Problem Set No 3

Econ 14.05:Intermediate Applied Macroeconomics

1. **The Solow model I:** Describe how, if at all,each of the following developments affects the break-even and actual investment lines in our basic diagram for the Solow model.Analyze the effect on the steady state values of k. Give intuition for your results.

   (a) The rate of depreciation falls

   (b) The rate of technological progressurises.

   (c) The production function is Cobb-Douglas, $f(k) = k^{\alpha}$, and capital's share,α, rises.

   (d) Workers exert more effort, so that output per unit of effective labor for a given value of capital per unit of effective labor is higher than before.(Hint:You can think about that as output per unit of effective labor being $Bf(k)$, and we are asking for what happens if B rises)

2. **Natural resources in the Solow model.**

   At least since Malthus, some have argued that the fact that some factors of production (notably land and natural resources) are available in finite supply m u s t eventually bring growth to a halt. This prediction is known as the Malthusian trap due to the pessimistic beliefs Malthus had.

   Let the production function be $Y = K^{(\alpha)}(AL)^{\beta}T^{(1-\beta-\alpha)}$, where T is the amount of land. Assume $\alpha > 0$, $\beta > 0$, and $\alpha + \beta < 1$. The factors of production evolve according to $\dot{K} = sY - \delta K$, $\dot{A} = gA$, $\dot{L} = nL$, and $\dot{T} = 0$.

   (a) Does this economy have a unique and stable balanced growth path? That is, does the economy converge to a situation in which each of Y, K, L, A, and T are growing at a constant (but not necessarily equal) rates? If so, what are those growth rates? If not, why not? [Hint: In order to simplify the problem redefine capital and output in terms of $(AL)^{(M)}T^{(N)}$, where$M = \frac{\beta}{1-\alpha}$ and $N = \frac{1-\alpha-\beta}{1-\alpha}$, i.e., set $y = \frac{Y}{\left((AL)^{(M)}T^{(N)}\right)}$ and $k = \frac{K}{\left((AL)^{(M)}T^{(N)}\right)}$.]

(b) In light of your answer, does the fact that the stock of land is constant imply that permanent growth rate is not possible?Explain intuitively.

(c) Under what circumstances will $\frac{Y}{L}$ grow over time, as opposed to shrinking steadily? Give your answer in terms of α, β, n and$g$.

(d) Was Malthus right? Discuss the results of the model above and their resemblance with Malthus predictions. How can an economy escape from the Malthusian trap?

3. **The Solow model II:**

(a) Consider an economy described by the following equations for output, capital accumulation, population growth and technological progress:

$$\begin{aligned} Y &= F(K, AL) \\ \dot{K} &= sY - \delta K \\ \dot{L} &= nL \quad \dot{A} = gA \end{aligned}$$

Where ,K, L, and A denote output, capital stock, population, and productivity respectively; $F(\,)$ is the production function (it satisfies the conditions in the text).Write down expressions for the output per effective worker (y) (where effective worker is defined as the product of the productivity level and the population: AL) as a function of the capital stock per effective worker, and for the evolution of the capital stock per effective worker ($k = K/AL$).

(b) Write down an expression for the rate of growth of capital per effective worker g_k.You should obtain that g_k is the difference between two expressions:a function of k and a constant.Draw a graph where you can observe g_k as a function of k. Hint: Draw the two expressions that you obtained separately, and show g_k as the difference between the two expressions. Notice that in class we talked about $\dot{k} = dk/dt$, and now you are asked to talk about $g_k = \dot{k}/k$.

(c) Consider now that the production function is the following:

$$Y = \min(aK, bAL)$$

where a, b are constants.Write down the expression for output per effective worker (y),as a function of capital per effective worker (k),and then draw a graph showing y as a function of k.

(d) Use your expression for y from the previous part, and the equation for g_k you derived in part 3a to obtain an equation for the rate of growth of capital per effective worker in this economy.As in part3a draw a graph where you can observe g_k as the difference between the two functions that form it.What determines whether this economy has a positive level of capital in steady state? What could an economy that is trapped in the zero capital equilibrium do to move to a better equilibrium?