18.03 Recitation Problems

Recitation 13, March 18, 2004

Impulse response = weight function and convolution

Convolution: $f(t) * g(t) = \int_0^t f(u)g(t-u)\,du$.

Definition: The weight function of an LTI operator $p(D)$ is the solution $w(t)$ of $p(D)x = \delta(t)$ with rest initial conditions.

The solution to $p(D)x = f(t)$ with rest initial conditions is $f(t) * w(t)$.

If $p(D) = mD + bI$, $w(t) = u(t)x(t)$ where x is the solution of the IVP $m\dot{x} + bx = 0$, $x(0) = 1/m$.

If $p(D) = mD^2 + bD + kI$, $w(t) = u(t)x(t)$ where x is the solution of the IVP $m\ddot{x} + b\dot{x} + kx = 0$, $x(0) = 0$, $\dot{x}(0) = 1/m$.

1. Find the weight function of the operator $D^2 + 2D + 2I$. Sketch it.

2. Use **(a)** and convolution to express the solution to $\ddot{x} + 2\dot{x} + 2x = f(t)$ with rest initial conditions for a general signal as an integral involving $f(t)$.

Then work this out explicitly for $f(t) = e^{-t}$, and sketch this solution.

3. Suppose that $w(t) \geq 0$ for all $t > 0$. If also the signal $f(t) \geq 0$ for $t > 0$, what can you say about the system response with rest initial conditions?

4. Compute $e^{at} * e^{bt}$ in case $a \neq b$.

5. Compute $e^{at} * e^{at}$.

6. What operator has weight function $u(t)$? How about $2e^{-2t}$?

[The solutions to I.19 will appear in the solution sheet to this problem list, but they are too hard to be appropriate for recitation work.]