Translations.

Now that we've learned the formal language of the sentential calculus, we want to see how we can use the formal language to help illuminate the structure of arguments in English. Our plan is to translate an English argument into the formal language by translating simple English sentences by atomic sentences of the formal language, then translating the connectives used to build compound English sentences out of simple ones into SC connectives. If all goes well, we'll get a translation from the English argument into an SC argument that is truth value preserving in the following sense: If we specify a N.T.A. for the formal language by saying an atomic sentence is true if and only if the simple English sentence it translates is true, then compound SC sentences will likewise be true under the N.T.A. when and only when the compound English sentences they translate are true.

In speaking of English sentences as being either true or false, I am being sloppy. The only kinds of sentences of which it makes sense to ask whether they are true or false are sentences that make assertions. Other sentences, like questions or commands or expletives, are neither true nor false. Even the sentences used to make assertions can't normally be said to be either true or false. The sentence "I just ate" isn't true or false. Whether a particular utterance of the sentence is true will depend upon when the sentence was uttered and by whom. An utterance of the sentence is true iff the speaker ate directly before the time of utterance. Similarly, "The cat is on the mat" is neither true nor false. To see whether an utterance of the sentence is true, one must look to the context of utterance to see what cat and mat are salient. It's only with respect to a particular context of utterance that a sentence can be said to be true or false, so that we shouldn't say that a sentence is true but only that it's true in a context. With that said, let me persist in my sloppy but convenient habit of simply saying that a sentence is either true or false.

The connectives of the formal language are all truth functional connectives, meaning that whether a compound sentence is true is determined by seeing whether its simple components are true. Connectives in English often fail to be truth functional. As an example, consider the English sentence "He hit me because I insulted him." In order for this sentence to be true, "He hit me" and "I insulted him" both have to be true. But knowing that "He hit me" and "I insulted him" are both true won't be enough to tell you whether "He hit me because I insulted him" is true. So "because" isn't truth functional.

The name of the game is to find, for each English connective, a truth functional connective that approximates its meaning as nearly as possible; recall that, because the connectives of SC are expressively complete, every truth functional connective can be represented in the formalism. The hope is that the approximation will be good enough so that the translation will take true sentences to true and false sentences to false.

"$\wedge$" is the catch-all connective. A large preponderance of English connectives are translated, with greater or less success, as "$\wedge$." Among them are the following:

and

both ... and
but
yet
still
however
so
and so
for
although
even though
since
moreover
thus
hence
;

There are significant differences in meaning among the words on this list.

He kissed me, so I let him stay the night.

means something very different from

He kissed me, yet I let his stay the night.

The first sentence suggests that his kissing me was a reason for letting him stay the night. The second suggests that his kissing me was a reason to not let him stay, though I let him stay anyway. The theory is that nuances of English speech are often lost when we translate, but raw truth value is preserved, so that ideas that are suggested or intimated but not directly stated by an English expression are lost in translation, but still the translation takes true sentences to true and false to false.

"$\vee$" is easier. It translates

or
either ... or

"$\neg$" translates "not," and a "$\neg$" can also be extracted from a host of other negative words, among them:

nobody
no one
no
nowhere
nothing
neither ... nor
never

"Neither Angela nor Beulah would help Jorge with his `problem'" is cashed out as "It's not the case that either Angela or Beulah would help Jorge with his `problem,'" which is translated "¬(A ∨ B)." Where "G" translates "I gave him money," "I gave him no money" is translated "¬G."

"→" is a little complicated. Most often, it translates "if" or "if ... then." "If you study, you'll pass" is translated "(S → P)." You can say the same thing with a different word order, so that "You'll pass if you study" is also translated "(S → P)." The rule is that the antecedent is whatever comes right after "if," whether or not it's the first clause in the sentence.

"Only if" is different from "if." "You'll pass if you study" is something you might say to someone who was doing well in the course, so that she had a good chance of passing even without studying, but, if she studies, she'll pass for sure. So "You'll pass if you study" is consistent with "(P ∧ ¬S)"; what it rules out is "(¬P ∧ S)."

"You'll pass only if you study" is something you might say to someone who's doing so badly in the course that there's no way she'll pass without studying, but she might fail even if she studies. That is, "You'll pass only if you study" is consistent with "(¬P ∧ S)" but inconsistent with "(P ∧ ¬S)."

"You'll pass if you study" is translated "(S → P)," while "You'll pass only if you study" is "(P → S)." "Only if you study will you pass" and "You will only pass if you study" are also translated "(S → P)." The rule is that, when you have an "only if" construction, the consequent is the clause that comes right after "if."

Other phrases that get translated by "→" include:

provided that
given that
assuming that
supposing that
even if
in case

"→" is sometimes read "implies," but this is a mistake. "Implies" is a transitive verb; what go before and after "implies" are names of sentences, as in

"No one went to the party" implies "Sam didn't go to the party."

"$\rightarrow$" is what logicians call a connective and what grammarians call a conjunction. What do on either side of "$\rightarrow$" are sentences, not names of sentences. Where "P" translates "Someone went to the party" and "S" translates "Sam went to the party," "$(\neg P \rightarrow \neg S)$" translates

> **If no one went to the party, then Sam didn't go to the party.**

"$\leftrightarrow$" translates the following:

> **if and only if**
> **when and only when**
> **just in case**

"You'll pass if and only if you study" can be expanded into a conjunction, "You'll pass if you study and you'll pass only if you study," which would be translated "$((S \rightarrow P) \wedge (P \rightarrow S))$." But the "if and only if" construction occurs often enough that it's worthwhile to be able to write it more concisely as "$(P' \leftrightarrow S)$," which is logically equivalent to "$((S \rightarrow P) \wedge (P \rightarrow S))$."

No one knows precise rules for doing translations. What I've been giving you are guidelines, but they don't constitute a precise recipe. You learn how to do translations by practice.* So let's look at some examples.

> **If he asks us to stay, we'll go, but we'll only go if he asks us to stay.**

This is a conjunction, whose first conjunct is an "if" conditional we can translate "$(S \rightarrow G)$," while the second is an "only if" conditional we can translate "$(G \rightarrow S)$." So the whole sentence is this:

$$((S \rightarrow G) \wedge (G \rightarrow S))$$

> **If you don't bribe the judge or the jury doesn't like you, you won't be acquitted.**

This is a conditional whose antecedent is a disjunction. "You don't bribe the judge," "The jury doesn't like you," and "You won't be acquitted" can all be translated as negations, say "$\neg B$," "$\neg L$," and "$\neg A$," respectively. So the whole sentence is this:

*** Gilbert Ryle made a big deal of the distinction between "knowing how" and "knowing that," the idea being that knowing how to swim, say, isn't just a matter of knowing that a bunch of propositions are true. Translation lore is a matter of knowing how.**

$((\neg B \vee \neg L) \rightarrow \neg A)$

Here's another sentence logically equivalent to the last one:

> **You will be acquitted only if you bribe the judge and the jury likes you.**

This is an "only if" conditional whose consequent in a conjunction. It's translation is this:

$(A \rightarrow (B \wedge L))$

> **I only like country and western music if I'm drunk, but if I'm drunk I like nothing better.**

This is a conjunction, whose first conjunct is an "only if" conditional,

(I like country and western music → I'm drunk)

The second conjunct is an "if" conditional:

(I'm drunk → I like nothing better than country and western music)

The negative word "nothing" tells us to translate the consequent of this conditional as a negation:

¬ I like something better than country and western music

Putting it all together, we get this:

((I like country and western music → I'm drunk) ∧
(I'm drunk → ¬ I like something better than country and western music))

or, putting in letters for the simple English sentences:

$((L \rightarrow D) \wedge (D \rightarrow \neg B))$

This illustrates a good technique for translating complicated sentences. Translate the sentence in stages. First find the main connective of the sentence. Then look at each of the components, and find the main connectives in each of them. Continue until you've gotten down to the simple sentences. You'll produce a sequence of hybrid English/sentential calculus sentences. As you go further along the sequence, you'll replace

more and more English connectives by SC connectives, until you've gotten them all; then put in letters for the simple English sentences. This is a good technique. A bad technique is to stare at the English sentence for five minutes, then, in a burst of energy, to write down a long formula.

The example illustrates something else. We commonly pretend that a correct translation only depends on the meanings of the English connectives, and not on the meanings of the other words in the sentence. That's not strictly correct. To translate "I like country and western music" properly, you have to know that there's one kind of music called "country and western music," rather that two kinds of music, called "country music" and "western music." If you didn't know this, you'd translate "I like country and western music" as "I like country music ∧ I like western music," "(C ∧ W)."

Here's an example of stage-by-stage translation:

> **If you want to try for slam, you should either bid four no trump or cue bid diamonds, but you should only bid four no trump if you want to try for slam.**
>
> **(If you want to try for slam, you should either bid four no trump or cue bid diamonds ∧ you should only bid four no trump if you want to try for slam)**
>
> **((You want to try for slam → you should either bid four no trump or cue bid diamonds) ∧ (you should cue bid diamonds → you want to try for slam))**
>
> **((You want to try for slam → (you should bid four no trump ∨ you should cue bid diamonds)) ∧ (you should cue bid diamonds → you want to try for slam))**
>
> **$((S \rightarrow (F \vee D)) \wedge (D \rightarrow S))$**

Some more examples:

> **You'll only get the job if you pass the exam, but even if you pass the exam, somebody else might get the job.**
>
> **(You'll only get the job if you pass the exam ∧ even if you pass the exam, somebody else might get the job)**

((You'll get the job $\rightarrow$ you pass the exam) $\wedge$ (you pass the exam $\rightarrow$ someone else might get the job))

((J $\rightarrow$ P) $\wedge$ (P $\rightarrow$ S))

Neither Muscles nor Slim told the cops a thing, but Spud sans like a canary

(Neither Muscles nor Slim told the cops a thing $\wedge$ Spud sand like a canary)

($\neg$ Either Muscles or Slim told the cops something $\wedge$ Spud sand like a canary)

($\neg$(Muscles told the cops something $\vee$ Slim told the cops something) $\wedge$ Spud sang like a canary)

($\neg$(M $\vee$ S) $\wedge$ C)

Neither a fear of punishment or a sense of mercy will keep John from killing his wife if he catches her with another man

(John catches his wife with another man $\rightarrow$ neither a fear of punishment or a sense of mercy will keep John from killing his wive)

(John catches his wife with another man $\rightarrow$ $\neg$ Either a fear of punishment or a sense of mercy will keep John from killing his wife)

(John catches his wife with another man $\rightarrow$ $\neg$(a fear of punishment will jeep John from killing his wife $\vee$ a sense of mercy will keep John from killing his wife))

(C $\rightarrow$ $\neg$(F $\vee$ M))

"Provided that" is translated "$\rightarrow$":

Provided that she gets through the border safely, Xochitl will be here on Tuesday.

is translated as follows, using "B" for "Xochitl gets through the border safely" and "X" for "Xochitl will be here on Tuesday":

$(B \rightarrow X)$

"Unless" is translated "If not," so that

Unless we leave right now, we'll miss the train.

is the same as

If we don't leave right now, we'll miss the train.

which is

$(\neg L \rightarrow M)$

I won't give you a loan unless I have some collateral.

is treated as

I won't give you a loan if I don't have some collateral.

$(\neg C \rightarrow \neg L)$

Angel will drive his truck to the picnic if we ask him to, but otherwise he'll go to his girlfriend's house in Sonoma.

The interesting thing here is "otherwise." The sentence amounts to the conjunction of two conditionals, where the antecedent of the second conditional is the negation of the antecedent of the first conditional:

(Angel will drive his truck to the picnic if we ask him to ∧, if we don't ask Angel to drive his truck to the picnic, Angel will go to his girlfriend's house in Sonoma)

((We ask Angel to drive his truck to the picnic → Angel will drive his truck to the picnic) ∧ (¬ we ask Angel to drive his truck to the picnic → Angel will go to his girlfriend's house in Sonoma))

$((A \rightarrow D) \wedge (\neg A \rightarrow G))$

If I win the lottery, I'll go to the South Seas, but otherwise, I'll see you at work on Monday.

$((W \rightarrow S) \wedge (\neg W \rightarrow M))$

If you have a disjunction of two simple sentences with the same verb but different subjects, you may, if you like, write the word "or" between the two subjects and write the verb only once. Instead of

Fritz gave Mei Sun the money or Martha gave Mei Sun the money.

we may say

Fritz or Martha gave Mei Sun the money

translating

$(F \vee M)$

Similarly, for conjunctions. Instead of

Aaron kissed Zaida and Brett kissed Zaida,

we may say

Aaron and Brett kissed Zaida

translated

$(A \wedge B)$

You can do the same thing with the verb:

Fred sang or Fred danced.

Fred sang or danced

$(S \vee D)$

Fred sang and Fred danced.

Fred sang and danced.

$(S \wedge D)$

You can do the same thing if the sentences only differ in the direct object:

The Hairyman chased Wiley or the Hairyman chased Ray.

The Hairyman chased Wiley or Ray

$(W \vee R)$

The Hairyman chased Wiley and the Hairyman chased Ray.

The Hairyman chased Wiley and Ray.

$(W \wedge R)$

You have to be careful about collapsed clauses. "Consuela knew that Mary or Sue had taken her jacket" doesn't mean "Consuela knew that Mary had taken her jacket or Consuela knew that Sue had taken her jacket." The former, but not the latter, would be true if she knew that one of them had taken her jacket but she didn't know which. "Bert and Ernie ate a pizza" doesn't mean the same as "Bert ate a pizza and Ernie ate a pizza"; it means that Bert and Ernie ate a pizza together. "George and Gracie were married" could mean the same as "George was married and Gracie was married," but it could also mean "George and Gracie were married to each other." The sentence is ambiguous, though you can usually tell from the context which was intended.

Sometimes the consolidation of repeated phrases introduces ambiguities, but other times it overcomes ambiguities. In the formal language, we have unique readability, but in English, we can sometimes build up the same sentence in two different inequivalent ways. Thus

(1) Fred will sing or Fred will dance and Ginger will dance

could be either of these:

(2) $(S \vee (F \wedge G))$

(3) $(S \vee F) \wedge G)$

Now if (1) is intended as a disjunction (2), it contains the sentence

(4) Fred will dance and Ginger will dance,

which can be rewritten as

(5) Fred and Ginger will dance,

giving us this version of (1):

(6) Fred will sing or Fred and Ginger will dance.

If (1) is intended as a conjunction (3), it will contain

(7) Fred will sing or Fred will dance,

which can be rewritten

(8) Fred will sing or dance,

so that (1) can be rewritten:

(9) Fred will sing or dance and Ginger will dance.

Thus (6) and (9) are unambiguous versions of (1). I believe this phenomenon was first pointed out by Quine.

"Either ... or" is generally nothing more than a stylistic variation of "or," but sometimes the "either" serves a bigger purpose. Consider

(10) Jones came and Smith stayed or Robinson left,

which is ambiguous between these:

(11) $((J \wedge S) \vee R)$

(12) $(J \wedge (S \vee R))$

"Either" is placed before the first disjunct. In (11), the whole sentence is a disjunction, and the first disjunct is "Jones came and Smith stayed," so that inserting "either" gives us

(13) Either Jones came and Smith stayed or Robinson left.

In (12), the disjunction is "Smith stayed or Robinson left," whose fist disjunct is "Smith stayed." So inserting "either" gives

(14) Jones came and either Smith stayed or Robinson left.

(13) and (14) are unambiguous versions of (10).

The same thing happens with "both."

(15) If both Jones or Smith and Robinson work late, we can finish the report on time.

and

(16) If Jones or both Smith and Robinson work late, we can finish the report on time.

are unambiguous versions of

(17) If Jones or Smith and Robinson work late, we can finish the report on time.

corresponding to

(18) $(((J \vee S) \wedge R) \rightarrow F)$

and

(19) $((J \vee (S \wedge R)) \rightarrow F)$

respectively.

Sometimes the ambiguity is harmless.

(20) Either Jack or Karen or LaVerne went up the hill

is ambiguous between

$$(J \vee (K \vee L))$$

and

$$((J \vee K) \vee L)$$

but since the two are logically equivalent, it doesn't matter. Translate (20) either way you like.

Now we know two important things. We know how to translate an English argument into SC, and we know how to test whether an SC argument is valid. Now we want to put those skills together. If the SC argument is valid, we can conclude that the

English argument was valid. (Though, let me remind you, from the fact that the SC argument is invalid, you can't tell one way or the other about the English argument.) So we have a method of showing that an English argument is valid.

Our first example is due to the Stoic philosopher Chryssipus, who used it to show that even dogs are capable of reasoning logically. According to Chryssipus, when a dog chasing a rabbit comes to a point where the path branches into three direction, the dog will first sniff along one of the paths, and, if he smells the rabbit, he'll dash after it. If not, he'll sniff along the second path, and, if he smell's the rabbit, he'll dash after it. But if he doesn't smell the rabbit down either of the first two paths, he'll rush down the third path without hesitating to smell, because he'll know that, if the dog hasn't gone down either of thee first two paths, he must have gone down the third. That is, the dog reasons as follows:

> **The dog went down either the first path or the second or the third.**
> **The dog didn't go down the first path.**
> **The dog didn't go down the second path.**
> **Therefore the dog went down the third path.**

In symbols,

$(F \vee (S \vee T))$
$\neg F$
$\neg S$
$\therefore\ T$

We can show that this argument is valid by the search-for-counterexample method, looking for a N.T.A. which makes the premises true and the conclusion false:

$\underline{(F \vee (S \vee T))\quad \neg F\quad \neg S\quad \therefore T}$
~~0 1 0 X 0 10 10 0~~

I have no idea whether dogs really do this.

Our next example is from *Catch 22* by Joseph Heller:

> **You will receive a medical discharge just in case you apply for one and you are judged to be insane. Applying for a medical discharge is regarded as a sign of sanity, so if you apply for a medical discharge, you will not be judged to be insane. Thus you will not receive a medical discharge.**

In symbols,

(D ↔ (A ∧ J))
(S ∧ (A → ¬J))
∴ ¬D

(D ↔ (A ∧ J))   (S ∧ (A → ¬J)) ∴¬D
~~**1 1 1 1 1   1 1   1 1 X1   01**~~

This example is from Michael Resnik's *Elementary Logic*:

> **If the horse loses a shoe, the owner will shoe the horse or hire someone to do so. Unless the owner does not shoe the horse, the owner will have a sore back. The race will be canceled if the owner has a sore back. So if the horse loses a shoe and the owner does not hire someone to shoe him, the race will be canceled.**

(L → (S ∨ H))
(¬¬S → B)
(B → C)
∴ ((L ∧ ¬H) → C)
(L → (S ∨ H)) (¬¬S → B) (B → C) ∴((L ∧ ¬H) → C)
~~**1 1 1 1 0   101 1 1   X 1 0   1 1 10   0 0**~~

Another example:

> **If !Kumsa captured the springbok he wounded yesterday, he would have brought it home and there would have been cries of joy from his hut, but otherwise he's still on the trail of the springbok. Therefore, since there weren't cries of joy from !Kumsa's tent, he must be still on the trail of the springbok.**

((S → (H ∧ J)) ∧ (¬S → T))
¬J
∴T

Notice that "since there weren't cries of joy from !Kumsa's tent" is a premise of the argument; it's evidence for the conclusion. The word "since" marks a premise.

((S → (H ∧ J)) ∧ (¬S → T))   ¬J   ∴T
~~**1 1   X 0   1 01 1 0   10   0**~~

The next example is taken from an old riddle:

If Smith is in the office, neither Jones nor Robinson is. If Robinson isn't in the office but Jones is, then Smith is in the office. Jones is in the office if and only if Robinson is not. Therefore Robinson, unlike Smith and Jones, is in the office.

(S → ¬(J ∨ R))
((¬R ∧ J) → S)
(J ↔ ¬R)
∴ (R ∧ (¬S ∧¬J))

(S → ¬(J ∨ R)) ((¬R ∧ J) → S) (J ↔ ¬R) ∴ (R ∧ (¬S ∧ ¬J))
~~0 1 0 0 1 1 1 0 1 0 0 1 0 1 1 0 10 X 10~~
~~1 1 X 11 0 10 11 1 1 1 1 1 0 0 0 1 1~~

This is from *Elementary Logic* by Benson Mates:

If the students are happy, the professor feels good. If the professor feels good, he is in no condition to lecture, and if the professor is in no condition to lecture, a test is given. The students are not happy if a test is given. So the students are not happy.
(H → P)
((P → ¬C) ∧ (¬C → T))
(T → ¬H)
∴ ¬H

(H → P) ((P → ¬C) ∧ (¬C → T)) (T → ¬H) ∴ ¬H
~~1 1 1 1 1 10 1 10 1 1 1 1 X1 01~~

Our last example is from Quine's Methods of Logic:

If Jones is ill or Smith is away, then neither will the Argus deal be concluded nor will the directors meet and declare a dividend unless Robinson comes to his senses and takes matters into his own hands. Consequently, if Smith is away and Robinson does not come to his senses, the Argus deal will not be concluded.

((J ∨ S) → (¬(R ∧ T) → ¬(A ∨ (M ∧ D))))
∴ ((S ∧ ¬R) → ¬A)

((J∨S) → (¬(R∧T) → ¬(A∨(M∧D)))) ∴ ((S∧ ¬R) → ¬A)
~~11 1 X 0 0 1 0 1 1 1 1 1 0 0 0 1~~