

## 11 FINS AND LIFTING SURFACES

Vessels traveling at significant speed typically use rudders, elevators, and other streamlined control surfaces to maneuver. Their utility arises mainly from the high lift forces they can develop, with little drag penalty. Lift is always defined to act in a direction perpendicular to the flow, and drag in the same direction as the flow.

### 11.1 Origin of Lift

A lifting surface is nominally an extrusion of a streamlined cross-section: the cross-section has a rounded leading edge, sharp trailing edge, and a smooth surface. The theory of lifting surfaces centers on the Kutta condition, which requires that fluid particle streamlines do not wrap around the trailing edge of the surface, but instead rejoin with streamlines from the other side of the wing at the trailing edge. This fact is true for a non-stalled surface at any angle of attack.

Since the separation point on the front of the section rotates with the angle of attack, it is clear that the fluid must travel faster over one side of the surface than the other. The reduced Bernoulli pressure this induces can then be thought of as the lift-producing mechanism. More formally, lift arises from circulation  $\Gamma$ :

$$\Gamma = \oint \vec{V} \cdot d\vec{s} \quad (127)$$

and then  $L = -\rho U \Gamma$ . Circulation is the integral of velocity around the cross-section, and a lifting surface requires circulation in order to meet the Kutta condition.

### 11.2 Three-Dimensional Effects: Finite Length

Since all practical lifting surfaces have finite length, the flow near the ends may be three-dimensional. Prandtl's inviscid theory provides some insight. Since bound circulation cannot end abruptly at the wing end, it continues on in the fluid, leading to so-called wing-tip vortices. This continuation causes induced velocities at the tips, and some induced drag. Another description for the wing-tip vortices is that the pressure difference across the surface simply causes flow around the end.

A critical parameter which governs the extent of three-dimensional effects is the aspect ratio:

$$AR = \frac{\text{span}}{\text{chord}} = \frac{\text{span}^2}{\text{area}} \quad (128)$$

The second representation is useful for non-rectangular control surfaces. The effective span is taken to be the length between the free ends of a symmetric wing. If the wing is attached to a wall, the effective span is twice the physical value, by reflection, and in this case the effective aspect ratio is therefore twice the physical value.

The aspect ratio is a strong determinant of wing performance: for a given angle of attack, a larger aspect ratio achieves a higher lift value, but also stalls earlier.

Lift is written as

$$L = \frac{1}{2} \rho U^2 A C_l \quad (129)$$

where  $A$  is the single-side area of the surface. For angles of attack  $\alpha$  below stall, the lift coefficient  $C_l$  is nearly linear with  $\alpha$ :  $C_l = C_l' \alpha$ , where  $C_l'$  is called the lift coefficient slope, and has one empirical description

$$C_l' = \frac{1}{\frac{1}{2\pi\alpha} + \frac{1}{\pi(AR)} + \frac{1}{2\pi(AR)^2}}, \quad (130)$$

where  $\alpha$  is in radians,  $\alpha \approx 0.90$ , and  $AR$  is the effective aspect ratio. When  $AR \rightarrow \infty$ , the theoretical and maximum value for  $C_l'$  is  $2\pi$ .

The lift generated on a surface is the result of a distributed pressure field; this fact creates both a net force and a net moment. A single equivalent force acts at a so-called center of action  $x_A$ , which depends mainly on the aspect ratio. For high  $AR$ ,  $x_A \approx c/4$ , measured back from the front of the wing. For low  $AR$ ,  $x_A \approx c/2$ .

### 11.3 Ring Fins

Ring fins are useful when space allows, since they are omnidirectional, and structurally more robust than cantilevered plane surfaces. The effective aspect ratio for a ring of diameter  $d$  is given as

$$AR = \frac{4d}{\pi c}. \quad (131)$$

The effective area of the ring is taken as

$$A_e = \frac{\pi}{2}dc, \quad (132)$$

and we thus have  $L = \frac{1}{2}\rho U^2 A_e C'_i \alpha$ , where one formula for  $C'_i$  is

$$C'_i = \frac{1}{0.63 + \frac{1}{\pi(AR)}}. \quad (133)$$