

12 PROPELLERS AND PROPULSION

12.1 Introduction

We discuss in this section the nature of steady and unsteady propulsion. In many marine vessels and vehicles, an engine (diesel or gas turbine, say) or an electric motor drives the propeller through a linkage of shafts, reducers, and bearings, and the effects of each part are important in the response of the net system. Large, commercial surface vessels spend the vast majority of their time operating in open-water and at constant speed. In this case, steady propulsion conditions are generally optimized for fuel efficiency. An approximation of the transient behavior of a system can be made using the quasi-static assumption. In the second section, we list several low-order models of thrusters, which have recently been used to model and simulate truly unsteady conditions.

12.2 Steady Propulsion of Vessels

The notation we will use is as given in Table 1, and there are two different flow conditions to consider. *Self-propelled* conditions refer to the propeller being installed and its propelling the vessel; there are no additional forces or moments on the vessel, such as would be caused by a towing bar or hawser. Furthermore, the flow around the hull interacts with the flow through the propeller. We use an **sp** subscript to indicate specifically self-propulsion conditions. Conversely, when the propeller is run in open water, i.e., not behind a hull, we use an **o** subscript; when the hull is towed with no propeller we use a **t** subscript. When subscripts are not used, generalization to either condition is implied. Finally, because of similitude (using diameter **D** in place of **L** when the propeller is involved), we do not distinguish between the magnitude of forces in model and full-scale vessels.

R_{sp}	N	hull resistance under self-propulsion
R_t	N	towed hull resistance (no propeller attached)
T	N	thrust of the propeller
ω_e	Hz	rotational speed of the engine
ω_m	Hz	maximum value of ω_e
ω_p	Hz	rotational speed of the propeller
λ		gear ratio
Q_e	Nm	engine torque
Q_p	Nm	propeller torque
η_g		gearbox efficiency
P_e	W	engine power
P_p	W	propeller shaft power
D	m	propeller diameter
U	m/s	vessel speed
U_p	m/s	water speed seen at the propeller
Q_m	Nm	maximum engine torque
\dot{f}	kg/s	fuel rate (or energy rate in electric motor)
\dot{f}_m	kg/s	maximum value of \dot{f}

Table 2: Nomenclature

12.2.1 Basic Characteristics

In the steady state, force balance in self-propulsion requires that

$$R_{ap} = T_{ap}. \quad (134)$$

The gear ratio λ is usually large, indicating that the propeller turns much more slowly than the driving engine or motor. The following relations define the gearbox:

$$n_a = \lambda n_p \quad (135)$$

$$Q_p = \eta_g \lambda Q_a,$$

and power follows as $P_p = \eta_g P_a$, for any flow condition. We call $J = U_p / n_p D$ the advance ratio of the prop when it is exposed to a water speed U_p ; note that in the wake of the vessel, U_p may not be the same as the speed of the vessel U . A propeller operating *in open water* can be characterized by two nondimensional parameters which are both functions of J :

$$K_T = \frac{T_a}{\rho n_p^2 D^4} \quad (\text{thrust coefficient}) \quad (136)$$

$$K_Q = \frac{Q_{pa}}{\rho n_p^2 D^5} \quad (\text{torque coefficient}). \quad (137)$$

The open propeller efficiency can be written then as

$$\eta_o = \frac{T_a U}{2\pi n_p Q_{pa}} = \frac{J(U) K_T}{2\pi K_Q}. \quad (138)$$

This efficiency divides the useful thrust power by the shaft power. Thrust and torque coefficients are typically nearly linear over a range of J , and therefore fit the approximate form:

$$K_T(J) = \beta_1 - \beta_2 J \quad (139)$$

$$K_Q(J) = \gamma_1 - \gamma_2 J.$$

As written, the four coefficients $[\beta_1, \beta_2, \gamma_1, \gamma_2]$ are usually positive, as shown in the figure.

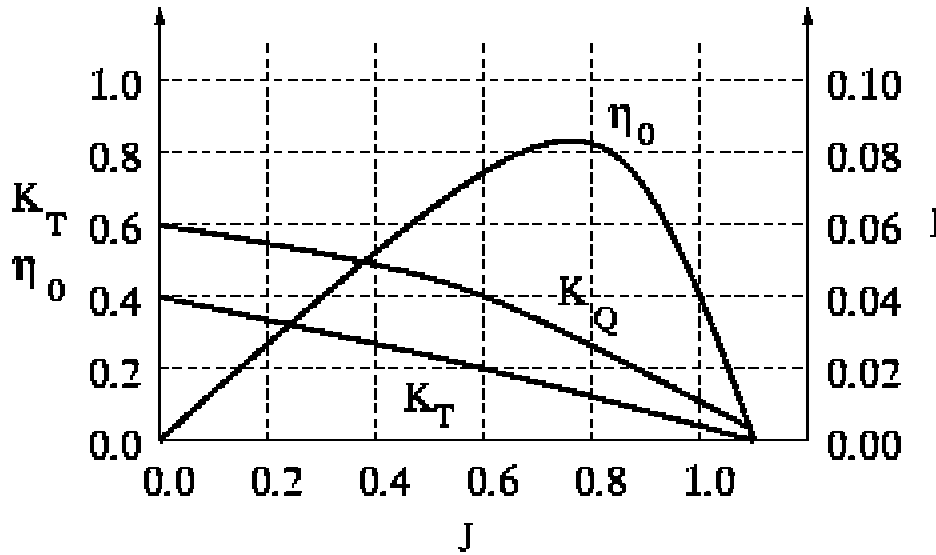


Figure 4: Typical thrust and torque coefficients.

We next introduce three factors useful for scaling and parameterizing our mathematical models:

$U_p = U(1 - w)$; w is referred to as the *wake fraction*. A typical wake fraction of 0.1, for example, indicates that the incoming velocity seen by the propeller is only 90% of the vessel's speed. The propeller is operating in a wake.

In practical terms, the wake fraction comes about this way: Suppose the open water thrust of a propeller is known at a given U and n_p . Behind a vessel moving at speed U , and with the propeller spinning at the same n_p , the prop creates some extra thrust. w scales U at the prop and thus J ; w is then chosen so that the open water thrust coefficient matches what is observed. The wake fraction can also be estimated by making direct velocity measurements behind the hull, with no propeller.

$R_t = R_{ap}(1 - t)$. Often, a propeller will increase the resistance of the vessel by creating low-pressure on its intake side (near the hull), which makes $R_{ap} > R_t$. In this case, t is a small positive number, with 0.2 as a typical value. t is called the *thrust deduction* even though it is used to model resistance of the hull; it is obviously specific to both the hull and the propeller(s), and how they interact.

The thrust deduction is particularly useful, and can be estimated from published values, if only the towed resistance of a hull is known.

$Q_p = \eta_R Q_{pr}$. The *rotative efficiency* η_R , which may be greater than one, translates self-propelled torque to open water torque, for the same incident velocity U_p , thrust T , and rotation rate η_p . η_R is meant to account for spatial variations in the wake of the vessel which are not captured by the wake fraction, as well as the turbulence induced by the hull. Note that in comparison with the wake fraction, rotative efficiency equalizes torque instead of thrust.

A common measure of efficiency, the quasi-propulsive efficiency, is based on the towed resistance, and the self-propelled torque.

$$\begin{aligned} \eta_{QP} &= \frac{R_t U}{2\pi n_p Q_{pr}} & (140) \\ &= \frac{T_o(1-t)U_p \eta_R}{2\pi n_p(1-w)Q_{pr}} \\ &= \eta_o \eta_R \frac{(1-t)}{(1-w)}. \end{aligned}$$

T_o and Q_{pr} are values for the inflow speed U_p , and thus that η_o is the open propeller efficiency at this speed. It follows that $T_o(U_p) = T_{ap}$, which was used to complete the above equation. The quasi-propulsive efficiency can be greater than one, since it relies on the towed resistance and in general $R_t > R_{ap}$. The ratio $(1-t)/(1-w)$ is often called the hull efficiency, and we see that a small thrust deduction t and a large wake fraction w are beneficial effects, but which are in competition. A high rotative efficiency and open water propeller efficiency (at U_p) obviously contribute to an efficient overall system.

12.2.2 Solution for Steady Conditions

The linear form of K_T and K_Q (Equation 140) allows a closed-form solution for the steady-operating conditions. Suppose that the towed resistance is of the form

$$R_t = \frac{1}{2} \rho C_r A_w U^2, \quad (141)$$

where C_r is the resistance coefficient (which will generally depend on Re and Fr), and A_w is the wetted area. Equating the self-propelled thrust and resistance then gives

$$\begin{aligned} T_{ap} &= R_{ap} \\ T_o &= R_t/(1-t) \\ K_T(J(U_p)) \rho n_p^2 D^4 (1-t) &= \frac{1}{2} \rho C_r A_w U^2 \\ (\beta_1 - \beta_2 J(U_p)) \rho n_p^2 D^4 (1-t) &= \frac{1}{2} \rho C_r A_w \frac{U_p^2}{(1-w)^2} \\ \beta_1 - \beta_2 J(U_p) &= \frac{C_r A_w}{2D^2(1-t)(1-w)^2} J(U_p)^2 \\ J(U_p) &= \frac{-\beta_2 + \sqrt{\beta_2^2 + 4\beta_1 \delta}}{2\delta}. \end{aligned} \quad (142)$$

The last equation predicts the steady-state advance ratio of the vessel, depending only on the propeller open characteristics, and on the hull. The vessel speed can be computed by recalling that

$J(U) = U/\eta_p D$ and $U_p = U(1 - \eta)$, but it is clear that we need now to find η_p . This requires a torque equation, which necessitates a model of the drive engine or motor.

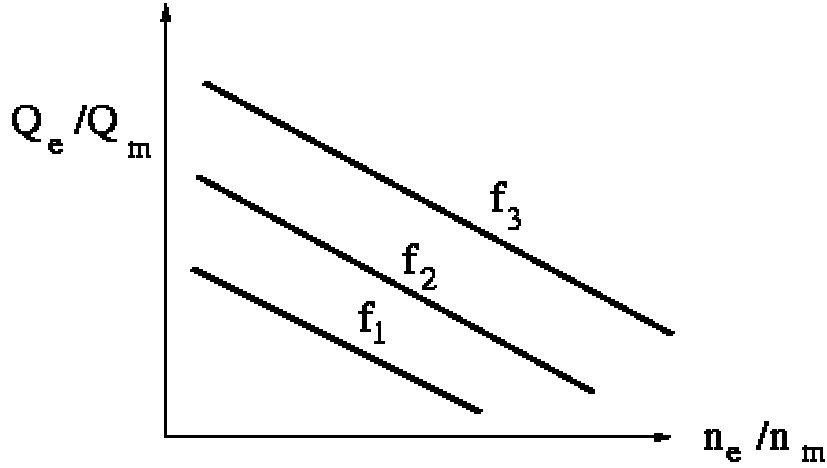


Figure 5: Typical gas turbine engine torque-speed characteristic for increasing fuel rates f_1, f_2, f_3 .

12.2.3 Engine/Motor Models

The torque-speed maps of many engines and motors fit the form

$$Q_e = Q_m F(f/f_m, n_e/n_m), \quad (143)$$

where F is the characteristic function. For example, gas turbines roughly fit the curves shown in the figure (Rubis). More specifically, if F has the form

$$\begin{aligned} F(f/f_m, n_e/n_m) &= -\left(\alpha \frac{f}{f_m} + b\right) \frac{n_e}{n_m} + \left(c \frac{f}{f_m} + d\right) \\ &= -\alpha_1 \frac{n_p}{n_m/\lambda} + \alpha_2. \end{aligned} \quad (144)$$

then a closed-form solution for n_e (and thus n_p) can be found. The manipulations begin by equating the engine and propeller torque:

$$\begin{aligned} Q_p(J(U_p)) &= \eta_E Q_{p,p}(J(U_p)) \\ \rho n_p^2 D^5 K_Q(J(U_p)) &= \eta_E \eta_g \lambda Q_e \\ \rho n_p^2 D^5 (\gamma_1 - \gamma_2 J(U_p)) &= \eta_E \eta_g \lambda Q_m F(f/f_m, n_e/n_m) \\ \frac{n_p^2}{(n_m/\lambda)^2} &= \frac{\eta_E \eta_g \lambda Q_m}{\rho D^5 (\gamma_1 - \gamma_2 J(U_p)) (n_m/\lambda)^2} \left(-\alpha_1 \frac{n_p}{(n_m/\lambda)} + \alpha_2 \right) \\ \frac{n_p}{(n_m/\lambda)} &= \frac{-\epsilon \alpha_1 + \sqrt{\epsilon^2 \alpha_1^2 + 4\epsilon \alpha_2}}{2}. \end{aligned} \quad (145)$$

Note that the fuel rate enters through both α_1 and α_2 .

The dynamic response of the coupled propulsion and ship systems, under the assumption of quasi-static propeller conditions, is given by

$$\begin{aligned} (m + m_a) \dot{u} &= T_{sp} - R_{sp} \\ 2\pi I_p \dot{n}_p &= \eta_g \lambda Q_e - Q_{p,p}. \end{aligned} \quad (146)$$

Making the necessary substitutions creates a nonlinear model with f as the input; this is left as a problem for the reader.

12.3 Unsteady Propulsion Models

When accurate positioning of the vehicle is critical, the quasi-static assumption used above does not suffice. Instead, the transient behavior of the propulsion system needs to be considered. The problem of unsteady propulsion is still in development, although there have been some very successful models in recent years. It should be pointed out that the models described below all pertain to open-water conditions and electric motors, since the positioning problem has been central to bluff vehicles with multiple electric thrusters. We use the subscript m to denote a quantity in the motor, and p for the propeller.

12.3.1 One-State Model: Yoerger *et al.*

The torque equation at the propeller and the thrust relation are

$$I_p \dot{\omega}_p = \lambda Q_m - K_\omega \omega_p |\omega_p| \quad (147)$$

$$T = C_T \omega_p |\omega_p| \quad (148)$$

where I_p is the total (material plus fluid) inertia reflected to the prop; the propeller spins at ω_p radians per second. The differential equation in ω_p pits the torque delivered by the motor against a quadratic-drag type loss which depends on rotation speed. The thrust is then given as a static map directly from the rotation speed.

This model requires the identification of three parameters: I_p , K_ω , and C_T . It is a first-order, nonlinear, low-pass filter from Q_m to T , whose bandwidth depends directly on Q_m .

12.3.2 Two-State Model: Healey *et al.*

The two-state model includes the velocity of a mass of water moving in the vicinity of the blades. It can accommodate a tunnel around the propeller, which is very common in thrusters for positioning. The torque equation, similarly to the above, is referenced to the motor and given as

$$I_m \dot{\omega}_m = -K_\omega \omega_m + K_n V - Q_p / \lambda \quad (149)$$

Here, K_ω represents losses in the motor due to spinning (friction and resistive), and K_n is the gain on the input voltage (so that the current amplifier is included in K_n). The second dynamic equation is for the fluid velocity at the propeller:

$$\rho A L \gamma \dot{U}_p = -\rho A \Delta \beta (U_p - U) |U_p - U| + T \quad (150)$$

Here A is the disc area of the tunnel, or the propeller disk diameter if no tunnel exists. L denotes the length of the tunnel, and γ is the effective added mass ratio. Together, $\rho A L \gamma$ is the added mass that is accelerated by the blades; this mass is always nonzero, even if there is no tunnel. The parameter $\Delta \beta$ is called the differential momentum flux coefficient across the propeller; it may be on the order of 0.2 for propellers with tunnels, and up to 2.0 for open propellers.

The thrust and torque of the propeller are approximated using wing theory, which invokes lift and drag coefficients, as well as an effective angle of attack and the propeller pitch. However, these formulae are static maps, and therefore introduce no new dynamics. As with the one-state model of Yoerger *et al.*, this version requires the identification of the various coefficients from experiments. This model has the advantage that it creates a thrust overshoot for a step input, which is in fact observed in experiments.