Problem Set No 4

Econ 14.05:Intermediate Applied Macroeconomics

1. **Human capital and growth:**

   It is frequently argued that education is beneficial for growth. The following simple model attempts to formalize this intuition. Production is determined by the function $Y = K^{\alpha}(L_Y h)^{\beta}$, where K is capital, L_Y is the amount of labor allocated to production, and h is the level of human capital of each worker(assumed identical for all workers). Capital evolves according to $\dot{K} = sY$. Each worker is endowed with one unit of labor (1"hour")of which she devotes a fraction$(1-e)$ to production activities, and a fraction e to education activities. The total number of workers evolves according to $\dot{L} = nL$. Finally, human capital per worker (h) depends on the amount of time each worker devotes to education, and on the existing level of human capital of each individual (teacher'suquality), according to $\dot{} = \phi(e)h$, where $\phi' > 0$, $\phi'' < 0$.

   (a) Write expressions for the rate of growth of K, L, Y, and h.

   (b) Under what conditions does this economy have a balanced growth path (BGP)? Determine the growth rate of capital$K$, output$Y$, and o u tput per worker in the BGP.

   (c) Assuming that the production function has constant returns to scale,what is the principal determinant of cross country differences in the growth of output per capita?

   (d) Write an expression for the logarithm of output per capital(Y/L) as a function of K, H, and e. Discuss the effects of an increase in e in the level of output per capita in the short and long run.

2. **The role of the Government in the Solow model:** This exercise has the purpose of make you think about how different governments, in particular benevolent or corrupt governments, can affect the long run equilibrium of a Solow economy. Let the production function be (in intensive form)$y = Bk^{\alpha}$. Assume the government taxes the private

production at a rate τ. This implies that private production is given now by $y = B(1- \tau)k^{\alpha}$.

(a) Derive the new equation for the path of capital and analyze the long run equilibrium for a given τ. How does τ affect the equilibrium? You don't need to worry about what the government does with what it earns, we will do that in the following parts of the exercise.

(b) Assume we have a benevolent government, and whatever the government earns in taxes it devotes to productive investments. In particular assume that government spending is $g = \tau k^{\alpha}$, and that $B = g^{\beta}$, i.e., government spending generates an externality to private production. Derive the new equation for the path of capital and analyze the long run equilibrium for a given τ, assuming $\beta = \frac{1-\alpha}{\alpha}$. What does that imply for the long run equilibrium? Do we have endogenous growth? What happens if $\beta < \frac{1-\alpha}{\alpha}$ for instance $\beta = \frac{1-\alpha'}{\alpha}$, $\alpha' > \alpha$?

(c) What is the tax rate τ that maximizes growth? What are the two effects here?

(d) Assume now we have a corrupt government that wastes everything it earns. In particular, assume $\beta = 0$. What does the presence of a corrupt government imply for the long-run equilibrium? Discuss how this model with different types of governments can explain why some countries do better than others. Can you think of particular examples?

3. **Appropriate technology and the role of education:** Even if it is true that all countries have access to the same technology, it does not need to be true that they know how to use it. In particular, you might think that to be able to understand certain technologies (like computers) you may need a certain level of education (human capital). Or to reap the benefit of all available technology you need not to be too backward. This exercise asks you to analyze these possibilities in the context of a model with externalities. Assume in the context of the model in section 3.2 in the book that $Y(t) = uA(t)(1- a_l)$, where we normalize $L(t) = 1$ and constant.

(a) Assume there is a world knowledge frontier given by $A^*(t) = A_0^* e^{g^* t}$, g^* constant. The country benefits from this frontier, but it benefits more the closest it is to the world frontier, i.e. $g_A = \frac{\dot{A}(t)}{A(t)} = u\frac{A(t)}{A(t)^*} g^*$, where we assume the country is backwards, $A(t) < A^*(t)$. That means that as long as the country is far from the world frontier it grows less. Do we have endogenous growth? Do we have convergence in the levels of technology.Does a country that starts behind always lag behind? Does it stay always at the same distance? Does she diverge? Hint: do as we did in class, find the growth rate for g_A.

(b) Some people argue it goes the other way around, the further you are from the world frontier of knowledge, the more you benefit from it. To analyze this case, assume $g_A = \frac{\dot{A}(t)}{A(t)} = \frac{A(t)^*}{A(t)} g^*$. What happens now?

(c) Part a. is rather pessimistic, b u it fails to account the fact that better education helps understand new technologies. Assume now that $g_A = \frac{\dot{A}(t)}{A(t)} = \frac{A(t)}{A(t)^*} g^* H(t)$, where H is human capital (or ed-u cation). Assume Human capital grows at rate e.Does Human capital help catching up with the world frontier?When is it the case?

(d) What should we expect in the real world, according to this exercise, in poor countries with little technology? Which mechanism do you think is a better description of the world (if not both),a or b? What could you tell a backward country to do in order to catch up?