Subject 240241 (Logic I) homework due in LEC #9

1. Use the search-for counterexample method to test the following sentences for validity:
   a) ((P ® Q) Ú (Q ® R))
   b) (((P Ú Q) ® (R Ú S)) ® ((P ® R) Ú (Q ® S)))
   c) (((P Ù Q) ® (R Ù S)) ® ((P ® R) Ù (Q ® S)))
   d) (((P ® Q) ® (R ® S)) ® (((P Ù Q) Ù R) ® S))

2. Use the search-for-counterexample method to test whether the following argument is valid:

   ((P « Q) Ú (P « R))
   (P ® (R ÚS))
   (P ® (U Ù ¬ W)
   \((Q Ù ¬ R) ® (S Ú (W ® (X Ú ¬Y))))

   How many lines are there in the truth table for this argument?

3. Use the search-for-counterexample method to test whether this set of sentences is consistent:
   { "(P « Q) Ú (P « R))," "(P ® (R Ú S))," "(P ® (U Ù ¬W))," "(Q Ù ¬R)," "W," "¬(X Ú ¬Y)," "S"}

4. Suppose that φ and ψ are SC sentences and that φ implies ψ. Show that, unless φ is tautological
   or ψ is contradictory, there exists a sentence θ such that every atomic sentence which occurs in θ occurs in both φ and ψ and such that φ implies θ and θ implies ψ. [Logic trivia: The analogue of this result for the predicate calculus is a famous theorem called the *Craig interpolation theorem*.]

5. Let's say that two SC sentences are *weakly equivalent* iff they have logically equivalent substitution instances.
   a) Which sentences are weakly equivalent to the tautologies?
   b) Which sentences are weakly equivalent to the contradictory sentences?
   c) Which sentences are weakly equivalent to the indeterminate sentences?