

Subject 24.241 (Logic I) homework due in LEC #7.

1. Give sentences with the following truth tables:

(a)	A B C	1 1 1 1	1 1 1 0	1 1 1 0
	A B C	1 1 0 0	1 1 0 0	1 1 0 0
	A B C	1 0 1 1	1 0 1 1	1 0 1 0
	A B C	1 0 0 0	1 0 0 1	1 0 0 1
	A B C	0 1 1 1	0 1 1 0	0 1 1 1
	A B C	0 1 0 0	0 1 0 0	0 1 0 1
	A B C	0 0 1 0	0 0 1 0	0 0 1 1
	A B C	0 0 0 0	0 0 0 0	0 0 0 0

2. Give sentences in disjunctive normal form that are logically equivalent to each of the following

sentences:

- (a) $(P \dot{\cup} (Q \ll R))$
- (b) $((P \dot{\cup} Q) \ll R)$
- (c) $((P \dot{\cup} Q) \ll (\neg P \dot{\cup} \neg Q))$

3. A sentence is said to be in *conjunctive normal form* iff it is a conjunction of one or more sentences each of which is a disjunction of one or more atomic sentences and negated atomic sentences. Find a sentence in conjunctive normal form that is logically equivalent to “ $(P \ll (Q \dot{\cup} R))$.” [Hint: First find a sentence in disjunctive normal form that is logically equivalent to the negation of the given sentence. Then apply de Morgan’s laws repeatedly.]

4. Which of the following sets of connectives are expressively complete? Explain your answers:

- (a) $\{\neg, \dot{\cup}, \dot{\cap}\}$, where $(\phi \dot{\cap} \psi)$ is true iff ϕ is true or ψ is false
- (b) $\{\neg, \dot{\cup}, \textcircled{\cap}\}$
- (c) {"NAND"}, where $(\phi \text{ NAND } \psi)$ is false iff ϕ and ψ are both true
- (d) {"NOR"}, where $(\phi \text{ NOR } \psi)$ is true iff neither ϕ nor ψ is true
- (e) {"XOR"}, where $(\phi \text{ XOR } \psi)$ is true iff ϕ is true or ψ is true but not both.

- 5. (a) Give a tautological substitution instance of “ $(P \ll (Q \dot{\cup} R))$.”
- (b) Give an inconsistent substitution instance of “ $(P \ll (Q \dot{\cup} R))$.”
- (c) Give an indeterminate substitution instance of “ $(P \ll (Q \dot{\cup} R))$.”