

## 18.03 Recitation Problems 16

April 6, 2004

**Laplace Transform: Second order equations (completing the square);  $t$ -shift formula; step and delta signals; weight and transfer; convolution; poles**

1. Solve  $\ddot{x} + 4\dot{x} + 5x = \sin(3t)$  with initial conditions  $x(0) = 1$ ,  $\dot{x}(0) = 2$ , using the Laplace transform.

Use complex coverup to find the coefficients in the partial fractions problem.

Check that the form of your answer agrees with what you expect from the old  $x_p + x_h$  story (but don't bother to find the constants).

2. Find the weight function for the operators  $2D + I$  and  $2D^2 + 4D + 4I$  by finding the inverse Laplace transform of the transfer function.

For each example, sketch the graph of  $|W(s)|$ . Describe where on that graph the graph of the "gain,"  $H(\omega) = |W(i\omega)|$ , is to be found. (Recall that  $H(\omega)$  is the amplitude of the sinusoidal solution to  $p(D)x = \cos(\omega t)$ .)

3. Solve the ODE  $\ddot{x} + 4x = u(t) - u(t - \pi)$  with rest initial conditions.