Problem Set No 5
Econ 14.05:Intermediate Applied Macroeconomics

1. **Borrowing and Lending in a 2-Period Model**

   Consider a consumer who lives 2 periods. Her intertemporal utility function is the following:
   $$U(c_1, c_2) = \log(c_1) + \beta \log(c_2),$$
   where c_1, c_2 denote consumption in the first and second period of her life respectively, and β is the individual discount factor $(0 < \beta < 1)$. The consumer receives an exogenous stream of income every period denoted by y_1 and y_2 respectively, $y_1 > y_2$.

   In the first period of her life, the consumer decides how much to consume and how much to save/borrow. However, in contrast with the cases you have analyzed so far, the interest rates at which she can lend/borrow are different. When the consumer saves she gets an interest rate of r_s in the next period, but when she borrows she has to pay an interest rate r_b on her loan. The interest rate on loans is higher than the interest rate on savings: $r_b > r_s$.

   (a) Write down the budget constraint faced by the consumer in each period. Distinguish between the cases in which she is a net saver or a net borrower. Write down the intertemporal budget constraint and draw it in the space (c_1, c_2).

   (b) Solve the maximization problem, and determine the consumption in the first period when the consumer is a net saver, and when the consumer is a net borrower.

   Hint: The way to solve this problem is to consider the solutions to two distinct problems with different interest rates, and then think about when it is that each one of the solutions applies.

   (c) Using the expressions obtained in the previous part, determine the range of values of the discount factor (β) for which the consumer is: (i) a net saver, (ii) a net borrower, (iii) neither a borrower nor a saver. Interpret briefly your findings.

   Hint: You should obtain a set of inequalities depending on the first and second period income, and the different interest rates.

   (d) The government introduces a proportional tax t_s on the interest earnings of savings. What happens to the net savings of an individual with discount factor larger that $y_2/(y_1(1 + r_d))$?. Analyze the possible cases. Note that you need to use the inequalities you obtained in the previous part (d).

2. **Taxation in the overlapping generations model**

In this questions we use the overlapping generations model to examine the impact of different forms of government taxation on the steady state and dynamics of the capital stock. Assume population is constant ($n = 0$), individuals live for two periods, working and saving in the first and living off capital in the second period. Assume that capital is the only asset and it is paid its marginal product. Moreover, normalize $A = 1$ and assume that there is no technological progress, i.e., $g = 0$.

The individual's utility function is

$$U = \log(c_{1,t}) + \log(c_{2,t+1}).$$

The production function in per capita terms is

$$y_t = k_t^\alpha.$$

(a) Assume that there exist a government who imposes a lump sum tax of an amount T_t on each time t young individual, and that each individual maximizes her utility subject to her budget constraint.

    i. Determine the intertemporal budget constraint of each individual.

    ii. Use the budget constraint to solve for the first period consumption c_{1t} and the first period saving, s_t.

(b) Using the saving function derived on part a) and the production function:

    i. Determine the relationship between k_{t+1} and k_t , and show it in a graph.

    ii. Write down the expression that implicitly defines the equilibrium capital stock k^*, assuming that the tax is constant: $T_t = T$

    iii. How many equilibria does the economy exhibit? Which ones are stable?

(c) The government now decides to use a proportional tax on labor income instead of a lump-sum tax. Therefore the income of each worker (after tax) is $w_t(1 - \tau_{wt})$, where τ_{wt} is the labor income tax.

    i. Determine the new relationship between k_{t+1} and k_t , and show it in a graph. *Hint*: You can use the expressions you derived above, and just replace $w_t(1 - \tau_{wt})$ for w_t, and remove the effect of the lump sum tax..

    ii. Write down the expression that implicitly defines the equilibrium capital stock k^*, assuming τ_{wt} is constant.

    iii. Assuming that the government sets τ_{wt} such that in equilibrium, the revenue is equal than with the lump-sum tax, that is $w^* \tau_w = T$, how does the new level of equilibrium capital per capita compare with the one obtained on part (b).

(d) Consider now the case in which the government decides to tax only saving income. Therefore, the after tax interest rate on savings is $r_{t+1}(1 - \tau_{rt+1})$. Again:

i. Determine the first period consumption c_{1t}, second period consumption c_{2t+1}, and the first period savings s_t. Compared to the non government case, Which of these variables is affected by the government policy?

ii. Determine the new relationship between k_{t+1} and k_t , and show it in a graph. *Hint*: You can use the expressions you derived above and just replace $r_{t+1}(1-\tau_{rt+1})$ for r_{t+1}.

iii. Write down the expression that implicitly defines the equilibrium capital stock k^*, assuming τ_{rt+1} is constant.

iv. Assuming that the government sets τ_{wt} such that in equilibrium, the revenue is equal than with the lump-sum tax, that is $s_t r_{t+1}\tau_{rt+1} = T$, how does the new level of equilibrium capital per capita compare with the one obtained on part (b)

(e) Comment: Given a fixed level of revenue in steady state.

i. Which taxation system (s) has (have) the highest equilibrium level of output?

ii. What characteristics of the model are crucial for the result that you obtained?. Explain the economic intuition behind your findings. **Note**: You have to answer this question in order to get significant credit for part e).

3. **Social Security in the Diamond model (based on Problem 2.16 in Romer's textbook). Part I: Pay as you go social security:**

Consider a Diamond economy where g is zero, production is Cobb-Douglas, and utility is logarithmic (see sections 2.9 and 2.10 in the textbook). Suppose the government taxes each young individual an amount T and uses the profits to pay benefits to old individuals; thus each old person receives (1+n)T.

(a) Write down the budget condition faced by an individual born at time t.

(b) Use the budget condition to determine the saving function of this individual.

Hint: You should get a function that depends on the wages and the social security tax in the following way: $s_t = B_t\omega_t - Z_t T$ where both B_t and Z_t are functions of the parameters of the model (n, ρ, A) and the interest rate at t+1 (r_{t+1})

(c) How if at all, does the introduction of the pay as you go social security affect the relation between k_t and k_{t+1} described by equation 2.59 in the textbook?

Hint: Use the individual saving function obtained in part b) and proceed as usual to determine the relationship between k_t and k_{t+1}.

(d) How, if at all, does the introduction of the pay as you go social security affect the balanced growth path value of k.

Hint: The key id to determine the sign of Z_t.

(e) If the economy is initially on a balanced growth path (BGP) that is dynamically efficient, how does a marginal increase in T affect the welfare of current and future generations? What happens if the initial BGP is dynamically inefficient?

4. **Social Security in the Diamond model (based on Problem 2.16 in Romer's textbook). Part II: fully funded social security:**

Consider a Diamond economy where g is zero, production is Cobb-Douglas, and utility is logarithmic (see sections 2.9 and 2.10 in the textbook). Suppose the government taxes each young person an amount T and uses the proceeds to purchase capital. Individuals born at T therefore receive $(1 + r_{t+1})T$ when they are old.

(a) Write down the budget condition faced by an individual born at time t.

(b) Use the budget condition to determine the saving function of this individual.

   Hint: You should again get a function that depends on the wages and the social security tax in the following way: $s_t = B_t \omega_t - Z_t T$ where both B_t and Z_t are functions of the parameters of the model.

(c) How if at all, does the introduction of the pay as you go social security affect the relation between k_t and k_{t+1} described by equation 2.59 in the textbook?

   Hint: Use the individual saving function obtained in part b) and proceed as usual to determine the relationship between k_t and k_{t+1}. Remember that in this case the government is also saving in behalf of the young individuals between t and t+1.

(d) How, if at all, does the introduction of the pay as you go social security affect the balanced growth path value of k.

(e) Compare the results obtained in problem 1.d, and 2.d. Explain intuitively why the results are different.