STATUS REPORT #6
ON
NONLINEAR AND ADAPTIVE CONTROL
NASA GRANT NAG 2-297
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SUBMITTED TO:
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SUMMARY

This status report overviews the research on Nonlinear and Adaptive Control carried out at the MIT Laboratory for Information and Decision Systems under NASA grant NAG 2-297 for the time period 1 July 1987 to 31 January 1988. Participating faculty were Professors Gunter Stein, Lena Valavani, and Michael Athans (principal investigator). The grant monitors are Dr. George Meyer (NASA Ames Research Center) and Mr. Jarrell R. Elliott (NASA Langley Research Center).

The primary thrust of the research is to conduct fundamental research in the theories and methodologies for designing complex high-performance multivariable feedback control systems; and to conduct feasibility studies in application areas of interest to our NASA sponsors that point out advantages and shortcomings of available control system design methodologies.

The theoretical research overviewed in this status report is focused on adaptive and nonlinear systems. On-going feasibility studies completed during this reporting period relate to the development of a rule-based decision aid which can provide automated help to the pilot of a C-130 aircraft in the case of elevator-jam failures. Significant progress in all areas has been accomplished during this reporting period.
1. ROBUST COMPENSATOR DESIGN

Research Goals. Our research to date has pinpointed the need for a good initial guess for an adaptive compensator, whose parameters are then updated, in real-time, by the adaptive algorithm. We are developing techniques that design the best (from the viewpoint of good command-following and disturbance-rejection) nonadaptive compensator for the given prior plant uncertainty information. It is yet unknown how to design such nonadaptive compensators that exhibit this property of "best" performance-robustness.

Such a robust design technique will prove useful in a number of ways. First, it will yield a systematic procedure for designing feedback systems for uncertain plants with performance guarantees. Thus, the feedback loop will be guaranteed to be stable and, in addition, will meet minimum performance specifications for all possible plant perturbations. Second, the solution of this robust design problem will also enable us to quantitatively address one of the most fundamental questions in adaptive control: what are the performance benefits of adaptive control? While much attention has been paid to the development of many specific adaptive algorithms, very little consideration has been given to this issue at the heart of the adaptive control problem. Practical adaptive systems rely upon external persistently exciting signals (to ensure good identification), slow sampling (which helps stability-robustness to unmodeled high frequency dynamics) in addition to extensive real-time computation (to provide safety nets and turn-off the adaptive algorithm when it exhibits instability). All these "gimmicks" degrade command-following and disturbance-rejection performance and tend to neutralize the hoped-for benefits of an adaptive compensator. In light of these circumstances it is imperative that the decision to use adaptive control, for a real engineering application, must be based upon a quantitative assessment of costs and benefits. One of the main goals of this research project is to quantitatively evaluate the performance benefits of an adaptive control system vis-a-vis the best fixed-parameter nonadaptive compensator for a linear plant. Note that for a nonlinear system the parameters of such compensators can be fine-tuned using gain scheduling.

Research Methodology. In his doctoral research Mr. David Milich, under the supervision of Professors Athans and Valavani, has examined design techniques which will yield the "best" fixed-parameter nonadaptive compensator for a plant characterized by significant structured, as well as unstructured, uncertainty. The "best" compensator is defined as the one that meets the posed performance (i.e. command-following, disturbance-rejection, insensitivity to sensor noise) specifications and stability-robustness over the entire range of possible plants.

Some of the key issues, and severe difficulties, in the design process have been
Conditions for stability-robustness and performance-robustness in the presence of significant structured and unstructured uncertainty have been developed. An a-priori magnitude bound, as a function of frequency, on the unstructured uncertainty is assumed known. In order to reduce the conservatism of the stability and performance conditions with respect to the structured uncertainty, directional information (in the complex plane) associated with the plant-parameter variations is exploited. Unfortunately, this directional information turns out to be closely associated with the so-called \textit{Real-}\(\mu\) \textit{problem}, i.e. the problem of calculating structured singular values for real -- rather than complex-valued -- plant modeling errors; this problem has been studied by Doyle and is generically very difficult. Its solution appears to be beyond the state of the art, at least in the near future.

The only reasonable alternative appears to be to translate the prior knowledge of structured uncertainty into an equivalent unstructured uncertainty. It is still a very hard problem to design a compensator with guaranteed performance characteristics in the presence of these modeling errors. We have transformed the problem into what Doyle calls the \(\mu\)-\textit{synthesis problem}, which unfortunately is also very hard to solve. From a technical point of view, the \(\mu\)-\textit{synthesis problem} involves a blend a co-prime factorizations, structured singular value theory, and \(H^\infty\)-optimization. Doyle has developed a method, called the D,K iteration, which converges to local minima.

\textbf{Recent Research Progress:} While the analysis aspect of LTI feedback design is well-established, the \(\mu\)-\textit{synthesis problem} remains open. The purpose of this research has been to develop a practical methodology (based on \(\mu\)) for the synthesis of robust feedback systems. That is, the design process will ensure the resulting feedback system is stable and performs satisfactorily in the event the actual physical plant differs from the design model (as it surely will). The motivation for an alternative to D,K iteration is due to the nonconvex nature of the \(\mu\)-\textit{synthesis problem}. Nonconvexity may lead to local minima, therefore it is essential that several independent methods be available to examine the problem.

Our research has produced a new approach to the design of LTI feedback systems. We call it the "Causality Recovery Methodology (CRM)". For a given plant, the Youla parameterization describes all stabilizing compensators in terms of a stable, causal operator \(Q\). LTI feedback design may be viewed as simply a procedure for choosing the appropriate \(Q\) to meet certain performance specifications. Thus, the design process imposes two constraints on the free parameter \(Q\): (1) stability and causality (i.e. \(Q\) must be an \(H_\infty\) function); (2) \(Q\) must produce a closed-loop system that satisfies some performance specification. The design objective of interest here
is performance robustness, which can be stated in terms of a frequency domain inequality using the structured singular value.

The CRM initially lifts the restriction of compensator causality and the synthesis problem with uncertainty is examined at each frequency. A feasible set of Q's in the space of complex matrices satisfying the performance specification is constructed. Causality is then recovered via an optimization problem which minimizes the Hankel norm (i.e. the measure of noncausality) of Q over the feasible set. If the problem is well posed (i.e. the performance specifications are not too stringent given the amount of modeling uncertainty), the resulting compensator nominally stabilizes the feedback system and guarantees robust stability and performance.

The theoretical foundation for the methodology have been established. Next, a research algorithm was written so that we can obtain numerical results. It was applied to two design examples to demonstrate its effectiveness. Excellent robust performance was obtained. However, the current generation of our CRM algorithms require very extensive off-line computational resources, because of the several optimization problems that must be solved to design the robust compensator.

**Documentation Status.** Only partial documentation exists [20] for this research. The Ph.D. thesis of Mr. Milich is scheduled for completion in February 1988 [29].
2. NONLINEAR CONTROL SYSTEMS.

A significant portion of the grant resources is devoted to the development of methodologies, theories, and design techniques that will advance the state of the art in multivariable control system design. During this reporting period we have made some significant progress in this area.

2.1 Systems with Multiple Saturation Nonlinearities.

Research Goals. The goal of this project is to develop new theory and methodologies for the analysis and synthesis of linear multivariable control systems that contain several saturation nonlinearities. We seek to develop modifications to the purely linear design methodologies, such as LQR, LQG, LQG/LTR, and H-∞ optimization, to explicitly take into account the problems associated with multiple saturation (magnitude and/or rate) nonlinearities in the control actuation channels.

There are several problems that can arise when a control system that has many saturation nonlinearities is designed by purely linear means. The most serious problem is that of stability; it is possible for a control system, which is stable when the actuators are not saturated, to become unstable when one or more controls become saturated. Such instability can happen if large command signals are applied or disturbances of large magnitude are present. The second class of problems are associated with performance. If the saturation limits are ignored in the purely linear design phase, it may happen that large crossover frequencies are specified by the designer. The actuators may not be able to provide the gain necessary to attain the required bandwidths; also, rate-limiting may not allow the physical controls to change as rapidly as a purely linear design demands. Hence, redesign must take place. However, in multivariable designs, by far the most serious degradation to the control system performance occurs because the saturation nonlinearities distort the direction of the commanded control. Changes in the direction of the control vector cause oscillatory responses which may be unacceptable from a performance viewpoint. Also, transient performance suffers when saturation nonlinearities interact with integrators in the control loop; the so-called reset windup phenomenon. Reset windup keeps the nonlinearities saturated longer than necessary, and as a consequence transient responses are characterized by large overshoots.

Research Methodology. Our plans are to examine these stability and performance problems associated with multiple saturations in a unified manner. Most of the existing theory is either too complex or incomplete. It is possible to deal with saturation nonlinearities using optimal control theory, and derive necessary conditions using Pontryagin's maximum principle; unfortunately, this only provides us with open-loop solutions through the solution of complex two point...
boundary value problems for high-order plants. Most other approaches are based upon Lyapunov theory, which does not capture in a straightforward way the input-output behavior necessary for design.

In our research to date, we have focused attention to the changes in the direction of the control signals that are induced by the saturating elements. The fact that we cannot deliver the "correct" magnitude should not produce any unpleasant effects except that the settling times should increase. What we want is to avoid is the highly oscillatory transients and unstable behavior. This appears to be more related to the changes in the directions of the control vectors.

Recent Research Progress. A major milestone was completed during this reporting period with the completion of the Ph.D. thesis of Petros Kapasouris, under the supervision of Professor Athans. We were able to come up with simple, yet elegant, ways of attacking the problem. The algorithms are different depending on whether or not the compensator is stable or unstable.

For closed-loop designs that use stable compensators to control stable plants, the concept is to have the command-following response of the MIMO system mimick, to the extent possible by the presence of the saturation nonlinearity, the transient response of the linear system. The idea is to monitor and adjust in real-time the tracking error vector, which acts as the input to the dynamic compensator so that the compensator never generates signals that will drive the system into saturation. In this manner, we are able to maintain the necessary "directional" properties of the design which are required to carry-out the \emph{approximate plant inversion} and substitution of the "desired" dynamics in the forward loop associated with modern multivariable design methodologies. Note that if we allow arbitrary saturation of the nonlinearities, the directional properties of the linear design become distorted; as a consequence, we destroy the approximate plant inversion property of our compensator. The method under study controls the signal levels so that the system always works in the linear region. \emph{This key idea appears to solve all at once the undesirable stability, performance, and reset-windup issues}. Of course, as to be expected, the speed of response (rise time, settling time etc) to commands of large magnitude is reduced compared to the design without saturation nonlinearities.

In order to implement this scheme one has to execute some off-line and some on-line computations. The off-line computations require the computation of the boundary of a convex compact set, with several nondifferentiable points. This set is defined over a Euclidean space whose dimension is that of the dynamic compensator. The on-line computations calculate a (pseudo)gradient vector to the boundary of the set, and adjust a scalar which reduces the instantaneous size of the tracking error vector. This causes the dynamic compensator to generate a control signal that never saturates.
We have used some linearized dynamics of the F-8 aircraft, to which we added a fictitious flaperon, to test these ideas. In this setting we command changes in both the flight path and pitch angles; these are to be controlled using the elevator and the flaperon. In this set of transient simulations the results show excellent nonlinear responses.

For feedback designs that contain open-loop unstable plants, or unstable compensators, it is important to limit the set of initial states, disturbances and commands so that the system can be stabilized. Assuming that the system is at rest and that the disturbance environment is such that the system can be stabilized, then the problem is to limit in an intelligent manner the size of the command (reference) vector. This is accomplished by a method that modulates the size of the command vector, and the rate at which it is applied, so that the controls do not saturate; eventually, the full command vector is applied. The nature of the computations is similar as in the open-loop stable case. However, the dimension of the underlying sets is now much larger.

We have used a model of the AFTI F-16 aircraft, which is open-loop unstable, to test the algorithm. As before, we are using the aircraft elevon and flaperon to control the pitch and flight path angles. Once more, the transient responses are excellent.

Similar ideas can be used to handle rate saturation, and simultaneous magnitude and rate saturation. Also, the same concepts can be used to ensure that certain state and/or output variables do not exceed prespecified limits (often introduced on the basis of safety considerations).

*Documentation Status.* Partial documentation of earlier research can be found in the paper by Kapasouris and Athans [5]. Full documentation can be found in the just completed doctoral thesis of P. Kapasouris [28]. Several papers are under preparation for publication.

### 2.2 Gain Scheduled Control Systems

Gain scheduling is a common engineering method used to design controllers for systems with nonlinear and/or parameter varying dynamics. In the nonlinear case, the dynamics are linearized at several operating points, and a linear compensator is designed for each linearized plant. The parameters of the compensator are then interpolated, or scheduled, in between operating points, thus resulting in a global compensator. The procedure for linear parameter varying dynamics is identical to that above, except that the linearization is omitted.
Research Goals. Despite the lack of a sound theoretical analysis, gain scheduling is a design methodology which is known to work in many engineering applications (e.g. jet engines, submarines, and aircraft). In the absence of such an analysis, a complete and systematic design methodology has yet to emerge. In its place, a collection of intuitive ideas has develop into heuristics for gain scheduled designs. Two common examples are: "the scheduling variable should vary slowly" and "the scheduling variable should capture the plant's nonlinearities." Thus, a sound analysis of various gain scheduling scenarios would prove very useful in better understanding these designs. Hopefully, this analysis would formalize the popular notions regarding the design of gain scheduled control systems. The analysis would then be used towards the ultimate goal to develop a complete and systematic gain scheduling design framework.

Recent Research Progress. This research is being carried out by Mr. Jeff Shamma under the supervision of Professor Athans. We have identified and analyzed three different gain scheduling scenarios: 1) Linear plants scheduling on an exogenous parameter, 2) Nonlinear plants scheduling on a reference input trajectory, and 3) Nonlinear plants scheduling on the plant output.

The first case of linear parameter varying plants can be described as follows. Using the gain scheduling procedure outlined above, the resulting closed-loop global design can be modeled as a linear parameter varying system. This feedback system has the property that for each frozen value of the parameter, the closed-loop dynamics have excellent feedback properties (by design), such as robust stability, robust performance, disturbance rejection etc. However, these properties need not carry over to the time varying case. In fact, even nominal stability can be lost in the presence of parameter time variations. Thus, we have developed sufficient conditions for stability and stability-robustness for linear parameter varying systems. More precisely, we have shown that stability and stability-robustness is maintained for sufficiently slow time variations. This is not surprising since the original local designs were based on line time-invariant approximations to the time varying plant. Research is ongoing regarding the possible conservatism of these stability tests. However, these tests have been used to guarantee stability of a gain scheduled design for the F-8 aircraft reported in [Stein et.al, "Adaptive Control Laws for F-8 Flight Test", IEEE Trans. on Auto. Control, Vol. AC-22, No. 5, October 1977].

Various additional insights have been obtained regarding the design for such parameter varying systems. Recall that in the case of nominal stability, it was shown that stability is maintained for sufficiently slow parameter variations. However, a quantitative statement of this condition reveals that the restrictions on the parameter variations critically depend on an overshoot-like property of the closed loop design.
This overshoot-like property is very sensitive to the scaling of the compensator state-variable. In fact, it is possible that a rescaling of compensator state-variables can significantly alter the stability properties of the resulting closed loop design. This distinction is important since only input/output aspects of the compensator (such as its frozen parameter frequency response) have been the focus of gain scheduling designs.

New insights have also been obtained in the analysis of the stability-robustness of the parameter varying system. The sufficient conditions for stability-robustness are very similar to their time-invariant counterparts in that they take the form of frequency-domain inequalities. However, these inequalities must be evaluated along a line parallel to the jω-axis in the left half s-plane. This implies that different information must be available regarding the nature of the unmodeled dynamics. In the absence of such information, it is shown that one can still use the time-invariant stability-robustness tests. However these tests must be satisfied with a greater degree of relative stability.

Guaranteed global properties for the cases of a nonlinear plant scheduling on either a reference trajectory or the plant output have also been analyzed. For such systems one has that, at each moment in time, the linearized closed loop system has excellent feedback properties. As in the parameter varying case, it is reasonable to ask under what conditions do these properties carry over to the global nonlinear case. In the case of scheduling on a reference trajectory, it was shown that these properties are maintained if 1) The reference command trajectory is sufficiently slow & 2) The reference command trajectory and corresponding reference control trajectory do not excite the unmodeled dynamics. In the case of scheduling on the plant output, it was shown that the various feedback properties are maintained if 1) The plant output is a naturally slow variable & 2) The plant output captures the bulk of plants nonlinearities.

The main idea behind all of these results may be summarized as follows. Gain-scheduled designs are based on linear time-invariant approximations of the true plant. If one wishes the feedback properties of the local designs to carry over to the global design, the true plant should not differ greatly from the approximate design plants. It turns out that in the case of scheduling on an exogenous parameter, this amounts to requiring the parameter to vary sufficiently slowly. In the case of a nonlinear plant scheduling on a plant output, this amounts to requiring the plant output to vary sufficiently slowly and capture the plant nonlinearities. Note that these are precisely the intuitive ideas which have guided existing gain-scheduled designs. However, this analysis has formalized these notions and transformed them into quantitative statements.
At present, we are investigating the design of gain scheduled compensators for linear parameter-varying plants. Initial research has shown that closed loop instability can occur even in the deceivingly innocent case of identical closed loop frequency responses for each value of the parameter. However, this instability was removed by exploiting the guaranteed stability properties of the time-varying Kalman filter. Thus, we are investigating to what extent this technique can be generalized and used to give new guaranteed stability/robustness closed loop properties.

Documentation Status. Only partial documentation of this research is available at present; see [26]. Mr. Shamma's doctoral thesis is scheduled for completion in the summer of 1988.

2.3 Sliding Mode Controllers for Multivariable Systems.

Sliding mode control is a technique within the variable structure methodologies which has been used to design SISO nonlinear systems, in controllable canonical form, and for a limited class of multivariable systems. Mr. Benito Fernandez, under the direction of Professor Hedrick, is developing a new methodology for designing nonlinear multivariable controllers using the sliding mode concepts, including guarantees of closed loop nominal stability, stability-robustness to unmodeled dynamics, and performance.

A major feature of the methodology is the relationship between the input-output linearization of invertible nonlinear systems and the sliding mode approach, when the error dynamics on the sliding mode surfaces are chosen to be linear and time-invariant.

3. FEASIBILITY STUDIES

3.1 Adaptive Redesign Strategies Following Failures

It is important to develop both high level (symbolic) and low level (quantitative) strategies for coping with control surface failures in aircraft. To compensate for a control surface failure, sufficient redundancy in the control authority must be provided by other control surfaces, thrust and moment producing mechanisms. To understand these issues, presently configured aircraft provide an opportunity for the development of such strategies.

Control failures in aircraft are not uncommon. Military aircraft can expect frequent damage to their control surfaces from enemy fire. However, even civil aircraft undergo such failures. A brief survey in [21] yielded almost 30 cases in which there were failures of controls other than engines. In all but five of these incidents, such malfunctions resulted in crashes, and loss of life to passengers and crew. In about half of these cases, the flight could have ended safely if the pilot had acted in a correct and timely manner; unfortunately, present procedures and training are inadequate to prevent many such accidents because corrective action must be taken extremely fast. What is needed is an automated means of helping the pilot to utilize the implicit multivariable redundancy of his many surfaces and thrust producing mechanisms so as to recover positive control of the aircraft.

The recently completed Ph.D. thesis of E. Wagner [21], under the supervision of Professor Valavani, has made important strides toward the development of an on-board automated aid advisory for a C-130 aircraft. A rule-based expert system was developed to handle elevator-jam failures for the C-130 aircraft and its value illustrated using extensive simulations. This expert system produces an intelligent guide to pre-simulations of alternative controls (elevator tab, collective ailerons, symmetric flaps and engine thrust) using a high fidelity model of the aircraft. Pre-simulation of a recovery strategy was crucial because (a) often even a few degrees of available deflections could make all the difference, and (b) side-effects of doing the wrong thing could be devastating. The rule-based system was programmed using the OPS5 program.

3.2 Multivariable Designs for the F-18 Aircraft.

Mr. Voulgaris, under the supervision of Professor Valavani, has been using the $H_\infty$ design methodology to design multivariable control systems using the dynamics of the F-18 aircraft provided to us by the NASA Langley Research Center. Further details will be provided in the next progress report.
PEOPLE

Professor Lena Valavani was appointed Boeing Assistant Professor of Aeronautics and Astronautics.

Professor Gunter Stein was re-appointed Associate Editor at Large of the IEEE Transactions on Automatic Control.

Professor Lena Valavani was re-appointed Associate Editor of the IEEE Transactions on Automatic Control.
PUBLICATIONS

The following publications have been supported in full or in part by NASA grant NAG 2-297 since its inception 1 June 1984. Copies of these publications have been transmitted to the grant monitors, NASA headquarters, and publications office as required.


32. M. Athans, G. Stein, and L. Valavani, "Status Report #6 on Nonlinear and