Semantics for the Sentential Calculus

***Linguistics*, the study of language, is divided into three parts. *Syntax* studies the internal structure of language. Syntax cares about when a string of words is a grammatical sentence, but it doesn't care at all what the sentence means. There are two parts to syntax, *phonetics*, which examines how words are built up out of simple sounds, and *grammar*, which looks at how sentences are built up out of words.**

***Semantics* looks outside the internal structure of language to ask what the sentences mean. It looks at the connections between the expressions of a language and the things or states of affairs that those expressions are about. It asks what an expression refers to and what would make a sentence true.**

Semantics still leaves people out of the picture. It asks what object a word refers to, without taking account of the people who use the word to refer to the object. To discuss the relation between language, the community of people who use the language, and the things they use the language to talk about is the province of *pragmatics.* Pragmatics pays particular attention to words like "this," "that," "here," and "now," which refer to different things on different occasions.

If someone asks you how your logic class is going, and you answer, "It's going OK now. The professor showed up at this morning's lecture completely sober," you have lead your interlocutor to believe that the professor has a drinking problem. But you haven't literally said that the prof is a heavy drinker. What you said will be literally true if the professor is a teetotaler. Still, though you haven't said it, you have suggested that the professor is a lush, because we usually only tell somebody something if the thing we are reporting is somehow noteworthy, and the fact that the professor came to class sober this morning is only noteworthy if the professor usually shows up drunk. The "implication" that the professor is a drunk occurs at a pragmatic level, not at a semantic level.

In the last chapter, we learned pretty much all there is to know about the syntax of languages of the sentential calculus. Whereas when you learn a natural language like Spanish or Nahuatl, you have to spend many tedious hours trying to learn the language's grammar, in the formal language there is no subjunctive mood or ablative case or pluperfect tense to worry about. That's one of the real benefits of a formal language: You can learn the grammar is just a few minutes.

We now turn to semantics.

> **Definition. A *truth assignment* for a language for the sentential calculus is a function that assigns a number, either 0 or 1, to each sentence.**

Our plan will be to assign the number 1 to sentences that are true and 0 to sentences that are false. There isn't anything about the notion of "truth assignment" that requires such a function to respect, in any way, the intended meanings of the connectives. That comes next:

> **Definition. A truth assignment $\Im$ for a language for the sentential calculus is a *normal truth assignment* (*N.T.A.*) just in case it satisfies the following conditions:**
>
> **A conjunction is assigned the value 1 if and only if both its conjuncts are assigned 1.**
>
> **A disjunction is assigned 1 if and only if one or both disjuncts are assigned 1.**
>
> **A conditional is assigned 1 if and only if either its antecedent is assigned 0 or its consequent is assigned 1 (or both).**
>
> **A biconditional is assigned 1 if and only if its components are both assigned the same value.**
>
> **A negation is assigned 1 if and only if the negatum is assigned 1.**
>
> **Definition. A sentence is said to be *true* under a N.T.A. if and only if it's assigned the value 1 by the N.T.A. A sentence that's assigned 0 by the N.T.A. is *false* under the N.T.A.**

This definition is handily summed up in the following table:

φ	ψ	$(\varphi \wedge \psi)$	$(\varphi \vee \psi)$	$(\varphi \rightarrow \psi)$	$(\varphi \leftrightarrow \psi)$	$\neg\varphi$
1	1	1	1	1	1	0
1	0	0	1	0	0	0
0	1	0	1	1	0	1
0	0	0	0	1	1	1

Using this table, we can determine for even the most complicated SC sentence whether or not it is true under a given N.T.A., provided we know which of the atomic sentence that occur within the sentence are true under the N.T.A. The method is to determine whether a complicated sentence is true by first determining whether its simpler components are true, then applying the table.

For example, to see whether "$(\neg(A \vee B) \rightarrow (A \wedge B))$" is true under an N.T.A. under which "A" and "B" are both true, note that "$(A \vee B)$" is true under the N.T.A., and so "$\neg(A \vee B)$" is false under the N.T.A.. "$(A \wedge B)$" will be true under the N.T.A., so that "(¬(A

∨ B) → (A ∧ B))" will be a conditional with a false antecedent and true consequent, and so true.

Next consider an N.T.A. under which "A" is true and "B" false. Under this N.T.A., "(A ∨ B)" be true and so "¬(A ∨ B)" will be false. "(A ∧ B)" will be false, and so "(¬(A ∨ B) → (A ∧ B))" will Be a conditional with a false antecedent and a false consequent, and so again true.

Now consider an N.T.A. under which "A" is false and "B" true. Under this N.T.A., "(A ∨ B)" will Be true and so "¬(A ∨ B)" will be false. "(A ∧ B)" will be false, and so, once again, "(¬(A ∨ B) → (A ∧ B))" will be a conditional with a false antecedent and a false consequent, and so true.

Finally, consider an N.T.A. under which "A" and "B" are Both false. Under this N.T.A. "(A ∨ B)" will be false, so that "¬(A ∨ B)" will be true. "(A ∧ B)" will be false. Thus under this N.T.A. "(¬(A ∨ B) → (A ∧ B))" will have a true antecedent and false consequent, and so it will be false.

Our results are nicely summarized in the following table:

A	B	(¬	(A	∨	B)	→	(A	∧	B))
1	1	0	1	1	1	1	1	1	1
1	0	0	1	1	0	1	1	0	0
0	1	0	0	1	1	1	0	0	1
0	0	1	0	0	0	0	0	0	0

This use of so-called *truth tables* to display the conditions under which a given sentence is true will prove to be extremely useful.

> Definition. An sentence is a *tautology* (or is *valid* or *necessary*) if and only if it is true under every N.T.A.. A sentence is a *contradiction* (or is *inconsistent* or *impossible)* if and only if it is false under every . A sentence is *indeterminate* (or *contingent* or *mixed*) if and only if it is true under some N.T.A.s and false under others.

Examples: "((P → Q) ∨ (Q → R))" is a tautology, as we can see by examining the following table; the main connective is "∨," and there are all "1"s under the main connective:

P	Q	R	((P → Q) ∨ (Q → R))
1	1	1	1 1 1 1 1 1 1
1	1	0	1 1 1 1 1 0 0
1	0	1	1 0 0 1 0 1 1
1	0	0	1 0 0 1 0 1 0
0	1	1	0 1 1 1 1 1 1
0	1	0	0 1 1 1 1 0 0
0	0	1	0 1 0 1 0 1 1
0	0	0	0 1 0 1 0 1 0

"¬((P → Q) ∨ (Q → R))" is a contradiction, as we can see from the following table; here the main connective is "¬," and there are all "0"s under the main connective:

P	Q	R	¬((P → Q) ∨ (Q → R))
1	1	1	0 1 1 1 1 1 1 1
1	1	0	0 1 1 1 1 1 0 0
1	0	1	0 1 0 0 1 0 1 1
1	0	0	0 1 0 0 1 0 1 0
0	1	1	0 0 1 1 1 1 1 1
0	1	0	0 0 1 1 1 1 0 0
0	0	1	0 0 1 0 1 0 1 1
0	0	0	0 0 1 0 1 0 1 0

"((P → Q) ∧ (Q → R))" is indeterminate, as we see from the following truth table, in which there are both "1"s and "0"s beneath the main connective, which is "∧":

P	Q	R	((P → Q) ∧ (Q → R))
1	1	1	1 1 1 1 1 1 1
1	1	0	1 1 1 0 1 0 0
1	0	1	1 0 0 0 0 1 1
1	0	0	1 0 0 0 0 1 0
0	1	1	0 1 1 1 1 1 1
0	1	0	0 1 1 0 1 0 0
0	0	1	0 1 0 1 0 1 1
0	0	0	0 1 0 1 0 1 0

A way to remember how to set up the columns of the truth table is to note that, if interpreted in binary notation, the numerals under the atomic sentences list the numbers from seven to zero, in decreasing order.

In general, whether a sentence is true under a given N.T.A. is determined by determining which of the sentential letters the sentence contains are true, and which are false. If a sentence contains n sentential letters, there will be 2^n ways to assign truth value to

the sentential letters, and so to test whether a sentence is a tautology, we have only to examine each of these 2^n possibilities. We can organize this investigation efficiently by writing a truth table. We produce an array of n columns, each headed by a sentential letter, and 2^n rows. At each position in this array we put either a "1" or an "0" in such a way that, the result will be the binary numerals for the natural numbers less than 2^n, in decreasing order. To the right of the column we write the sentence we want to investigate, and underneath the sentential letters and connectives we write "1"s and "0"s as prescribed by the definition of N.T.A. If we wind up with all "1"s under the main connective, we shall conclude that the sentence we started with was a tautology. Similarly, if we wind up with all "0"s, we shall conclude that the sentence we started with was contradictory, whereas if we wind up with mixed "1"s and "0"s we shall conclude that the sentence we started with was indeterminate.

Example: "((A → B) ∨ (¬A → B))" is a tautology, because it has the following truth table:

A	B	((A → B) ∨ (¬A → B))
1	1	1 1 1 1 01 1 1
1	0	1 0 0 1 01 1 0
0	1	0 1 1 1 10 1 1
0	0	0 1 0 1 10 0 0

SC Theorem 1. An sentence is a tautology if and only if its negation is a contradiction.

Proof: For any sentence φ,

φ is a tautology
iff φ is true under every N.T.A.
iff ¬φ is false under every N.T.A.
iff ¬φ is contradictory.☒

Here "iff" is an abbreviation for "if and only if."

SC Theorem 2. An SC sentence is contradictory iff its negation is a tautology.

Proof: For any sentence φ,

φ is contradictory
iff φ is false under every N.T.A.
iff ¬φ is true under every N.T.A.
iff ¬φ is a tautology.☒

SC Theorem 3. An SC sentence is indeterminate iff its negation is indeterminate.

Proof: For any sentence φ

φ is indeterminate
iff there are some N.T.A.s under which φ is true and other N.T.A.s under which φ is false
iff there are some N.T.A.s under which $\neg\varphi$ is false and other N.T.A.s under which $\neg\varphi$ is true false
iff $\neg\varphi$ is indeterminate.☒

SC Theorem 4. A conjunction is tautological iff both its conjuncts are tautological.

Proof: For any sentences φ and ψ,

$(\varphi \wedge \psi)$ is tautological
iff under every N.T.A. $(\varphi \wedge \psi)$ is true
iff under every N.T.A. φ is true and ψ is true
iff under every N.T.A. φ is true and under every N.T.A. ψ is true
iff φ is tautological and ψ is tautological.☒

SC Theorem 5. For any SC sentences φ and ψ, if φ is a contradiction or ψ is a contradiction, then $(\varphi \wedge \psi)$ is a contradiction.

Proof: The argument breaks up into two (not necessarily exclusive) cases:

Case 1: φ is a contradiction. Then φ is false under every N.T.A. So $(\varphi \wedge \psi)$ is false under every N.T.A.. So $(\varphi \wedge \psi)$ is a contradiction.

Case 2: ψ is a contradiction. Then ψ is false under every N.T.A. So $(\varphi \wedge \psi)$ is false under every N.T.A.. So $(\varphi \wedge \psi)$ is a contradiction.

So, in either case, $(\varphi \wedge \psi)$ is a contradiction.☒

Notice that the converse to SC Theorem 5 ("For any SC sentences φ and ψ, if $(\varphi \wedge \psi)$ is a contradiction, then either φ is a contradiction or ψ is a contradiction.") is false. For example "$(P \wedge \neg P)$" is a contradiction, even though neither "P" nor "$\neg$P" is a contradiction.

SC Theorem 6. A conditional is contradictory iff its antecedent is a tautology and its consequent is a contradiction.

Proof: Take a conditional $(\varphi \rightarrow \psi)$. We have

$(\varphi \rightarrow \psi)$ is a contradictory
iff under every N.T.A. $(\varphi \rightarrow \psi)$ is false
iff under every N.T.A. φ is true and ψ is false
iff under every N.T.A. φ is true and under every N.T.A. ψ is false
iff φ is a tautology and ψ is a contradiction.☒

Definition. Two sentences are *logically equivalent* iff they are true under precisely the same N.T.A.s.

You can test whether two sentences are logically equivalent by writing out their truth tables. The sentences will be logically equivalent iff the columns of "1"s and "0"s under the main connectives of the two sentences are the same.

Example: The two sentences "$(P \vee (Q \wedge R))$" and "$((P \vee Q) \wedge (P \vee R))$" are logically equivalent, since their truth table is

P	Q	R	((P ∨ (Q ∧ R))	((P ∨ Q) ∧ (P ∨ R))
1	1	1	1 1 1 1 1	1 1 1 1 1 1 1
1	1	0	1 1 1 0 0	1 1 1 1 1 1 0
1	0	1	1 1 0 0 1	1 1 0 1 1 1 1
1	0	0	1 1 0 0 0	1 1 0 1 1 1 0
0	1	1	0 1 1 1 1	0 1 1 1 0 1 1
0	1	0	0 0 1 0 0	0 1 1 0 0 0 0
0	0	1	0 0 0 0 1	0 0 0 0 0 1 1
0	0	0	0 0 0 0 0	0 0 0 0 0 0 0

SC Theorem 7. Two SC sentences φ and ψ are logically equivalent iff the biconditional $(\varphi \leftrightarrow \psi)$ is a tautology.

Proof: φ is logically equivalent to ψ
iff φ and ψ are assigned the same truth value by every N.T.A.
iff $(\varphi \leftrightarrow \psi)$ is true under every N.T.A.
iff $(\varphi \leftrightarrow \psi)$ is a tautology.☒

SC Theorem 8 (Augustus de Morgan). $\neg(\varphi \lor \psi)$ is logically equivalent to $(\neg\varphi \land \neg\psi)$.

Proof: For any N.T.A. $\Im$, we have

$\Im(\neg(\varphi \lor \psi)) = 1$
iff $\Im(\varphi \lor \psi) = 0$
iff $\Im(\varphi) = 0$ and $\Im(\psi) = 0$
iff $\Im(\neg\varphi) = 1$ and $\Im(\neg\psi) = 1$
iff $\Im((\neg\varphi \land \neg\psi)) = 1$.☒

SC Theorem 9 (also due to de Morgan). $\neg(\varphi \land \psi)$ is logically equivalent to $(\neg\varphi \lor \neg\psi)$.

Proof: For any N.T.A. $\Im$, we have

$\Im(\neg(\varphi \land \psi)) = 1$
iff $\Im((\varphi \land \psi)) = 0$
iff $\Im(\varphi) = 0$ or $\Im(\psi) = 0$ (or both)
iff $\Im(\neg\varphi) = 0$ or $\Im(\neg\psi) = 1$ (or both)
iff $\Im((\neg\varphi \lor \neg\psi)) = 1$☒

This theorem tells us, for instance, that "¬(A ∧ ¬B)" is logically equivalent to "(¬A ∨ ¬¬B)". Now consider the two sentences "((C ↔ (¬D ∨ (A → ¬(A ∧ ¬B)))) ∧ (C ∨ D))" and "((C ↔ (¬D ∨ (A → (¬A ∨ ¬¬B)))) ∧ (C ∨ D))". The two sentences are just alike except that an occurrence within one of them of the subsentence "¬(A ∧ ¬B)" has been replaced by an occurrence of the logically equivalent subsentence "(¬A ∨ ¬¬B)", and it is easily shown, by using truth tables, that the two sentences are logically equivalent. This phenomenon is perfectly general, as the following theorem shows; even though the theoremm is more-or-less obvious, its proof is long and intricate, so I won't give the proof:

SC Theorem 10. Let $\Im$ be a N.T.A. and let φ, ψ, χ, and θ be SC sentences such that $\Im(\varphi) = \Im(\psi)$ and such that χ and θ are just alike except that some occurrences of φ as a subsentence of χ have been replace by occurrences of ψ as subsentences of θ. The $\Im(\chi) = \Im(\theta)$.

As an immediate corollary, we have this:

SC Theorem 11. If the SC sentences φ and ψ are logically equivalent and if χ and θ are just alike except that some occurrences of φ as a subsentence of χ have been replaced by occurrences of ψ as a subsentence of θ, then χ and θ are logically equivalent.

Definition. An SC sentence *implies* (or *entails*) a sentence ψ iff there is no N.T.A. under which φ is true and ψ is false. In other words, φ implies ψ iff ψ is true under every N.T.A. under which φ is true.

Example: "$\neg(P \vee Q)$" implies "$\neg(P \wedge Q)$." We can see this by observing that, whenever a "1" appears under the main connective in the truth table for "$\neg(P \vee Q)$" (namely, in the fourth row), a "1" also appears under the main connective in the truth table for "$\neg(P \wedge Q)$":

P	Q	$\neg(P \vee Q)$	$\neg(P \wedge Q)$
1	1	0 1 1 1	0 1 1 1
1	0	0 1 1 0	1 1 0 0
0	1	0 0 1 1	1 0 0 1
0	0	1 0 0 0	1 0 0 0

SC Theorem 12. For any sentences φ and ψ, φ implies ψ iff the conditional $(\varphi \rightarrow \psi)$ is a tautology.

Proof: φ implies ψ

iff there is no N.T.A. under which φ is true and ψ is false
iff there is no interpretation under which $(\varphi \rightarrow \psi)$ is false
iff $(\varphi \rightarrow \psi)$ is a tautology.☒

SC Theorem 13. A contradiction implies every sentence.

Proof: Suppose that φ is a contradiction. Take an arbitrary sentence ψ. We want to see that φ implies ψ. Since φ is a contradiction, there is no N.T.A. under which φ is true. So there is no interpretation under which φ is true and ψ is false. That is, φ implies ψ.☒

SC Theorem 14. A tautology is implied by every sentence.

Proof: Suppose that ψ is a tautology. Take a sentence φ. We want to see that φ implies ψ. Since ψ is a tautology, there is no N.T.A. under which ψ is false. So there is no N.T.A. under which φ is true and ψ is false. That is, φ implies ψ.☒

SC Theorem 15. Two sentences are logically equivalent iff each implies the other.

Proof: Take sentences φ and ψ. We have:

φ is equivalent to ψ
iff φ and ψ are true under precisely the same N.T.A.s

iff φ is true under every N.T.A. under which ψ is true and ψ is true under every N.T.A. under which φ is true
iff φ implies ψ and ψ implies φ.☒

A Distinction to Keep in Mind: Truth and falsity for SC sentences are relative notions. An SC sentence is true under a particular N.T.A. or false under a particular N.T.A.. It makes no sense to say, simply, that an SC sentence is true or false. The notions of tautology, contradiction, logical equivalence, and implication are absolute notions. To be a tautology is to be true under every N.T.A.. That a sentence is a tautology is a property of the sentence, not relative to any particular N.T.A.. It makes no sense to say that a sentence is a tautology under an N.T.A., or that it is a contradiction under an N.T.A., or that one sentence implies another under an N.T.A., or that one is logically equivalent to another under an N.T.A..☒

Definition. An *SC argument* is a finite sequence of SC sentences. The last member of the sequence is the *conclusion* and the earlier members are the *premisses*. The argument is valid iff there is no N.T.A. under which all the premisses are true and the conclusion is false.

For example, the sequence

(A ∨ B)
(B → C)
¬C
∴ A

is an SC argument whose premisses are "(A ∨ B)," "(B → C)," and "¬C" and whose conclusion is "A." It's valid, as we can see from examining the following truth table:

A	B	C	(A ∨ B)	(B → C)	¬C	A
1	1	1	1	1	0	1
1	1	0	1	0	1	1
1	0	1	1	1	0	1
1	0	0	1	1	1	1
0	1	1	1	1	0	0
0	1	0	1	0	1	0
0	0	1	0	1	0	0
0	0	0	0	1	1	0

We see that in every row in which there is a "1" under all three of the premisses (namely, in row 4) there is also a "1" under the conclusion. This means that under every N.T.A. under which all three of the premisses are true the conclusion is also true, that is, the argument is valid.

Let's look at another SC argument. Consider

$(A \vee B)$
$(C \to B)$
$\neg C$
$\therefore A$

Looking at the truth table,

A	B	C	$(A \vee B)$	$(C \to B)$	$\neg C$	A
1	1	1	1	1	0	1
1	1	0	1	1	1	1
1	0	1	1	0	0	1
1	0	0	1	1	1	1
0	1	1	1	1	0	0
0	1	0	1	1	1	0
0	0	1	0	0	0	0
0	0	0	0	1	1	0

we see that in the sixth row all three premisses are true even though the conclusion is false. So the argument is not valid.

SC Theorem 16. An SC argument

φ_1
φ_2
.
.
.
φ_n
$\therefore \psi$

is valid iff the conjunction $(\varphi_1 \wedge (\varphi_2 \wedge \ldots \wedge \varphi_n)\ldots)$ implies ψ.

Proof: The argument is valid
iff there's no N.T.A. under which all the premisses are true and the conclusion false
iff there's no N.T.A. under which the conjunction $(\varphi_1 \wedge (\varphi_2 \wedge \ldots \wedge \varphi_n)\ldots)$ of the premisses is true and the conclusion ψ is false
iff the conjunction $(\varphi_1 \wedge (\varphi_2 \wedge \ldots \wedge \varphi_n)\ldots)$ implies the conclusion ψ.☒

SC Theorem 17. An SC argument

φ_1
φ_2
.
.
.
φ_n
$\therefore\ \psi$

is valid iff the conditional $((\varphi_1 \wedge (\varphi_2 \wedge \ldots \wedge \varphi_n)\ldots) \rightarrow \psi)$is a tautology.

Proof: The argument is valid
iff the conjunction $(\varphi_1 \wedge (\varphi_2 \wedge \ldots \wedge \varphi_n)\ldots)$ implies the conclusion ψ [by SC Theorem 16]
iff the conditional $((\varphi_1 \wedge (\varphi_2 \wedge \ldots \wedge \varphi_n)\ldots) \rightarrow \psi)$ is a tautology [by SC Theorem 12].☒

Thus the argument

$(A \vee B)$
$(B \rightarrow C)$
$\neg C$
$\therefore\ A$

is valid because the conditional

$$(((A \vee B) \wedge ((B \rightarrow C) \wedge \neg C)) \rightarrow A)$$

is a tautology, as we can see from its truth table:

A	B	C	$(((A \vee B)$	$\wedge$	$((B \rightarrow C)$	$\wedge$	$\neg C))$	$\rightarrow$	$A)$
1	1	1	1	0	1	0	0	1	1
1	1	0	1	0	0	0	1	1	1
1	0	1	1	0	1	0	0	1	1
1	0	0	1	1	1	1	1	1	1
0	1	1	1	0	1	0	0	1	0
0	1	0	1	0	0	0	1	1	0
0	0	1	0	0	1	0	0	1	0
0	0	0	0	0	1	1	1	1	0

Similarly, the argument

(A ∨ B)
(C → B)
¬C
∴ A

is invalid, as we can see by observing that the conditional

(((A ∨ B) ∧ ((C → B) ∧ ¬C)) → A)

is not a tautology; look at its truth table:

A	B	C	(((A ∨ B)	∧	((C → B)	∧	¬C))	→	A)
1	1	1	1	0	1	0	0	1	1
1	1	0	1	1	1	1	1	1	1
1	0	1	1	0	0	0	0	1	1
1	0	0	1	1	1	1	1	1	1
0	1	1	1	0	1	0	0	1	0
0	1	0	1	1	1	1	1	0	0
0	0	1	0	0	0	0	0	1	0
0	0	0	0	0	1	1	1	1	0

Thus we can use the method of truth tables to test whether an SC argument is valid. This gives us a method for showing that an English argument is valid: Translate the English argument into the language of the sentential calculus. If the SC argument you get is valid, the original English argument was valid. for example, the English argument

Either Jack or Jill went up the hill.
If Jill went up the hill, so did Clarissa.
Clarissa didn't go up the hill.
Therefore Jack went up the hill.

is valid. It's valid because it translates into the valid SC argument

(A ∨ B)
(B → C)
¬C
∴ A

This method works because good translation is *truth-value preserving*. That is, if we take a body of English discourse and correctly translate it into the language of the sentential calculus, then an N.T.A. which counts a sentential letter as true iff the simple English sentence it translates is true will also make a compound SC sentence true iff the

compound English sentence it translates is true. Thus if an English argument with true premisses translates into a valid SC argument, then an N.T.A. which makes a sentential letter true iff the simple English sentence it translates is true will count the premisses of the SC argument as true. So, since the SC argument is valid, the conclusion of the SC argument will be true under the N.T.A.. So, since translation is truth-value preserving, the conclusion of the English argument must have been true.

The method doesn't work both ways. If you take an English argument and translate it into a valid SC argument, you can be sure the English argument is valid, but if you take an English argument and translate it into an invalid SC argument, you can't be sure that the English argument was invalid. Thus the valid English argument

Socrates is a man.
All men are mortal.
Therefore Socrates is mortal.

translates into the invalid SC argument

P
Q
∴ R

The method will only show a valid English argument to be valid if the reason the English argument is valid is because of the way compound English sentences that occur within the argument are built up out of simple English sentences; that's because the only thing the sentential calculus looks at is how compound sentences are built out of simple sentences. But the English argument might also be valid on account of the way the simple sentences it contains are constructed, in which case to see the argument is valid you need a deeper level of logical analysis that looks inside the simple sentences.

Definition. A sentence φ is a *logical consequence* of a set of sentences Γ iff there is no N.T.A. which assigns the value T to all the members of Γ and assigns F to φ.

Definition. A set of sentences is *consistent* iff there is at least one N.T.A. by which all its members are assigned T.

Here are some facts about these two notions, which I'll state without proof, since these proofs are starting to get kind of tedious:

φ is a logical consequence of $\{\gamma_1,\gamma_2,...,\gamma_n\}$ if and only if the argument with $\gamma_1,\gamma_2,...,\gamma_n$ as premisses and with φ as conclusion is valid.

A sentence is a logical consequence of the empty set iff it's valid.

A sentence is a tautology iff it is a logical consequence of every set of sentences.

Each member of a set of sentences is a logical consequence of that set of sentences.

If every member of Δ is a logical consequence of Γ and φ is a logical consequence of Δ, then φ is a logical consequence of Γ.

If Δ is a subset of Γ and φ is a logical consequence of Δ, then φ is a logical consequence of Γ.

For any sentence ψ and set of sentences Γ, ψ is a logical consequence of Γ if and only if Γ and $\Gamma \cup \{\psi\}$ have precisely the same logical consequences.
$(\varphi \wedge \psi)$ is a logical consequence of Γ iff φ and ψ are both logical consequences of Γ.

$(\varphi \rightarrow \psi)$ is a logical consequence of Γ iff ψ is a logical consequence of $\Gamma \cup \{\varphi\}$.

$\{\gamma_1,\gamma_2,...,\gamma_n\}$ is inconsistent iff $(\gamma_1 \wedge (\gamma_2 \wedge ... \wedge \gamma_n)...)$ is an inconsistent sentence.

If Γ is an inconsistent set of sentences, then every sentence is a logical consequence of Γ.

A set of sentences Γ is inconsistent iff $(P \wedge \neg P)$ is a logical consequence of Γ.

A set of sentences Γ is inconsistent iff every sentence is a logical consequence of Γ.

If Δ is inconsistent and $\Delta \subseteq \Gamma$, then Γ is inconsistent.

φ is a logical consequence of Γ iff $\Gamma \cup \{\neg\varphi\}$ is inconsistent.