# Introduction to Modeling and Simulation,

**MIT** − 1.992, 2.993, 3.04, 10.94, 18.996, 22.091
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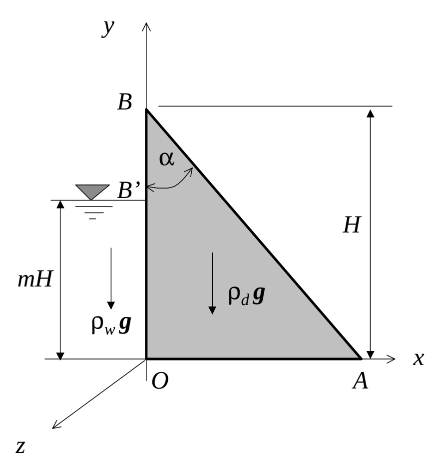
**Water Filling of a Gravity Dam** We consider a gravity dam of triangular shape (Height $H$, Top angle $\alpha$) clamped on its base $OA$ (zero displacement prescribed), see figure below. The down-stream surface $AB$ is stress free. The upstream wall $OB$ is subjected to the water pressure (body force $\rho_w g$) over a height $OB' = mH$, where $m \in [0, 1]$. The remaining surface $B'B$ is stress free. For the purpose of analysis, we assume that the gravity dam is composed of a linear isotropic elastic material ($E, \nu$ or $\lambda, \mu$) with volume mass $\rho_d$. With regard to the dimension of the gravity dam in the $Oz-$direction, the problem can be treated as a plane strain problem with regard to the plane parallel to $Oxy$.

We want to evaluate the displacement of the dam along the upstream wall $OB$ with water filling $m \in [0, 1]$.



1. **First approximation (In CLASS)**: We consider a displacement field of the form:

$$\boldsymbol{\xi}' = a_0 \frac{y}{H}\mathbf{e}_x + b\frac{y}{H}\mathbf{e}_y$$

Using the displacement based variational method, give a first approximation of the displacement along $OB$. Show that the displacement based variational approach can be recast in the format of the finite element method:

$$k_{ij} q_j = F_j$$

where $k_{ij}$ are the components of the stiffness matrix, $q_j$ are the unknown nodal displacements, and $F_j$ are nodal forces. Determine $k_{ij}$ and $F_j$ for the given approximation of the displacement $\boldsymbol{\xi}'$ in the gravity dam.

2. **Finite Element Approximation**: Consider $m = 1$, and $\rho_d = 0$. By using finite elements, show that the horizontal displacement solution converges with increasing number of elements. To this end, normalize the obtained displacement at $y = H$ by:

$$\Xi = \frac{\xi_x(x = 0, y = H)}{\left(\frac{\rho_w g H^2}{3\mu \tan\alpha}\right)}$$

with $\mu$ the shear modulus. Represent your results in form of a graph giving $\Xi$ as a function of the number of elements over $OB$.