22 CARTESIAN NAVIGATION

The bulk of our discussion on maneuvering and control has assumed that the necessary system states can be measured. The marine engineer is in fact faced with choices between many different basic sensor packages, notably compasses, paddle wheels, inertial navigation units, rate gyros, and depth guages, for example. These listed sensors are *self-contained* and rely primarily on the physical properties of the natural environment. There is also a class of *distributed* sensor systems; these generally involve an array of communicating elements, located remotely from the vehicle. We present the fundamental concepts behind two methodologies in this second class: the global positioning system (GPS) and acoustic navigation, both of which can provide high-accuracy absolute Cartesian navigation.

22.1 Acoustic Navigation

Consider a *transponder* A, which can transmit an acoustic signal, and also measure, with microsecond accuracy, the time to receive a reply. Next, place a *responder* B at a distance R away from A; the job of B is just to transmit a signal whenever it receives one, with a (short) predictable response time T_t. Thus, the elapsed time T between a tranmission and consequent reception at A is

$$T = T_t + 2R/c_w,$$

where c_w is the speed of sound in water, about 1450m/s. The range R follows by inversion:

$$R = \frac{(T - T_t)c_2}{2}.$$

Suppose that the location of B is known; then a given measurement of R places A on a sphere around B. It is a case of three unknowns (x, y, z) and one equation:

$$(x_A - x_B)^2 + (y_A - y_B)^2 + (z_A - z_B)^2 = R^2.$$

The introduction of a another responder C (typically listening and responding at a different frequency than B) places A on the intersection of two spheres, i.e., a circle. There are two equations, but still three unknowns. When A, B, and C lie in a nearly horizontal plane, then the intersection circle lies in a vertical plane; the addition of a depth sensor to our suite would allow us to pin A's location at one of two points on the circle. Finally, if we know which side A is on, and do not allow for abrupt crossovers, then we have a functional set of measurements for acoustic navigation with just two responders. The *baseline* is the line connecting responders B and C; when A is near this baseline, positional accuracy will be very poor since the two solution spheres are tangent.

Better and better performance can be obtained by increasing the number of responders, and consequently of the baselines and spheres. With three responders, for example, the intersection of a sphere (responder D) and a circle (responders B and C) is two points. Here there are three equations and three unknowns, but the nonlinearity of the equations leads to the non-uniqueness in the solution. A fourth responder or a depth transducer would be needed to completely constrain the solution.

The above discussion is a minimum conceptual explanation of acoustic navigation. There are many other pieces to the approach, including an account of the variation of sound speed c_w with water depth, obtaining the Cartesian locations of the responder network, and handling various geometric configurations that give rise to poor or degenerate solutions.

There are two common configurations used for acoustic navigation, named for the length of the baselines relative to the target (transponder A) range.

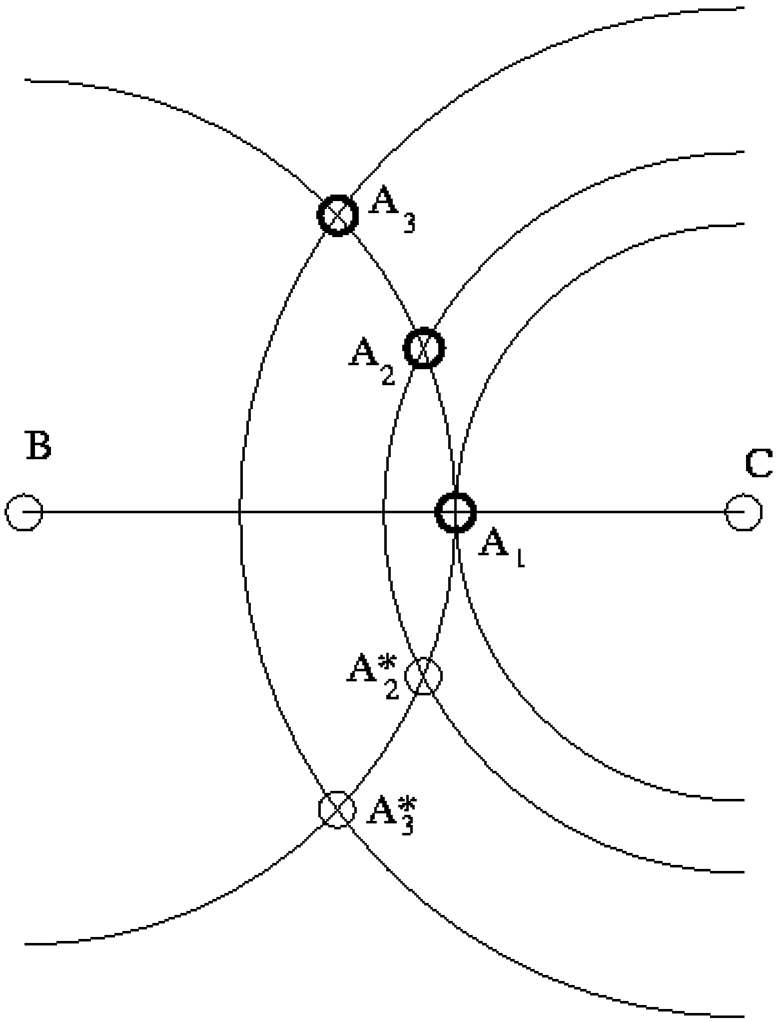

Figure 15: *Left: General configuration of a long-baseline acoustic system, with target transponder A and fixed responders B and C. The position A_1 is on the baseline and has poor accuracy in the direction normal to the baseline; in contrast, A_3 is well-posed. A_3^* is an additional solution to the two-responder problem. Right: General configuration of a short-baseline system.*

(Ultra) Short-Baseline

The geometry of SBL or USBL puts very short baselines between the responders compared to the target distance. For instance, the fixed net is often attached to a vessel or other structure, with baseline lengths on the order of 10-100m. Typical frequencies in use are around 100kHz, with a working range of 100-500m to the target. The wavelength of a 100kHz signal is about 1cm and 5cm is a reasonable estimate of the accuracy for these systems.

Long-Baseline

Long-baseline systems typically involve a larger responder net, and the target ranges are similar to the baseline lengths. Very large systems utilize frequencies of 10-15kHz (for a placement accuracy around 2-5m), and may have ten-kilometer baselines. Frequent sources of error in long-baseline systems are variations of c_w, and also in multipath. In the latter condition, false signals can be caused by reflections off the seafloor and the surface; these signals are typically eliminated by rejecting those receptions which are outside a very tight and slowly moving window on travel time

T. Sometimes, bottom topography can shield a direct acoustic path, and the only receptions available are via multipath!

22.2 Global Positioning System (GPS)

The GPS system is the most powerful system publicly available for absolute position reference above water. It is similar in concept to the acoustic navigation systems described above, but with one fundamental difference: only the one-way travel time from each satellite is measured at the target. The accuracies achieved with GPS are therefore strongly dependent on the accuracy of time-keeping.

The core of the system is a network of 24 satellites arranged in six planes (4 per plane) inclined at 55 degrees from the equatorial plane, at an altitude of about 20,000km. The altitude is chosen carefully to correspond with a 12-hour orbit; geosynchronous orbit is much higher, at around 40,000km. The orbit lines over a *non-rotating* earth are thus near-sinusoids that reach lattitudes of 55°N and 55°S, and the six lines are spaced 60° apart in longitude. With reference to a *rotating* Earth, the spatial frequency is doubled, so that the longitudinal distance between tracks is 30 degrees, or 1800 nautical miles. Each satellite transmits a regular signal which contains, among other items, the time of its transmission, and the three-dimensional location of the satellite (the *ephemeris*). It is from these two pieces of information, from many satellites, that triangulation and navigation are performed at the target.

For the ideal case that the time bases of the target and the satellites are exactly aligned, triangulation of the type described for acoustic navigation is possible. In practical terms, the speed of light $c = 3 \times 10^8 m/s$implies that a 1ns timing error will cause a 0.3m range error. For this reason, each of the satellites carries four atomic clocks on board, good to 1ns per day. The time base is continually monitored and updated from a ground station, and accurate to within 1ns. The ephemeris of each satellite is also monitored and updated from the ground station, using a combination of least-squares analysis of past data (1 week), and a Kalman filter to predict the future ephemeris. The accuracy of satellite position is better than 10m.

We now come to the last thorny issue: how to make target receivers that don't require atomic clocks! The solution is very clever, and illustrated for the case of two-dimensions; the arguments for three dimensions are the same. In two dimensions, the range measurements from two sources locate the target on one of two points (the intersection of two circles). Suppose that the target clock is too slow by an amount $\tilde{t}$, so that the range estimate is

$$r + \tilde{r} = c(t + \tilde{t}).$$

The estimated range circles are too large, and the estimated location of the target too far from the baseline. Introduce a third satellite now so that the intersections of the circle pairs now occur at six points. Three of these are very close, and indicate the approximate true solution. The trick is to find the correction for $\tilde{t}$that puts the three close solutions onto a single point. This single point is near the centroid of the three approximate points, and represents the best position solution. The target time base can be kept up to date by performing this check for every set of signals. Hence, the two-dimensional timing problem is solved with three satellites; the three-dimensional timing problem is solved with four satellites.

The position specification for GPS is 25m, at the 95'th percentile. This is a remarkable feat, given that it is an absolute measure over the entire surface and atmosphere of our 6000km-radius Earth. Major sources of error include: clock base and satellite navigation ($\approx 2.5m$), ionosphere and troposphere electromagnetic variations ($\approx 2.5m$), and inaccuracies in the receiver and multipath ($\approx 2m$). GPS is also subject to *selective availability* or S/A, the addition of a slow-varying random component in the satellite ephemeris data. Selective availability is controlled by the United States Department of Defense, and degrades the position specification to 100m (typical). The advent of *differential GPS* solved most of the S/Aconcerns of American allies, by providing high-accuracy navigation in local areas. The idea here is that a stationary target can detect the effects of S/A (since it is not moving) and then transmit the corrections over a small geographical area. Many current GPS receivers are capable of decoding these local corrections, which are then applied to their own satellite navigation processing. Differential GPS typically provides 1-2m accuracy.

Finally, as with the acoustic navigation systems described, an independent altitude measurement will enhance the accuracy of GPS, essentially reducing a three-dimensional to a two-dimensional problem.