Application of Damping in High-rise Buildings

by

Jeremiah C. O’Neill Jr.

B.S. in Civil and Environmental Engineering
Lafayette College

Submitted to the Department of Civil and Environmental Engineering
in partial fulfillment of the requirements for the degree of

Master of Engineering in Civil and Environmental Engineering
at the
Massachusetts Institute of Technology

June 2006

All rights reserved

The author hereby grants to MIT permission to reproduce and to distribute publicly paper and
electronic copies of this thesis document in whole or in part in any medium now known or
hereafter created

Signature of Author...........................

Department of Civil and Environmental Engineering
May 12, 2006

Certified by ........................................

Jerome J. Connor
Professor of Civil and Environmental Engineering
Thesis Supervisor

Accepted by ........................................

Andrew J. Whittle
Chairman, Departmental Committee for Graduate Students
Application of Damping in High-rise Buildings

by

Jeremiah C. O’Neill Jr.

Submitted to the Department of Civil and Environmental Engineering on May 12, 2006 in Partial Fulfillment of the Requirements for the Degree of Master of Engineering in Civil and Environmental Engineering

ABSTRACT

The outrigger structural system has proven to be an efficient lateral stiffness system for high-rise buildings under static loadings. The purpose of this thesis is to research the incorporation of viscous dampers into the outrigger system to improve the dynamic performance. This study will be conducted on a typical high-rise structure in Boston, MA in attempt to find realistic results.

This thesis will utilize two analysis models for the study: a simplified single degree of freedom model and a more sophisticated computer model constructed with the structural analysis software, SAP2000. The models will be used to assess the effect that increasing damping or changing damper locations has on the dynamic performance of the structure. Furthermore, the constructability issues of each damping configuration will be identified and discussed.

Thesis Supervisor: Jerome J. Connor
Title: Professor of Civil and Environmental Engineering
ACKNOWLEDGEMENTS

First and foremost, I would like to thank my parents for giving me the opportunity to have the education I have had. I truly appreciate your dedication to providing your children with the best education possible.

I am also indebted to thank everyone who has helped with this thesis, particularly Professor Connor. He has shared an exceptional amount of knowledge throughout the whole year, and without it, this thesis would have never been successful.

I also need to thank those that made this year an enjoyable experience and my life more interesting then damping in high-rise buildings. So thank you Jill and all of you Meng kids.
# TABLE OF CONTENTS

## CHAPTER 1 INTRODUCTION

1.1 Thesis Objective ................................................................. 7
1.2 Outrigger Structure Basics ............................................... 8
1.3 Introduction to Damping .................................................... 9
1.4 Building Description ....................................................... 11

## CHAPTER 2 NUMERICAL ANALYSIS

2.1 The Concept ........................................................................ 13
2.2 Numerical Derivation ....................................................... 14
2.3 Differential Equation Solution .......................................... 16

## CHAPTER 3 STATIC ANALYSIS

3.1 Introduction ......................................................................... 18
3.2 Static Wind Loads and Design Criteria ............................... 18
3.3 Simple Cantilever SDOF Analysis ....................................... 19
3.4 Simple Cantilever SAP2000 Static Analysis ....................... 21
3.5 Outrigger SDOF Static Analysis .......................................... 22

## CHAPTER 4 DYNAMIC ANALYSIS

4.1 Introduction ......................................................................... 25
4.2 Brief Vortex Shedding Analysis ........................................ 25
4.3 Dynamic Wind Loads and Design Criteria .......................... 27
4.4 Determination of Damping Coefficients .............................. 31
4.5 SDOF Dynamic Analysis .................................................. 33
   4.5.1 Purpose .......................................................................... 33
   4.5.2 Analysis Results ............................................................. 34
      4.5.2.1 No damping .............................................................. 34
      4.5.2.2 Equivalent damping of 10% .................................... 36
      4.5.2.3 Effectiveness of damping ........................................ 39
4.6 SAP2000 Dynamic Analysis of Configuration A ............... 40
   4.6.1 Concept .......................................................................... 40
   4.6.2 Analysis Results ............................................................. 41
      4.6.2.1 No damping .............................................................. 41
      4.6.2.2 Equivalent damping of 10% .................................... 43
      4.6.2.3 Equivalent damping of 20% .................................... 45
   4.6.3 Why is the equivalent damping so high? ......................... 46
   4.6.4 Construction considerations ......................................... 47
4.7 SAP2000 Dynamic Analysis of Configuration B ............... 48
   4.7.1 Concept .......................................................................... 48
   4.7.2 Analysis Results ............................................................. 49
   4.7.3 Constructability Concerns .............................................. 52
4.8 SAP2000 Dynamic Analysis of Configuration C ............... 53
   4.8.1 Concepts ......................................................................... 53
   4.8.2 Analysis Results ............................................................. 54
LIST OF FIGURES

Figure 1.1. Typical outrigger design (Hoenderkamp, 2003) ........................................... 9
Figure 1.2. Viscous damper (Taylor Devices, Inc.) .......................................................... 10
Figure 1.3. Building Dimensions ...................................................................................... 11
Figure 1.4. Cross-section of building with outrigger and damper layout ......................... 12
Figure 2.1. Continuous outrigger model ............................................................................. 13
Figure 3.1. Bending rigidity versus outrigger column area ............................................. 23
Figure 3.2. Optimum outrigger location (Smith, 1991) .................................................... 24
Figure 4.1. Transfer function for displacement (Connor, 2003) ....................................... 27
Figure 4.2. Transfer function for acceleration (Connor, 2003) ....................................... 27
Figure 4.3. Forcing function for SAP2000 model ............................................................. 29
Figure 4.4. Forcing function for SDOF system ................................................................. 30
Figure 4.5. SDOF system ................................................................................................. 34
Figure 4.6. Top floor displacement with no damping and loading A ............................... 35
Figure 4.7. Top floor acceleration with no damping and loading A .................................. 36
Figure 4.8. Top floor displacement with 10% damping and loading A ............................. 37
Figure 4.9. Top floor acceleration with 10% damping and loading A .............................. 38
Figure 4.10. Dynamic model Configuration A ................................................................... 41
Figure 4.11. Top floor displacement with no damping and loading A ............................. 42
Figure 4.12. Top floor acceleration with no damping and loading A ............................... 43
Figure 4.13. Top floor displacement with 10% damping and loading A .......................... 44
Figure 4.14. Top floor acceleration with 10% damping and loading A ............................ 44
Figure 4.15. Damper force (kN) ..................................................................................... 47
Figure 4.16. Dynamic model Configuration B ................................................................... 49
Figure 4.17. Damper stroke under load B with no outrigger columns ............................. 51
Figure 4.18. Damper stroke under load B with outrigger columns .................................. 52
Figure 4.19. Dynamic model Configuration C ................................................................... 54
Figure 4.20. Displacement at top and bottom of damper ............................................... 55
Figure 4.21. Displacement at top and bottom of damper ............................................... 56

LIST OF TABLES

Table 3.1. Wind Pressures ................................................................................................. 18
Table 3.2. Model parameters ........................................................................................... 19
Table 4.1. Human perception levels of motion ................................................................. 31
Table 4.2. C values for corresponding equivalent damping ............................................ 33
Table 4.3. Top floor displacement and acceleration for steady state response ............... 36
Table 4.4. Top floor displacement and acceleration for 10% equivalent damping .......... 38
Table 4.5. Top floor displacement and acceleration for 10% equivalent damping .......... 45
Table 4.6. Top floor displacement and acceleration for 20% equivalent damping .......... 45
Table 4.7. Comparison of displacement and acceleration of Configurations A and B .... 52
CHAPTER 1 INTRODUCTION

As cities continue to grow and land value continues to increase, buildings are being built taller in an effort to maximize rentable space. The result is tall buildings on small lots of land, which leads to higher building aspect ratios (height-to-width). Furthermore, material advancements have yielded higher strength steels, but the modulus of elasticity has remained constant. As materials advance in this way and building become more slender the design shifts from strength based design to motion based design. This means that the critical design is not the strength of the structural members supporting the gravity loads, but the stiffness of the structure resisting the lateral loads. When exposed to high winds, high-rise buildings may experience drifts on the magnitude of meters, which causes concerns for serviceability and human comfort. For this reason, one of the largest problems facing engineers today is drift control in high-rise structures. This thesis will explore drift and acceleration control with viscous damping in an outrigger structural system.

1.1 Thesis Objective

The outrigger structural system has proven to be an effective structural design, but as in all slender buildings it can be susceptible to high drifts and consequently high accelerations under dynamic loading. Passive damping has proven to be an efficient way to reduce these unwanted effects. There are many forms of passive damping, but most commonly damping systems come in the form of viscous damped braced frames or tuned mass dampers. As will be explained in the following section, viscous dampers are most efficiently placed in a location where their differential velocity is maximized. Following this concept, it seems only reasonable that a damper would be most effective at the end of
an outrigger where the displacement is magnified, but there is little to no research on this application. This thesis will explore the effectiveness and feasibility of implementing a viscous damping system in the outrigger structural system to help control building motion and acceleration. This will be done by running analyses for a typical 40-story building in the Boston area.

1.2 Outrigger Structure Basics

In order to meet building owner's demands for open space, building designers have been locating lateral systems (braced steel frames, concrete shear walls, etc) in the center of buildings where they can be disguised by mechanical shafts. This system uncouples the core from the exterior columns, and uses the core as the only resistance for the lateral forces. The outrigger design enables a building to activate its total width when resisting lateral loads by coupling the core and the columns. The design consists of a braced steel or reinforced concrete central core with cantilevered outriggers connecting to exterior columns, as shown in Figure 1.1. When subjected to a lateral force the central core is subjected to bending deformation which causes the outrigger to rotate. The rotation of the outrigger is resisted by the exterior columns, which imparts a concentrated bending moment that reduces the moment in the core and at the base. The addition of outriggers provides no additional shear resistance, so the core must be designed to resist the full shear force. The advantages of the outrigger structural systems as stated by The Council on Tall Buildings and Urban Habitat are summarized below.

- "Core overturning moments and their associated induced deformation can be reduced through the "reverse" moment applied to the core at each outrigger intersection.
- Significant reduction and possibly the complete elimination of uplift and net tension forces throughout the columns and foundation system.
- The exterior column spacing is not driven by structural considerations and can easily mesh with aesthetic and functional considerations.
Exterior framing can consist of “simple” beam and column framing without the need for rigid-frame-type connections, resulting in economies.

For rectangular buildings, outriggers can engage the middle columns on the long faces of the building under the application of wind loads in the more critical direction.” (Kowalczyk, 1995)

The main drawback of the outrigger system is the inference with interior space. Each outrigger is typically one or two stories deep, which makes the outrigger floors unrentable space. Most commonly, a mechanical floor is located at the outrigger level, but in tall buildings the most cost effective location for a mechanical floor is the basement so The Council on Tall Buildings and Urban Habitat proposed alternative ways of incorporating these systems.

- “Locating outriggers in the natural sloping lines of the building profile.
- Incorporating multilevel single diagonal outriggers to minimize the member’s interference on any single level.
- Skewing and offsetting outriggers in order to mesh with the functional layout of the floor.” (Kowalczyk, 1995)

![Figure 1.1. Typical outrigger design (Hoenderkamp, 2003)](image)

### 1.3 Introduction to Damping

In structural building design, damping is the process by which a structure dissipates the energy input by external forces. Viscous damping, which will be the focus
in this paper, operates by forcing a fluid through an orifice in turn creating a resisting force. Figure 1.2 shows an example of 1.5 million pound viscous damper from Taylor Devices, Inc.

![Viscous Damper](image)

**Figure 1.2. Viscous damper (Taylor Devices, Inc.)**

The major benefit of viscous dampers is that they are dependent on the velocity of the structure, which is completely out of phase from the maximum displacements (also bending and shear stresses) in a building. Equation (1.1) defines the expression for the damper force

\[ F_{\text{damper}} = c \dot{u} \]  

(1.1)

where \( c \) is the damping coefficient that describes a damper and \( \dot{u} \) is the velocity. Consider a building that is excited laterally by an earthquake. As the structure oscillates, the columns reach their maximum stress when the displacement has reached its maximum. At this point the structure instantaneously stops, meaning there is no velocity, and therefore the force in the viscous dampers is zero. As the structure rebounds the dampers reach a maximum output at the point of maximum velocity, which is also the point of zero displacement and therefore a point of zero lateral stress in the columns. The result is that viscous dampers can reduce the overall displacement and acceleration without increasing the stresses in the main lateral force resisting system.
1.4 Building Description

This thesis will analyze a 40 story outrigger structure in Boston, MA which is shown in Figure 1.3. The floor height will be 3.5 meters giving a total building height of 140 meters. The building will be 30 meters wide, in order to achieve an aspect ratio of about H/5. The construction method is assumed to be concrete slab/metal deck with steel beams and columns, therefore the total weight of the building will consist of the weight of concrete in slabs and the weight of structural steel. The mass was taken from published data for an existing 40-story building in Boston, MA, with a total mass of 55,800 tons and a typical floor area of 2090 m². (McNamara, 2000) After scaling the mass by the floor area being used in this thesis (900 m²) the mass of the building is taken as 24,000 tons, giving \( \rho = \frac{m}{H} = 170,000 \frac{kg}{m} \). Lastly, Figure 1.4 shows a cross-section through the building showing the layout of outriggers and where dampers will be added. There are eight outriggers cantilevering from the central core and connecting to different configurations of columns and viscous dampers.

![Figure 1.3. Building Dimensions](image)
Figure 1.4. Cross-section of building with outrigger and damper layout
CHAPTER 2 NUMERICAL ANALYSIS

2.1 The Concept

In order to explore the effect of dampers in the response of a structure with outriggers, a single-degree of freedom (SDOF) model of the system was derived. The structure was first simplified to a simple continuous cantilevered beam with infinitely rigid outrigger arms that are attached to a spring and damper, as is shown in Figure 2.1. The next step was to derive the equation of motion for the SDOF system, which was done by transforming the continuous beam equations into a set of discrete equations.

Figure 2.1. Continuous outrigger model
2.2 Numerical Derivation

The derivation followed the procedure presented by Connor (2003) for the fundamental mode response of a continuous beam. The derivation begins by expressing the displacement and rotation in terms of generalized coordinates.

\[ u = q(t)\phi(x) \quad \text{(2.1)} \]

\[ \beta = q(t)\phi'(x) \quad \text{(2.2)} \]

where \( \phi(x) \) is the fundamental mode shape and \( q \) is the modal amplitude parameter. The analysis will consider the cantilever beam as a pure bending beam, meaning that there will be no shear deformation. Therefore,

\[ \phi(x) = \left( \frac{x}{H} \right)^2 \quad \text{(2.3)} \]

\[ \phi'(x) = \frac{2x}{H^2} \quad \text{(2.4)} \]

The bending deformation \( \chi \) is related to \( q \) by

\[ \chi = \beta, x = \frac{2}{H^2} q \quad \text{(2.5)} \]

Using The Principle of Virtual Displacements (Bathe, 1995) the equilibrium equation can be written in terms of \( q(t) \). The procedure begins by expressing the displacement variables (\( u, \beta \)) in terms of generalized coordinates (\( q(t) \)). A virtual displacement is defined as a displacement distribution created when the generalized coordinate is excited by a small amount, \( \delta q \). This principle is based on the requirement that the work done by the external loads during the virtual displacement must be equal to the work done by the internal forces during the same virtual displacement. The mathematical form is
\[ \int_{0}^{H} M \delta \chi \, dx = \sum \text{Force} \ast \text{displacement} + \sum \text{Moment} \ast \text{rotation} \]  \hspace{1cm} (2.6)

where \( M \) is the internal moment, \( \text{Force} \) is the externally applied loading and inertial forces, and \( \text{Moment} \) is the externally applied moment by the outriggers. So this expression becomes

\[ \int_{0}^{H} M \delta \chi \, dx = \int_{0}^{H} b \delta u \, dx - \left| \vec{M} \delta \beta \right|_h \]  \hspace{1cm} (2.7)

\[ b = \vec{b} - m\ddot{u} = \vec{b} - \rho\phi\ddot{q} \]  \hspace{1cm} (2.8)

Assuming that both columns and dampers attached to the outrigger work in compression and tension, the counter moment can be expressed as

\[ \vec{M} = 2ek_{\text{column}}e \sin \beta \big|_h + 2ec_{\text{damper}}e \sin \beta \big|_h = \frac{4e^2hk_{\text{column}}}{H^2} q + \frac{4e^2hc_{\text{damper}}}{H^2} \dot{q} \]  \hspace{1cm} (2.9)

\[ M = D_B \chi = D_B \frac{2}{H^2} q \]  \hspace{1cm} (2.10)

where \( \vec{b} \) is a prescribed loading, \( m\ddot{u} \) is the inertial forces of the building, \( k_{\text{column}} \) is the axial stiffness of the columns attached to the outriggers, \( c_{\text{damper}} \) is the damping coefficient of the dampers attached to the outriggers, and \( D_B \) is the bending rigidity of the building core. Although the bending rigidity of the building core would typically vary with the height, this analysis assumes \( D_B \) to be constant throughout the building height. Also, the expression for the applied moment from the outriggers, \( \vec{M} \), assumes that the outriggers are infinitely rigid, which overestimates the counter moment, but makes a simplified
solution. Using the definition of \( u \) given in equation (2.1), the virtual terms can be related to \( \delta q \).

\[
\delta u = \delta q \phi 
\]  
\( (2.11) \)

\[
\delta \beta = \delta q \frac{2x}{H^2} 
\]  
\( (2.12) \)

\[
\delta \gamma = \delta q \frac{2}{H^2} 
\]  
\( (2.13) \)

After substituting equations (2.11), (2.12), and (2.13) into equation (2.7) the equilibrium equation becomes

\[
\int_{0}^{H} 4D_B q \delta q dx = \int_{0}^{H} b \phi \delta q dx - \frac{2xM}{H^2} \delta q 
\]  
\( (2.14) \)

and can be simplified to

\[
\int_{0}^{H} \frac{4D_B}{H^4} q dx = \int_{0}^{H} b \phi dx - \frac{2xM}{H^2} 
\]  
\( (2.15) \)

Following integration and substitution of terms the final equilibrium equation of motion is expressed as

\[
\frac{\rho H}{5} \ddot{q} + \frac{8e^2 h^3 c_{\text{damper}}}{H^4} \dot{q} + \left[ \frac{8e^2 h^3 k_{\text{column}}}{H^4} + \frac{4D_B}{H^3} \right] q = \frac{\bar{b}H}{3} 
\]  
\( (2.16) \)

### 2.3 Differential equation solution

Equation (2.16) is expressed in the form of \( \ddot{q} + \bar{c} \dot{q} + \bar{k} q = \bar{p} \) which can be solved using one of the internal differential equation solvers in MATLAB, such as ode45. The ode45 function is used to solve initial value ordinary differential equations, using 4th and 5th order Runge-Kutta methods to integrate the equations. The function requires three
main inputs: a function describing the differential equations, a vector indicating the time of integration, and a vector defining the initial conditions of the system.

The ode45 function is setup to handle first-order equations and so the second-order differential equation must be converted into an equivalent series of first-order equations, as shown below

\[ y_1 = q \]  \hspace{1cm} (2.17)

\[ y_2 = \dot{y}_1 = \dot{q} \]  \hspace{1cm} (2.18)

\[ \dot{y}_2 = \ddot{q} = \frac{p - ky_1 - cy_2}{m} = \frac{p - kq - c\dot{q}}{m} \]  \hspace{1cm} (2.19)

Next the time vector is created, which specifies the initial and final times that MATLAB will integrate between. Lastly, the initial conditions of the system must be defined, which in this case are \( q_0 = 0 \) and \( \dot{q}_0 = 0 \). Once these three inputs are defined the ode45 function can be run. The results give a vector of the time values, the displacement, and the velocity. Knowing that the acceleration is the derivative of the velocity, a FOR loop was created in MATLAB to calculate the slope between each point of the velocity vector. The MATLAB code used for this analysis can be found in APPENDIX A.
CHAPTER 3  STATIC ANALYSIS

3.1 Introduction
Before moving to a complicated dynamic analysis, a static analysis was used to verify that the simplified models were providing proper results. The static analysis is used to calibrate the stiffness parameters of the models, and to investigate the effect of certain parameters in the outrigger model.

3.2 Static Wind Loads and Design Criteria
The static wind loading was taken directly from the Massachusetts Building Code, and is shown in Table 3.1. These wind loadings are based on a 145 km/hr wind speed, which is a 100-year wind, and an exposure factor of B, which is for areas where ½ mile upwind is continuous urban development, forest, or rolling terrain. Under the 100-year wind the structure should be designed to have no serviceability problems, therefore the static deflection at the top of the structure was limited to \( \frac{H}{500} \).

<table>
<thead>
<tr>
<th>Height (m)</th>
<th>Reference Wind Pressures (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-30</td>
<td>1005</td>
</tr>
<tr>
<td>31-45</td>
<td>1245</td>
</tr>
<tr>
<td>46-61</td>
<td>1436</td>
</tr>
<tr>
<td>62-76</td>
<td>1628</td>
</tr>
<tr>
<td>77-91</td>
<td>1772</td>
</tr>
<tr>
<td>92-122</td>
<td>1963</td>
</tr>
<tr>
<td>123-140</td>
<td>2202</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td><strong>1607</strong></td>
</tr>
</tbody>
</table>
3.3 **Simple Cantilever SDOF Analysis**

For the purpose of this thesis, as described earlier, the building being analyzed is 140 meters tall therefore H is set constant at 140. The analysis will also assume that there are two braced frames and outriggers in each direction, as shown in Figure 1.4, therefore each set will resist \( \frac{1}{2} \) the lateral forces. This assumption set the wind load (w) constant at the average of the pressures in Table 3.1 (about 1600 Pa) times \( \frac{1}{2} \) the building width (15 meters), so w was taken as about 24,000 N/m. Lastly, the mass density of the building was held constant as 170,000 kg/m as explained earlier. Since the SDOF system is representative of one outrigger system (1/2 of the total system), the mass density used in the analysis is \( \frac{1}{2} \) of the total. The values used in the analysis are summarized in Table 3.2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>H (m)</td>
<td>140</td>
</tr>
<tr>
<td>( \rho ) (kg/m)</td>
<td>85000</td>
</tr>
<tr>
<td>w (N/m²)</td>
<td>24000</td>
</tr>
<tr>
<td>e (m)</td>
<td>15</td>
</tr>
</tbody>
</table>

The first step was to run the analysis as a simple cantilevered beam which is representative of a braced frame core structure with no outriggers. This model can be used as a control to compare the sensitivity of certain changes. The stiffness of this structure is dependent on the provided bending rigidity, \( D_B \). To determine \( D_B \) for this system the maximum deflection constraint was applied to the top of the structure,

\[
q^* = \frac{H}{500} = 0.28m
\]

The expression for \( D_B \) for a cantilever beam with a static uniform loading is derived by Connor (2003) as
\[ D_b = \frac{b[H-x]^2}{2\chi^*} \]  

(3.1)

where \( b \) is the uniform loading, \( x \) is the distance up the building, and \( \chi^* \) follows equation (2.5) as

\[ \chi^* = \frac{2}{H^2}q^* \]  

(3.2)

Since the SDOF solution is derived based on a constant \( D_b \), equation (3.1) is evaluated at the point \( x = \frac{H}{2} = 70m \) to average the typical rigidity distribution. This value of \( D_b \) is about \( 2.0 \times 10^7 \) and is used for the bending rigidity of the full height. When the MATLAB code is run using these values and ignoring the outriggers a maximum static deflection of .38 meters is found and a period of 6 seconds. The deflection is higher than predicted by the calculations above, and the period is higher than expected. The expected period is \( .1N \) where \( N \) is the number of stories. Using this rule of thumb, this structure should have a period of around 4 seconds. The cause of these inaccuracies is the assumption of a constant \( D_b \). The derivation is based on a specified mode shape, \( \phi \), as shown in equation (2.3). This mode shape can only be obtained by a bending rigidity that varies with \( x \). By implementing a constant bending rigidity, a new fundamental mode shape is created.

Instead of solving for \( D_b \) with an equation that assumes a varying \( D_b \), \( D_b \) will be solved for directly using equation (2.16). It is known that the static displacement is

\[ u_{\text{static}} = \frac{P}{k} = \frac{bH}{\frac{3}{8e^2h^2k_{\text{column}}} + \frac{4D_b}{H^3}} \]  

(3.3)
Since in this case the outrigger is being ignored the equation is reduced to

\[ u_{\text{static}} = \frac{bH^4}{12D_B} \]  

(3.4)

Again specifying \( q^* = \frac{H}{500} = .28m \), \( D_B \) is determined as \( 2.75 \times 10^{12} \). When entering this value into MATLAB, \( u_{\text{max}} \) is found to be .28 meters, as expected, and the period is calculated as 4.8 seconds, which is closer to the expected value. These results were considered acceptable for static response, so the next step was to build a model in SAP2000 for further verification.

### 3.4 Simple Cantilever SAP2000 Static Analysis

In order to obtain more accurate results, a model was constructed in the structural analysis program SAP2000. The braced core was represented as a single cantilevered element with properties representing that of a braced frame building. The mass, loading, and height are those specified in Table 3.2. The stiffness was determined by using the conventional equation for deflection at the top of a cantilever beam subjected to a uniform load and limiting deflection as shown below.

\[ u_{\text{max}} = .28m \]  

(3.5)

\[ u_{\text{max}} = \frac{wH^4}{8EI} \]  

(3.6)

Solving equations (3.5) and (3.6) for \( EI \), the bending rigidity is found to be \( D_B = 4.1 \times 10^{12} \). Upon running the analysis it is found that \( u_{\text{max}} = .28m \) as expected, and the fundamental mode is found to have a period of 5.0 seconds, which is acceptable. Verifying these results with the SDOF model, it is noticed that the bending rigidity is of the same magnitude but is about 50% less than the bending rigidity needed in the SAP
model. This inconsistency is again the cause of the mode shape assumption as described above. The fundamental period of both models is comparable, about 5 seconds.

3.5 Outrigger SDOF Static Analysis

In the static case two parameters, the column stiffness \( k_{\text{column}} \) and the outrigger height (h), will have an effect on the stiffness of the SDOF system. The length of the cantilevering outrigger, \( e \), would also effect the stiffness, but in this case the building dimensions are fixed at 30 meters making \( e \) an irrelevant parameter. The first portion of this section will explore the sensitivity of the aforementioned parameters.

To understand the effect of changing \( k_{\text{column}} \) a comparison can be made between how an increase in \( k_{\text{column}} \) decreases the necessary \( D_B \). The column is treated as a simple axial member with stiffness, \( k_{\text{column}} \), expressed as

\[
k_{\text{column}} = \frac{AE}{h}
\]  
(3.7)

For the static case, \( D_B \) can be expressed as a function of the column area

\[
D_B = \frac{H^3}{4} \left[ \frac{bh}{3q^*} - \frac{8e^2h^2AE}{hH^4} \right]
\]  
(3.8)

In this equation \( H, e, q^* \), and \( b \) were defined earlier. \( E \) is the elastic modulus of steel, \( 210 \times 10^9 \frac{N}{m^2} \), and \( h \) will vary from \( \frac{1}{4} \) points of the full height (35m, 70m, 105m, and 140m).

Figure 3.1 shows the linear relationship between bending rigidity and outrigger column area. As expected, when the column area increases the necessary bending rigidity decreases. This plot also shows that if enough column stiffness is provided a
point is reached where zero bending rigidity is required. But by comparing the column stiffness needed for each plot to reach zero bending rigidity it is clear that there is an inversely proportional relationship between outrigger height and column stiffness. That is, as the height of the outrigger decreases the column stiffness must increase to obtain an equal bending rigidity.

![Figure 3.1. Bending rigidity versus outrigger column area](image)

These results are approximate as many simplifications have been made in this SDOF model. First, the continuous model shown in Figure 2.1 is an infinite degree of freedom system which is converted to a SDOF by lumping all mass, stiffness, and loading resulting in the model shown in Figure 4.5. This model can give a good first approximation, but fails to capture the response of the entire system. For instance, the
analysis above showed that if enough stiffness is provided in the columns with the outrigger at $\frac{1}{2}$ the building height, zero bending rigidity is required. This analysis fails to realize that when the outrigger is at $\frac{1}{2}$ the building height there is $\frac{1}{2}$ of the building above the outrigger acting as a cantilever, where bending rigidity must be provided. Also, as mentioned earlier, the SDOF model assumed a pure bending beam meaning that there would be no shear deformation. One must realize that this analysis has been focusing solely on bending rigidity, but there must be sufficient shear rigidity to resist the shear in the building, because an outrigger system will not provide any.

Smith (1991) has derived the equations that define an outrigger under uniform static loads. Using these equations Figure 3.2 was derived to show the optimum location of a building with one outrigger. In the figure $\omega$ represents a ratio that characterizes the flexibility of the outrigger; it increases as the flexibility increases. For the case of this thesis, the assumption will remain that the outrigger is infinitely rigid, therefore $\omega$ can be assumed as zero. This assumption creates an optimal outrigger location of about $\frac{1}{2}$ the total building height, which will be used from this point forward.

![Figure 3.2. Optimum outrigger location (Smith, 1991)](image-url)
CHAPTER 4 DYNAMIC ANALYSIS

4.1 Introduction

The next step is to look at the dynamic response of the SDOF system. Due to the high period of the structure an earthquake will have little effect on the structure because it will not excite it at resonance. A more critical loading is a periodic wind loading that could potentially excite the fundamental mode of the structure. In fact most design codes state that any building with an aspect ratio greater than 4 or 5 will behave dynamically when subjected to wind (Smith, 1991), which validates this dynamic analysis. This loading could occur in two forms: vortex shedding or fluctuating wind gusts. The main dynamic analysis will focus on fluctuating wind gusts, but the next section is a brief analysis of the effects of vortex shedding.

4.2 Brief Vortex Shedding Analysis

The phenomenon of vortex shedding creates a response that is perpendicular to the wind loading, a cross-wind response. This type of response is difficult to quantify due to the sensitivity to building geometry, building density, turbulence factors, and upwind buildings effects. (Smith, 1991) If the proper conditions exist a specific wind speed can create building oscillation at the fundamental frequency. This effect can be estimated by calculating the vortex shedding frequency as expressed below.

\[ f = \frac{S_r V}{D} \quad (4.1) \]

where \( S_r \) is the Strouhal number which is taken as .2, \( V \) is the wind velocity, and \( D \) is the width of the building perpendicular to the wind. To find the critical wind speed the vortex shedding frequency is set to the natural frequency of the building, which is the
case that the building is excited at resonance. Evaluating equation (4.1) gives a critical velocity of 108 km/hr, which is below the 100 year wind of 145 km/h, meaning this building is vulnerable to vortex shedding at its natural period. Damping can help control the problem, but nevertheless displacement and acceleration will be magnified. Figure 4.1 and Figure 4.2 show the transfer functions for displacement and acceleration, $H_1$ and $H_2$. These transfer functions make it possible to find the dynamic response as taken from Connor (2003)

\[ u_{\text{dynamic}} = H_1 u_{\text{static}} \]  
\[ a_{\text{dynamic}} = H_2 \frac{\dot{p}}{m} \]  

Connor (2003) also states that the maximum values for $H_1$ and $H_2$ when damping values are small are

\[ H_1 \big|_{\text{max}} = H_2 \big|_{\text{max}} = \frac{1}{2\xi} \]  

These maximum values occur at resonance meaning that when vortex shedding occurs at the buildings natural frequency the transfer function will equal equation (4.4). If an assumed value of $\xi = .1$ is used, the amplification of the static response will be 5.

Considering that damping of structures rarely exceeds 10% this can be considered the best case scenario. The other option is to change the aerodynamics of the building so that vortex shedding does not occur at any wind speed.
4.3 Dynamic Wind Loads and Design Criteria

The second type of dynamic wind is periodic gusts. Wind is a turbulent flow that is characterized by random fluctuations in velocity and pressure. Standard static wind design is based on an average wind speed which is taken over different time durations. If
the wind speeds are looked at as a function, certain periodic gusts within the wide spectrum of wind may find resonance with natural frequency of the building, and although the total force caused by that particular gust frequency would be much less than the static design load for the building, dangerous oscillations may be set up. In order to get a true response of the structure a detailed wind prediction would be conducted, which would assess the local wind climate and surrounding topography to create a wind velocity spectrum. This spectrum tends to be random in amplitude and is spread over a wide variety of frequencies. Although the spectrum tends to be random, if any bands of the spectrum fall near the natural frequency of the building a resonant response can occur. (Smith, 1991) Typically, the structure would be subjected to a wind tunnel test or a wind velocity spectrum to determine if there was a dynamic response, but since these are not available for this thesis a forcing function will be created.

The goal in designing the forcing function was to make a somewhat realistic representation of wind loading. Due to the sporadic nature of wind, it would be unrealistic to subject the structure to a series of strong wind gusts with the natural period of the structure. Instead, the function was created to represent two successive wind gusts as impulse loads that had periods close to the natural period of the structure, which is shown in Figure 4.3.
The function runs linearly from zero to the peak amplitude in 2.5 seconds and then returns to zero in another 2.5 seconds, leaving a total duration of 5 seconds; this is repeated twice. This function was easily input in the SAP2000 models, but the function was idealized as sinusoidal function for simplicity in the SDOF analysis. The function was taken as a piece-wise function as described below and shown in Figure 4.4

\[ F = \sin(\Omega t) \quad 0 \leq t \leq \frac{\pi}{\Omega} \]  

\[ F = -\sin(\Omega t) \quad \frac{\pi}{\Omega} < t \leq \frac{2\pi}{\Omega} \]  

\[ F = 0 \quad t < \frac{2\pi}{\Omega} \]
The analysis will focus on two amplitudes: the 100-year wind (loading A hereon) and $\frac{1}{2}$ the 100-year wind (loading B hereon). The 100-year wind applied in a dynamic fashion can be considered as an extreme loading, in fact it may be unreasonably high. Considering the return period of this amplitude is 100 years, the chance of the 100-year wind occurring as two periodic gusts has an even smaller likelihood. For this reason, the $\frac{1}{2}$ 100-year amplitude, which is a more realistic loading, will also be applied.

The design criteria for the dynamic wind loading will not be as severe as the static wind loading due to probabilistic nature of the event. Instead of limiting maximum displacements to $\frac{H}{500}$, the criteria will be lowered to $\frac{H}{300}$. This criterion will ensure that
there is no damage to the non-structural elements during the dynamic loading. Secondly, under the dynamic effects the designer must consider human perception of movement. The topic of human comfort has been widely studied, but still there are no accepted standard. (Smith, 1991) Table 3.1 shows how humans will perceive different levels of acceleration where the critical design zone is defined as .1 to .25 m/s². This is the range that humans begin to perceive the accelerations, but the criteria cannot simply be to limit maximum accelerations to .2 m/s². The reason is that it can be acceptable if a structure has accelerations greater than the acceleration criteria on occasion, because it will only cause momentary discomfort for the occupants. Instead, the problem is when accelerations continuously exceed the criteria. For this reason the acceleration design criteria will use engineering judgment to determine the critical acceleration, and that must be limited to .2 m/s² during the dynamic response.

<table>
<thead>
<tr>
<th>Acceleration (m/sec^2)</th>
<th>Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>Humans cannot perceive motion</td>
</tr>
<tr>
<td>0.05-0.10</td>
<td>Sensitive people can perceive motion; hanging objects may move slightly</td>
</tr>
<tr>
<td>0.1-0.25</td>
<td>Majority of people will perceive motion; long-term exposure may produce motion sickness</td>
</tr>
<tr>
<td>0.25-0.4</td>
<td>Desk work becomes difficult of almost impossible</td>
</tr>
<tr>
<td>0.4-0.5</td>
<td>People strongly perceive motion; difficult to walk</td>
</tr>
<tr>
<td>0.5-0.6</td>
<td>Most people cannot tolerate motion and are unable to walk naturally</td>
</tr>
<tr>
<td>0.6-0.7</td>
<td>People cannot walk or tolerate motion</td>
</tr>
<tr>
<td>&gt;0.85</td>
<td>Objects may begin to fall and people may be injured</td>
</tr>
</tbody>
</table>

4.4 Determination of Damping Coefficients

Typically damping of a structure is defined by an equivalent damping ratio, $\xi$, which is a percentage of critical damping. The typical steel structure has around 1% natural material damping, but through the application of damping devices this percentage
can be greatly increased. For the outrigger system the damping parameter $\bar{c}$ is taken from the derivation in equation (2.16) as

$$
\bar{c} = \frac{8e^2 h^2 c_{\text{damper}}}{H^4}
$$

(4.8)

It is also known that a system governed by $\ddot{m}\dot{q} + \ddot{c}q + \ddot{k}q = \ddot{p}$ has a damping ratio of

$$
\xi = \frac{\ddot{c}}{2\sqrt{km}}
$$

(4.9)

By equating equations (4.8) and (4.9) the damping coefficient for the dampers can be expressed as a function of $\xi$.

$$
c_{\text{damper}} = \frac{2\sqrt{km} \xi H^4}{8e^2 h^2}
$$

(4.10)

where $\ddot{k}$ and $\ddot{m}$ are taken from equation (2.16) as

$$
\ddot{k} = \frac{8e^2 h^2 k_{\text{column}}}{H^4} + \frac{4D_s}{H^3}
$$

(4.11)

$$
\ddot{m} = \frac{\rho H}{5}
$$

(4.12)

Using expression (4.10) damping coefficient values were calculated for given damping ratios. The damping coefficients were inputted for the dampers in the analysis program to obtain a damped dynamic response. In order to verify the calculations, a logarithmic decrement analysis was used to determine the damping ratio from the dynamic response. The method relates the change of amplitude between two successive peaks of a free vibration damped response to the equivalent damping ratio as shown below

$$
\xi = \frac{1}{2\pi j} \ln \left( \frac{u_i}{u_{i+1}} \right)
$$

(4.13)
where \( j \) is the number of cycles between peaks, \( u_i \) is the amplitude of the first peak, and \( u_{i+j} \) is the amplitude of the second peak. This analysis verified that equation (4.10) correctly estimated the damping coefficients. Table 4.2 shows the coefficient of damping needed to achieve several equivalent damping ratios for a system with a fundamental period of 5 seconds.

<table>
<thead>
<tr>
<th>( \xi )</th>
<th>( c ) (kN(s/m))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02</td>
<td>5183</td>
</tr>
<tr>
<td>0.04</td>
<td>10366</td>
</tr>
<tr>
<td>0.06</td>
<td>15549</td>
</tr>
<tr>
<td>0.08</td>
<td>20732</td>
</tr>
<tr>
<td>0.1</td>
<td>25916</td>
</tr>
<tr>
<td>0.2</td>
<td>51831</td>
</tr>
</tbody>
</table>

4.5 SDOF Dynamic Analysis

4.5.1 Purpose

The SDOF model, as shown in Figure 4.5, will be evaluated using the derived equation of motion in equation (2.16) to determine the displacements and accelerations of the system under dynamic loading. The results will be used to provide a general understanding of the effectiveness of dampers in an outrigger structure and to verify the findings of the SAP2000 analyses to follow.
4.5.2 Analysis Results

4.5.2.1 No damping

The first analysis was run with zero damping to note the response and use the results as a comparison point for further tests. Figure 4.6 shows the displacement at the top of the structure due to the dynamic loading of the 100-year wind. Since there is no damping in the structure the conditions present at the end of the forcing function (t=10 seconds) create a typical response of a free vibration system with no damping. This produces a maximum displacement of 0.79 meters which is about 60% greater than the displacement criteria of $\frac{H}{300}$. Figure 4.7 shows the acceleration at the top floor with respect to time. The same free vibration behavior is noted with a maximum acceleration of 1.11 m/s$^2$ that occurs at t= 8 seconds, which is 5.5 times the allowable acceleration of 0.2 m/s$^2$. Since this is a steady-state response the maximum acceleration occurs continuously and therefore is the critical acceleration. For an amplitude of $\frac{1}{2}$ the 100-
year wind the plots retain the same shape but the amplitudes are reduced by 50%. This
gives a maximum displacement under loading B of .4, which already meets the criteria.

Again this analysis shows the undamped dynamic response of the system, which
makes it evident that the dynamic response needs to be reduced in order to meet the set
acceleration criteria. These results are summarized in Table 4.3 and will be used as a control to measure the effects of adding damping to the system.

Figure 4.6. Top floor displacement with no damping and loading A
4.5.2.2 Equivalent damping of 10%

Now that the undamped response is understood, the analyses will proceed with damping to note the reductions of displacements and accelerations. Figure 4.8 and Figure 4.9 show the SDOF response to the dynamic function with amplitude of the 100-year wind with a 10\% equivalent damping ratio, and Table 4.3 summarizes the results.

It should be noted that, during the 10 seconds of the forcing function, the building has a greater response in the direction of the loading, whereas the undamped model had equal response in both directions. The addition of damping allows dissipation of strain...
energy during the building's rebound from the force, which causes the unsymmetrical response plot. Once the forcing function ends, the system begins free vibration about the x-axis.

Figure 4.8. Top floor displacement with 10% damping and loading A
The addition of dampers has reduced the displacement by about 30% so that the system easily meets the displacement criteria of \( \frac{H}{300} \) under loading B. The results give a maximum acceleration of 0.315 m/s\(^2\) and a critical acceleration of about 0.29 m/s\(^2\) for loading B, meaning that the accelerations are still 30% higher then the set criteria.

The dampers have proven to greatly reduce the dynamic effects, but in order to meet the acceleration criteria the equivalent damping must be increased further. The SDOF analysis will stop here as the results are enough for verifying the SAP2000 model.
and gaining a basic understanding of the damped outrigger dynamic performance. As the SAP2000 model is expected to yield the most accurate results, a more comprehensive analysis will be carried out.

4.5.2.3 Effectiveness of damping

This section will briefly use the SDOF model to compare the effectiveness of dampers as a means for controlling dynamic response versus the other possible alternatives: mass and stiffness.

By setting the equivalent damping to zero, the SDOF equations of motion can be used to find what change in mass and stiffness are needed to obtain the same response as the 15% damped system. The analysis yields that either mass needs to be increased by a factor of 2.4 or the stiffness increase by 55% to achieve the same results. Increasing the mass has the cost of the actual material added for the mass, but also the added structural steel needed to carry the heavier loads to the foundations. Increasing the stiffness by 55% is a more difficult to put into perspective, but with this stiffness the static displacement becomes .1 m which corresponds to a displacement criteria of $\frac{H}{1400}$. This criterion is about three times more than the static displacement criteria stated. To truly compare the three alternatives a cost estimate would need to be conducted, but by using the reasoning stated above it appears that the eight dampers providing 15% equivalent damping would be more cost effective.
4.6 SAP2000 Dynamic Analysis of Configuration A

4.6.1 Concept

As described earlier, an outrigger is traditionally used to provide a counter-moment to the central core, which in turn reduces the necessary bending rigidity. This concept utilizes the moment arm to provide a large resisting force with a comparatively small amount of material. Configuration A will look at the dynamic effect of installing a damper in parallel with the stiffness element at the end of the outrigger, which is the same model that was generalized in the SDOF system. The damper is connected to the end of the outrigger and the ground, whereas in typical applications a damper is connected to two adjacent floors. In the typical setup, the damper only experiences the differential movement between the two floors. In this application, since the ground is a fixed point the differential movement in the damper will be the full magnitude of the vertical movement of the outrigger. Remembering that the damping force is proportional to the velocity, this setup should be highly efficient for a damper.

Figure 4.10 is representative of the model created in the structural analysis program SAP2000. The system was subjected to the wind loading function shown in Figure 4.3 with amplitudes of both the 100-year wind and \( \frac{1}{2} \) the 100-year wind. The axial members connected to the outrigger were assigned an area of 171 \( \text{cm}^2 \), the bending rigidity of the central core was calibrated to meet the 100 year wind static loading requirements as described before, giving \( D_B = 3.08 \times 10^{12} \).
4.6.2 Analysis Results

4.6.2.1 No damping

The first analysis was run with the zero damping to note the response and use it as a comparison for further test. Figure 4.11 shows the displacement at the top of the structure due to the dynamic loading of the 100-year wind. Similar to the results found in the SDOF system, the system has forced vibrations during the 10 seconds of the forcing function followed by constant free vibration. Again, almost matching the SDOF system, there is a maximum displacement of .78 m as compared to the SDOF maximum displacement of .79 m. Figure 4.12 shows the acceleration at the top floor with respect to
time. The same free vibration behavior is noted with a maximum acceleration of 1.25 m/s$^2$, which is about a 10% difference from the maximum acceleration found in the SDOF model. As stated earlier, for an amplitude of $\frac{1}{2}$ the 100-year wind the plots would retain the same shape but the amplitudes would be cut in $\frac{1}{2}$. Continuing the comparison of the SAP2000 plots and the SDOF system plots, they appear to be very close in shape and amplitude. The main difference is the noise in the acceleration plot of the SAP2000 model. In comparison to the SDOF acceleration plot, Figure 4.7, the SAP2000 response is not as smooth as it has micro-vibrations. This problem is the cause of the SAP2000, an infinite degree of freedom system, being mildly excited at a higher mode causing small vibrations along the fundamental response. The SDOF model, by definition, only has one mode and therefore cannot experience effects from higher frequencies. Overall, the results are close enough to verify that both models are creating accurate outputs.

![Figure 4.11. Top floor displacement with no damping and loading A](image)

Figure 4.11. Top floor displacement with no damping and loading A
Figure 4.12. Top floor acceleration with no damping and loading A

4.6.2.2 Equivalent damping of 10%

Following the procedure of the SDOF analysis, the SAP2000 model was next analyzed with 10% equivalent damping. Figure 4.13 and Figure 4.14 show the system response to the dynamic function with amplitude of the 100 year wind with a 10% equivalent damping ratio (c=25916 kN*s/m), and Table 4.3 summarizes the results.
Figure 4.13. Top floor displacement with 10% damping and loading A

Figure 4.14. Top floor acceleration with 10% damping and loading A
Again, the plots from the SDOF model and the SAP2000 model are almost identical in shape and magnitude. Since both the undamped and now damped responses have matched in both models, the conclusion can be drawn that both models are accurate. As was already stated in the SDOF section, the dampers have provided a large reduction in displacements and accelerations, but the accelerations are still 35% greater than the allowable, so the damping must be increased.

### 4.6.2.3 Equivalent damping of 20%

Table 4.6 shows the displacements and accelerations for loadings A and B with 20% equivalent damping. This damping value lowers the displacement in both loadings well below the specified criteria, and gives a maximum acceleration value that is just below the specified criteria when subjected to loading B. This means that if the $\frac{1}{2}$ 100-year loading was the design dynamic loading, this structure would need 20% equivalent damping to meet the design criteria.

Table 4.5. Top floor displacement and acceleration for 10% equivalent damping

<table>
<thead>
<tr>
<th>$\xi$ = .1</th>
<th>A-</th>
<th>B-</th>
</tr>
</thead>
<tbody>
<tr>
<td>100-year wind amplitude</td>
<td>1/2 100-year wind amplitude</td>
<td>Criteria</td>
</tr>
<tr>
<td>Top floor disp. (m)</td>
<td>0.54</td>
<td>0.27</td>
</tr>
<tr>
<td>Top floor accel (m/s²)</td>
<td>0.60</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Table 4.6. Top floor displacement and acceleration for 20% equivalent damping

<table>
<thead>
<tr>
<th>$\xi$ = .2</th>
<th>A-</th>
<th>B-</th>
</tr>
</thead>
<tbody>
<tr>
<td>100-year wind amplitude</td>
<td>1/2 100-year wind amplitude</td>
<td>Criteria</td>
</tr>
<tr>
<td>Top floor disp. (m)</td>
<td>0.41</td>
<td>0.21</td>
</tr>
<tr>
<td>Top floor accel (m/s²)</td>
<td>0.36</td>
<td>0.18</td>
</tr>
</tbody>
</table>
4.6.3 Why is the equivalent damping so high?

Although 20% equivalent damping is typically considered as uneconomically high, this system obtains this equivalent damping rather easily due to the amplification of velocities and displacements. Looking back at Equation (1.1), the damper resisting force is the product of velocity and the damper coefficient, c. Therefore, there are two ways to increase the equivalent damping of a system, that is, the more traditional method, increase the damping coefficient of the damper or increase the velocities.

According to Taylor Devices Inc., “In theory, there is no limit to the c value in practice, you are limited by how small you can machine the actual orifice.” That said, although there is no theoretical minimum, at a point there is an economical minimum where the manufacturing of a single damper will be more expensive then just increasing stiffness. Also, if a designer is trying to achieve a certain damping force at a low velocity Equation (1.1) would suggest an increase c, but, according to Taylor Devices, Inc., at low velocities there may not be enough energy flux to overpower the structures inherent structural damping. It is clear that increasing the damping coefficient may not always be the most efficient answer to dynamic problems.

On the other hand the outrigger system effectively magnifies the rotations of the core to a point where a high damping force can be achieved with lower damping coefficients. This results in a higher equivalent damping ratio at a lower damping coefficient and therefore a lower cost. Figure 4.15 shows the force in the damper during the excitation of the 100-year wind, which records a maximum force of about 2113 kN (about 475 kips). Taylor Devices, Inc. states that they manufacture viscous dampers with force capacities ranging from 2 kips to 2,000 kips, which places this damper force on the low side of the range. Therefore we can conclude that this application of dampers
creating a damping ratio of 20% is not unreasonable, in fact the damping ratio could be increased.

4.6.4 Construction considerations

The above results have proven that adding dampers to the outrigger system is an efficient way to reduce the dynamic effects, but the constructability of such a system must be looked at. The system analyzed above utilizes a damper that connects between the ground and the outrigger at mid-height (70 m). Although the damper will not physically be 70 m in length, it is critical that the damper is connected directly to the ground to maximize the displacements and velocity. If the damper were connected to a floor below the outrigger the damper would only be subjected to the differential movement and velocities between the two stories, which would dramatically reduce the damper effectiveness.

Figure 4.15. Damper force (kN)
This setup would have the damper connected directly to the outrigger and an element (possibly a column) that connected the damper to the foundation. The element that connects the damper to the ground must span close to 70 meters, and will be subjected to large tension and compression loads. The member must be designed to have minimal axial deformation because the internal axial deformations will decrease the displacement and velocity experienced by the damper. A column would be the best choice, but should be braced at each floor to prevent buckling. If the compression forces become a design problem the system can be altered to only activate the tension side of the building. This would enable cables to be used as the connecting element. If cables were used, only the damper subjected to tension in the cable would work at any given time, therefore the damper would need to be engineered with double the damping coefficient to achieve the same equivalent damping.

4.7 SAP2000 Dynamic Analysis of Configuration B

4.7.1 Concept

As stated earlier, a damper is most effective in a location that maximizes the stroke or differential displacement between the two connection points. Configuration A applied the damper in parallel with the axial member, which in effect limited the differential displacement of the damper to the axial strain in the axial member. In an attempt to increase the displacement at the damper the axial column was removed from the model, as can be seen in Figure 4.16. By doing this the central core must be restored to the full bending rigidity for a cantilever beam, $D_B = 4.1 \times 10^{12}$, to meet the static loading criteria.
4.7.2 Analysis Results

Figure 4.17 and Figure 4.18 show the displacement of the damper in Configuration A and B when subjected to the \( \frac{1}{2} \) 100-year dynamic wind loading. When compared it is clear that Configuration B has increased the displacement in the damper by about .5 cm, which proves the hypothesis stated above.
Table 4.7 The increased damper stroke further decreases the displacements and accelerations as shown in Table 4.7. The new configuration led to a decrease in top-floor displacement of about 10% and a decrease in acceleration of around 20%. This
configuration yields an improved damper performance, but this may not be economical. A cost analysis would need to be conducted comparing the cost associated with increasing the bending rigidity of the core in Configuration B versus the cost associated with increasing the damping coefficient in Configuration A to achieve the results of Configuration B.

Figure 4.17. Damper stroke under load B with no outrigger columns
2.40.4

Figure 4.18. Damper stroke under load B with outrigger columns

Table 4.7. Comparison of displacement and acceleration of Configurations A and B

<table>
<thead>
<tr>
<th>c=5,1831 kN-s/m</th>
<th>Configuration A</th>
<th>Configuration B</th>
<th>Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top floor disp. (m)</td>
<td>0.21</td>
<td>0.19</td>
<td>0.467</td>
</tr>
<tr>
<td>Top floor accel (m/s²)</td>
<td>0.22</td>
<td>0.18</td>
<td>0.2</td>
</tr>
</tbody>
</table>

4.7.3 Constructability Concerns

Again, the feasibility of implementing a structural system must be looked at realistically no matter how favorable the theoretical results are. The main concern with this system is the differential movement between the outrigger and the floor at that level. Although this analysis has assumed the outrigger to be infinitely rigid, the reality of the problem is that the outrigger will deflect up and down. This movement will be too large for any gravity structural system to be connected to the outrigger. The implementation of this system must allow for the outrigger to move independently of the surrounding
structural systems. This could be achieved using a form of an expansion joint between the two structural systems, but further research should be conducted.

4.8 **SAP2000 Dynamic Analysis of Configuration C**

4.8.1 **Concepts**

Configuration C is shown in Figure 4.19. This alteration is a traditional outrigger structure with dampers running from the middle outriggers to another set of outriggers at the top of the building. This configuration allows the stiffness elements to be included in the bottom half of the building giving the benefit of a reduction in the central core bending stiffness. The theory is that the dampers will be subjected to the differential movement between the top floor and the middle floor which should be similar to the differential movement in Configuration B, while at the same time the central core stiffness is reduced. The stiffness elements were input identical to Configuration A; the central core bending rigidity was $D_a = 3.08 \times 10^{12}$ and the axial members had an area of 171 cm$^2$. 
4.8.2 Analysis Results

Initially, the dampers were assigned a damping coefficient of 51,831 kN-s/m which equated to 20% equivalent damping in Configuration A. Upon subjecting the model to the dynamic loading with 100-year wind amplitude, the results showed that the equivalent damping was much lower than Configuration A. Figure 4.20 shows the top displacement of the damper (larger amplitude) and the bottom displacement of the damper (smaller displacement). By using the logarithmic decrement analysis as described earlier, the equivalent damping is calculated to be close to 2%, which is about 10 times less than that noticed in Configuration A.
The reason is clear; the differential movement between the top and bottom of the damper is small. In Configuration A the damper was anchored directly to the ground (a fixed point), therefore the total displacement in the damper was equal to the displacement in the top of the damper. In this case, the damper is connected to a spring which under loading deforms; therefore the displacement in the damper is equal to the difference between the top displacement and the axial displacement in the stiffness element. In this particular setup, the supporting axial member displaces about 80% of the top displacement, so the damper is only subjected to 20% of the top displacement. In order to make this system more efficient the axial members must be stiffened.

The area of the axial elements is increased from 171 cm$^2$ to 806 cm$^2$. The bending rigidity of the core is then reduced to satisfy the static deflection criteria, resulting in $D_s = 1.05 \times 10^{12}$. The resulting displacements at the top and bottom of the
damper are shown in Figure 4.21. The axial deformation of the supporting stiffness element has been reduced to about 40% of the top displacement of the damper, meaning that the damper experiences about 60% of the top deflection. The result is an equivalent damping ratio of about 12%, which is about a 6 times increase. Still the equivalent damping ratio is much less than that found in Configuration A due to the axial deformation effects. An increase in the damping coefficient would eventually increase the equivalent damping to 20%, but at a higher cost.

![Figure 4.21. Displacement at top and bottom of damper](image)

**4.8.3 Constructability Considerations**

The same problem that faced Configuration B will affect this setup. There will be movement in the outrigger at the roof that will be higher than allowable tolerances in the floor. As proposed before, a form of an expansion may solve the problem, but the roof framing will need to be uncoupled from the outrigger. Another issue may be the stiffness
element in the bottom half of the building. At its maximum, when subjected to the \( \frac{1}{2} \) 100-year loading, the axial element connecting the outrigger to the ground experiences 3,500 kN (770 kips) of force. When this element is subjected to these forces in compression buckling may become a design problem. Again if buckling begins to control the design, the elements could be assumed to only work in tension, which reduces the efficiency of the system, but removes the large compression element.
CHAPTER 5 CONCLUSION

This thesis has attempted to show that the application of damping in an outrigger structure is not only effective but sensible. The inherent outrigger configuration creates amplified movements and velocities which are ideal conditions for dampers. This characteristic leads to higher equivalent damping ratios when using damping coefficients that typically produce lower damping ratios. These reasons make the outrigger structure an ideal system for the integration of viscous damping.

The results have shown that when a buildings dynamic response needs to be reduced, damping is a much more effective solution than increases in stiffness or mass. This thesis looked at three possible configurations. Configuration B produced the largest decrease in the dynamic effects, but needed an increased central core bending rigidity due to the elimination of the stiffness element. Considering configuration B did not radically improve the dynamic response of configurations A and C, the argument can be made that the savings in damper size will not offset the extra expenses on stiffness. Both configurations A and C provide a damping element and a stiffness element, which makes the comparison quite simple. Configuration A gave the best dynamic response, and therefore is the most efficient way to implement dampers into the outrigger structure. The results show that 20% equivalent damping can be easily achieved, and reduce undamped dynamic displacements by about 75% and undamped accelerations by almost 85%.

For these reasons, the suggestion of this thesis are that Configuration A be used for the reduction of the dynamic response in an outrigger structural system. The system works most efficiently with both dampers working (one in tension and one in
compression) during the dynamic excitation, but the system can be altered to deactivate the compression side of the system allowing the long compression elements to be replaced with tension elements.
CHAPTER 6 References


(D. Taylor, personal communication, May 8, 2006)


APPENDIX A MATLAB CODES

A.1 ode45 MATLAB code

Code is used for solving ordinary differential equation with the forcing function specified in Section 4.3.

clear all;

H1=140 %total building height
h1=70 %height of outrigger
el=15 %dist from build center to edge of outrigger
ro=85000 %mass density of building (kg/m)
Aco=.0171 %area of outrigger columns (m^2)
zeta=0 %equivalent damping ratio
w=24000 %uniform wind load
Omega=.6 %forcing frequency (rads/s)
Db=1.94075E+12 %Bending rigidity (N-m^2)
Ec=210000000000; %E steel (N/m^2)

kc=Aco*Ec/h1; %column stiffness (N/m)
P=w*H1/3; %equivalent amplitude (N)
m=(ro*H1)/5 %modal mass (kg)
k=((8*el^2*h1^2*kc)/H1^4+(4*Db)/H1^3); %modal stiffness (N/m)
wn = sqrt(k/m) %natural frequency (rad/sec)
Period = 2*pi/wn %natural period (s)
c=zeta*2*wn*m; %damping coefficient (kg s/m)

Staticdisp=P/k %static displacement
t_final = 30; %calculation time
t_span=[0,t_final]; %time span

% initial conditions
u_0 = 0; %initial displacement
udot_0 = 0; %initial velocity
y0 = [u_0,udot_0]; %form a vector(array) of initial conditions

% ODE45 solver
[t,y] = ode45('forcedsub',t_span,y0); %ode45 calling forcesub funct

u = y(:,1); % displacement
udot = y(:,2); % velocity

a(1)=0; %first point for acceleration array
for i=1:size(udot,1)-1 %Acceleration array
    a(i+1)=(udot(i+1)-udot(i))/(t_final/size(udot,1));
end

% results
max_u=max(abs(u)) %maximum displacement
max_a=max(abs(a)) %maximum acceleration
A.2 sub-function defining the second-order differential equation

Defines the second-order differential equation as a system of first-order differential equations for use in ode45

function ydot = forcedsub(t,y,m,k,c)

H1=140; %total building height
h1=70; %height of outrigger
e1=15; %dist from build center to edge of outrigger
ro=85000; %mass density of building (kg/m)
Aco=0.0171; %area of outrigger columns (m^2)
zeta=.15; %equivalent damping ratio
w=12000; %uniform wind load
Omega=.6; %forcing frequency (rads/s)
Db=1.94075E+12; %Bending rigidity (N-m^2)
Ec=210000000000; %E steel (N/m^2)

kc=Aco*Ec/h1; %column stiffness (N/m)
P=w*H1/3; %equivalent amplitude (N)
m=(ro*h1)/5; %modal mass (kg)
k=((8*e1^2*h1^2*kc)/H1^4+(4*Db)/H1^3); %modal stiffness (N/m)
wn=sqrt(k/m); %natural frequency (rad/sec)
c=zeta*2*wn*m; %damping coefficient (kg s/m)

ydot1 = y(2);
if t<pi/Omega
    ydot2 = (-k*y(1)-c*y(2)+P*sin(Omega*t))/m; %funct for 1st wind gust
elseif t>pi/Omega && t<2*pi/Omega
    ydot2 = (-k*y(1)-c*y(2)-P*sin(Omega*t))/m; %funct for 2nd wind gust
else
    ydot2 = (-k*y(1)-c*y(2))/m; %free-vibration response
end

ydot = [ydot1;ydot2];