Solutions to Recitation 17 Problems

1. For (i), $F(s) = \dfrac{1}{s^2+1} + \dfrac{s}{s^2+4}$, so poles are located at $\pm i, \pm 2i$.

   For (ii), $F(s) = \dfrac{1}{s} + \dfrac{1}{(s+1)^2+1}$, so poles are at $0, -1 \pm i$.

   For (iii), $F(s) = \dfrac{4e^{-100s}}{s} + \dfrac{1}{s+1}$, so poles are at $0, -1$.

2. For (i), for example, $F(s) = \dfrac{1}{s-1} + \dfrac{1}{s+1}$ will give us the desired pole diagram, so the corresponding $f(t)$ would be $e^t + e^{-t}$. A very "cheap" way of creating a second such function would be to make $f(t)$ discontinuous: i.e. let $f(t)$ as above for all $t \neq 0$ and $f(0) = 100{,}000{,}000$. A slightly more sophisticated way would be to use $F(s) = \dfrac{1}{s-1} + \dfrac{2}{s+1}$, which would give us $f(t) = e^t + 2e^{-t}$. An even more complicated way would be to use $F(s) = \dfrac{1}{(s-1)^2} + \dfrac{1}{s+1}$. Now for this, $f(t) = te^t + e^{-t}$.

   For (ii), we can take $F(s) = \dfrac{1}{s-i}$, which makes $f(t) = e^{it}$. Note that $f(t)$ in this case cannot be a real-valued function, since the pole diagram is not symmetric around the x-axis. The second candidate can be made just like (i): make it discontinuous, $f(t) = 2e^{it}$, or $f(t) = te^{it}$.

   For (iii), since the poles appear in conjugate pairs, we can take $F(s) = \dfrac{1}{s^2+\pi^2} + \dfrac{1}{(s+1)^2+\pi^2}$, whose corresponding $f(t) = \dfrac{1}{\pi}\sin(\pi t) + \dfrac{1}{\pi}e^{-t}\sin(\pi t)$. Again, for the second candidate, we can create discontinuity, adjust the constants in front to something other than $\frac{1}{\pi}$, or use cos instead of sin on one or both of the terms.

3. $h = 1 \div 5 = 0.2$, $t_0 = 0$, so $t_n = n \cdot h = 0.2n$. We have $\dot{x} = 1 - tx = F(t, x)$. The Euler method approximates the value of x by assuming that the derivative stays constant for the period of h, i.e. $x_{n+1} = x_n + h \cdot F(x_n, t_n)$ (remember $F(x_n, t_n)$ is the value of the derivative at t_n). So now we can fill out the table:

n	t_n	x_n	$F(t_n, x_n)$	$hF(t_n, x_n)$
0	0	0	1	0.2
1	0.2	0.2	0.96	0.192
2	0.4	0.392	0.843	0.169
3	0.6	0.561	0.664	0.133
4	0.8	0.694	0.445	0.089
5	1.0	0.783		

   This is a linear equation, with $\rho = e^{t^2/2}$, so the precise answer is $e^{-t^2/2}\int_0^t e^{u^2/2}du$ (note that this automatically satisfies the initial condition). If you use a calculator to compute $x(1)$ numerically, we get 0.725, so Euler method produces an 8% error.