18.03 Recitation Problems 17

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Laplace Transform: Meaning of the pole diagram.

Euler's Method

1. Sketch the pole diagrams of $F(s)$ if $f(t)$ is: (i) $\sin(t) + cos(2t)$; (ii) $e^t + e^{-t}\sin(t)$; (iii) $4u(t-100) + e^{-t}$.

2. For each of the following pole diagrams, name two functions whose Laplace Transform exhibits them. (i) $\{1, -1\}$; (ii) $\{i\}$; $\{\pi i, -\pi i, -1+\pi i, -1-\pi i\}$.

3. Set up Euler's method to compute the value at $t = 1$ of the solution to the ODE $\dot{x} + tx = 1$ with initial value $x(0) = 0$. Use five steps and keep three decimal places in the computation. You should set up a table like this, which works for the general first order ODE $\dot{x} = F(t, x)$:

n	t_n	x_n	$F(t_n, x_n)$	$h \cdot F(t_n, x_n)$
0				
1				
...				

Things to think about: what is the stepsize h? What is t_0? What is t_{n+1} in general? What is x_0? How can we compute x_n from the information on the previous line?

Aside: Express the value $x(1)$ you estimated in Problem **1** as a definite integral (albeit one you can't evaluate in elementary functions).