Solutions to Exam 2
Econ 14.05

1. **Solow model with endogenous population growth**

   (a) This is the standard Solow model seen in class with no depreciation ($\delta = 0$). The evolution of capital per effective worker is given by:

$$\dot{k} = sk^{\alpha} - (n+g)k$$

   (b) The derivative of n with respect to $\hat{y}$ corresponds to:

$$\begin{aligned} \frac{dn}{d\hat{y}} &= -n_1 \left(\frac{\alpha - 1}{\alpha}\right) \hat{y}^{\frac{-1}{\alpha}} \\ &= n_1 \left(\frac{1-\alpha}{\alpha}\right) \hat{y}^{\frac{-1}{\alpha}} > 0, \end{aligned}$$

   so, an increase in $\hat{y}$ increases the rate of population growth. Population will be constant if $n = 0$, therefore, the condition for constant population is

$$\begin{aligned} n_0 - n_1 \hat{y}^{\frac{\alpha-1}{\alpha}} &= 0 \\ \hat{y} &= \left(\frac{n_0}{n_1}\right)^{\frac{\alpha}{\alpha-1}}. \end{aligned}$$

   As population growth is increasing in $\hat{y}$, if income per worker lies below this level, population falls, while if $\hat{y}$ is above this level population increases. The economic interpretation of this level of income per worker is that it corresponds to a minimum subsistence level. If people has less income than that, either the death rate increases or the birth rate (controlling for birth mortality) falls, reducing population levels (negative growth). The opposite would happen if $\hat{y}$ is above subsistence.

   (c) If there is no productivity growth, and $A = 1$, output (and capital) per worker is equal to output per effective worker. The equation for the evolution of capital per effective worker is then:

$$\dot{k} = sk^{\alpha} - (n_0 - n_1 \hat{y}^{\frac{\alpha-1}{\alpha}})k,$$

   but now $\hat{y} = y$, so

$$\dot{k} = sk^{\alpha} - (n_0 - n_1 (k^{a})^{\frac{\alpha-1}{\alpha}})k.$$

   So, a steady state with zero growth of capital per worker will be characterized by

$$\begin{aligned} sk^{\alpha} - (n_0 - n_1 (k^{a})^{\frac{\alpha-1}{\alpha}})k &= 0 \\ sk^{\alpha} + n_1 k^{a} &= n_0 k \\ (s + n_1)k^{\alpha} &= n_0 k. \end{aligned}$$

   Which has the same form than the equation of the Solow model with no depreciation, and no productivity growth, replacing s by $s + n_1$. Therefore, the reduced form effect of the increase in population growth is similar to an increase in the savings rate. The intuition behind this result is that, as demonstrated in part (b), in this model population starts growing only when output per worker (and implicitly, capital per worker) achieves the minimum subsistence level. For capital below this minimum level, population is actually falling, so in this region accumulation of capital per worker would be higher for a given savings rate. In other words, the break-even investment curve has moved downwards. The situation can be depicted both as an increase in the savings rate or as an fall in the break-even investment (Figures 1 and 2 respectively)

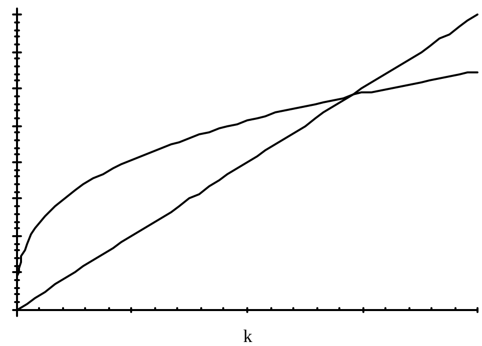

Figure 1:

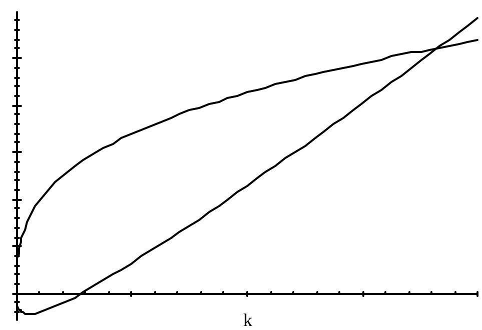

Figure 2:

(d) The comparison is straightforward. The equations defining the steady state in each case are:

$$k_0^* = \left(\frac{s+n_1}{n_0}\right)^{\frac{1}{1-\alpha}},$$
$$k_1^* = \left(\frac{s}{n_0}\right)^{\frac{1}{1-\alpha}}.$$

Where it is clear that $k_0^* > k_1^*$. The situation is easily depicted in Figure 3, which adds Figure 1 the curve sk^α (dotted line).

The intuition behind the result was already explained in part (c).

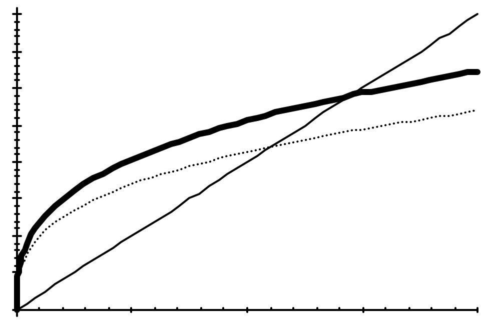

Figure 3:

(e) The difference between this case and the case analyzed in (c) and (d), is twofold: (i) output per worker is not equal to output per effective worker; (ii) there is a productivity growth term in the equation of the evolution of capital per effective worker. Under these considerations, the evolution of capital per effective worker is given by:

$$\begin{aligned}
\dot{k} &= sk^{\alpha} - (n_0 - n_1 \hat{y}^{\frac{\alpha-1}{\alpha}} + g)k \\
&= sk^{\alpha} - (n_0 - n_1 (Ay)^{\frac{\alpha-1}{\alpha}} + g)k \\
&= sk^{\alpha} - (n_0 - n_1 (Ak^{\alpha})^{\frac{\alpha-1}{\alpha}} + g)k \\
&= sk^{\alpha} - (n_0 + g)k + n_1 A^{\left(\frac{\alpha-1}{\alpha}\right)} k^{\alpha} \\
&= (s + n_1 A^{-\left(\frac{1-\alpha}{\alpha}\right)})k^{\alpha} - (n_0 + g)k,
\end{aligned}$$

and the growth rate of capital per effective worker corresponds to

$$\frac{\dot{k}}{k} = (s + n_1 A^{-\left(\frac{1-\alpha}{\alpha}\right)})k^{\alpha-1} - (n_0 + g).$$

To understand the evolution over time, consider first an instant in time, with a given level of A, at which capital per effective worker is growing, that is $(s + n_1 A^{-\left(\frac{1-\alpha}{\alpha}\right)})k^{\alpha-1} > (n_0 + g)$. For a given A we can plot the usual graph that shows the growth rate as the difference between $s_0 f(k)/k$ and $n + g + \delta$, but considering that in this case $s_0 = s + n_1 A^{-\left(\frac{1-\alpha}{\alpha}\right)}$, and that $\delta = 0$. This can be observed in Figure 4 (dotted line). The difference between these two curves gives us the growth rate. At the initial level of capital, the growth rate is positive. However, in the next instant, A will be higher, that is, the $(s + n_1 A^{-\left(\frac{1-\alpha}{\alpha}\right)})k^{\alpha}$ curve will move inwards (remember $\alpha < 1$). Therefore, the growth rate of capital will not fall more than in the standard case. Moreover, as A increases, the $(s + n_1 A^{-\left(\frac{1-\alpha}{\alpha}\right)})k^{\alpha}$ curve converges to the sk^{α} curve. Therefore, aymptothically, this economy has the same equilibrium than the economy with a constant population growth (bold line). The intuition is that productivity growth keeps raising the level of output per worker, therefore the level of output per *effective* worker consistent with the subsistence level converges to zero.

2. **Transmission of Technologies to the South trough Learning by Doing from Northern Technologies:**

(a) The North is just a Solow economy with growth rate $g = Ba_{LN}L_N$, and consequently we now that in the long run this is going to be the growth rate of output per worker.

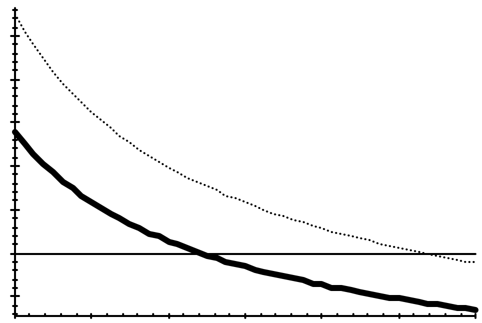

Figure 4:

(b) Take the time derivative to get

$$\dot{Z}(t) = \frac{A_N(t)\dot{A}_S(t) - A_S(t)\dot{A}_N(t)}{A_N(t)^2},$$

and using the expressions given

$$\dot{Z}(t) = \frac{A_N(t)\mu a_{LS} L_S[A_N(t) - A_S(t)] - A_S(t)(Ba_{LN}L_N A_N(t))}{A_N(t)^2} =$$

$$\mu a_{LS} L_S \left[1 - \frac{A_S(t)}{A_N(t)}\right] - \frac{A_S(t)}{A_N(t)} Ba_{LN}L_N =$$

$$\mu a_{LS}L_S - \mu a_{LS}L_S Z(t) - Z(t)Ba_{LN}L_N = \mu a_{LS}L_S - (\mu a_{LS}L_S + Ba_{LN}L_N)Z(t).$$

Remember we assume $Z \leq 1$. To find the steady state value set this expression equal to 0 to get

$$Z^* = \frac{\mu a_{LS}L_S}{\mu a_{LS}L_S + Ba_{LN}L_N},$$

and you can easily see this is a stable value. For $Z(t) > Z^*$, the negative term is bigger than the positive and thus $\dot{Z}(t) < 0$. For $Z(t) < Z^*$ the opposite happens.

(c) We have just shown that Z is constant in the long run, which implies that A_S is growing at the same rate than A_N. Thus in the long run, the South is a Solow economy with the same growth rate than the North. Notice that a_{LS} does not affect the growth rate in the South, it only matters the number of people the North has in R&D. This looks like an unrealistic feature of this model. However, it is the North that is creating new technologies while the South just copies them. So, the restriction on the technology created is the labor force in the North, not that in the South.

(d) As always,

$$\dot{k}_N(t) = \frac{\dot{K}(t)}{A_N(t)L_N(t)} - \frac{K(t)}{A_N(t)L_N(t)}\frac{\dot{A}_N(t)}{A_N(t)} =$$

$$s_N y_N(t) - Ba_{LN}L_N k_N(t) \Leftrightarrow$$

$$\dot{k}_N(t) = s_N k_N^{\alpha}(1 - a_{LN})^{\alpha} - Ba_{LN}L_N k_N(t).$$

Similarly for the South (remember $\frac{\dot{A}_N(t)}{A_N(t)} = \frac{\dot{A}_S(t)}{A_S(t)}$ in the long run)

$$\dot{k}_S(t) = s_S k_S^\alpha (1 - a_{LS})^\alpha - B a_{LN} L_N k_S(t),$$

and using the fact that $a_{LN} = a_{LS}$ and $s_N = s_S$

$$\dot{k}_S(t) = s_N k_S^\alpha (1 - a_{LN})^\alpha - B a_{LN} L_N k_S(t)$$

we have the same equation for both countries, which tell us that in the long run $k_S^* = k_N^*$ and $y_S^* = y_N^*$, or which is the same

$$\frac{y_N^*}{y_S^*} = 1 \Leftrightarrow \frac{Y_N/A_N L_N}{Y_S/A_S L_S} = 1 \Leftrightarrow$$
$$\frac{Y_N/L_N}{Y_S/L_S} = \frac{A_N}{A_S}$$

i.e. the ratio of output per workers is the same as the ratio of technologies. Using the expression for the ratio of technologies

$$\frac{Y_S/L_S}{Y_N/L_N} = \frac{\mu a_{LS} L_S}{\mu a_{LS} L_S + B a_{LN} L_N},$$

which is smaller than one, output per person will always be lower in the South. There is no convergence in levels. Notice that the bigger is imitation (labor devoted to it, a_{LS}), the closer will be the two levels.

(e) Imitation always benefit the South while the North does not care, it does not affect him. Thus from this exercise we could conclude that intellectual property rights should not be enforced as they just decrease imitation. But this model misses an important characteristic of R&D: researchers do research for profit. Moreover, research is always uncertain and requires high initial investments. So, without property rights protection profits would be zero and there would be no research. The North would not grow without innovations. And if the North does not innovate, the South can't copy!!

(a) According to Jones, the main characteristics of the evolution of the World income distribution between 60's and 80's are that:

   i. There is some mobility mainly reflected in catching up at the top and divergence at the bottom, generating a double hump shape
   ii. Distribution is relatively stable, distribution have not spread
   iii. There have been miracles and disasters; miracles have high investment rate while disasters have a low investment rate (on average). There seems to be no particular difference in the initial position for average miracles versus average disasters. There also seem to be no clear pattern of factor accumulation: in some miracles labor force participation has increased and in others decreased.

(b) According to Hall and Jones, in a pure accounting sense, the main proximate source of differences in output per worker across countries are differences in productivity. However, they claim that the ultimate cause of output per worker differences are differences in social infrastructure. According to their view, differences in social infrastructure determine both differences in factor accumulation (physical and human capital per worker), and productivity differences.

(c) Engerman and Sokoloff argue that the differences in economic performance in the New World can be explained by a mechanism that has its origin in the differences in climate across former colonies. The mechanism they have in mind is the following: differences in climate determine the type of crop that can be most profitably raised in a given location. For technological reasons, the explotation of different crops requires different scales of production, and productive system. Differences in the scale of production were associated with differences in the concentration of land ownership. These differences translated into differences in political power, which affected earlier institutions, which tended to preserve the differences in wealth and political power in time. The last chain of the argument is that differences in wealth concentration translate into differences in growth. Less concentrated wealth foments innovation, among other reasons, because of the existence of large markets for standard products.

A possible way of testing this theory is to observe whether it is the case that early institutions preserved themselves by comparing for example, the degree of political integration today in countries that, as colonies, were used to produce large scale crops.

(d) Acemoglu et.al. argue that the current differences in income across former colonies are due to the type of institutions that European settlers established in colonial times. They distinguish between productive and extractive institutions, and argue that European colons established productive institutions (good institutions) only in those places in which they could settle, while in places in which settler's mortality was too high they established institutions aimed only to extract resources. As in Engerman and Sokoloff's (ES), these early institutions tend to preserve themselves, affecting current institutions, and current performance. The main difference between ES and Acemoglu et al. is the source of the difference in initial institutions. While in ES these are determined by the productive structure, in Acemoglu et al. they are determined by settler's mortality.

(e) Young's methodology consists in directly estimating TFP growth for a sample of countries. To this end he fits a regression, explaining the average growth of output per worker for the period 1970-1985 as a function of the average growth of capital per worker across countries. The TFP growth of each country is associated with the estimated residual. Young's main conclusion is that the levels of TFP growth in the East Asian Tiger Economies are not abnormal, and that most of the growth of these countries during the last 30 years was due to a large mobilization of resources. In particular, investment was high compared to the rest of the world, and labor force participation increased dramatically during the period.