Subject 24.241. Logic I. Homework due in LEC #15.

1. Derive "$((\forall x)(Fx \rightarrow Ga) \leftrightarrow ((\exists x)Fx \rightarrow Ga))$" from the empty set.

2. Derive "$((\exists x)(Fx \rightarrow Ga) \leftrightarrow ((\forall x)Fx \rightarrow Ga))$" from the empty set.

3. Derive "$((\exists x)(Ga \rightarrow Fx) \leftrightarrow (Ga \rightarrow (\exists x)Fx))$" from the empty set.

4. Give a derivation of "$(\neg(\exists x)Fx \leftrightarrow (\forall x)\neg Fx)$" from the empty set without using the rule (QE). [Note: The idea of this derivation can be generalized to show that rule (QE) is superfluous.]

For the next five problems, let Ω be a set of sentences with these three properties:

a) Ω is d-consistent, that is, we cannot find sentences $\omega_1, \omega_2, \ldots, \omega_n$ in Ω such that the sentence $\neg(\omega_1 \wedge (\omega_2 \wedge \ldots \wedge \omega_n)\ldots)$ is derivable from the empty set.
b) For any sentence φ, either φ or $\neg\varphi$ is an element of Ω.
c) For any formula φ, if $(\exists x)\varphi$ is in Ω, then there is a constant c such that $\varphi^{x}/_{c}$ is in Ω.

5. Show that, for each formula φ, there is a constant c such that the conditional $((\exists x)\varphi \rightarrow \varphi^{x}/_{c})$ is in Ω.

6. Show that, for any sentences φ and ψ, $(\varphi \rightarrow \psi)$ is in Ω if and only with either $\psi \in \Omega$ or $\varphi \notin \Omega$.

7. Show that, for any sentences φ and ψ, $(\varphi \leftrightarrow \psi)$ is in Ω if and only either both φ and ψ are in Ω or neither is.

8. Show that, for any formula φ, $(\forall x)\varphi$ is in Ω iff, for every constant c, $\varphi^{x}/_{c}$ is in Ω.

9. Show that, if φ is derivable from Ω — that is, there is a derivation of φ whose premiss set is included in Ω — then φ is an element of Ω.