8 STREAMLINED BODIES

8.1 Nominal Drag Force

A symmetric streamlined body at zero angle of attack experiences only a drag force, which has the form

$$F_A = -\frac{1}{2}\rho C_A A_o U^2. \qquad (93)$$

The drag coefficient C_A has both pressure and skin friction components, and hence area A_o is usually that of the wetted surface. Note that the A-subscript will be used to denote zero angle of attack conditions; also, the sign of F_A is negative, because it opposes the vehicle's x-axis.

8.2 Munk Moment

Any shape other than a sphere generates a moment when inclined in an inviscid flow. d'Alembert's paradox predicts zero net *force*, but not necessarily a zero moment. This *Munk moment* arises for a simple reason, the asymmetric location of the stagnation points, where pressure is highest on the front of the body (decelerating flow) and lowest on the back (accelerating flow). The Munk moment is always destabilizing, in the sense that it acts to turn the vehicle perpendicular to the flow. Consider a symmetric body with added mass components A_{xx} along the vehicle (slender) x-axis (forward), and A_{zz} along the vehicle's z-axis z(up). We will limit the present discussion to the vertical plane, but similar arguments can be used to describe the horizontal plane. Let α represent the angle of attack, taken to be positive with the nose up - this equates to a negative pitch angle ϕ in vehicle coordinates, if it is moving horizontally. The Munk moment is:

$$\begin{aligned} M_m &= -\frac{1}{2}(A_{zz} - A_{xx})U^2 \sin 2\alpha \\ &\simeq -(A_{zz} - A_{xx})U^2\alpha. \end{aligned} \qquad (94)$$

$A_{zz} > A_{xx}$ for a slender body, and the negative sign indicates a negative pitch with respect to the vehicle's pitch axis. The added mass terms A_{zz} and A_{xx} can be estimated from analytical expressions (available only for regular shapes such as ellipsoids), from numerical calculation, or from slender body approximation (to follow).

8.3 Separation Moment

In a viscous fluid, flow over a streamlined body is similar to that of potential flow, with the exceptions of the boundary layer, and a small region near the trailing end. In this latter area, a helical vortex may form and convect downstream. Since vortices correlate with low pressure, the effect of such a vortex is stabilizing, but it also induces drag. The formation of the vortex depends on the angle of attack, and it may cover a larger area (increasing the stabilizing moment and drag) for a larger angle of attack. For a small angle of attack, the transverse force F_n can be written in the same form as for control surfaces:

$$\begin{aligned} F_n &= \frac{1}{2}\rho C_n A_o U^2 \\ &\simeq \frac{1}{2}\rho \frac{\partial C_n}{\partial \alpha}\alpha A_o U^2. \end{aligned} \qquad (95)$$

With a positive angle of attack, this force is in the positive z-direction. The zero-α drag force F_A is modified by the vortex shedding:

$$\begin{aligned} F_a &= -\frac{1}{2}\rho C_a A_o U^2 \text{ , where} \\ C_a &= C_A \cos^2\phi. \end{aligned} \qquad (96)$$

The last relation is based on writing $C_A(U\cos\phi)^2$ as $(C_A\cos^2\phi)U^2$, i.e., a decomposition using apparent velocity.

8.4 Net Effects: Aerodynamic Center

The Munk moment and the moment induced by separation are competing, and their magnitudes determine the stability of a hullform. First we simplify:

$$F_a = -\gamma_a$$
$$F_n = \gamma_n \alpha$$
$$M_m = -\gamma_m \alpha.$$

Each constant γ is taken as positive, and the signs reflect orientation in the vehicle reference frame, with a nose-up angle of attack. The Munk moment is a pure couple which does not depend on a reference point. We pick a temporary origin O for F_n however, and write the total pitch moment about O as:

$$\begin{aligned} M &= M_m + F_n l_n \\ &= (-\gamma_m + \gamma_n l_n)\alpha. \end{aligned} \tag{97}$$

where l_n denotes the (positive) distance between O and the application point of F_n. The net moment about O is zero if we select

$$l_n = \frac{\gamma_m}{\gamma_n}, \tag{98}$$

and the location of O is then called the aerodynamic center or AC.

The point AC has an intuitive explanation: it is the location on the hull where F_n would act to create the total moment. Hence, if the vehicle's origin lies in front of AC, the net moment is stabilizing. If the origin lies behind AC, the moment is destabilizing. For self-propelled vehicles, the mass center must be forward of AC for stability. Similarly, for towed vehicles, the towpoint must be located forward of AC. In many cases with very streamlined bodies, the aerodynamic center is significantly *ahead* of the nose, and in this case, a rigid sting would have to extend at least to AC in order for stable towing. As a final note, since the Munk moment persists even in inviscid flow, AC moves infinitely far forward as viscosity effects diminish.

8.5 Role of Fins in Moving the Aerodynamic Center

Control surfaces or fixed fins are often attached to the stern of a slender vehicle to enhance directional stability. Fixed surfaces induce lift and drag on the body:

$$\begin{aligned} L &= \frac{1}{2}\rho A_f U^2 C_l(\alpha) \simeq \gamma_L \alpha \\ D &= -\frac{1}{2}\rho A_f U^2 C_d \simeq -\gamma_D \text{ (constant)} \end{aligned} \tag{99}$$

These forces act somewhere on the fin, and are signed again to match the vehicle frame, with $\gamma > 0$ and $\alpha > 0$. The summed forces on the body are thus:

$$\begin{aligned} X &= F_a - |D|\cos\alpha + |L|\sin\alpha \\ &\simeq -\gamma_a - \gamma_D + \gamma_L \alpha^2 \\ Z &= F_n + |L|\cos\alpha + |D|\sin\alpha \\ &\simeq \gamma_n \alpha + \gamma_L \alpha + \gamma_D \alpha. \end{aligned} \tag{100}$$

All of the forces are pushing the vehicle up. If we say that the fins act a distance l_f behind the temporary origin O, and that the moment carried by the fins themselves is very small (compared to the moment induced by Ll_f) the total moment is as follows:

$$M = (-\gamma_m + \gamma_n l_n)\alpha + (\gamma_L + \gamma_D) l_f \alpha. \tag{101}$$

The moment about O vanishes if

$$\gamma_m = \gamma_n l_n + l_f(\gamma_L + \gamma_D). \tag{102}$$

The Munk moment γ_m opposes the aggregate effects of vorticity lift γ_n and the fins' lift and drag $\gamma_L + \gamma_D$. With very large fins, this latter term is large, so that l_f might be very small; this is the case of AC moving aft toward the fins. A vehicle with excessively large fins will be difficult to turn and maneuver.

Equation 102 contains two length measurements, referenced to an arbitrary body point O. To solve it explicitly, let l_{fn} denote the (positive) distance that the fins are located behind F_n; this is likely a small number, since both effects usually act near the stern. We solve for l_f:

$$l_f = \frac{\gamma_m + \gamma_n l_{fn}}{\gamma_n + \gamma_L + \gamma_D}. \tag{103}$$

This is the distance that AC is located forward of the fins, and thus AC can be referenced to any other fixed point easily. Without fins, it can be recalled that

$$l_n = \frac{\gamma_m}{\gamma_n}.$$

Hence, the fins act directly in the denominator to shorten l_f. Note that if the fins are located forward of the vortex shedding force F_n, i.e., $l_{fn} < 0$, l_f is reduced, but since AC is referenced to the fins, there is no net gain in stability.

8.6 Aggregate Effects of Body and Fins

Since all of the terms discussed so far have the same dependence on α, it is possible to group them into a condensed form. Setting $\hat{F}_a$ and $\hat{F}_n$ to account for the fuselage and fins, we have

$$\begin{aligned} X &= \hat{F}_a \simeq -\frac{1}{2}\rho \hat{C}_a \hat{A}_o U^2 \\ Z &= \hat{F}_n \simeq \frac{1}{2}\rho \hat{C}_n' \hat{A}_o U^2 \alpha \\ M &= -\hat{F}_n x_{AC} \simeq -\frac{1}{2}\hat{C}_n' \hat{A}_o U^2 x_{AC} \alpha. \end{aligned} \tag{104}$$

8.7 Coefficients Z_w, M_w, , and M_q for a Slender Body

The angle of attack α is related to the cross-body velocity w as follows:

$$\begin{aligned} \alpha &= -\tan^{-1}\left(\frac{w}{u}\right) \\ &\simeq -\frac{w}{U} \text{ for } U >> w. \end{aligned} \tag{105}$$

We can then write several linear hydrodynamic coefficients easily:

$$\begin{aligned} Z_w &= -\frac{1}{2}\rho \hat{C}_n' \hat{A}_o U \\ M_w &= \frac{1}{2}\rho \hat{C}_n' \hat{A}_o U x_{AC}. \end{aligned} \tag{106}$$

The rotation of the vessel involves complex flow, which depends on both w and q, as well as their derivatives. To start, we consider the contribution of the fins only - slender body theory, introduced shortly, provides good results for the hull. The fin center of pressure is located a distance l_f aft of the body origin, and we assume that the vehicle is moving horizontally, with an instantaneous pitch angle of θ. The angle of attack seen by the fin is a combination of a part due to θ and a part linear with q:

$$\alpha_f \simeq -\theta + \frac{l_f q}{U} \tag{107}$$

and so lateral force and moment derivatives (for the fin alone) emerge as

$$\begin{aligned} Z_q &= -\frac{1}{2}\rho C_l' A_f U l_f \\ M_q &= -\frac{1}{2}\rho C_l' A_f U l_f^2. \end{aligned} \tag{108}$$