10.450 Process Dynamics, Operations, and Control
Lecture Notes 2

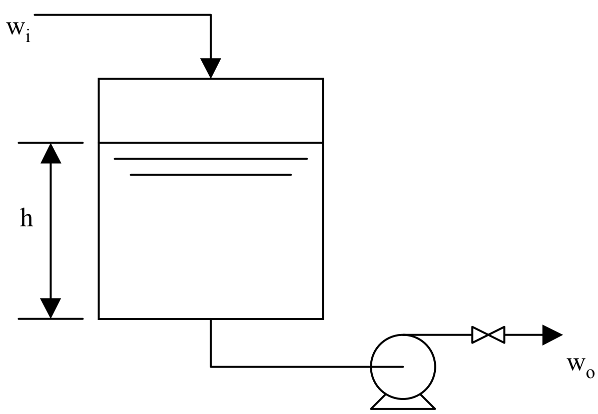
Chapter 2. Dynamic system

2.0 Context

In this chapter, we define the term 'system' and how it relates to 'process' and 'control'. We will also show how a simple dynamic system responds to several disturbances.

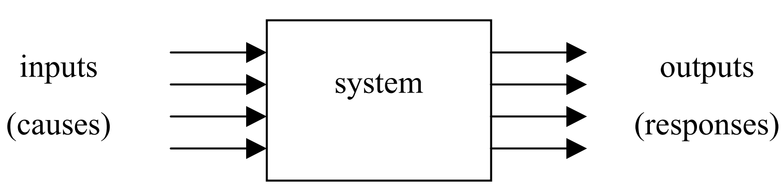
2.1 System

In Chapter 1, we introduced a process - a surge tank with pumped outlet - that was subject to disturbances in time. We thought of the process as a collection of equipment and other material, marked off by a boundary in space, communicating with its environment by energy and material streams.



'Process' is a good notion; another useful notion is that of 'system'. <u>A system is some collection of equipment and operations, usually with a boundary, communicating with its environment by a set of inputs and outputs.</u> By these definitions, a process is a type of system, but *system* is more abstract and general. For example, the system boundary is often tenuous: suppose that our system comprises the equipment in the plant and the controller in the central control room, with radio communication between the two. A physical boundary would be in two pieces, at least; perhaps we should regard this system boundary as partly physical (around the chemical process) and partly conceptual (around the controller).

Furthermore, the inputs and outputs of a system need not be material and energy streams, as they are for a process. System inputs are "things that cause" and outputs are "things that respond".



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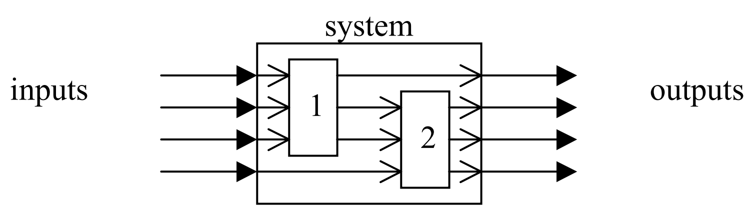
To approach the problem of controlling our surge tank process, let's think of it in system terms: the primary output is the liquid level h -- not a stream, certainly, but an important response variable of the system. Disturbances are of course inputs, and so the stream w_i is an input. And peculiar as it first seems, the outlet flow w_o is also an input, because it influences the liquid level, just as does w_i.

The point of all this is to look at a single schematic and know how to view it as a process, and as a system. View it as a process (w_o as an output) to write the material balance and make fluid mechanics calculations. View it as a system (w_o as an input) to analyze the dynamic behavior implied by that material balance and make control calculations.

2.2 Systems within systems

We call something a system and identify its inputs and outputs as a first step toward understanding, predicting, and influencing its behavior. We recognize that our understanding may improve if we determine some of the structure within the system boundaries; that is, if we identify some *component systems*. Each of these, of course, would have inputs and outputs, too.



Considering the relationship of these component systems, we recognize the existence of *intermediate variables* within a system. Neither inputs nor outputs of the main system, they connect the component systems. Intermediate variables may be useful in understanding and influencing overall system behavior.

When we add a controller to a process, we create a single time-varying system; however, it is useful to keep process and controller conceptually distinct as component systems. This is because relatively few control schemes (relationships between process and controller) suffice for myriad process applications. Using the terms we defined in Chapter 1, we represent a control scheme called *single-loop feedback control* in this fashion:

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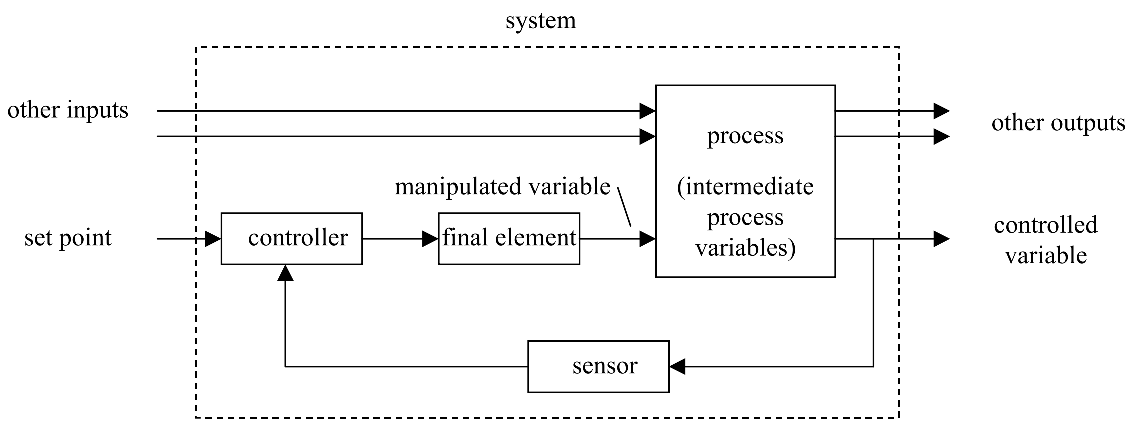
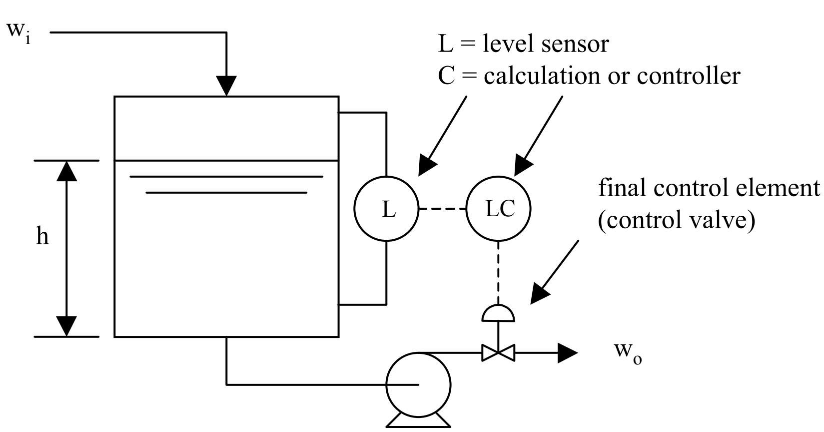


Inside the block called "process" is the physical process, whatever it might be, and the block is the boundary we would draw if we were doing an overall material or energy balance. HOWEVER, we remember that the inputs and outputs for the block are NOT necessarily the same as the material and energy streams that cross the process boundary. From among the outputs, we may select a controlled variable (V_C), and provide a suitable sensor to measure it. From the inputs, we choose a manipulated variable (V_M) and install an appropriate final control element. The measurement is fed to the controller, which decides how to adjust V_M to keep V_C at the set point. Other inputs are disturbances that affect V_C, and so require action by the controller.

We keep in mind this feedback control scheme, and how it relates the controller to the process, when we represent the equipment in schematic form, as with the surge tank of Chapter 1.



We'll have much more to say about feedback control later. For now, it's important to think of a chemical process as a dynamic system that responds in particular ways to its inputs. We attach other dynamic systems (sensor, controller, etc.) to that process in single-loop feedback scheme and arrive at a new dynamic system that responds in different ways to the inputs. If we do our job well, it responds in <u>better</u> ways, so to justify all the trouble.

To do our job well, we must understand more about system dynamics -- how systems behave in time. That is, we must be able to describe how important output variables react to arbitrary disturbances.

2.3 Dynamics of a tank, without any control

From Chapter 1, our process model was

$$\rho A \frac{dh}{dt} = w_i - w_o \qquad h(0) = h_o$$
$$h(t) = h_o + \frac{1}{\rho A}\int_0^t \left(w_i(t) - w_o(t)\right)dt \tag{2.3.1}$$

Now mindful of our system concepts, we recognize h(t) as the output and $w_i(t)$-$w_o(t)$ as the input. Indeed the flow rates are separate inputs, but our model of the process indicates that they always influence the output liquid level by their difference. For convenience, let us represent this difference as x(t). Our model (2.3.1) captures the system dynamics; it tells us how the output h(t) responds in time to input disturbances x(t). We now integrate (2.3.1) for several specific cases of x(t).

2.4 Response to rectangular pulse at time t_d

Let the tank be operating at steady state, so that the flows are initially balanced, and x is zero. Suppose that at time t_d, extra liquid is injected into the feed stream: mass M is added over time interval Δt before the inlet flow returns to normal. We can idealize this as a rectangular pulse.

$$x(t) = \begin{cases} 0, & 0 \le t < t_d \\ \dfrac{M}{\Delta t}, & t_d \le t \le t_d + \Delta t \\ 0, & t_d + \Delta t < t \end{cases} \tag{2.4.1}$$

Inserting the disturbance (2.4.1) into process model (2.3.1), we compute the response.

$$h(t) = \begin{cases} h_o, & 0 \le t < t_d \\ h_o + \dfrac{M}{\rho A \Delta t}(t - t_d), & t_d \le t \le t_d + \Delta t \\ h_o + \dfrac{M}{\rho A}, & t_d + \Delta t < t \end{cases} \tag{2.4.2}$$

As shown in Figure 2-1, the level never recovers its former value h_o; the temporary disturbance has had a permanent effect on the output.

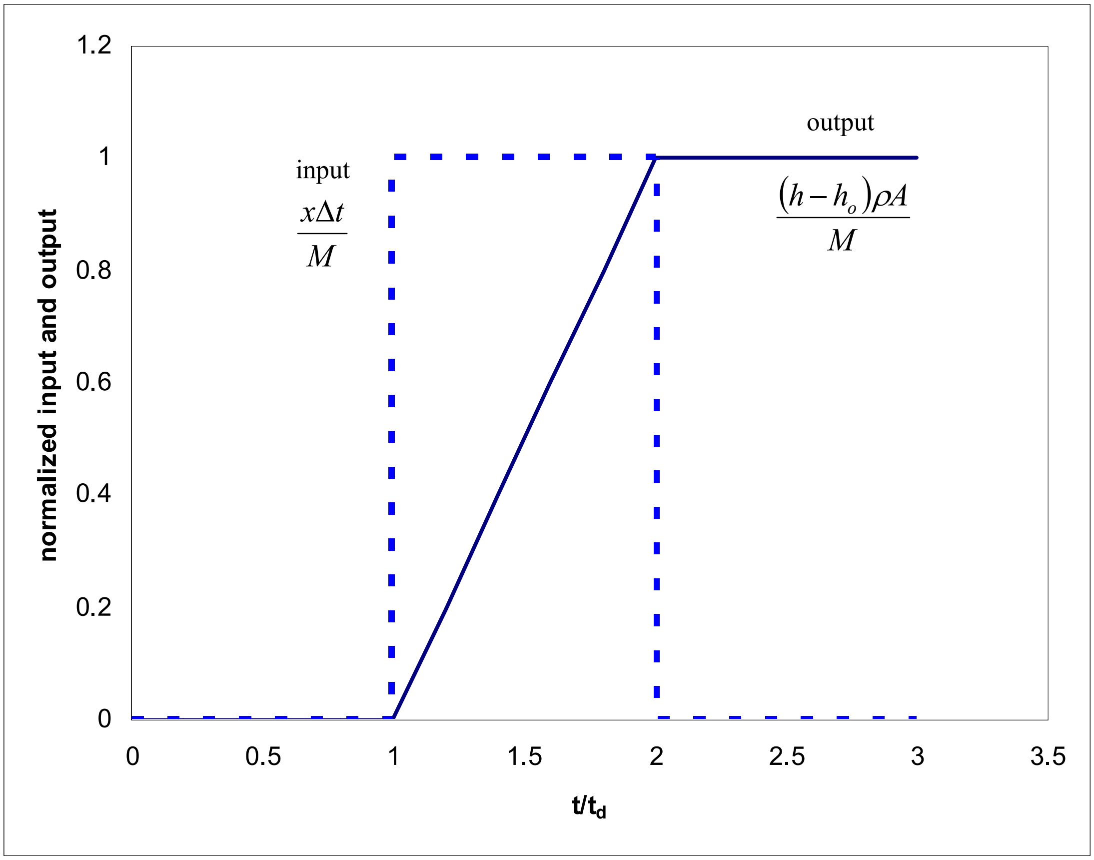


Figure 2-1. The pulse width Δt has arbitrarily been set equal to the onset time t_d.

2.5 Impulse at time t_d

If in (2.4.1) we decrease the time interval of fluid injection, we finally arrive at an infinite flow rate in an infinitesimal time interval, delivering extra mass M to the fluid. Thus we introduce the mathematical *delta function* -- a singularity at time t_d. Its value is infinite there and zero elsewhere. Its integral over all time is 1. It has the *dimension* of reciprocal time, although it's not clear whether the actual *units* of a single time point are at all significant.

$$\begin{aligned} x(t) &= M\delta(t-t_d) \\ h(t) &= h_o, \qquad 0 \le t < t_d \\ &\quad h_o + \frac{M}{\rho A}, \qquad t_d \le t \end{aligned} \tag{2.5.1}$$

Again, the level never recovers from the brief disturbance.

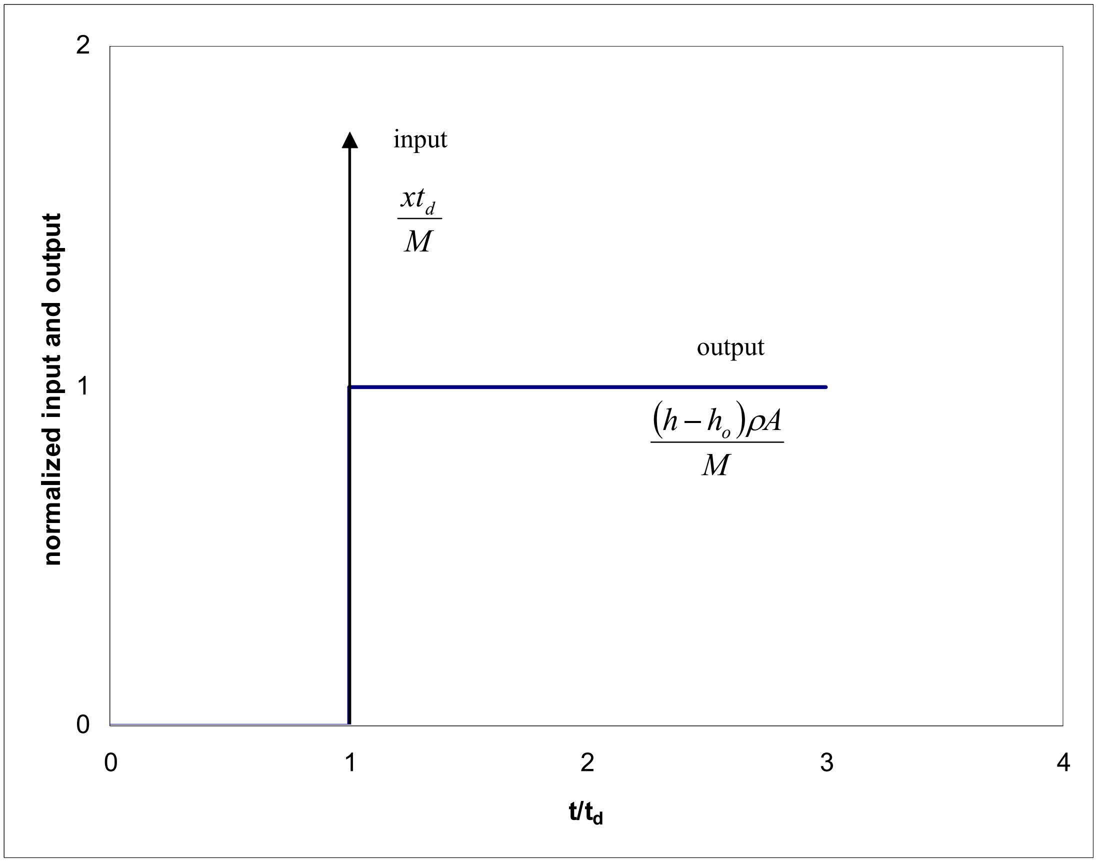


Figure 2-2. The impulse is shown as a vertical arrow at the time of its action. The input flow x(t) has been made non-dimensional by multiplying by t_d.; however, the sense of the plot would be unchanged if this were not done.

2.6 Step at time t_d

The dimensionless unit step function is zero before t_d and one thereafter. We use it to represent a sudden, permanent change in the inlet flow rate.

$$
\begin{aligned}
&x(t) = \Delta w u(t - t_d) \\
&h(t) = h_o, \qquad 0 \le t < t_d \\
&\quad h_o + \frac{\Delta w}{\rho A}(t - t_d), \quad t_d \le t
\end{aligned}
\tag{2.6.1}
$$

The permanent change in the input has caused the output to rise without limit. This is certainly reason to consider adding a control system.

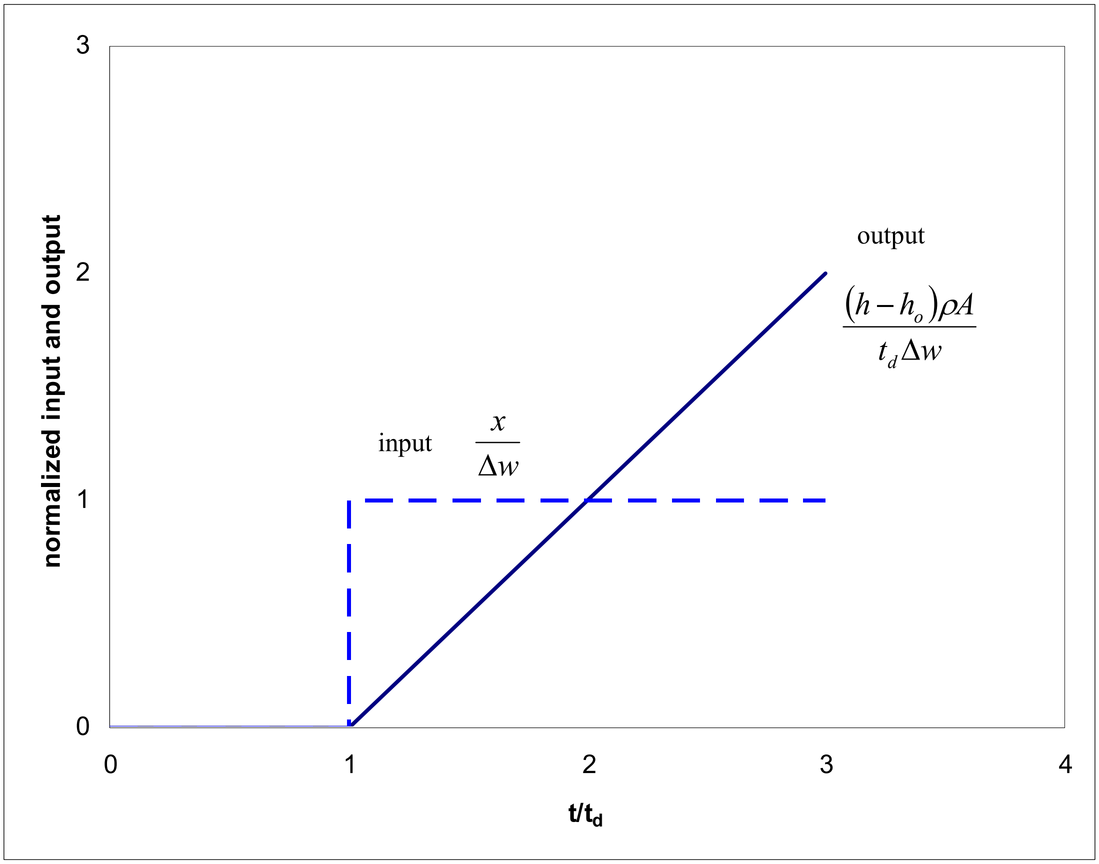


Figure 2-3. Of course, the model ceases to be applicable when the liquid level reaches the top of the tank.

2.7 Sine

At time t_d, the inlet flow begins to oscillate with radian frequency ω.

$$\begin{aligned} x(t) &= \Delta w \sin(\omega t - \omega t_d) \\ h(t) &= h_o + \frac{\Delta w}{\rho A \omega}\left(1 - \cos(\omega t - \omega t_d)\right) \end{aligned} \qquad (2.7.1)$$

The liquid level oscillates at the frequency of the disturbance. Its response is delayed, in that the level reaches its peak some time after the inlet flow has peaked. Notice that the amplitude of oscillation decreases as the frequency increases. This indicates that the tank cannot follow fast changes.

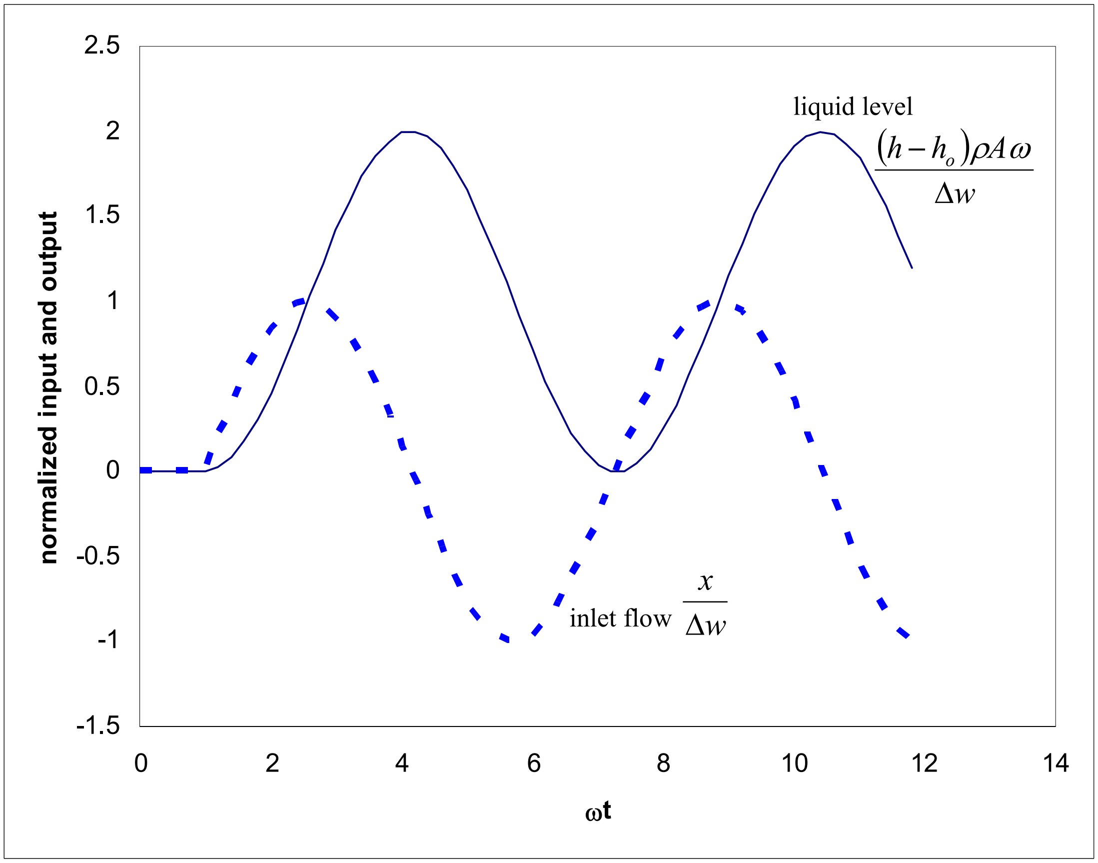


Figure 2-4. The onset of disturbance ωt_d has arbitrarily been set to 1.

More detail

Response to a sine disturbance has two parts – the initial transient, and a recurring oscillation. We can recast the response in this form by using the sum-of-angles formula to write the cosine as a sine that includes a phase angle.

$$h(t) = h_o + \frac{\Delta w}{\rho A\omega} + \frac{\Delta w}{\rho A\omega}\sin\left(\omega(t-t_d) - \frac{\pi}{2}\right) \qquad (2.7.2)$$

Equation (2.7.2) shows that the output lags the input by $\pi/2$ radians, or 90°. The liquid level differs at most from its initial value by twice the amplitude. It either exceeds or stays below the initial level according to the sign of Δw; that is, whether the flow initially increased or decreased.

2.8 Typical disturbances

Knowing how a system responds to disturbances is a prerequisite for controller design. We will use the impulse, step, and sine disturbances repeatedly to test various dynamic systems. While we can never test our control designs against every conceivable disturbance, testing against these standard ideal disturbances will usually tell us what we need to know.