidence in the recent literature supports this belief. For example, Davis and Haltiwanger [1992] show that the job reallocation, defined as the sum of job creation and job destruction, in US manufacturing exhibits significant counter-cyclical time variations. As the changes in employment status tend to be highly correlated with individual earnings over business cycles, recessions would be associated with greater income risks. More direct evidence on the counter-cyclical income risk is provided by Storesletten, Telmer and Yaron [2004], who show idiosyncratic labor income risk increases by 80% in recessions.

finance their expenditures.

This paper is motivated by this tension, and tries to reconcile these seemingly contradictory observations. Through a series of comparative statics, I first show that expected consumption growth decreases with lower income growth, while it increases with greater income uncertainty. Then, using a regime switching income process where recessions are associated with greater income variance and lower income growth, I consider both effects. The results suggest that expected consumption growth can indeed be lower in recession, even with greater income variance.

The underlying mechanism behind the results is as follows. With new parameter values in the income process, the optimal consumption rule and the implied expected consumption growth locus would accordingly change, and asset holding positions are also adjusted. Hence, in order to compare the expected consumption growth associated with different parameter values in the income process, one needs to consider these two factors: the shape and location of expected consumption growth locus, and the average asset holding.

Now, I briefly summarize the results from individual experiments. First, with lower income growth, the associated expected consumption growth curve shifts down. At the same time, the lower income growth, equivalent to less future resources or potential buffer stock, strengthens precautionary saving motives, leading to increases in average wealth. So expected consumption growth decreases at the average wealth level. Second, with greater income variance, expected consumption growth shifts up too, while the average wealth increases (as consumers would want to hold more assets as buffer against greater uncertainty), so the net effect is in principle ambiguous. For all the parameter values tried, however, the effect of upshifting locus seems to dominate, and the expected consumption growth increases. Finally, when I consider the joint effect of income growth and variance using a regime-switching income process, the net effect is ambiguous due to the switching moments in the income process (as income growth is lower, but more uncertain, in recessions). The expected consumption growth thus is determined by the relative importance of income growth and variance. This paper shows that if income growth is relatively low and/or uncertainty is not too high in recessions, it is possible for expected consumption growth to be lower in recessions (even with greater income uncertainty).

The discussion above implies that expected consumption growth is implicitly a function of several factors, such as income growth and asset holding, as well as income uncertainty. This result seems inconsistent with the conventional argument which claims that uncertainty is the sole determinant of expected consumption growth, and that there is a positive correlation between two. This paper shows the issue lies in the linearized Euler equations, which have been thought of as an analytical rationale for the conventional argument. The linearized Euler equations, which link expected consumption growth to a conditional second moment in consumption growth, implicitly assume that the second moment is the only determinant of expected consumption growth, and it can fully reflect a consumer's optimal behavior (including endogenous asset holding adjustment) in response to the change in income process parameters.

This paper points out the potential pitfalls of this argument, and shows that inferences based on the linearized Euler equations can be misleading. In addition, this paper explores the sources of biases due to the method associated with linearization, such as failing to take into account additional constraints, among others.

The remaining part of the paper is organized as follows. Section 2 presents a main model to investigate the question and reports results from comparative studies to see how expected consumption growth would change with income process parameters. Section 3 then addresses the issues associated with the linearized Euler equations on which the conventional argument has been based, and explores potential sources of bias. Section 4 ends with some concluding remarks.

2.2 Expected Consumption Growth in a Simple Consumer's Problem

Consider the following standard buffer-stock/precautionary saving problem. A consumer with isoelastic preference wishes to maximize the stream of his expected utility:

$$\max E_0 \left[\sum_{t=0}^{\infty} \beta^t U(C_t) \right] \quad (2.1)$$

with

$$U(C_t) = \begin{cases} \frac{C_t^{1-\gamma}-1}{1-\gamma}, & \gamma \neq 1 \\ \ln C_t, & \gamma = 1 \end{cases}$$

subject to:

$$W_{t+1} = (1+r)(W_t - C_t) + Y_{t+1}, \quad (2.2)$$

$$A_t \geq 0, \quad (2.3)$$

$$A_0 \geq 0, \text{ and } \lim_{t \rightarrow \infty} (1+r)^{-t} A_t \geq 0, \quad (2.4)$$

where C_t denotes consumption, $W_t = A_t + Y_t$ is wealth, or “cash on hand” (the sum of asset, A_t , and income, Y_t), and $(1+r)$ is the gross interest rate. The inequalities in (3) and (4) state liquidity constraint, and initial and terminal conditions for asset holding, respectively.

I first assume a simple income process with a constant mean growth. Specifically, the income process (in terms of log) is given by:

$$\ln Y_t = \ln P_t + \ln U_t,$$

$$\Delta \ln P_t = \ln G + \ln N_t.$$

In words, consumer’s labor income, Y_t , consists of permanent, P_t , and transitory component, U_t ; the evolution of permanent income is determined by constant growth factor, G , and permanent income shock, N_t . It will often be more convenient to write the process in terms of log difference, or growth rate:

$$\begin{aligned} \Delta \ln Y_{t+1} &= \ln G + \ln N_{t+1} + \ln U_{t+1} - \ln U_t \\ &= g + n_{t+1} + u_{t+1} - u_t, \end{aligned}$$

where $g \equiv \ln G$, $n_t \equiv \ln N_t \sim (-\frac{1}{2}\sigma_n^2, \sigma_n^2)$, and $u_t \equiv \ln U_t \sim N(-\frac{1}{2}\sigma_u^2, \sigma_u^2)$. This type of specification for income process is reasonable, as this is commonly used in the precautionary savings literature, and seems consistent with microeconomic studies of individual earning process. Finally, I impose the following impatience condition to ensure the unique solution

to the problem:

$$\beta(1+r)E_t[(GN_{t+1})^{-\gamma}] = \beta(1+r)E_t[e^{-\gamma(g+n_{t+1})}\mathbf{1}] < 1. \quad (2.5)$$

The optimal consumption behavior then is characterized by the familiar Euler equation:

$$C_t^{-\gamma} = \max \left\{ W_t^{-\gamma}, \beta(1+r)E_t[C_{t+1}^{-\gamma}] \right\}. \quad (2.6)$$

As is well known in the literature, if one assumes stochastic labor income, there is no known analytical solution for optimal consumption policy in the case of CRRA utility.² Therefore, researchers who want to explore the effect of (income) uncertainty on consumption behavior (either quantitatively or qualitatively) usually have to rely on approximation, or solve the model numerically.

This paper first numerically solves the consumer's problem and compares the result with the approximated solution later. In the numerical solution case, the optimal normalized consumption, $c_t = C_t/P_t$, can be expressed as a function of normalized wealth, $w_t = W_t/P_t$: $c_t = c(w_t)$, and is shown in Figure 1.A. With this optimal consumption policy at hand, one can easily calculate the expected consumption growth as a function of normalized wealth:

$$\begin{aligned} E_t \left[\frac{C_{t+1}}{C_t} \right] &= E_t \left[\frac{c_{t+1}P_{t+1}}{c_t P_t} \right] \\ &= E_t \left[\frac{e^{(g+n_{t+1})} c(w_{t+1})}{c(w_t)} \right] \\ &= \frac{1}{c(w_t)} E \left[e^{(g+n_{t+1})} c \left(\frac{(1+r)(w_t - c(w_t))}{e^{(g+n_{t+1})}} + e^{u_{t+1}} \right) \right], \end{aligned}$$

which is shown in Figure 1.B.³ The graph indicates that the expected consumption growth is strictly decreasing in (normalized) wealth: as poor consumers have a lesser ability to hedge shocks to income, precautionary motives work to depress consumption and increase the expected consumption growth more for poor people than for wealthier people.⁴

²More generally, this is true for the class of Hyperbolic Absolute Risk Aversion (HARA) preference. See Carroll and Kimball [1996].

³The parameter values used are: $\gamma = 2$, $\delta = 0.01$, $r = 0.075$, $g = 0.005$, $\sigma_n = 0.03$, and $\sigma_u = 0.12$, and all the numerical procedures used in this paper are contained in Appendix.

⁴See Carroll [2004] for formal and extensive discussion on the shape of this expected consumption growth

Another feature of the graph is that, at low wealth levels where consumers are liquidity-constrained, their consumption is simply set equal to their wealth: $c_t = c(w_t) = w_t$, and for this wealth range, the expected consumption growth is an explicit function of income growth and wealth:

$$\begin{aligned} E_t \left[\frac{C_{t+1}}{C_t} \Big|_{c_t=w_t} \right] &= E_t \left[e^{(g+n_{t+1})} \frac{e^{-(g+n_{t+1})}(1+r)(w_t - c_t) + e^{u_{t+1}}}{w_t} \right] \\ &= \frac{e^g}{w_t}. \end{aligned}$$

Therefore, given a (normalized) wealth, the (expected) income growth is the sole determinant of the expected consumption growth. To see this result better, the following discussion is helpful. Notice that, combining the impatience condition (5) and the ‘unconstrained’ Euler equation (6), one can get $E_t [(P_{t+1}/P_t)^{-\gamma}] < E_t [(C_{t+1}/C_t)^{-\gamma}]$. So in the absence of non-borrowing, this expression induces the expected consumption growth to be lower than the (permanent) income growth through borrowing and consuming more (than income) today. However, when borrowing is not allowed, consumption is bounded above by the current wealth. So for this wealth range, a consumer’s endogenous factor, especially impatience in preference which otherwise would give more freedom for dynamic optimal behavior and allow for more consumption, is checked, and consumption growth is more dependent on exogenous factors such as income growth.⁵

Although the problem does not yield an analytically tractable solution, it is still possible to investigate how the expected consumption growth would change with different parameter values, in particular, of income growth and shock. Below I do a few such simple experiments and present the results.

2.2.1 Comparative Study I: The Effect of Income Growth

In the first experiment, I consider the case where the income growth is lower than its benchmark value, $g_L < g_H = g$, while keeping the other parameters in the model unchanged. The usual argument in precautionary saving theory does not give any direct prediction for

curve and its implications.

⁵Figure 2 plots a sample simulation of wealth and expected consumption growth in this case.

the result of this experiment because it presupposes that income growth does not matter in determining the expected consumption growth, which is mainly determined by future (consumption) uncertainty.

Figure 3 plots the two expected consumption growth loci, each corresponding to high/low (permanent) income growth rate.⁶ The expected consumption growth curve with lower growth lies below the high growth counterpart for all wealth levels. To see this result better, first note that the expected consumption growth can be decomposed into two parts, the expected income growth and the expected normalized consumption growth:

$$\begin{aligned}
E_t [\Delta \ln C_{t+1}] &= E_t [\ln c_{t+1} P_{t+1} - \ln c_t P_t] \\
&= E_t [\ln P_{t+1}] - \ln P_t + E_t [\ln c_{t+1}] - \ln c_t \\
&= g + E_t [\Delta \ln c_{t+1}].
\end{aligned}$$

Using this, the difference in two expected consumption growths associated with two different income growth rates, g_H and g_L , can be written as the sum of the difference in income growth rates and the difference in two expected normalized consumption growths:

$$\begin{aligned}
&E_t [\Delta \ln C_{t+1}|_{g=g_H}] - E_t [\Delta \ln C_{t+1}|_{g=g_L}] \\
&= (g_H - g_L) + (E_t [\Delta \ln c_{t+1}|_{g=g_H}] - E_t [\Delta \ln c_{t+1}|_{g=g_L}]).
\end{aligned}$$

At low wealth levels where consumption is constrained to current wealth, two normalized consumption policies are again the same, so the difference in expected consumption growth is simply determined by the gap between two income growth rates, $(g_H - g_L)$. As wealth increases and the constraint starts to be non-binding, the difference in two normalized consumptions comes in and plays a role: as wealth increases, the difference between two normalized consumptions, $(E_t [\Delta \ln c_{t+1}|_{g=g_H}] - E_t [\Delta \ln c_{t+1}|_{g=g_L}])$, becomes larger, so the overall gap becomes smaller. For sufficiently large levels of wealth, each expected normalized consumption growth converges to $(\beta(1+r))^{1/\gamma} - g_H$ and $(\beta(1+r))^{1/\gamma} - g_L$, respectively, and both of the expected consumption growth curves converge to $(\beta(1+r))^{1/\gamma}$, which is the

⁶The high income growth rate is set to $g_H = 0.002$ (0.2%).

consumption growth in the certainty equivalent case (not shown in the figure).

The new expected consumption growth locus itself, however, does not tell us what the expected consumption growth would be with lower growth rate, as it is a function of wealth. So in order to pin down the expected consumption growth, we need to characterize the wealth dynamics. One useful concept for this is the target wealth, w^* such that if $w_t = w^*$, then $E_t[w_{t+1}] = w^*$, and the wealth fluctuates around this stable point (see Carroll [1997, 2004]). In this paper, I instead use the average wealth, which is simply calculated as the simulation average. Conceptually, this is quite similar to the target wealth, although they are not exactly the same. So we can compare the expected consumption growth at the average wealth, associated with a different income growth rate.

How would the average wealth change when the income growth is lower? Notice lower income growth rate implies, given the same degree of future income uncertainty, a consumer's present discount value of future income is also lower, and he has relatively less buffer to insure himself against future uncertainty. Therefore, there is relatively more incentive to save, and he wants to consume less and hold more asset. This 'growth effect' associated with lower income growth will further decrease the expected consumption growth, which is already lower at a given wealth, by increasing the average asset holding to a higher level.

The simulation result in Table 2 shows that the mean wealth is indeed higher and the expected consumption growth decreases with lower income growth.⁷

2.2.2 Comparative Study II: The Effect of Income Uncertainty

What would happen to the expected consumption growth when there is greater income uncertainty? The conventional argument would predict that the expected consumption growth will increase, too. In the second experiment, I consider the variance/uncertainty effect and see how expected consumption growth would change with more risky income stream: $\sigma_{n|H} > \sigma_{n|L} = \sigma_n$ (permanent income shock) and $\sigma_{u|H} > \sigma_{u|L} = \sigma_u$ (transitory income shock).

As before, we can numerically solve the problem, and Figure 4 shows the two expected

⁷In this case and all the following tables, I report results such as the mean (normalized) wealth, the expected consumption growth at that wealth level, and average expected consumption growth across consumers. Each simulation uses 100,000 consumers.

consumption growth curves, each corresponding to high and low variances of permanent income shock.⁸ Again, at low wealth levels, the expected consumption growth is constrained by the income growth. However, as the constraint becomes non-binding, the expected consumption growth becomes higher with higher income variance, which seems consistent with the conventional argument.

Again, an important consideration is the average asset position. It is not hard to conjecture that the average asset position is higher with higher income variance. This idea is essentially the flip side of the higher income growth case: Given the same income growth, greater future uncertainty will induce consumers to engage in more precautionary behavior and save more.

In sum, with the greater income uncertainty, the expected consumption growth shifts up, while the average wealth increases. Therefore, it is in principle impossible to anticipate whether the expected consumption growth would go up with greater income risk.

As the simulation result in Table 2 indicates, the upshift effect (gap between two curves) seems to be large enough and to dominate the effect from the higher wealth position, that the expected consumption growth is higher at the average normalized wealth (albeit larger) than in the smaller variance case.⁹ The discussion for transitory shocks is essentially similar, although the quantitative result is different.

2.2.3 Comparative Study III: Regime Switching Income Case

Discussion in the previous subsections shows that the expected consumption growth depends on income growth as well as its variance. An immediately interesting question then is: what will happen when we incorporate both factors together in the determination of expected consumption growth? In particular, what would happen when we combine high income growth and small variance, or low income growth and high variance? I do this analysis with a simple regime-switching type income process, the key feature of which is that the mean and variance of income growth are stochastic, dependent upon the aggregate business cycle

⁸I set the parameter value to $\sigma_{n|H} = 0.05$.

⁹An issue one may wonder about is: can the expected consumption growth at average wealth be lower with higher variance? Again, as we lack an analytical expression, it is not easy to answer. However, I obtained the same qualitative results with several other parameter values of income variance.

regime. This type of income process provides a natural framework to think about the above issue, especially in terms of (counter-) cyclical/time-varying risks.

Assume now the dynamics of income fluctuations are characterized by the following regime-switching type process:

$$\Delta \ln Y_t = \alpha_0 + \alpha_1 S_t + n_t + u_t - u_{t-1}, \quad (2.7)$$

where $n_t \equiv \ln N_t \sim (-\frac{1}{2}\sigma_{n|S}^2, \sigma_{n|S}^2)$, $u_t \equiv \ln U_t \sim (-\frac{1}{2}\sigma_{u|S}^2, \sigma_{u|S}^2)$, $\sigma_{n|S}^2 = S_t\sigma_{n|E}^2 + (1-S_t)\sigma_{n|R}^2$, $\sigma_{u|S}^2 = S_t\sigma_{u|E}^2 + (1-S_t)\sigma_{u|R}^2$, and $S_t = 0$ (recession) or 1 (expansion). The switches between two regimes are governed by the following transition probabilities:

$$\Pr[S_t = 1|S_{t-1} = 1] = p, \quad \Pr[S_t = 0|S_{t-1} = 0] = q. \quad (2.8)$$

Suppose that $\sigma_{n|R} > \sigma_{n|E}$, $\sigma_{u|R} > \sigma_{u|E}$, and $p > q > 1/2$ (these are empirically supported assumptions; see Storesletten, Telmer and Yaron [2004]). Then, the income process in (7) and (8) implies that recessions are associated with greater risk, as $Var_t[\Delta \ln Y_{t+1}|S_t = 0] = \sigma_{n|R}^2 + \sigma_{u|R}^2 + \alpha_1^2 q(1-q) > Var_t[\Delta \ln Y_{t+1}|S_t = 1] = \sigma_{n|E}^2 + \sigma_{u|E}^2 + \alpha_1^2 p(1-p)$. Hence, according to the conventional argument, the expected consumption growth should be higher in recession.

In the regime-switching income case, one also can numerically solve the problem and calculate the exact expected consumption growth rates in each business cycle regime. It is straightforward to show that the expected consumption growth in expansion/recession is:

$$E_t \left[\frac{C_{t+1}}{C_t} \middle| E \right] = \frac{pE [c(w_{t+1|EE}, E)e^{g_{t+1|E}}] + (1-p)E [c(w_{t+1|ER}, R)e^{g_{t+1|R}}]}{c(w_t, E)}$$

and

$$E_t \left[\frac{C_{t+1}}{C_t} \middle| R \right] = \frac{(1-q)E [c(w_{t+1|RE}, E)e^{g_{t+1|E}}] + qE [c(w_{t+1|RR}, R)e^{g_{t+1|R}}]}{c(w_t, R)},$$

where $g_{t+1|S_{t+1}} = \alpha_0 + \alpha_1 + n_{t+1|S_{t+1}}$, and $w_t|S_t S_{t+1} = (1+r)(w_t - c(w_t, S_t))e^{-g_{t+1|S_{t+1}}} + e^{u_{t+1|S_{t+1}}}$. Notice that the expected consumption growth in this case depends on two arguments, normalized wealth and business cycle regimes: $E_t [C_{t+1}/C_t|S_t] = \chi(w_t, S_t)$.

Figure 5 plots the expected consumption growth rates as a function of wealth and business

cycle regime.¹⁰ Again, for the liquidity constrained part, the expected consumption growth is governed by the expected income growth, $\alpha_0 + (1 - p)\alpha_1$ and $\alpha_0 + (1 - q)\alpha_1$, respectively, so the expected consumption growth is higher in expansion for a given wealth. However, as wealth increases and the constraint becomes relatively irrelevant, the two curves cross; above that point, given a wealth level, the expected consumption growth becomes higher in recession due to the large variance.

So the ultimate effect of recession on the expected consumption is again ambiguous, and it depends on the relative strength of the “growth effect” and the “variance/uncertainty effect” (and the average wealth conditional on each regime). The simulation result in Table 3 shows that, as opposed to the conventional argument, the expected consumption growth is lower on average (and the mean asset holding is higher) in recession.

However, it seems that this result is not robust and is dependent on parameter values. For example, with a relatively low income growth in expansion, and/or a relatively larger income variance in recession, the uncertainty effect may dominate the growth effect, and the expected consumption growth may be higher in recessions. As an illustrative example, when I use $\sigma_{n|E} = 0.05$, and $\sigma_{u|R} = 0.09$, the expected consumption growth is higher in recession (0.98%) than in expansion (0.41%) at the mean wealth conditional on the regime.

The comparison of the expected consumption growth over two regimes is not a full description. A more interesting exercise with the regime switching income process is to see the dynamics of wealth and the corresponding expected consumption growth over business cycles. As Hwang [2006] shows, a consumer’s behavior over a typical regime switching-type cycle can be described as follows. On the onset of a recession, he starts accumulating assets by decreasing consumption sharply, and wealth keeps increasing as the recession continues. Then, as the recession ends and the economy switches back to an expansion, he uses the accumulated asset to finance his consumption boost. Hence, as soon as a boom is announced, consumption jumps up and wealth begins to decline.

Then, what are the implied dynamics of expected consumption growth? As a recession starts, the expected consumption growth declines as wealth accumulates. Then, when the

¹⁰In the calibration, I also assume the following impatience condition: in either regime, $\beta(1 + r)E_t[e^{\gamma(\alpha_0 + \alpha_1 S_{t+1} + u_{t+1})} | S_t] < 1$. The parameter values used are: $\alpha_0 = -0.01$, $\alpha_1 = 0.03$, $p = 0.95$, $q = 0.8$, $\sigma_{n|E} = 0.02$, $\sigma_{n|R} = 0.03$, and $\sigma_{u|E} = \sigma_{u|R} = 0.12$.

economy switches to an expansion regime, the expected consumption growth picks up until the expansion ends (see Figure 6, which plots a sample time-series simulation of the cross section average of normalized wealth and associated with expected consumption growth). Notice that the dynamics of the expected consumption growth are essentially the mirror image of those of wealth, and there is an asymmetric pattern in each regime. The simulation result shows that the differences in wealth and expected consumption growth are often bigger within a regime (i.e., 1st period vs. last period of a certain regime) than across two regimes, further illustrating that wealth dynamics over business cycles are another key factor to determine the expected consumption growth.

2.3 Revisiting Approximated Euler Equations

The previous section shows that expected consumption growth may depend on income growth as well as income variance, and that it can be lower even with greater income uncertainty when income growth is relatively lower. This result seems inconsistent with the conventional argument that claims the future uncertainty is the sole determinant of the expected consumption growth, and there is a positive correlation between the two.

How can we reconcile this result with the traditional argument? What if any is the conventional argument missing then? To answer these questions, it is useful to go over the Euler equations on which the usual argument in the literature has been based.

2.3.1 Two Linear (Approximated) Euler Equations

The Euler equations used in the literature regarding this issue can be classified into two broad categories. First, with the assumptions that preference is represented by a CRRA utility function and shocks to consumption are log-normally distributed, a simple manipulation of the standard Euler equation leads to the following expression for expected consumption growth:

$$E_t[\Delta \ln C_{t+1}] = \frac{1}{\gamma}(r - \delta) + \frac{\gamma}{2} \text{Var}_t[\Delta \ln C_{t+1}]. \quad (2.9)$$

Second, another common approach is to second-order Taylor expand the standard, or unconstrained, Euler equation around the point at which the expected consumption growth

rate is 0, or around the current consumption to obtain:¹¹

$$E_t \left[\frac{C_{t+1} - C_t}{C_t} \right] \approx \frac{1}{\gamma} \left(\frac{r - \delta}{1 + r} \right) + \left(\frac{1 + \gamma}{2!} \right) E_t \left[\left(\frac{C_{t+1} - C_t}{C_t} \right)^2 \right]. \quad (2.10)$$

Expressions (9) and (10) have been believed to deliver an analytical rationale for intuitive account of consumers' behavior with prudence under uncertainty, and the conventional argument in the literature goes as follows. When consumers expect future income tends to be more risky so that the expected consumption growth is also likely to exhibit more variations (assuming that future uncertainty can be reasonably captured by either the conditional variance in (9), or the conditional expectation of squared consumption growth in (10)), they decrease the current consumption and save more. The result then is that the expected consumption growth increases when their future income is more uncertain.¹²

Although the argument above does sound valid, a careful inspection of the linearized Euler equations would reveal potential pitfalls.¹³ First, notice that the conditional expectation in (10) is not the variance of the expected consumption growth; instead what the approximation actually implies is:

$$E_t \left[\frac{C_{t+1} - C_t}{C_t} \right] \approx \frac{1}{\gamma} \left(\frac{r - \delta}{1 + r} \right) + \left(\frac{\gamma + 1}{2} \right) Var_t \left[\frac{C_{t+1} - C_t}{C_t} \right] + \left(\frac{\gamma + 1}{2} \right) \left(E_t \left[\frac{C_{t+1} - C_t}{C_t} \right] \right)^2.$$

¹¹From here, by further assuming $\left(E_t \left[\frac{C_{t+1} - C_t}{C_t} \right] \right)^2$ is sufficiently small and negligible, one may derive a similar expression to (9): $E_t \left[\frac{C_{t+1} - C_t}{C_t} \right] \approx \frac{1}{\gamma} \left(\frac{r - \delta}{1 + r} \right) + \left(\frac{1 + \gamma}{2!} \right) Var_t \left[\frac{C_{t+1} - C_t}{C_t} \right]$.

¹²This intuition has appeared in some popular graduate level macroeconomics textbooks (such as Deaton [1991] and Romer [2001]) as well as in academic journals as one of key elements in prudent consumers' behavior. In addition, these two expressions (or their variations) are widely used in asset pricing and empirical studies on precautionary saving behavior. The literature in this line of research is vast, and limited examples include: Skinner [1988], Dynan [1993], Guiso, Jappelli and Terlizzese [1992], and Carroll and Samwick [1997] with cross-sectional data; Carroll [1992], Hahn and Steigerwald [1999] using aggregate time series data. On the other hand, several authors attempt to quantify the effects of precautionary savings motives in consumption fluctuations based on a generalized version of (10). For example, Gourinchas and Parker [2001] use the approximation method similar to (10) to characterize the empirical importance of precautionary savings in the U.S. consumption data. Parker and Preston [2002], again using a generalized but essentially similar method to (10), try to decompose the changes in average consumption growth rate into four proximate causes such as intertemporal substitution, changes in the preferences, incomplete market for consumption insurance, and unpredictable portion.

¹³For the sake of convenience, I call both (9) and (10) 'linearized' Euler equations, although Expression (9) does not involve any approximation, and is derived exactly.

So, in addition to the variance of expected consumption growth, there is an extra term, and the expectation on the right hand side of (10) may not be a reasonable measure of future uncertainty.

Second, the implicit assumption behind the argument, based on the linearized Euler equations above, is that the second moment, either the conditional variance in (9) or the expectation of squared expected consumption growth in (10), can reasonably capture the income uncertainty. But as the previous section shows, it seems that the second moment is not able to properly reflect the consumer's behavior (especially adjustment in the asset position) in response to new income process parameters, even in a qualitative sense. In particular, as the regime-switching income example indicates, greater income uncertainty in recession may be translated into lower variance, rather than greater variance, in consumption growth.

2.3.2 How Accurate Are the Linearized Euler Equations?

The discussion above assumes that errors from linearization are not serious, so that the linearized Euler equations themselves are reasonably accurate. However, a careful comparison of the expected consumption growth curve using linearization with the true one would reveal that there are non-negligible biases in linearization. In order to see the difference between them, I numerically calculate Expressions (9) and (10), and plot them as a function of wealth. In doing so, the expectations on right hand side of (9) and (10) are numerically evaluated using the optimal consumption policy from the baseline model (using the baseline parameter values).

Figure 7 plots the expected consumption growth curves using two linearized Euler equations along with the true curve. First, in the case of Expression (9), the Euler equation with log-normal assumption, the curve significantly underestimates the true one, especially for low wealth levels. In addition, the variations in the conditional variance term seem to be too small to account for the actual change in the expected consumption growth associated with different wealth levels. Hence, the graph suggests that there is more than the variance of expected consumption growth to explain the actual changes in the expected consumption growth, and Expression (9) seems to fail to capture these potentially important factors.

When I consider Expression (10), the second-order Taylor approximated Euler equation, there is no significant improvement either. The approximation seems clearly better than (9), but it still significantly underestimates the true curve at low wealth levels, although the biases seem to disappear as wealth increases. Finally, even when I include higher (up to fourth) order terms in the Taylor expansion, the picture does not change very much, and large biases remain (see Figure 8).

What goes wrong with these linearized Euler equations? First, Expression (9) relies on the critical assumption that the consumption process is log-normally distributed. However, normality tests using consumption data (either actual or simulated data) typically reject the distributional assumption. So this may be thought of as a special case, although useful in that it yields a closed form expression.

Second, in both cases, it seems apparent that the main culprit of this bias is the presence of liquidity constraint. Note that, in deriving either (9) or (10), the approximation essentially assumes that the standard Euler equation, $U'(C_t) = \beta(1+r)E_t[U'(C_{t+1})]$, holds every period, and that there is no explicit consideration to incorporate endogenous responses to exogenous factors (such as possible liquidity constraints or the nature of the income process) in optimal consumption behavior. However, consumers in this model are expected to occasionally undergo some periods when they are credit constrained. In such cases, this standard, or unconstrained, Euler equation is clearly violated, and they are forced to consume less than they would have desired otherwise. And once consumption is cut down significantly due to the credit constraint, the next period's consumption tends to return to the mean or normal level, and the expected change in consumption would be higher than the unconstrained case. This may partly explain why the linearizations tend to underestimate the true expected consumption growth for low wealth levels, by wrongly assuming a lower curvature of the consumption function, which in turn implies lower expected consumption growth.

2.3.3 Another Example: the Possibility of Zero Income

One may think the results above, inconsistent with conventional beliefs, are due to the presence of additional constraints, such as the liquidity constraint. However, this result is not specific to the liquidity constraint case. As long as current consumption does not exceed

current wealth, similar results would arise even without credit constraint. One good example is the case in which there is a small positive probability of a catastrophic event such as zero income (Carroll [1992, 1997], and Gourinchas and Parker [2001]). In this case, as Schechman [1976] and Zeldes [1989] have pointed out, along with the assumption that consumption must remain strictly positive, the zero lower support of the income process implies consumers will never borrow, i.e., $C_t = C(W_t) < W_t$, for all W_t . So this amounts to the fact that a consumer would impose additional self-generated constraint as a precautionary measure against the extremely adverse shock, although there is no such explicit constraint in the consumer's problem.

In the case of possible zero income, I also conduct the same experiments as in Section 2.¹⁴ The simulation results reported in Table 4 deliver the qualitatively same message as in the liquidity constraint. First, for the benchmark parameter values, both the expected consumption growth and the asset holding are higher compared to the credit constraint case, indicating the precautionary saving motive is stronger with the possible zero income. Second, lower income growth (higher income variance) leads to higher (lower) average asset holdings and lower (higher) expected consumption growth. Third, for the regime switching income case, qualitative results are also largely similar; the expected consumption growth is lower during recessions.

Figures 9 and 10 show that, with the possibility of zero income, the overall shape of the expected consumption growth is quite similar to the liquidity constraint case, although it is somewhat smoother, and that biases from the linearized Euler equations are still huge, especially for low wealth levels.

2.4 Concluding Remarks

This paper shows how the expected consumption growth will change with income growth and variance, and why the expected consumption growth can be lower in recession, even with greater income uncertainty. I then compare these results to the usual argument in the literature, based on the linear approximation of the Euler equation.

¹⁴The probability of the zero income, ρ is set to 0.0005.

A few recent papers in the literature already address the issues associated with linear-approximated Euler equations (Carroll [2001], and Ludvigson and Paxson [2001], for example). Their main argument is that there is no way to consistently estimate and/or recover the structural parameters such as the risk aversion coefficient using the linear-approximated Euler equation such as (10). Ludvigson and Paxson [2001] conjecture that the reason for this bias is due to the omitted higher order terms in (10). On the other hand, rather than the issue of consistent estimation of the risk aversion parameter as in aforementioned papers, this paper is mainly concerned with the quantitative/qualitative effect of various factors such as income growth (as well as income uncertainty) on expected consumption growth. And I show that the linearized versions of the Euler equation may not be useful for investigating these issues, therefore present some caveats.

A few future research topics are in process. While this paper mainly focuses on the effect of properties in the income process on expected consumption growth, there are certainly several other factors and parameters that would affect the expected consumption growth, such as the degree of risk aversion, intertemporal substitution, impatience, and credit market tightness, among others. Exploring their effects seems a natural next step. Also, while the numerical solution algorithm allows us to obtain the actual optimal policy without any approximation when an analytical solution is not available, the underlying mechanism behind the result often remains a black box, and we may still prefer approximation so one can study other issues and obtain analytical insights. So it seems clear that developing an alternative approximation method as an analytical tool to take into account possible and/or additional constraints in characterizing the optimal behavior will be a fruitful direction for future research.

2.5 Appendix

Most of the numerical procedure used in this paper are very similar to those employed in Hwang [2006], and interested readers may refer to that paper. This appendix instead addresses the numerical calculation not covered in that paper.

Several points in this paper, I need to calculate the expectations such as $E_t \left[\left(\frac{C_{t+1} - C_t}{C_t} \right)^m \right]$. The general expression for m 's moment of conditional consumption growth is:

$$\begin{aligned}
& E_t \left[\left(\frac{C_{t+1} - C_t}{C_t} \right)^m \right] \\
&= E_t \left[\left(\frac{e^{(g+n_{t+1})} c(w_{t+1})}{c(w_t)} - 1 \right)^m \right] \\
&= \frac{1}{c(w_t)^m} E \left[\left(e^{(g+n_{t+1})} c \left(\frac{(1+r)(w_t - c(w_t))}{e^{(g+n_{t+1})}} + e^{u_{t+1}} \right) - c(w_t) \right)^m \right] \\
&= \frac{1}{c(w_t)^m} \int \int \left(e^{(g+n_{t+1})} c \left(\frac{(1+r)(w_t - c(w_t))}{e^{(g+n_{t+1})}} + e^{u_{t+1}} \right) - c(w_t) \right)^m d\Phi_n d\Phi_u.
\end{aligned}$$

where $\Phi_{(\cdot)}$'s are normal cdfs associated with each shock.

I approximate this double integrals using Gauss-Hermite quadrature:

$$\frac{1}{c(w_t)^m} \frac{1}{\pi} \sum_{k,l} \left(e^{(g+\sqrt{2}\sigma_n n_k)} c \left(\frac{(1+r)(w_t - c(w_t))}{e^{(g+\sqrt{2}\sigma_n n_k)}} + e^{\sqrt{2}\sigma_u u_l} \right) - c(w_t) \right)^m \omega_{k,l}.$$

where n_k/u_l and ω_k/ω_l are Gauss-Hermite quadrature nodes and weights associated with each node, respectively.

In the case of zero income, the expression is modified as follows

$$\begin{aligned}
& E_t \left[\left(\frac{C_{t+1} - C_t}{C_t} \right)^m \right] \\
&= E_t \left[\left(\frac{e^{(g+n_{t+1})} c(w_{t+1})}{c(w_t)} - 1 \right)^m \right] \\
&= \frac{1}{(c(w_t))^m} \left\{ (1-\rho) E \left[\left(e^{(g+n_{t+1})} c \left(\frac{(1+r)(w_t - c(w_t))}{e^{(g+n_{t+1})}} + e^{u_{t+1}} \right) - c(w_t) \right)^m \right] \right. \\
&\quad \left. + \rho E \left[\left(e^{(g+n_{t+1})} c \left(\frac{(1+r)(w_t - c(w_t))}{e^{(g+n_{t+1})}} \right) - c(w_t) \right)^m \right] \right\}.
\end{aligned}$$

Once we obtain the above expectations, the calculation of the conditional variance of

expected consumption growth is straightforward

$$\text{Var}_t \left[\frac{C_{t+1}}{C_t} - 1 \right] = E_t \left[\left(\frac{C_{t+1}}{C_t} - 1 \right)^2 \right] - \left(E_t \left[\frac{C_{t+1}}{C_t} - 1 \right] \right)^2 .$$

The expectations in the regime-switching income process and in the case of zero income possibility are calculated in a similar manner.

Bibliography

- [1] Carroll, Christopher [1992], “The Buffer-Stock Theory of Saving: Some Macroeconomic Evidence”, *Brookings Papers on Economic Activity* 2, pp. 61-135.
- [2] Carroll, Christopher [2001], “Death to the Log-Linearized Consumption Euler Equation ! (And Very Poor Health to the Second Order Approximation)”, *Advances in Macroeconomics* Vol. 1: No. 1, Article 6.
- [3] Carroll, Christopher [2001], “A Theory of the Consumption Function, With and Without Liquidity Constraint”, NBER Working Paper 8387.
- [4] Carroll, Christopher [2004], “Theoretical Foundations of Buffer Stock Saving”, NBER Working paper 10867.
- [5] Carroll, Christopher and Miles Kimball [1996], “On the Concavity of the Consumption Function,” *Econometrica* 64, pp. 981-992.
- [6] Carroll, Christopher and Andrew Samwick [1997], “The Nature of Precautionary Wealth,” *Journal of Monetary Economics*, pp. 41-72.
- [7] Deaton, Angus [1991], *Understanding Consumption*, Oxford University Press.
- [8] Deaton, Angus and Guy Laroque [1992], “On the Behavior of Commodity Prices,” *Review of Economic Studies*, pp. 1-23.
- [9] Dynan, Karen [1993], “How Prudent Are Consumers?”, *Journal of Political Economy* 101, pp.1104-13.
- [10] Gourinchas, Pierre-Olivier, and Jonathan Parker [2001], “The Empirical Importance of Precautionary Savings”, *American Economic Review Papers and Proceedings*, pp. 406-412.

- [11] Gourinchas, Pierre-Olivier, and Jonathan Parker [2002], "Consumption over Life-Cycle", *Econometrica*, pp. 47-89.
- [12] Guiso, Luigi, Tulio Jappelli and Daniele Terlizzese [1992], "Earning Uncertainty and Precautionary Saving," *Journal of Monetary Economics*, pp. 307-337.
- [13] Hahm, Joon-Ho and Douglas Steigerwald [1999], "Consumption Adjustment under Time-Varying Income Uncertainty", *The Review of Economics and Statistics*, pp. 32-40.
- [14] Hwang, Youngjin [2006], "Regime-Switching Fluctuations, Aggregate and Individual Consumption", manuscript.
- [15] Ludvigson, Sydney and Christina Paxson [2001], "Approximation Bias in Linearized Euler Equations", *The Review of Economics and Statistics*, pp. 242-256.
- [16] Parker, Jonathan and Bruce Preston [2002], "Precautionary Saving and Consumption Fluctuations," NBER Working Paper 9196.
- [17] Romer, David [2001], *Advanced Macroeconomics*, 2nd edition, McGraw Hill.
- [18] Schechtman, Jack [1976], "Income Fluctuation Problem", *Journal of Economic Theory*, pp. 218-241
- [19] Skinner, Jonathan [1988], "Risky Income, Life Cycle Consumption, and Precautionary Savings," *Journal of Monetary Economics* 22, pp. 235-255.
- [20] Zeldes, Steven [1989], "Optimal Consumption with Stochastic Income: Deviations from Certainty Equivalence," *Quarterly Journal of Economics*, pp. 275-298

Table 1 Parameter Values of the Models

| symbol | numerical value | description |
|---|-----------------|--|
| <i>common (preference) parameters</i> | | |
| γ | 2 | (constant relative) risk aversion coefficient |
| δ | 0.01 | discount rate; β (discount factor) = $1/(1 + \delta) = 0.9852$ |
| r | 0.0075 | risk-free real interest rate |
| b | 0.0 | borrowing limit (fraction of permanent income) |
| <i>linear income process with constant growth</i> | | |
| g | 0.005 [0.002] | (constant) mean income growth rate |
| σ_n | 0.03 [0.05] | std. dev. of permanent income shock |
| σ_u | 0.12 [0.18] | std. dev. of transitory income shock |
| ρ | 0.0005 | probability of zero income |
| <i>regime switching income process</i> | | |
| α_0 | -0.01 | income growth rate in recession |
| α_1 | 0.03 | $\alpha_0 + \alpha_1$ is income growth rate in expansion |
| p | 0.95 | retention probability of expansion |
| q | 0.8 | retention probability of recession |
| $\sigma_{n E}$ | 0.02 | std. dev. of permanent income shock (expansion) |
| $\sigma_{n R}$ | 0.03 | std. dev. of permanent income shock (recession) |
| $\sigma_{u E}$ | 0.12 | std. dev. of transitory income shock (expansion) |
| $\sigma_{u R}$ | 0.12 | std. dev. of transitory income shock (recession) |

Note: the values in the brackets are the one used in comparative studies.

Table 2 Expected Consumption Growth

| | mean wealth | $E_t \left[\frac{C_{t+1}}{C_t} - 1 \right]$ | |
|---------------------------|----------------|--|--------|
| | | A | B |
| baseline | 1.2443 | 0.058% | 0.558% |
| $g = g_L$ | 1.4051 | 0.018% | 0.216% |
| $\sigma_n = \sigma_{n H}$ | 1.3389 | 0.250% | 0.526% |
| $\sigma_u = \sigma_{u H}$ | 1.4605 | 0.065% | 0.606% |

Note: table reports results from simulations with 100,000 consumers. Under the column, $E_t [C_{t+1}/C_t - 1]$, A is the expected consumption growth at the mean (normalized) wealth, and B is the average expected consumption growth across simulations.

Table 3 Regime-Switching Income Process

| A. across two regimes | | | |
|-----------------------|--------------------|--------------------|--------------------|
| at mean wealth | | average | |
| $S_t = 0$ | $S_t = 1$ | $S_t = 0$ | $S_t = 1$ |
| 1.1812 [0.186%] | 1.1188 [0.270%] | 1.266% | 1.829% |
| B. within each regime | | | |
| $S_t = 0$ | | $S_t = 1$ | |
| 1st quarter | last quarter | 1st quarter | last quarter |
| 1.1146 [0.181%] | 1.2083 [0.147%] | 1.2008 [0.180%] | 1.1150 [0.277%] |

Note: table reports average wealth and expected consumption growth at that wealth level (in brackets) over two regimes using simulations with 100,000 consumers.

Table 4 Expected Consumption Growth with Zero Income Possibility

| A. Linear Income Process | | | |
|------------------------------------|--------------------|--|--------------------|
| | mean wealth | $E_t \left[\frac{C_{t+1}}{C_t} - 1 \right]$ | |
| | | A | B |
| baseline | 1.3920 | 0.165% | 0.640% |
| $g = g_L$ | 1.5940 | 0.045% | 0.237% |
| $\sigma_n = \sigma_{n H}$ | 1.5210 | 0.287% | 0.616% |
| $\sigma_u = \sigma_{u H}$ | 1.5796 | 0.095% | 0.635% |
| B. Regime-Switching Income Process | | | |
| at mean wealth | | average | |
| $S_t = 0$ | $S_t = 1$ | $S_t = 0$ | $S_t = 1$ |
| 1.2991 [0.321%] | 1.2262 0.684%] | 1.288% | 1.874% |
| $S_t = 0$ | | $S_t = 1$ | |
| 1st quarter | last quarter | 1st quarter | last quarter |
| 1.2244 [0.297%] | 1.3225 [0.373%] | 1.3160 [0.302%] | 1.2210 [0.707%] |

Note: see the notes to Tables 2 and 3.

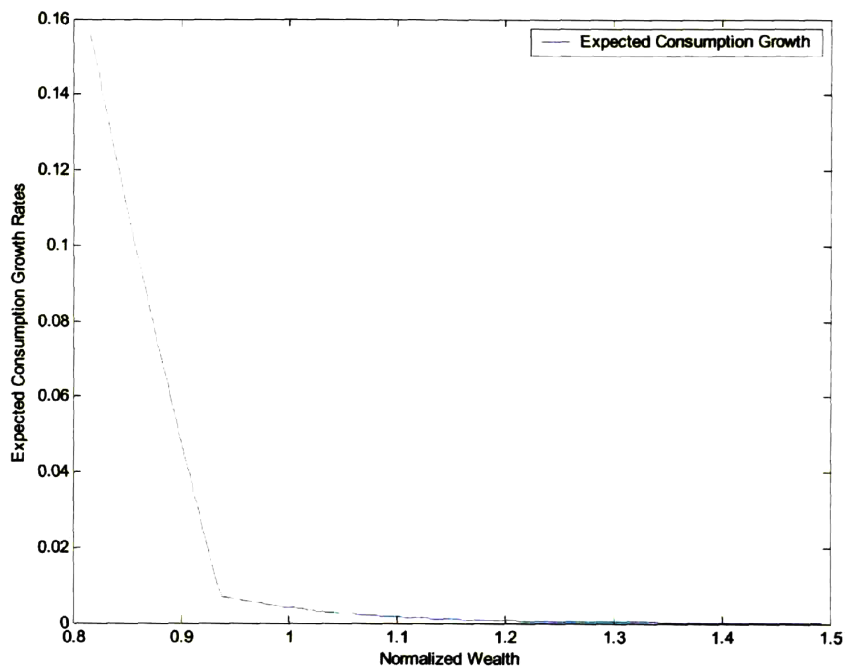


Figure 1 Expected Consumption Growth: Baseline Parameters

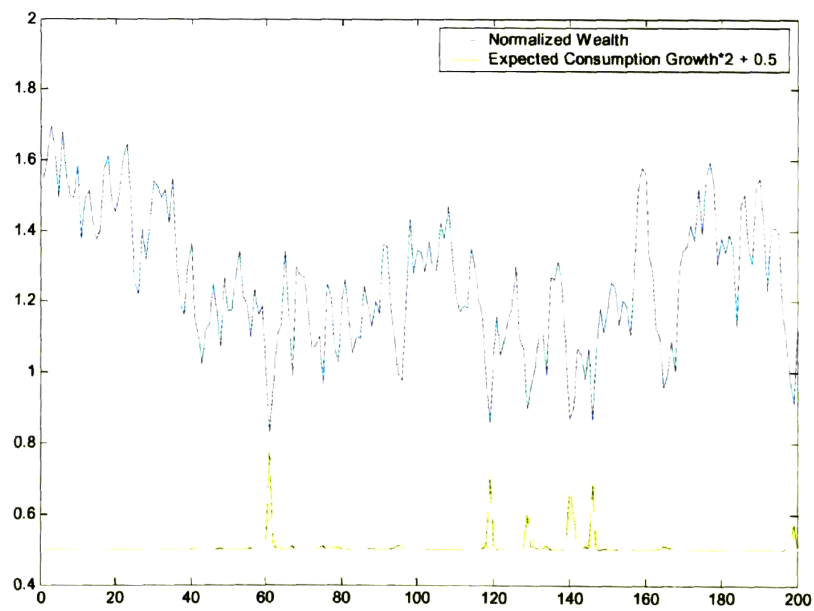


Figure 2 A Simulation of Expected Consumption Growth: Linear Constant Growth Income Process

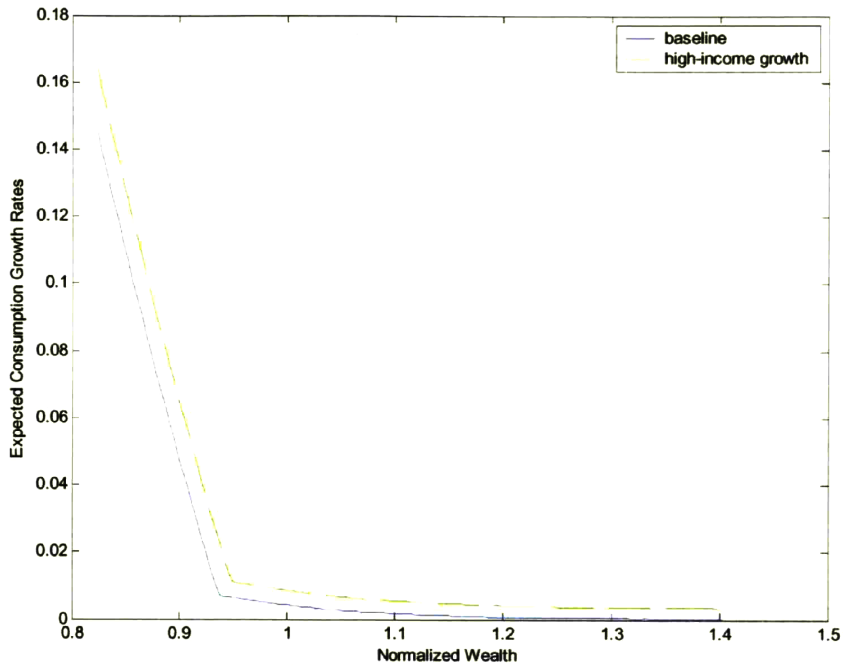


Figure 3 Effect of Increased Income Growth

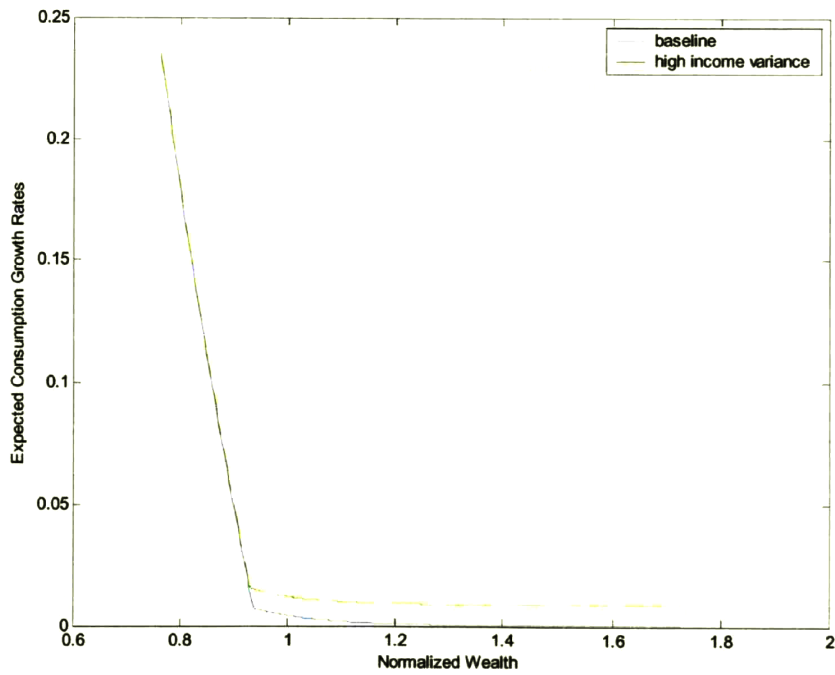


Figure 4 Effect of Increased Income Variance

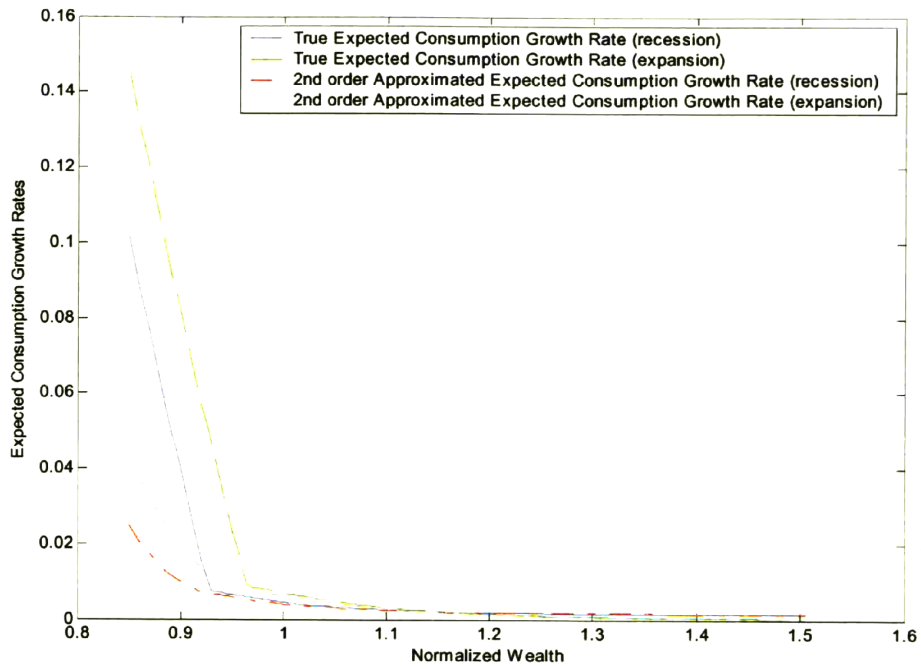


Figure 5 Expected Consumption Growth in Regime-Switching Income Process

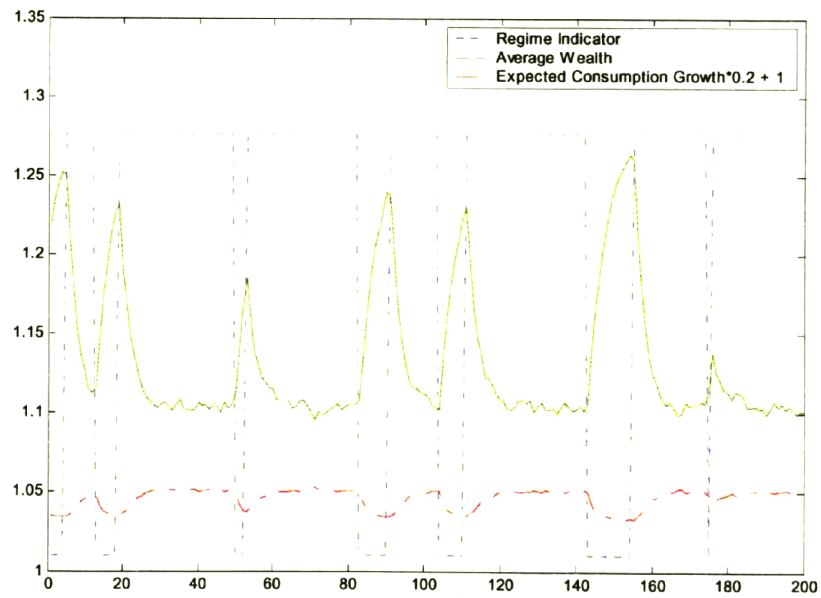


Figure 6 A Simulation of Expected Consumption Growth: Regime-Switching Income Process

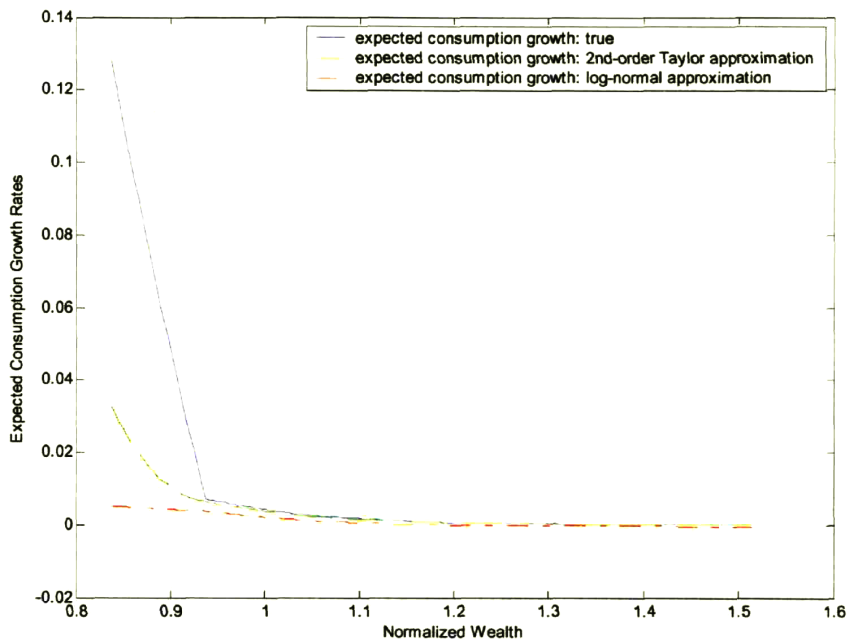


Figure 7 Expected Consumption Growth: True versus Two Approximations

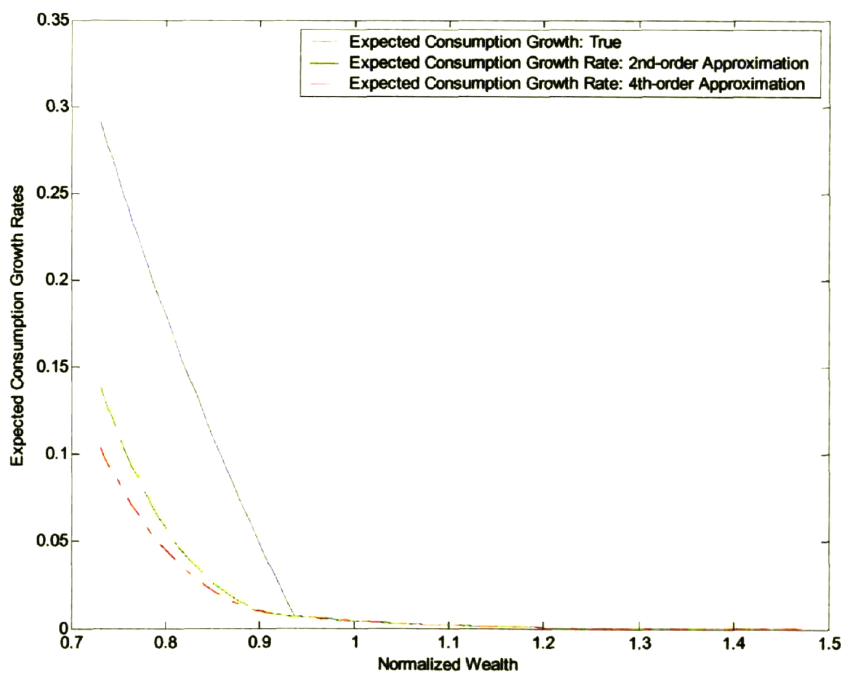


Figure 8 Expected Consumption Growth: Higher-order Taylor Approximation

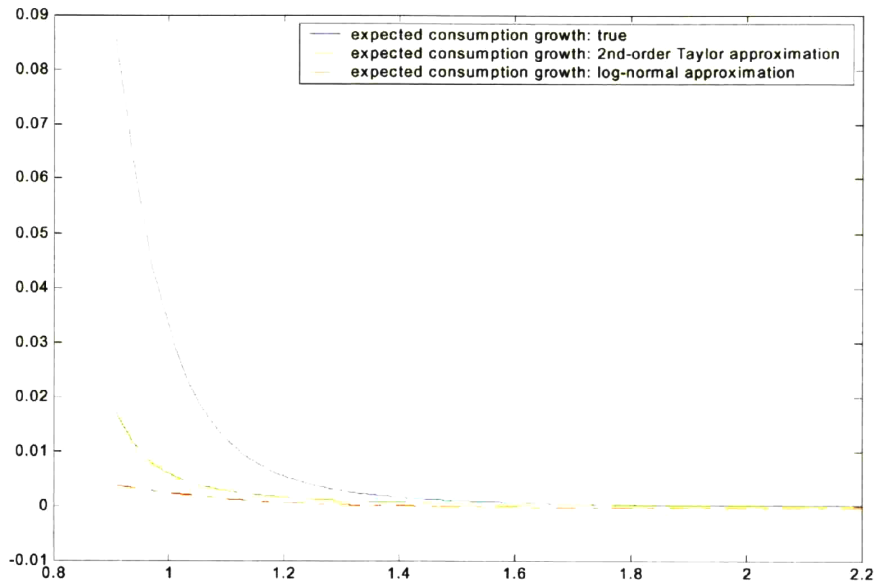


Figure 9 Expected Consumption Growth: True versus Two Approximations (with the Possibility of Zero Income)

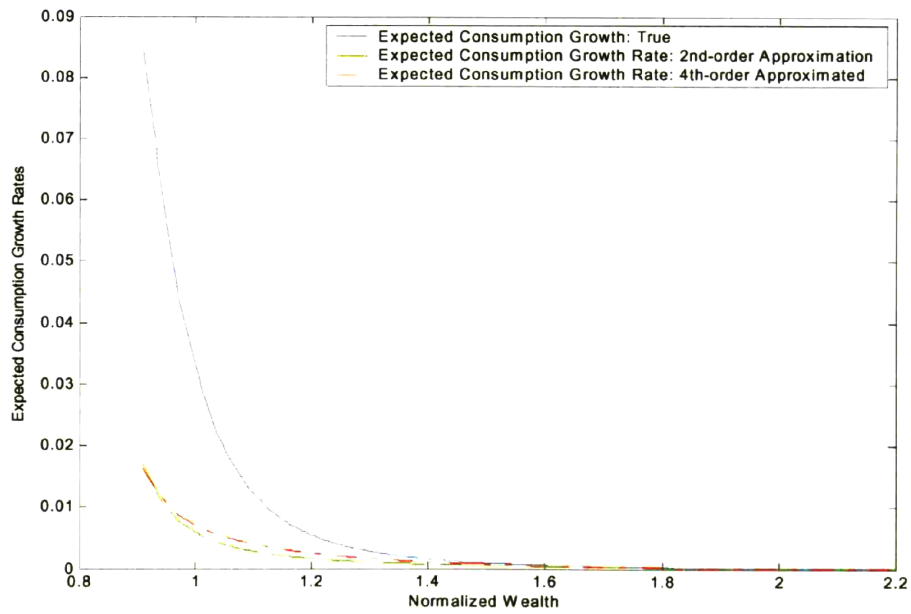


Figure 10 Expected Consumption Growth: Higher-order Taylor Approximations (with the Possibility of Zero Income)

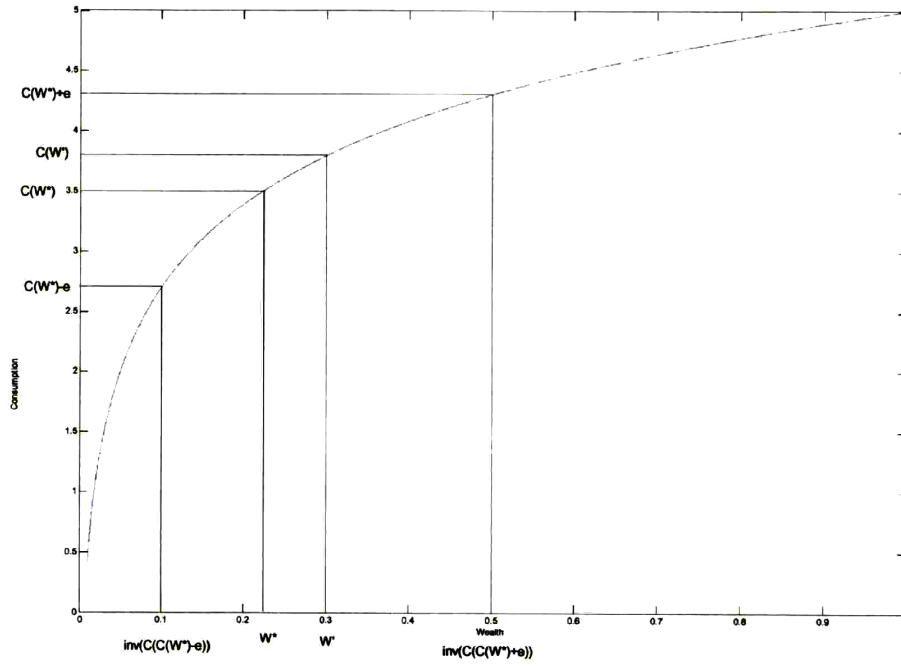


Figure 11 Bias due to Linear Approximation

Chapter 3

Estimating Preference Parameters: Simulated Method of Moment Approach

3.1 Introduction

Since the seminal work of Kydland and Prescott [1982], dynamic stochastic general equilibrium (DSGE) models have become a main workhorse in modern macroeconomics. The main attraction of DSGE models is that they take the intertemporally optimized model and specify explicitly the objectives and constraints faced by agents, and then compute the prices and allocations resulting from their market interaction in an uncertain environment.

To date, calibration is the most common approach in the literature to investigate the empirical properties of DSGE models. In calibration, the value of the structural parameters is assigned to either those estimated in microeconomic studies or those computed using long-run averages of aggregate data. Based on the optimal decision rule, the model then is simulated using a series of shocks, and the unconditional moments of the simulated variables are computed and compared with the ones of actual data. The model is usually evaluated in terms of a metric, usually the distance between these two sets of moments, to compare the outcome of the model to a set of “stylized facts.”

Although calibration is a very useful tool for understanding the dynamic properties of

DSGE models, there are some potential problems with this procedure and advantages in fully-fledged econometric estimations. First, parameter estimates are obtained by imposing some particular restrictions relevant to the model on the data. This addresses the concern that the assumptions of the DSGE model, which usually employs a representative agent framework, might be inconsistent with those employed by the micro studies that produced parameter estimates used in calibration. Second, the estimation of the DSGE model often allows one to obtain estimates of parameters that might be hard to estimate using disaggregated data alone. Third, standard tools of model selection and evaluation can be readily applied.¹ All this is valuable and useful information in the construction of more realistic economic models.

Indeed, there are a large number of studies in this line of research, and the estimation procedures usually include Generalized Method of Moments (GMM) and Maximum Likelihood Estimation (MLE).²

However, there are a few issues in each of these methods. First, in the case of GMM, estimation often suffers from weak identification/instruments, and can yield biased estimates when applied to conditional moments of the data. GMM estimates also may have poor small sample properties when the instruments are weak. Second, for MLE, we need some distributional assumptions, which can often be quite strong. Another common difficulty associated with MLE is (stochastic) singularity; as models generally contain more observable endogenous variables than exogenous shocks, it is often necessary to add additional stochastic elements into the model.³ Finally, as typical DSGE models do not yield closed-form solutions, estimating model parameters in the context of DSGE set-up in both approaches has usually relied on the linearization of first-order conditions and constraints around the model's steady states to transform the model into an estimable form. Although this approach is attractive in that it allows us to convert the complicated non-linear model and the resulting decision rules

¹For example, one can test the residuals for serial correlation and neglected autoregressive pattern and/or conditional heteroskedasticity, compare the root mean square error (RMSE) of the DSGE model with that of another DSGE model or a vector autoregression (VAR), perform tests of parameter stability, or directly test some of the model's identification assumptions.

²Selected examples include Christiano and Eichenbaum [1992], Burnside, Eichenbaum and Rebelo [1993], Ambler, Guay and Phaneuf [1999], and Barun [1996] for GMM; McGrattan [1994], Hall [1996], McGrattan, Rogerson, and Wright [1997], and Ireland [1999, 2001] for MLE.

³A typical strategy in the literature to circumvent this issue is to add measurement error or additional sources of shocks, or assume that some variables (e.g., capital stock) in the model are latent. But these procedures are sometimes arbitrary, and have no convincing theoretical support.

to analytically tractable and easily estimable forms, the cost associated with linearization might be non-negligible. In particular, when there are non-linear features in underlying stochastic processes, or in agent optimization problems, they would be lost in linearization.⁴

This paper uses the Simulated Method of Moments (SMM) technique to estimate some structural parameters such as the representative agent's preference parameters in the context of standard DSGE. This method was first introduced by McFadden [1989] and Pakes and Pollard [1989] for the estimation of discrete choice models in an i.i.d. environment, and Lee and Ingram [1991] and Duffie and Singleton [1993] extend it to the time-series cases.

The key idea of SMM is to find parameter estimates which minimizes an appropriately defined criterion function, usually the difference between the moments of actual data and the simulated data. This method is particularly useful when the theoretical moments do not yield analytical expression, or are very hard to compute.

The SMM estimation approach, based on a numerical solution algorithm, offers several advantages over traditional methods such as GMM or MLE, which directly estimate a (linearized) optimality condition. First, the model allows us to directly incorporate an underlying process and constraint into the estimation procedure while preserving its (possibly non-linear) nature, which would be lost in a linearization.⁵ Second, when it comes to the moment condition consideration, this approach does not necessarily rely on instruments, which may cause some issues in a GMM framework, and we can compare the estimation results depending on the selection of moment conditions. Third, we do not necessarily need the additivity assumption of shocks, as the estimation can be based on the moment conditions.⁶

The organization of this paper is as follows. Section 2 presents the baseline macroeconomic and econometric model, and Section 3 discusses the (numerical) implementation of the model.

⁴For example, Carroll [2001], Ludvigson and Paxon [2001], and Adda and Cooper [2003] show that, in the context of a simple precautionary saving model, it is impossible to obtain consistent estimates of the risk aversion parameter using the linearized Euler equation, due to the omitted higher-order terms, non-linear income process, or inequality constraint such as a borrowing limit.

⁵In this sense, the most closely related research to this paper is Valderrama (2002). He shows, applying the Efficient Method of Moment methodology to estimate the model parameters, that standard real business cycle models cannot replicate nonlinear features of the data. However, he assumes the underlying process is still log-linear. To the best of my knowledge, this paper is the first to estimate the preference parameters using SMM in the context of DSGE while explicitly incorporating a non-linear underlying process such as the regime-switching one.

⁶There are, of course, some disadvantages with SMM. For example, when we are using unconditional moment conditions, it may be hard to prove parameters are actually identified. Also SMM can suffer from long computing time.

After presenting the baseline model and estimation results, I consider two extensions in Section 4, by incorporating the agent’s learning about the underlying aggregate states due to incomplete information, and by allowing for the preference parameters to vary depending on the aggregate states. These are considered important factors in recent business cycle and asset pricing research. The last section presents conclusions.

3.2 The Model

This section presents the baseline model. I first consider a simple stochastic general equilibrium “macroeconomic model” to describe the dynamics of the economy. Building on this, I then set out an “econometric model” to estimate the baseline model’s parameters.

3.2.1 A Macroeconomic Model

Consider the following simple dynamic stochastic general equilibrium model. There is a representative consumer in the economy, who wishes to maximize the sum of discounted expected utility:

$$\max E_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma} \right] \quad (3.1)$$

where C_t is consumption, $\beta \in (0, 1)$ is subjective discount factor, and γ is (constant) risk aversion parameter.

The consumption good (which is the only good in the economy) is produced according to the production function $Y_t = A_t K_t^\alpha L_t^{1-\alpha}$, where K_t is the aggregate capital stock and L_t is the aggregate labor input, which is normalized to 1. The accumulation of capital is determined by

$$K_{t+1} = (1 - \delta)K_t + I_t, \quad (3.2)$$

where I_t is the investment and δ is the depreciation rate. The resource constraint is given by

$$C_t + I_t = Y_t. \quad (3.3)$$

Therefore, combining (2), (3), and the production function, we have

$$C_t + K_{t+1} - (1 - \delta)K_t = A_t K_t^\alpha. \quad (3.4)$$

The model economy is perturbed by exogenous technology shocks, and the conventional specification in the literature is to assume the technological progress is driven by the following log-linear AR(1) type process,

$$\begin{aligned} \ln A_{t+1} &= \rho \ln A_t + \varepsilon_{t+1}, \\ \varepsilon_{t+1} &\sim N(0, \sigma^2), \end{aligned} \quad (3.5)$$

with highly persistent autoregressive coefficient (ρ being close to one). Alternatively, one may consider the non-linear regime-switching process; specifically, productivity can take one of two values, μ_0 or $\mu_0 + \mu_1$, along with small stationary fluctuations around each value:⁷

$$\begin{aligned} \ln A_{t+1} &= \mu_0 + \mu_1 S_{t+1} + \tilde{\varepsilon}_{t+1}, \\ \tilde{\varepsilon}_{t+1} &\sim N(0, \tilde{\sigma}^2), \end{aligned} \quad (3.6)$$

where S_t can take either the value of 0 (recession) or 1 (expansion), and the switches between two regimes are governed by the transition probabilities: $\Pr[S_{t+1} = 1 | S_t = 1] = p$, and $\Pr[S_{t+1} = 0 | S_t = 0] = q$.

With some appropriate parameter values, it can be that both the unconditional means and standard deviations are the same in the two technology processes, so if we focus on the first two moments in cyclical behavior of the variables, a usual linearization technique might not be a bad approximation. However, there are a few dimensions in which the two processes differ: i.e., the higher moments such as skewness and kurtosis, and the type, persistence, and size of underlying shocks. These are all potentially important aspects in business cycle research, but it is not clear if the conventional linearization can properly incorporate these features into the solution and estimation procedure. In addition, in the regime-switching process, due to the non-linear nature of the underlying process, there is no clear notion of

⁷Examples that adapted this specification are Saito [2005], and Van Nieuwerburgh and Veldkamp [2006].

steady state, making the use of traditional linearization methods around it more questionable.

3.2.2 An Econometric Model

In total, there are 6 or 9 parameters in the model, depending on the underlying stochastic technology process: $\Theta = \{\beta, \gamma, \alpha, \delta, \rho, \sigma\}$, or $\Theta = \{\beta, \gamma, \alpha, \delta, p, q, \mu_0, \mu_1, \tilde{\sigma}\}$. Among these, I limit our interest to consumer's preference parameters, $\theta = \{\beta, \gamma\}$. This decision is based on the following considerations among others. First, I want to avoid potential bias due to the joint estimation of parameters in the exogenous structure of the economy (such as production function, resource constraint, and technology process) and the preference parameters which govern the agent's endogenous response to them. Second, when we consider the full set of parameters in the model, an under-identification issue may arise for some moment conditions we consider below. Because one of the purposes of this paper is to see the sensitivity of the choice of moment condition for the estimation result, I decide to limit the subset of parameters. Finally, when I consider a full or larger set of parameters, the computational burden can be substantial, especially in calculating the gradient in the estimation step, which is often quite time-consuming.

Now I briefly discuss the actual econometric model to estimate the model's parameters, θ . To make the presentation concise and easier, I define some notations:

- $\{x_t\}_{t=1}^T$: a sequence of observations of actual data.
- $\{\tilde{\eta}_t^{(r)}\}_{t=1}^{RT}$: a sequence of simulated shocks (drawn once and for all).⁸
- $\{\tilde{x}_t(\tilde{\eta}_t^{(r)}), t = 1, \dots, T; \theta^{(r)}\}_{r=1}^R$: a set of R series of simulated data, each has the length of T artificial data (conditional on the vector of parameters from the previous iteration).
- $m(\tilde{x}_t)$: simulated moments.
- $m(x_t)$: moments using actual data.

The simulated method of moment estimator, $\hat{\theta}_{SMM}$, is defined by

$$\hat{\theta}_{SMM} = \arg \min_{\theta} D'W^{-1}D, \quad (3.7)$$

⁸Throughout the paper, I distinguish simulated variables by putting tilde ($\tilde{}$) over them from actual data.

where D is an m -dimensional distance function:

$$D = \frac{1}{T} \sum_{t=1}^T m(x_t) - \frac{1}{RT} \sum_{t=1}^{RT} m(\tilde{x}_t, \theta),$$

and W is some positive definite (optimal) weighting matrix. As the theoretical moments, $E[m(x_t, \theta)]$, do not yield analytical expressions, I approximate them using the simulated moments, $E[m(x_t, \theta)] = (\frac{1}{RT}) \sum_{t=1}^{RT} m(\tilde{x}_t, \theta)$.

Then, under some regularity conditions (Duffie and Singleton [1993]), we have the following properties:

- $\hat{\theta}_{SMM}$ is consistent.
- $\sqrt{T}(\hat{\theta}_{SMM} - \theta) \rightarrow N(0, \Omega)$.

where

$$\begin{aligned} \Omega = & \left(1 + \frac{1}{R}\right) \left[E \left(\frac{\partial m(\tilde{x}_t)'}{\partial \theta} \right) W^{-1} E \left(\frac{\partial m(\tilde{x}_t)}{\partial \theta'} \right) \right]^{-1} \\ & \times \left[E \left(\frac{\partial m(\tilde{x}_t)'}{\partial \theta} \right) W^{-1} \Sigma(\theta) W^{-1} E \left(\frac{\partial m(\tilde{x}_t)}{\partial \theta'} \right) \right]^{-1} \left[E \left(\frac{\partial m(\tilde{x}_t)'}{\partial \theta} \right) W^{-1} E \left(\frac{\partial m(\tilde{x}_t)}{\partial \theta'} \right) \right]^{-1}, \end{aligned} \quad (3.8)$$

and $\Sigma(\theta)$ is the covariance matrix of $1/\sqrt{T} \left(\frac{1}{T} \sum_{t=1}^T m(x_t) - \frac{1}{RT} \sum_{t=1}^{RT} m(\tilde{x}_t, \theta) \right)$. When the weighting matrix is chosen optimally, $W^* = \Sigma$, the covariance matrix Ω is simplified to:

$$\Omega = \left(1 + \frac{1}{R}\right) \left(E \left[\frac{\partial m(\tilde{x}_t)'}{\partial \theta} \right] (W^*)^{-1} E \left[\frac{\partial m(\tilde{x}_t)}{\partial \theta'} \right] \right)^{-1}. \quad (3.9)$$

There are a few issues as to the selection of moment conditions. Ideally, the moment conditions need to be picked so that they uniquely identify the model parameters. However, this is not straightforward in this SMM estimation problem, and may be hard to check.⁹ So I rather rely on the usual moments considered in the DSGE literature and the some key properties in actual aggregate data.

⁹For this issue, one may refer to the methods (in the context of Indirect Inference and Efficient Method of Moments) proposed by Gourieoux, Monfort and Renault [1993], Smith [1993], and Gallant and Tauchen [1996]. In generating moment conditions, they propose using the score of the density, generated by an auxiliary model which has an analytical expression, or is easy to simulate.

First, it is useful to look at stylized facts from the data. Table 2 reports some statistics for consumption, output, and investment, to be more precise, their cyclical components calculated by applying the Hodrick-Prescott (HP) filter to the logarithm of actual data. Many features documented in the literature are readily visible, such as consumption smoothing (standard deviation of consumption is less than that of output), and greater relative variability of investment (its volatility is greater than output). Also, consumption seems to be highly correlated with its lagged values and output. Notice that even after HP-detrended, data exhibit some mild negative skewness.

Referring to these features of the data, I consider the following three sets of moment conditions:

$$[\text{MC1}] : \text{s.d.}(c_t)/\text{s.d.}(y_t), \text{s.d.}(c_t)/\text{s.d.}(i_t), \text{corr}(y_t, c_t), \text{corr}(c_t, c_{t-1}), \text{corr}(y_t, c_{t-1});$$

$$[\text{MC2}] : \text{s.d.}(c_t)/\text{s.d.}(y_t), \text{s.d.}(c_t)/\text{s.d.}(i_t), \text{corr}(y_t, c_t), \text{corr}(c_t, c_{t-1}), \text{corr}(y_t, c_{t-1}), \\ \text{skewness}(c_t), \text{kurtosis}(c_t);$$

$$[\text{MC3}] : \text{s.d.}(c_t)/\text{s.d.}(y_t), \text{s.d.}(c_t)/\text{s.d.}(i_t), \text{corr}(y_t, c_t), \text{corr}(c_t, c_{t-1}), \text{corr}(y_t, c_{t-1}), \\ \text{skewness}(c_t), \text{kurtosis}(c_t), \text{corr}(c_t, c_{t-2}), \text{corr}(y_t, c_{t-2});$$

where lower case variables are constructed using the same detrending method as in Table 2.

Distinction between these three sets of moment conditions is designed to investigate the effects of including potentially irrelevant or insufficient moments on the estimation results. The first set of moment condition [MC1] is to capture key stylized facts of business cycles commonly used in the model evaluation step in the DSGE literature, such as relative volatilities between consumption, output, and investment, and comovement, and first order serial correlation. In the second set [MC2], higher moments such as skewness and kurtosis are included to take into account asymmetry and disruptive changes in the endogenous consumption dynamics, often observed in business cycles, to which typically little attention has been paid in the real business cycle literature.¹⁰ The third set [MC3], which further includes

¹⁰ A few exceptions are Veldarrama [2002], and Van Nieuwerburgh and Veldkamp [2006].

the second order serial correlation, is intended to consider the effect of the possible difference in persistence between linear and non-linear nature of the underlying shock generation mechanism.

3.3 Implementation of the Model

This section addresses several issues in the numerical solution and estimation procedure. First, I assign standard values in the literature on non-preference parameter values: depreciation rate is set to $\delta = 0.015$, and the capital share in the Cobb-Douglas production function, $\alpha = 0.4$. For the linear AR(1) technology process, I follow the literature, and parameter values are set to the standard values; the first order autocorrelation coefficient, $\rho = 0.95$, and the standard deviation of shock $\sigma = 0.007$. For the regime-switching model, I refer to Saito [2005] and Van Nieuwerburgh and Veldkamp [2006] and assign $\mu_0 = -0.021$, $\mu_1 = 0.046$, and $\tilde{\sigma} = 0.015$. Note that these parameter values imply that the unconditional first two moments are the same in both processes. These parameter values are fixed throughout the entire numerical solution and estimation procedure.

3.3.1 Computation and Estimation

This subsection discusses the numerical procedure to solve and estimate the model. The algorithm for solution computation and estimation is as follows:

1. Start with initial guess, $\hat{\theta}^0 = (\hat{\beta}^0, \hat{\gamma}^0)$.
2. Draw $\{\eta_t^{(r)}\}_{t=1, r=1}^{RT}$ once and for all.
3. Do r 'th iteration. The r 'th iteration consists of two parts: numerical solution and simulation ((a) and (b)), and estimation ((c) and (d)):
 - (a) For a given $\hat{\theta}^{(r-1)}$, numerically solve the model in Section 2 to find the optimal policy.
 - (b) Given the optimal policy obtained in the previous step, compute $(100+T)$ points of simulated series of consumption, output, capital, and investment, i.e., $\left\{ \tilde{x}_t \left(\eta_t^{(r)}; \hat{\theta}^{(r-1)} \right) \right\}_{t=1}^T$;

discard the first 100 observations, and the last T data points are retained (T is set to the same as the number of sample observations).

(c) Using the simulated and actual data, compute the distance function,

$$D^{(r)} = \frac{1}{T} \sum_{t=1}^T m(x_t) - \frac{1}{TR} \sum_{r=1}^{TR} m(\tilde{x}_t(\eta_t^{(r)}, \hat{\theta}^{(r-1)}))$$

and find $\hat{\theta}^{(r)} = \arg \min_{\theta} D^{(r)'} W^{-1} D^{(r)}$.

(d) Update $\hat{\theta}^{(r)}$ and return to step 3.(a).

4. Once all R iterations are completed, report $\hat{\theta}^R$, along with robust asymptotic standard errors.

In actual implementation, I use the following quarterly U.S. data: per capita nondurable and service good consumption, non-residential fixed investment, and gross domestic output for 1947:I - 2005:IV. For the numerical algorithm to solve the macroeconomic model, I use the value function iteration method; starting from an arbitrary guess of a value function over a discretized space, the Bellman equation is iterated until it converges. As to the econometric part of implementation: First, to ensure $\beta \in (0, 1)$ and $\gamma > 0$, I estimate $\bar{\beta} = 1 / (1 + \exp(\beta))$ and $\bar{\gamma} = \exp(\gamma)$, and then recover standard errors using the delta method. In calculation of $\Omega(W_T)$, $\frac{\partial m(\tilde{x}_t; \theta)}{\partial \theta'}$ is obtained numerically by averaging over $T \times R$ points of simulated data, and the optimal weighting matrix, Ω^* , is estimated as the long-run covariance matrix using both the Bartlett and the Parzen window with four lags.¹¹ Whenever the iteration fails to converge (because the objective function is too flat, for example), the replication is discarded. The number of simulations, R , is set to 200.¹² The more details about computations are contained in Appendix.

¹¹The results are quite similar in several cases, and I report the results using the Bartlett window. In doing so, I also use a multi-step iteration method by setting the initial matrix to the identity.

¹²Although using more simulations is straightforward, the efficiency gain from increasing R seems to be small compared to the difference in the estimates and the computation time. For example, it takes more than 60 hours to complete the estimation of the model, while the efficiency gain solely due to the increase in the number of simulations from 200 to 1000 is as small as $\sqrt{1 + 1/200} - \sqrt{1 + 1/1000} = 0.001997$.

3.3.2 Results

The estimation results are reported in Table 3. Each cell contains the estimate of the parameters for each model and moment condition, along with asymptotic standard error (in parenthesis), and the last column reports p values for over-identification test.¹³

First, for the linear model, the estimates of the discount factor are close to 1 with fairly high precision, and are similar to the values usually assigned in the calibration procedure or actual estimates using micro data in the literature. It also seems that the estimation results are not very sensitive to the choice of moment conditions. However, the risk aversion parameter, whose estimates are around 1, is not estimated precisely in general, and exhibits some differences depending on the moment condition, indicating higher moments, and degree of persistence in data may be important information in the estimation of the risk aversion coefficient.

For the non-linear regime-switching model, the differences in estimates are a bit more pronounced across the moment conditions, and the discount factor and risk aversion parameters are somewhat lower (by 10% or 20%) when I consider the standard moment condition commonly used in the DSGE literature (MC1). In addition, the risk aversion parameter seems to be higher than the linear technology case, but not statistically different due to the large standard errors.

The overall results indicate that the nature of the underlying process that generates the business cycles and the selection of moment conditions in the estimation step may be important considerations in the estimation of the parameter in the model.

3.4 Two Extensions

This section considers two extensions of the basic model presented in Section 2. The recent business cycle literature suggests that the learning about latent variables and output fluctuations are intimately related. For example, Evans, Honkapohja and Romer [1998] and Kasa [1995] show that business cycles can be produced by learning, rather than by technological fluctuations. Cagetti, Hansen, Sargent and Williams [2001] consider a model with agents who

¹³For this, I use the chi-square test proposed by Hansen [1982], $T(1 + 1/R) [D(\theta)'W^{-1}D(\theta)] \rightarrow \chi^2(m - n)$, where m is the number of moment conditions in the model, and n is the number of parameters to estimate.

are uncertain about the drift of technology and solve a filtering problem, while Van Nieuwerburgh and Veldkamp [2006] show that an otherwise standard DSGE model, when embedded with individual learning about unobserved productivity, can provide a richer explanation to business cycle asymmetry.

In the first extension, following this line of the literature, I treat the aggregate business cycle regime, S_t in the regime switching model, as unobserved, and allow for the agent to learn about it using available data. Specifically, I assume that, the agent updates his belief about the latent regime every period using realized productivity data in a dynamic Bayesian learning fashion,¹⁴ and based on the inferred regime, he then makes the optimal consumption/investment decision. In this case, as the agent's learning often involves delay and/or misperception about the recurrently shifting regime, and the optimal decision depends on not only the state variables but also his belief, learning will have direct effects on the output dynamics and the consumption behavior accordingly. For instance, one may conjecture that the distribution of simulated series would be less skewed and platykurtic (having a flattened shape). The persistence of endogenous variables would also change.

In the second extension, I allow for parameters to be regime dependent, i.e., $\theta = \{\beta_E, \beta_R, \gamma_E, \gamma_R\}$. Since its inception, the calibration practices in DSGE have assumed that underlying parameters of the model are constant. However, there is no convincing reason to believe this is the case. In addition, a few recent studies suggest that, with more flexible specification of preference (e.g., endogenous risk aversion (Campbell and Cochrane [1999]), and state-dependent preference (Melino and Yang [2003], and Danthine, Donaldson, Giannikos, and Guirguis [2004])), we can potentially explain some puzzling phenomena in finance, e.g., equity premium puzzle.

For both extensions, the remaining simulation and estimation steps are the same as in the main model in Section 2, and more details about the macroeconomic models and its solution methods are contained in Appendix.

Panels C and D in Table 2 report the estimation result. First, for the case of incomplete information about the business cycle regime, the results are largely similar to the bench-

¹⁴One may think of the inference problem using output data because the technology is not directly observable. However, as long as the capital stock is observable, the nature of the inference problem is essentially the same.

mark case. However, the effect of inclusion of skewness/kurtosis matters less compared to the complete information case, illustrating that learning may be an important factor in the parameter estimation. Again, the risk aversion parameter is estimated poorly. Next, as to the state-dependent preference extension, an interesting result is that the consumers tend to discount more and be less risk averse in expansions, which suggests the sharp drops in endogenous variables such as consumption in recession times may be partly due to the change in their preferences as well, although the differences are not big enough.

3.5 Conclusion

This paper uses the SMM technique to estimate a representative agent's preference parameters such as risk aversion and discount factor in the context of standard DSGE. One of the novel elements is, by fully utilizing the possible non-linearity in the underlying stochastic process in the optimal solution and the estimation procedure, this approach allows us to investigate several issues which other alternatives may not properly consider.

The results suggest that higher moments of the data such as skewness and kurtosis, and higher order cross-/auto-correlation between variables can be important considerations in the estimation of such parameters. In addition, the effects of learning about the unobserved business cycle regime and possible regime-dependent preferences on the estimation results are discussed.

There are a few remaining issues and extensions. First, this paper only uses the HP filter to extract the cyclical component of the data. However, several papers argue the properties of cyclical data and estimates can be sensitive depending on the filtering method (for example, see Canova [1999] and Christiano and den Haan [1996]). Thus, it seems necessary to check the robustness of results for various detrending methods.¹⁵

Second, the endogenous labor decision is completely ignored in this paper. One of the major problems with the standard real business cycle model is that it is not very successful in explaining some cyclical features in labor input data, such as that labor input is nearly as volatile as output, and that labor input and productivity are essentially uncorrelated, at

¹⁵For example, there was a clear pattern of excessive skewness and kurtosis in consumption data when the first difference filter is used.

least in the U.S. By allowing labor elasticity to be aggregate state-dependent as in one of the extensions in this paper, we may find some insight into the issue.

3.6 Appendix

The Bellman equations for the problem in the main text can be written as follows: First, for the linear AR(1) process,

$$\begin{aligned} V(K_t, A_t, A_{t-1}) &= \max\{U(C_t) + \beta E_t[V(K_{t+1}, A_{t+1}, A_t)]\} \\ &= \max\{U[A_t K_t^\alpha + (1 - \delta)K_t - K_{t+1}] + \beta E_t[V(K_{t+1}, A_{t+1}, A_t)]\} \end{aligned}$$

In the numerical solution procedure, I follow the finite state approximation technique proposed by Tauchen [1986]. Since the technology process follows a first order AR process, it follows that $\ln A_t = a_t \sim N(0, \varphi^2)$, where $\varphi^2 = \sigma^2/(1 - \rho^2)$. I approximate this with a discrete first-order Markov process in which log technology takes one of M discrete values, φz_m , where z_m is a discretized standard normal variable. The transition probability between the aggregate states m and n , $\tilde{\mathbf{P}}_{m,n}$, is calculated as:

$$\begin{aligned} \tilde{\mathbf{P}}_{m,n} &= \Pr(\varphi z_n \geq a_t \geq \varphi z_{n-1} | \varphi z_m \geq a_{t-1} \geq \varphi z_{m-1}) \\ &= \frac{1}{\varphi \sqrt{2\pi}} \int_{\varphi z_{m-1}}^{\varphi z_m} e^{-\frac{x^2}{2\varphi^2}} \left\{ \Phi\left(\frac{\varphi z_n - \phi x}{\varphi}\right) - \Phi\left(\frac{\varphi z_{n-1} - \phi x}{\varphi}\right) \right\} dx \end{aligned}$$

using this the Bellman equations then can be expressed

$$V(K_t, a_t) = \max\{U[e^{a_t} K_t^\alpha + (1 - \delta)K_t - K_{t+1}] + \beta \tilde{\mathbf{P}} E_t[V(K_{t+1}, a_{t+1})]\}$$

where ν is a vector of values the productivity can take in the discrete approximation. In practice, I use $M = 13$, and one can obtain the value function by iterating this system of Bellman equations until the convergence happens.

For the other processes, the nature of problems is quite similar, hence I briefly sketch the Bellman equation in each case along with other discussion when necessary. For the regime-switching process, the Bellman equations in each regime are:

$$\begin{aligned} V(K_t, E) &= \max\{U[A_t K_t^\alpha + (1 - \delta)K_t - K_{t+1}|E] \\ &\quad + \beta E_t[pV(K_{t+1}, E) + (1 - p)V(K_{t+1}, R)]\}, \end{aligned}$$

and

$$V(K_t, R) = \max\{U[A_t K_t^\alpha + (1 - \delta)K_t - K_{t+1}|R] \\ + \beta E_t[(1 - q)V(K_{t+1}, E) + qV(K_{t+1}, R)]\}.$$

These two equations then can be compactly written as

$$\begin{bmatrix} V(K_t, E) \\ V(K_t, R) \end{bmatrix} = \max \left\{ \begin{bmatrix} U[\cdot|E] \\ U[\cdot|R] \end{bmatrix} + \beta E_t \begin{bmatrix} p & 1-p \\ 1-q & q \end{bmatrix} \begin{bmatrix} V(K_{t+1}, E) \\ V(K_{t+1}, R) \end{bmatrix} \right\}$$

where $U[\cdot|S] = U[A_t K_t^\alpha + (1 - \delta)K_t - K_{t+1}|S]$.

When the consumer's problem involves learning process due to latent aggregate regime, I discretize the agent's belief space with $M = 11$ points, and assume he updates his belief about the aggregate regime at time t , μ_{nt} , using the Bayes rule.¹⁶ The Bellman equations then are:

$$\begin{bmatrix} V(K_t, \mu_{1t}) \\ \vdots \\ V(K_t, \mu_{Nt}) \end{bmatrix} = \max \left\{ \begin{bmatrix} U[\cdot|\mu_{1t}] \\ \vdots \\ U[\cdot|\mu_{Nt}] \end{bmatrix} + \beta E_t \begin{bmatrix} \pi_{1t} & 1 - \pi_{1t} \\ \vdots & \vdots \\ \pi_{Nt} & 1 - \pi_{Nt} \end{bmatrix} \begin{bmatrix} V(K_{t+1}, 0) \\ \vdots \\ V(K_{t+1}, 1) \end{bmatrix} \right\}$$

where $U[\cdot|\mu_{nt}] = U[A_t K_t^\alpha + (1 - \delta)K_t - K_{t+1}|\mu_{nt}]$ and $\pi_{nt} = p\mu_{nt} + (1 - q)\mu_{nt}$.

In the regime dependent preference case, the Bellman equations are written in a similar way:

$$\begin{bmatrix} V(K_t, E) \\ V(K_t, R) \end{bmatrix} = \max \left\{ \begin{bmatrix} U[\cdot|E, \gamma_E] \\ U[\cdot|R, \gamma_R] \end{bmatrix} + E_t \begin{bmatrix} p & 1-p \\ 1-q & q \end{bmatrix} \begin{bmatrix} \beta_E V(K_{t+1}, E) \\ \beta_R V(K_{t+1}, R) \end{bmatrix} \right\}$$

where $U[\cdot|S, \gamma_S] = U[A_t K_t^\alpha + (1 - \delta)K_t - K_{t+1}|S, \gamma = \gamma_S]$.

¹⁶See Hwang [2006] for more details.

Bibliography

- [1] Adda, Jérôme and Russell Cooper [2003], *Dynamic Economics: Quantitative Methods and Applications*, The MIT Press.
- [2] Ambler, Guay and Phaneuf [1999], “Wage Contracts and Labor Adjustment Cost as Endogenous Propagation Mechanism”, manuscript.
- [3] Braun [1994], “Tax Disturbances and Real Economic Activity in the Post war United States”, *Journal of Monetary Economics*, pp. 441-462.
- [4] Burnside, Craig, Martin Eichenbaum, and Sergio Rebelo [1993], “Labor Hoarding and the Business Cycle”, *Journal of Political Economy*, pp. 245-273.
- [5] Cagett, Hansen, Sargent and Williams [2001], “Robustness and Pricing with Uncertain Growth”, *Review of Financial Studies*, pp. 363-404.
- [6] Canova, Fabio [1998], “Detrending and business cycle facts,” *Journal of Monetary Economics*, pp. 475-512.
- [7] Campbell, John and John Cochrane [1999], “By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior”, *Journal of Political Economy*, pp. 205-251.
- [8] Carrasco, Marine and Jean-Pierre Florens [2002], “Simulation Based Method of Moments and Efficiency”, *Journal of Business & Economic Statistics*, pp. 482-492.
- [9] Carroll, Christopher [2001], “Death to Log-Linearized Euler Equation (And Very Poor Health to the Second-Order Approximation)”, *Advances in Macroeconomics*, Vol. 1: No. 1, Article 6.

- [10] Christiano, Lawrence and Martin Eichenbaum [1992], "Current Real Business Cycle Theories and Aggregate Labor Market Fluctuations", *American Economic Review*, pp. 430-450.
- [11] Danthine, Jean-Pierre, John Donaldson, Christos Giannikos, and Hany Guirguis [2004], "On the Consequences of State Dependent Preferences for the Pricing of Financial Assets", *Finance Research Letters*, pp.143-153.
- [12] Duffie Darrell and Kenneth Singleton [1993], "Simulated Moments Estimation of Markov Models of Asset Prices", *Econometrica*, pp. 929-952.
- [13] Eichenbaum, Martin and Lars Hansen and Kenneth Singleton [1988], "A Time Series Analysis of Representative Agent Models of Consumption and Leisure Choice under Uncertainty," *The Quarterly Journal of Economics*, pp. 51-78.
- [14] Evans, Honkapohja and Romer [1998], "Growth Cycles", *American Economic Review*, pp. 495-515.
- [15] Gallant, Ronald and George E. Tauchen [1996], "Which Moments to Match?" *Econometric Theory*, pp.657-681.
- [16] Hall, George [1996], "Overtime, Effort, and the Propagation of Business Cycle Shocks," *Journal of Monetary Economics*, pp. 139-160.
- [17] Heaton, John [1993], "An Empirical Investigation of Asset Pricing with Temporally Dependent Preference Specifications", *Econometrica*, pp. 681-717.
- [18] Ireland, Peter [2001], "Sticky-Price Models of the Business Cycle: Specification and Stability," *Journal of Monetary Economics*, pp. 3-18.
- [19] Kasa, Kenneth [1995], "Signal Extraction and the Propagation of Business Cycles", Federal Reserve Bank of San Francisco Working Paper.
- [20] Lee, Bong-Soo and Beth Fisher Ingram [1991], "Simulation Estimation of Time-Series Models," *Journal of Econometrics*, pp. 197-205.

- [21] McFadden, Daniel [1989], "A Method of Simulated Moments for Estimation of Discrete Response Models Without Numerical Integration", *Econometrica*, pp. 995-1026.
- [22] McGrattan, Ellen [1994], "The Macroeconomic Effects of Distortionary Taxation", *Journal of Monetary Economics*, pp. 573-601.
- [23] McGrattan, Ellen, Richard Rogerson, and Randall Wright [1997], "An Equilibrium Model of the Business Cycle with Household Production and Fiscal Policy", *International Economic Review*, pp. 267-90.
- [24] Melino, Algeno and Alan Yang [2003], "State Dependent Preferences Can Explain the Equity Premium Puzzle", *Review of Economic Dynamics*, pp. 806-830.
- [25] Pakes, Ariel and David Pollard [1989], "Simulation and the Asymptotics of Optimization Estimators," *Econometrica*, pp. 1027-1057.
- [26] Saito, Masashi [2005], *Models of Business Cycles and Monetary Policy under Imperfect Information*, Boston University, PhD dissertation.
- [27] Tauchen, Geroge [1986], "Finite State Markov Chain Approximations to Univariate and Vector Autoregressions," *Economic Letters*, pp. 177-181.
- [28] Valderrama, Diego [2002], "Statistical Nonlinearities in the Business Cycle: a Challenge for the Canonical RBC Model", Federal Reserve Bank of San Francisco Working Paper.
- [29] Van Nieuwerburgh, Stijn and Laura Veldkamp [2006], "Learning Asymmetries in Real Business Cycles", *Journal of Monetary Economics*, forthcoming.

Table 1 Model Parameter Values

| symbol | numerical value | description |
|------------------|-----------------|---|
| β | estimated | discount factor |
| γ | estimated | (constant) risk aversion parameter |
| α | 0.4 | capital share in production function |
| δ | 0.015 | depreciation rate |
| ρ | 0.95 | $AR(1)$ coefficient in technology process |
| σ | 0.007 | std. dev. in linear process |
| p | 0.95 | expansion regime retention probability |
| q | 0.72 | recession regime retention probability |
| μ_0 | -0.020 | mean productivity in recession |
| $\mu_0 + \mu_1$ | 0.046 | mean productivity in expansion |
| $\tilde{\sigma}$ | 0.015 | std. dev. in regime-switching process |

Table 2 Moment from Actual Data

| | | |
|----|-----------------------------|---------|
| M1 | s.d.(c_t)/s.d.(y_t) | 0.4708 |
| M2 | s.d.(c_t)/s.d.(i_t) | 0.1603 |
| M3 | skewness(c_t) | -0.0227 |
| M4 | kurtosis(c_t) | 2.9713 |
| M5 | corr(c_t, c_{t-1}) | 0.8317 |
| M6 | corr(c_t, c_{t-2}) | 0.6285 |
| M7 | corr(y_t, c_{t-1}) | 0.6765 |
| M8 | corr(y_t, c_{t-2}) | 0.4954 |
| M9 | corr(c_t, y_t) | 0.7855 |

Notes: The sample period is 1947:I - 2005:IV.

Table 3 Estimation of the Models

| A. Linear AR(1) | β | γ | OID |
|---------------------|------------------|------------------|-------|
| MC1 | 0.965 (0.086) | 0.863 (0.434) | 0.054 |
| MC2 | 0.985 (0.076) | 1.034 (0.574) | 0.102 |
| MC3 | 0.970 (0.088) | 0.942 (0.456) | 0.110 |
| B. Regime-Switching | β | γ | OID |
| MC1 | 0.926 (0.053) | 1.193 (0.623) | 0.078 |
| MC2 | 0.968 (0.058) | 1.473 (0.758) | 0.069 |
| MC3 | 0.998 (0.072) | 1.356 (0.845) | 0.085 |

Notes: Standard error are reported in parentheses. The last column reports p-value of over-identification test.

Table 3 Estimation of the Models (continued)

| C. Regime-Switching (Incomplete Information) | β | γ | | | OID |
|---|-------------------|-------------------|------------------|------------------|-------|
| MC1 | 0.977 (0.076) | 0.975 (0.424) | | | 0.055 |
| MC2 | 0.986 (0.105) | 1.246 (0.643) | | | 0.087 |
| MC3 | 0.980 (0.106) | 1.191 (0.545) | | | 0.094 |
| D. State-Dependent Preference | β_E | β_R | γ_E | γ_R | OID |
| MC1 | 0.9652 (0.104) | 0.9835 (0.095) | 1.220 (0.239) | 1.352 (0.389) | 0.074 |
| MC2 | 0.9764 (0.116) | 0.9932 (0.108) | 1.232 (0.327) | 1.547 (0.603) | 0.093 |
| MC3 | 0.9635 (0.100) | 0.9829 (0.099) | 1.356 (0.237) | 1.476 (0.561) | 0.104 |

Notes: Standard error are reported in parentheses. The last column reports p-value of over-identification test.