ON THE DYNAMICS OF DRAW TEXTURING

by

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ABSTRACT

This study is concerned with the mechanics and dynamics of a process for increasing bulk, softness, cover and stretch of thermoplastic continuous filament yarns. In particular this investigation focuses on the interactions between the feed yarn and the texturing machine in the draw texturing process. It treats the cases of both steady state and transient operation. The degree of filament drawing, the cold, hot, cooling and post spindle twist distributions, the threadline tension, and the threadline torque are shown to possess a non-linear relationship dictated by machine-material parameters. This study presents a tractable model for determining analytically threadline torque, tension, and twist distributions, given information on machine settings and on material properties. Considerable attention is devoted to the determination of twist distribution in the threadline during transient operation.

The results of numerous observations of threadline response were compared with the data generated from the analysis of threadline operation. In general, the agreement between theory and experiment is good.

Thesis Supervisor: Stanley Backer
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to Sue and Kara
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Chapter 1
INTRODUCTION

Texturing is a process whereby yarn stretch, bulk, absorbency, and improved hand are provided through the introduction of "permanent" crimp into otherwise straight continuous filaments. Fabrics made from textured yarns have captured a large share of the textile apparel market. The popularity of garments made from textured yarns results from improved wear performance, freedom from wrinkling, and high bulk or cover power.

Yarn texturing can be accomplished by any one of at least a dozen techniques; however, most texturing is accomplished by the false twist process. This single process sequentially twists, heat sets, and untwists a thermoplastic filament bundle in a continuous operation. A schematic diagram showing the yarn path through a typical pin-spindle false twist machine is pictured in Figure 1.1.

Yarn is removed from the supply package, fed at a controlled rate over the heater, through the false twist pin-spindle, and then onto a takeup package. Twisted yarn moving between the feed rollers and spindle is heat set before passing through the rotating spindle.

The elements of false twisting can be presented rather simply. If a stationary multifilament yarn is held at both
Figure 1.1

PIN-SPINDLE FALSE TWIST
TEXTURING MACHINE
ends and twisted by a spindle at its center, equal amounts of twist with opposite directions of spirality will be imparted to each side, as seen in Figure 1.2.

If each half of the yarn is considered separately, it is observed that the twist is real; however, the algebraic sum of twist throughout the yarn is zero. Now with the twisting element rotating continuously and yarn moving through it, the twist on the downstream side of the spindle is carried away through the take-up rollers and twist from the upstream side of the twister moves across the spindle to the downstream side. Once a yarn segment has moved through the twisting element, the twisting direction relative to that yarn segment is reversed and the segment is untwisted. At the same time, more twist is inserted at the feed rollers so that at steady state operation, a stationary zone of twisted yarn appears upstream of the spindle and twist free yarn occupies the downstream zone. This condition is illustrated in Figure 1.3.

Twisting can be performed either by a pin-spindle or by a friction bushing. The yarn paths on each such spindle are
shown in Figure 1.4. For the pin-spindle system operating at steady state the segment of yarn which is about to enter the spindle generally rotates once for each spindle rotation. In the bush system the yarn will rotate \( \frac{R_{\text{bushing}}}{R_{\text{yarn}}} \) times for each bush rotation, provided that no rotational slippage occurs. However, in virtually all friction twisting, rotational slippage does occur at the spindle. The ratio of bushing to yarn rotation varies from machine to machine and from time to time on a given machine. This thesis deals with the pin-spindle twisting system which dominates in the U.S. textile industry at present.

In the pin-spindle system, a heater is placed in the upstream zone, between the feed rollers and the spindle, with enough space left for the yarn to cool before being untwisted at the spindle. This upstream zone is termed the texturing zone, in which continuous twisting, heat setting and cooling are carried out, followed by untwisting downstream of the spindle. The presence of the heater causes two levels of twist to occur in the threadline in the texturing zone for reasons to be discussed later. However, threadline tension and torque along the texturing zone are uniform. A schematic diagram of the threadline characteristics during steady state operation is shown in Figure 1.5.

Until recently, most feed yarns for texturing were manufactured by fiber producers, via extrusion and drawing.
Figure 1.4

PIN-SPINDLE

YARN

FRICTION BUSHING

YARN
Figure 1.5

Typical Threadline Characteristics
Texturing of this fully drawn yarn was then undertaken by the textile manufacturer. In 1966 Mey(1) reported on studies of the combination of drawing and texturing processes. By 1972 selected American and European fiber producers began to integrate drawing processes with false twist texturing. This innovation is called false twist draw-texturing or simply, draw texturing.

There are two basically different methods of draw texturing: the sequential process and the simultaneous process. In the sequential process, the drawing operation, which precedes false twisting, is carried out in a separate zone on the machine. Since the local processes of drawing and texturing are thus isolated in the sequential process, the resulting yarns are essentially the same as those produced from previously drawn yarn by the conventional texturing process. In the simultaneous process, the yarn moves into the cold portion of the texturing zone where it receives some cold twist. At or near the entrance to the heater further twisting occurs and at this point the yarn draws, both actions taking place simultaneously.

Yarns produced in this way show several distinct differences from conventional false twist textured yarns and from those draw textured yarns produced by the sequential process, the most important difference being that fabrics made from simultaneously draw textured yarns have a more uniform dyed
appearance than those made from conventional false twist textured yarns.\(^{(2)}\) Thwaites shows that this superiority is associated with the modified light reflectance of the simultaneously drawn yarns\(^{(2)}\).

This thesis will deal primarily with that simultaneous draw texturing process which employs pin-spindle twisting.

One problem encountered with the simultaneous draw texturing process is that when essentially unoriented amorphous polyester is used as the feed yarn, the filament bundle loses strength and/or the separate filaments fuse together as soon as the yarn is heated above its glass transition temperature for several seconds \(^{(3)}\). As a result, "stringup" of the texturing machine becomes very difficult. A solution to this problem is to use a relatively amorphous, yet somewhat pre-oriented, polyester feed yarn, which does not soften and lose its strength as rapidly upon heating. This preoriented polyester yarn (POY PET) can be produced simply by high speed winding of the freshly extruded filament bundle, without the provision of a separate drawing operation. Thus, there is a major economic advantage in the production of partially oriented yarn (rather than conventionally drawn yarn) and in its usage as feed stock for simultaneous draw texturing processes. For the latter methods employ yarns which have been extruded and partially drawn in a single process while conventional texturing requires yarns which are drawn to their final
draw ratio in a process separate from the extrusion process.

A successful draw texturing operation depends on the quality of its feed yarn as well as on the control of its process to produce consistently uniform quality yarn on an hour to hour and day to day basis. Once a draw-texturing machine is adjusted for a given operation, the textile manufacturer expects it to produce large quantities of yarn possessing uniform crimp with no residual twist. In mechanical terms this implies threadline torque and tensions which are constant in time from spindle position to position in a given machine and from machine to machine. Any variation experienced in generalized threadline forces (tension or torque) may cause variation in crimp level or in subsequent dyeing behavior of the output yarn or it may introduce real twist in the post spindle region as a consequence of periodic twist slippage past the rotating spindle. The textured yarn producer must make certain decisions as to the setting of machine and/or material parameters, on the basis of resulting product quality. As of this time, such decisions are made on the basis of preliminary experimental trials or in accordance with past production experience. This thesis seeks to provide a quantitative basis for anticipating the effect of variations in feed material properties and of machine parameters on the draw texturing process during both steady and non-steady operation.
2.1 INTRODUCTION

False twist draw texturing is a textile process which manifests strong interactions between feed material and machine. Once the texturing machine is set for given feed and take-up roller velocities, spindle speed, and texturing zone heater temperature, the yarn being processed reacts to the given machine conditions by twisting, drawing, uptwisting, heat setting and untwisting. The degree of filament drawing, the cold, hot and cooling twist levels, the threadline tension, and the threadline torque are determined by a complex non-linear relationship of machine-material parameters. A block diagram of the process is shown in Figure 2.1. A full understanding of this diagram must await the detailed analysis of texturing mechanics to be presented later, but it suffices at present to emphasize certain essential features of its construction. Note first that all mass flow moves from left to right, i.e. from feed roller to take up roller. Second, the twist originates at the spindle and moves to its left, upstream, or to its right, downstream. Finally, generalized threadline forces operating at each station of the texturing zone interact directly.

This section provides a tractable model for analytically
SCHEMATIC DIAGRAM OF FALSE TWIST TEXTURING

Figure 2.1
determining threadline torque, tension, and twist distribution, given information on machine settings and on material properties. Later, these analytical solutions will be compared with experimental data to test the validity of the model. In addition, a parametric survey will be presented to show the effect of different feed yarn properties on threadline behavior.

2.2 METHOD OF SOLUTION

The false twist draw texturing process can be divided into zones as shown in Figure 2.2.

![Figure 2.2](image)

\[ V_0 = \text{Feed roller linear velocity} \]
\[ V_2 = \text{Linear velocity of cold twisted yarn} \]
\[ V_3 = \text{Linear velocity of hot twisted yarn} \]
\[ V_4 = \text{Take up roller linear velocity} \]
\[ W_s = \text{Angular velocity of false twist spindle} \]

In this device, yarn enters the cold zone, twists, tra-
verses to the heater, simultaneously draws and uptwists, cools, progresses to the spindle, crosses the spindle and untwists. Between the feed rollers and the spindle, threadline torque and tension are considered uniform since for our particular system, negligible friction occurs between the yarn and the heater plate. Because of the extremely small yarn masses involved inertial effects are considered negligible for the rotating threadline. Justification of this assumption is given in the Appendix 1. Since the threadline torque and tension are uniform along the texturing zone, we can chose any yarn cross section in this zone for the purpose of analyzing tension and torque. For this analysis we must determine the local material-system constitutive relations, the local yarn geometry, and the system compatibility at each cross section.

As a first approach, the pin-spindle texturing machine can be thought of as a device that deforms yarn at both cold and hot temperatures. The feed rollers, the take up rollers, and the rotating false twist spindle are the deforming devices; the heater is the temperature treatment device. Generalized threadline forces, i.e. torque and tension, are consequences of the deformation and temperature treatment. During steady state operation, the condition of 100% pin-spindle efficiency is defined as the condition when the cooling yarn segment about to enter the spindle, rotates once for every turn of
that spindle. It is assumed that the hot yarn segment up-
stream of the cooling zone rotates at this same velocity
since experimental observations of twist distribution in the
threadline show relatively little difference between hot and
cooling zone twist levels \(^4\). The system can also be
thought of as running at steady state if there is continual
rotational slippage of the cooling yarn relative to the spin-
de. For example, in the case of 80% efficiency, the cooling
yarn segment will rotate 0.8 times for every turn of the
spindle. This analysis will consider the case for 100%
efficiency; however, if the efficiency were given, that case
could be easily handled with the present analysis.

During steady state operation, the yarn in the cooling
zone progresses downstream one twist pitch for each spindle
rotation. As this yarn moves through the spindle, the spin-
de untwists one turn of twist and the given pitch length is
untwisted completely. In the case of non-steady operation
this condition of one twist pitch of linear motion per spin-
dele revolution need not prevail. Treatment of the non-
steady state condition follows in Chapter 3.

As the untwisted filaments enter the cold zone, they are
strained and twisted, the net result being that the fil-
ament bundle contracts longitudinally. This contraction
results in a reduction of yarn linear velocity according to
continuity requirements. The mechanics of cold zone twist
contraction is examined in Section 2.3.2.

The physical motivation for the yarn to twist upon entry into the cold zone is the existence of a threadline torque field. In Section 2.3.4 we show that cold zone tension is a function of cold zone twist and cold filament strain. We can express cold zone yarn tension as a function of certain unknowns of the system:

$$T_{\text{cold zone}} = F_1 (e_{\text{cold}}, P_c)$$  (2-1)

where $e_{\text{cold}} = \text{cold filament strain}$

$P_c = \text{cold zone twist (radians/unit twisted length)}$

Cold zone yarn torque has one component due to filament tension and one due to filament bending and filament twisting. Since the filaments in the twisted yarn are constrained to lie approximately in right cylindrical helices, they are bent and twisted about their individual axes. The filament bending and torsional moments resulting from this deformation can be resolved along the yarn axis so as to determine their contribution to total threadline torque. Platt et al.\textsuperscript{5} has analytically determined this relation, showing that yarn torque due to fiber torsion and bending is a function of the torsional rigidity of the fiber, its bending rigidity, and its geometrical path in the yarn structure. Accordingly, we can express cold zone yarn torque due to fiber bending and torsion as
\[ M_t \text{ cold zone (bending, torsion)} = F_2 (EI, GI, P_c) \] (2-2)

where \( EI = \) fiber bending rigidity
\( GI_p = \) fiber torsional rigidity
\( P_c = \) cold zone twist

Filament tension can be readily determined from filament strain and can then be radially resolved and multiplied by the appropriate radius (corresponding to the filament position in the yarn structure) to determine the individual tensile component of yarn torque. This tensile component can be expressed as:

\[ M_t \text{ cold zone (tension)} = F_3 (e_{\text{cold}}, P_c) \] (2-3)

Combining (2-2) and (2-3) we can express the total torque in a yarn section from the cold zone as

\[ M_{tc} = F_4 (P_c, e_{\text{cold}}, EI, GI_p) \] (2-4)

\( EI \) and \( GI_p \) are known properties of the filaments. \( P_c \) and \( e_{\text{cold}} \) are unknown and must be determined. Therefore there are two equations with four unknowns relating to the threadline in the cold zone, i.e.:

\[ M_{tc}, T_c, e_c, P_c \]

In order to generate more independent simultaneous equations it is necessary to choose another threadline cross section with different geometry, different material-system con-
stitutive relations, and different system compatibility. Towards this end, a cross section of the hot drawn yarn at the base of the threadline drawing "neck" is analyzed. In section 2.3.6 we demonstrate that hot zone filament tension is a function of both hot and cold zone yarn twist, cold and hot yarn linear velocity, and the hot draw material-system constitutive relation. On the assumption that the material constitutive relation, MCR, for the hot drawing yarn can be determined, hot yarn tension can be expressed as:

\[ T_{\text{hot zone}} = F_5(V_2, V_3, P_c, P_h, \overline{MCR}) \quad (2-5) \]

where
- \( V_2 \) = linear velocity of the cold yarn
- \( V_3 \) = linear velocity of the hot yarn
- \( P_c \) = cold zone twist (radians/unit twisted length)
- \( P_h \) = hot zone twist (radians/unit twisted length)

Taking the component of filament hot tension perpendicular to the yarn axis we can determine the contribution of tension to yarn torque in the hot zone:

\[ M_t \text{ (tension)} = F_6(V_2, V_3, P_c, P_h) \quad (2-6) \]

This derivation is presented in Section 2.3.7.

The objective of the false twist texturing process is to temporarily deform thermoplastic filaments into a twisted configuration and then to stress relax them before untwisting by passing the twisted yarn over a heater whose temper-
ature is above the crystal melting temperature of the feed yarn. If this process is to be effective, components of yarn torque due to fiber bending and fiber torsion should be relaxed out over the heater. If this relaxation occurs, as Naar\(^6\) points out, the torque in the yarn over the heater should be due primarily to individual fiber tension. This assumption is to some extent justified when one looks at the twist distribution of the yarn over the heater. Once the torque due to cold fiber torsion and cold fiber bending is relaxed, one observes a dramatic twist crankup at the heater entrance which is necessary to keep the torque constant. Total torque in the hot zone is primarily the torque due to fiber tension and accordingly can be expressed as:

\[ M_{\text{t hot zone}} = M_{\text{t hot zone (tension)}} \]  \hspace{1cm} (2-7)

By equating (2-1) and (2-5) and equations (2-4) and (2-7), which corresponds to equalizing tension and torque through the threadline, we can reduce our system of equations to two equations with five unknowns. \(EI\) and \(GI_p\) are assumed given. These are shown for

**Equal Tension in Cold and Hot Zones**

\[ 0 = F_1(e_{\text{cold}}, P_c) - F_5(V_2, V_3, P_c, P_h) \]  \hspace{1cm} (2-8)
Equal Torque in Cold and Hot Zones

$$0 = F_4 (e_{\text{cold}}, P_c, EI, G, I_p) - F_7 (V_o, V_0, P_c, P_h) \quad (2-9)$$

In Section 2.3.3 we show how cold filament strain is analytically related to cold yarn contraction and to the ratio of feed roller velocity and translational velocity of the cold twisted yarn. Cold strain can be expressed as:

$$e_{\text{cold}} = F_8 (V_2, V_0, P_c) \quad (2-10)$$

where

- $V_2 = \text{Linear velocity in the cold zone}$
- $V_0 = \text{Linear velocity of the feed roller}$
- $P_c = \text{Cold twist (radians/twisted length)}$

As indicated, for steady state operation, the hot twisted yarn segment progresses downstream one pitch length during the time it takes for the spindle to rotate one revolution. Therefore the linear velocity of the twisted cooling yarn segment, $V_b$, as it enters the spindle is:

$$V_b = \frac{1}{2\pi P_s} \frac{1}{2\pi \omega_s} = \frac{\omega_s}{P_s} \quad (2-11)$$

where $\omega_s = \text{Spindle speed, (radians per second)}$

It has been observed that the difference between local twist in the cooling (therefore setting) zone, $P_s$, and the local twist at the exit of the heater, $P_h$, is negligible. Likewise the difference between the linear velocity of the
threadline as it leaves the heater and as it enters the spindle is considered to be negligible. This permits a re-writing of equation (2-11) in the form

\[ P_h = \omega_s / V_3 \]  

(2-12)

The spindle speed is a given quantity, therefore we need to determine the threadline velocity at the heater exit \( V_3 \). It is shown in Section 2.3.9 that the given value of take up roll velocity can be used to determine \( V_3 \).

With the appropriate mathematical substitutions the system of equations outlined in this section can be reduced to three nonlinear simultaneous equations with three unknowns:

\[ 0 = u(V_2, e_c, P_c) - v(V_2, P_c) \]  

(2-13)

\[ 0 = x(V_2, e_c, P_c) - y(V_2, P_c) \]  

(2-14)

\[ 0 = z(V_2, e_c, P_c) \]  

(2-15)

where \( e_c \) is \( e_{\text{cold}} \)

Equations (2-13), (2-14), and (2-15) are solved by the computer subroutine "YARN" which is listed in Appendix 5. The three equations are reduced to a single equation with the unknown, \( P_c \), and the value of \( P_c \) satisfying the single equation is determined. Given the value of \( P_c \) for a given set of machine parameters, the corresponding values of \( V_2 \) and \( e_c \) are then determined. When the solution for \( P_c, e_c, \) and
V is found, they are substituted back into equations (2-1), (2-5), (2-4) and (2-7) to determine the threadline tension and threadline torque.

In summary, this method of solution makes possible the determination of the threadline tension, threadline torque, and threadline twist distribution given the feed roller and take up roller velocities, spindle speed, and the material properties of the feed yarn. It remains to provide specific relationships for cold and hot tension, cold and hot torque, hot twist, hot twist contraction, and cold twist contraction. This is done in Section 2.3.
2.3 STEADY STATE THREADLINE MECHANICS

2.3.1 Continuous Twisting of Filament Yarns

As untwisted feed yarn is delivered by the feed rollers into the texturing zone and is exposed to the threadline torque, the yarn uptwists until its torque reaches that of threadline. As the yarn uptwists, its filaments are extended, partially because of the threadline tension and partially because of the tortuous path lengths the filaments are forced to follow in conforming to the geometry of twists. Filaments lying on relatively high helix angles in the outer radial layers of the yarn, are prone to extend to a greater extent than filaments located closer to the yarn center. The presence of such differential strain was first recognized by Morton (7) who suggested that the magnitude of the differential strains would be reduced with the occurrence of interchange between outer and inner filaments. A continuing interchange of this nature tends to equalize filament tensions throughout the yarn during steady state twisting. This cyclical interchange of fiber position is known as radial migration. Backer & Yang (4) demonstrated that radial migration does occur at the entrance to the texturing zone at the feed rollers. On the basis of these observations Naar (6) assumed that the filaments in the cold yarn segment of the texturing zone are under equal tension. We adopt this assumption as part of our model system.
2.3.2 Cold Zone Twist Contraction

When yarn is twisted under a given tension it will contract because of the longer path lengths that the outer filaments must follow. The resulting contraction is quantitized by a "contraction factor" defined as:

\[ C_r = \frac{\text{Length of Zero Twist Yarn}}{\text{Length of Twisted yarn}} \] (2-16)

The length of a projected segment of a filament lying in a twisted yarn is compared with the same length of filament in an untwisted yarn in Figure 2.3. The contraction factor of the filament yarn segment of Figure 2.3 is obviously sec θ.

Figure 2.3
Hearle's Model of Twist Contraction

Consider a twisted yarn of length \( h \) as shown in Figure 2.4.

Figure 2.4

Hearle (8) assumes that because of radial migration the difference in the path length of twisted filaments in a given segment of yarn vs. the length of the twisted yarn is spread uniformly over all the filaments in a relatively long segment of yarn. He shows that the distribution of lengths in a short segment is linear and therefore the contraction factor equals the arithmetic average of the minimum filament length (equal to yarn length) and maximum filament length (for outside layer). The contraction factor for this yarn model is
where $\theta$ is the surface helix angle of the yarn.

**Alternative Model of Twist Contraction**

Hearle's model of twist contraction essentially averages the length of the filaments in a yarn segment over its cross sectional area. Alternatively, we can average the filament lengths over the yarn radius. This method will disproportionately weigh the average towards a lower value than that obtained from Hearle's model since there are fewer fibers in central yarn radial positions compared to the number of fibers in outer radial positions. The alternative model for twist contraction is formulated as

$$C_r = \frac{1}{2} \left[ \ln \left( \tan \theta + \sec \theta \right) + \sec \theta \right]$$

(2-18)

The predicted value of contraction factor using either model must be compared with the measured values. Experimental values of twist contraction in the cold zone are compared to the predicted values using both models below for five different cold segments derived using different machine twists.
### COLD YARN TWIST CONTRACTION

<table>
<thead>
<tr>
<th>Machine Twist</th>
<th>Measured*</th>
<th>Hearles' Model</th>
<th>Alternative Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>1.053</td>
<td>1.091</td>
<td>1.063</td>
</tr>
<tr>
<td>60</td>
<td>1.139</td>
<td>1.190</td>
<td>1.136</td>
</tr>
<tr>
<td>65</td>
<td>1.158</td>
<td>1.207</td>
<td>1.148</td>
</tr>
<tr>
<td>70</td>
<td>1.165</td>
<td>1.213</td>
<td>1.153</td>
</tr>
<tr>
<td>80</td>
<td>1.191</td>
<td>1.247</td>
<td>1.178</td>
</tr>
</tbody>
</table>

Comparison of the measured values to both models indicates that the alternative model for twist contraction in the cold zone predicts closer values to the measured ones. Nonetheless, both models will be included separately in the steady state prediction of threadline behavior (i.e., threadline tension, threadline torque, cold threadline twist, and hot threadline twist.)

*By untwisting in a twist tester, yarn segments which had been twisted in the texturing apparatus. The tension of untwisting equalled the texturing tension.
2.3.3 Flow Continuity in Cold Zone

During steady state operation of the texturing process, the product of the linear density of the yarn and the local translational yarn velocity (mass flow rate) must be constant at any station in the texturing threadline, i.e.

\[ D \cdot V |_{X} = \text{constant} \]  \hspace{1cm} (2-19)

where
- \( D \) = Yarn denier (linear density)
- \( V \) = Linear velocity of yarn
- \( X \) = Linear location on threadline, i.e. its station

The linear density of a textile yarn is usually expressed in terms of its denier, defined as the weight in grams of 9000 yarn meters.

The machine configuration between the feed rollers and the cold zone is illustrated in Figure 2.5

Figure 2.5
At each of these two stations, the mass flow rate is constant, i.e.

$$V_{D_0} = V_{D_2}$$  \hspace{1cm} (2-20)

where

$$V_0 = \text{Linear velocity of feed roller and of the feed yarn passing through}$$

$$V_2 = \text{Linear velocity of twisted yarn under threadline tension}$$

$$D_0 = \text{Denier of feed yarn under no tension}$$

$$D_2 = \text{Denier of twisted yarn under threadline tension}$$

The contraction factor is defined as:

$$C_r = \frac{\text{untwisted yarn length}}{\text{twisted yarn length}} \bigg|_{\text{constant filament strain}}$$  \hspace{1cm} (2-21)

The denier ratio for constant filament strain as specified for station 2 is:

$$\frac{D_2}{D_0} = \frac{\text{grams/twisted yarn length}}{\text{grams/untwisted yarn length}} \bigg|_{\text{constant filament strain}}$$  \hspace{1cm} (2-22)

where $e_{\text{cold}}$ is the filament strain in the cold zone at station 2.

Substituting (2-20) into (2-22) we obtain

$$e_{\text{cold}} = \frac{V^2}{V_0^2} C_r - 1$$  \hspace{1cm} (2-23)
2.3.4 **Cold Zone Tension**

Given the tensile strain level of the filaments in the cold zone, we use the measured constitutive relationship to determine filament tension. The corresponding yarn tension in the cold zone entrance at station #2 is then determined by resolving the filament tensions in the direction of the yarn axis by multiplying filament tension by the cosines of local helix angles. Thus the cold yarn tension, $T_c$, is simply expressed as:

$$T_c = E_c e_c A_f \sum N_i \cos \theta_i$$

where

- $E_c$ = Cold filament extensional modulus
- $e_c$ = Cold Filament strain (assumed uniform for all filaments)
- $A_f$ = Cross sectional area of filament
- $N_i$ = Number of filaments in $i^{th}$ radial layer
- $\theta_i$ = $i^{th}$ radial layer helix angle

Equation (2-24) is fully developed in Chapter 3.

2.3.5 **Cold Zone Torque**

Naar proposes that yarn torque at the entrance to the cold zone in the false twist texturing threadline is simply due to components of individual filament bending moments, filament torsional moments, and filament tension. Thus he neglects transverse interfiber stresses and interfiber fric-
tion. We assume that interfiber transverse stresses or inter-
fiber friction do not play a significant role in developing
yarn torque at the cold zone entrance. However, as is dis-
cussed later, transverse interfiber stresses and friction do
contribute to the high values of incremental torsional rigid-
ity observed in the yarn segment downstream of the cold zone
entrance and in the cooling yarn segment downstream of the
heater. If the filaments do not exceed their elastic limit
during their traverse of the cold zone, they should not stress
relax significantly before reaching the heater and the cold
yarn twist level should remain uniform.

Actually, there is some stress relaxation of the cold fil-
aments, which results in the yarn slowly gaining twist as it
moves downstream towards the heater. For simplicity of
analysis, this slight uptwist will be neglected.

For the case of elastic fiber, Platt et al. (5) have shown
that the components of yarn torque due to fiber bending and
fiber torsion are:

\[
M_t \text{ cold zone (bending)} = \frac{EI \sin^3 \theta}{R} \quad (2-25)
\]

\[
M_t \text{ cold zone (torsion)} = GI_p P_c \cos^3 \theta \quad (2-26)
\]

where

- \( EI \) = Filament bending modulus
- \( GI_p \) = Filament torsional modulus
- \( \theta \) = Helix angle of filament in yarn
- \( R \) = Radial position of filament in yarn
\[ P_c = \text{Cold twist (radians/twisted length)} \]

The yarn torque due to filament bending and filament torsion is simply the summation of the individual components of torque from each filament. The components of individual filament tension that are perpendicular to the yarn axis are multiplied by their local radial position in the yarn to determine the tensile contribution of the filaments to the yarn torque. This can be expressed as:

\[ M_t \text{ cold zone (tension)} = E c c_f \sum_i N_i R_i \sin \theta_i \]  
(2-27)

The total torque in the cold zone is then:

\[ M_t \text{ cold zone} = EI \sum_i N_i \sin^3 \theta_i + GI \sum_i \frac{P_c N_i \cos^3 \theta}{R_i} + E c c_f \sum_i N_i R_i \sin \theta_i \]  
(2-28)

Equation (2-28) is fully developed in Chapter 3.

2.3.6 Hot Zone Tension

The threadline tension at the hot zone entrance is due to the components of filament drawing forces resolved in the direction of the yarn axis. These drawing forces are determined by the filament draw ratio and the material-system constitutive behavior. The draw ratio of each filament is dependent on its radial position, on the amount of uptwist taking place at the heater entrance and on the ratio of yarn velocity before and after the drawing neck. Brookstein(9)
has shown, in studies of sequential cross sections through the necking zone, that there is little opportunity for additional radial migration at this point in the threadline. However, the radial migration pattern developed at the cold zone entrance persists. Therefore the filament whose position, upon entering the hot zone, is on the outside yarn layer, stays on the outside while it is drawn. From this we deduce that filaments on the outside of the neck have a greater path length to follow and therefore draw more than filaments centrally located in the hot yarn.

On the other hand, because of the radial migration which took place at the cold zone entrance, interchange of radial position will occur for each filament along its length as it enters the hot zone. This interchange assures that each filament will be subjected to different draw ratios along its length. However, the average draw ratio in a long yarn segment is equal for all filaments. Figure 2.6 shows an example of how draw ratio may vary along a yarn length for two different filaments.

Figure 2.6
The filament drawing extension in the hot zone can be determined from its path length as is shown in Figure 2.7.

Figure 2.7

Let

\[ V_2 \Delta t = \text{Length of yarn entering the neck in time } \Delta t \]
\[ V_3 \Delta t = \text{Length of yarn leaving the neck in time } \Delta t \]

Each \((V_2 \cdot \Delta t)\) yarn length contains \((V_2 \sec \theta \cdot \Delta t)\) filament length. As this length moves through the hot yarn neck during \(\Delta t\) it is drawn to the new length \((V_3 \sec \theta \cdot \Delta t)\). Thus the filament drawing extension in the neck is simply:

\[
\text{Extension} = \frac{V_3 \sec \theta \Delta t - V_2 \sec \theta \Delta t}{V_2 \sec \theta \Delta t} = \mathbf{(2-29)}
\]

\[
= \frac{V_3 \sec \theta}{V_2 \sec \theta} \frac{3^3}{2} - 1 = \mathbf{(2-30)}
\]

It should be pointed out that filament drawing extension expressed in eq. (2-29) does not include the cold zone strain referred to in (2-23). For it can be shown that the cold modulus of the filament far exceeds its "drawing modulus" and thus the levels of filament extension present in the cold
zone are about two orders of magnitude lower than the drawing extensions. Therefore neglect of cold strain is justified when determining filament drawing extension.

Having determined filament drawing extension by means of eq. (2-30), we must now develop information on its constitutive relationship. And from the latter we will be able to calculate filament drawing tensions.

Since the constitutive behavior of fibrous polymers incorporates the influence of temperature and strain rate, it was deemed advisable to measure the local extension behavior under the same machine conditions as in actual texturing. The MITEX False Twisting Apparatus was accordingly used as a continuous extensometer by simply bypassing the pin spindle and measuring the threadline tension for different draw ratios and temperatures. The load-extension data were fit to polynomial expressions through regression analysis and the expression shown below forms the material-system constitutive relationships used in determining filament tensions in the hot draw-uptwist zone:

\[
\text{Filament Tension} = A (\text{Draw Ratio})^2 + B (\text{Draw Ratio})^3
\]

(2-31)

The data on which (2-31) is based, are presented in Appendix 3. Finally the threadline tension at the hot zone entrance can be determined from knowledge of hot zone geometry,
filament extensions, and material-system constitutive relations. Hot zone tension is expressed simply, as:

\[ T_{\text{hot zone}} = \sum_i \text{Tension}_i N_i \cos \theta_i \]  

(2-32)  

Substituting (2-31) into (2-32) we obtain

\[ T_{\text{hot zone}} = \sum_i \left( A (\text{Draw Ratio}_i)^2 + B (\text{Draw Ratio}_i)^3 \right) N_i \cos i \theta \]  

(2-33)  

where

- \( N_i \) = Number of filaments in \( i^{th} \) radial layer
- \( \theta_i \) = Helix angle of the \( i^{th} \) radial layer
- \( \text{Draw Ratio}_i \) = Draw Ratio of filaments in the \( i^{th} \) radial yarn layer
- \( A, B \) are experimentally determined material-system properties.

2.3.7 **Hot Zone Torque**

The heater is provided in the texturing zone to effect heat setting of the thermoplastic feed yarn. In mechanical terms, this takes place as a result of hot relaxation of cold zone fiber stresses. As indicated, the filament stresses in the cold zone included bending, torsional and tensile stresses. But as the above mentioned components of threadline torque are heat relaxed, the line torque cannot be kept constant unless the yarn uptwists, which according to all observations it does. This uptwisting incurs some additional filament
bending and torsion strains but the resulting stresses are negligible due to the extremely low hot moduli. For the case of the added tensile strains, due to drawing the filament tensions increase significantly to provide the requisite threadline tension. Accordingly, the assumption in our model is that threadline torque at the heater entrance is based solely on filament tensile forces. Thus hot yarn torque can be expressed as:

\[ M_t \text{ hot zone} = \sum_{i} \text{Load}_i N_i R_i \sin \theta_i \]  

or substituting (2-31) into (2-34)

\[ M_t \text{ hot zone} = \sum_{i} \left( A \text{ Draw Ratio}_i^2 + B \text{ Draw Ratio}_i^3 \right) N_i R_i \sin \theta_i \]  

2.3.8 Calculation of Twist

As indicated previously, there are essentially two levels of twist in the texturing threadline: the cold zone twist (which is a fraction of the hot zone twist) and the hot zone twist. We assume that the hot zone twist and the cooling zone twist are equal.

If the hot zone rotational velocity is \( W_h \) and the cold zone rotational velocity \( W_c \) then the relative rotational velocities before and after the hot zone entrance is \( W_h - W_c \) and this quantity determines the rate of uptwist at this
station according to

\[ P_h^{\text{incremental}} = \frac{W_h - W_c}{V_3} \]  

(2-36)

where \( W_h \) = Angular velocity of hot twisted yarn segment (radians/second)

\( W_c \) = Angular velocity of cold twisted yarn segment (radians/second)

\( V_3 \) = Linear velocity of hot twisted yarn

\( P_h \) (incremental) = hot uptwist/hot twisted length (radians/twisted length)

The hot incremental twist is added to the twist that already exists in the cold zone due to the angular velocity of the cold zone

\[ P_c = \frac{W_c}{V_2} \]  

(2-37)

where \( P_c \) = Cold twist (radians/cold twisted length)

\( V_2 \) = Linear velocity of cold twisted yarn segment

The cold twist per unit cold twisted length is reduced as the cold yarn segment is extended by the ratio \( V_2/V_3 \) so that the cold twist per unit hot twisted length, \( P_{c/h} \), is

\[ P_{c/h} = \frac{P_c V_2}{V_3} = \frac{W_c V_2}{V_2 V_3} = \frac{W_c}{V_3} \]  

(2-38)

The total hot twist per unit hot twisted length is obtained by adding (2-36) and (2-38)
\[ P_h = \frac{W_c}{V_3} + \frac{W_h - W_c}{V_3} = \frac{W_h}{V_3} \] (2-39)

where \( P_h \) = Hot twist per unit hot twisted length

2.3.9 Calculation of Hot Twisted Yarn Velocity

Experimental observation of the twist distribution on the spindle surface indicates that the twisted yarn in the texturing zone generally untwists at the last quadrant of the spindle surface. From this fact it can be assumed that the tension in the yarn during untwisting is approximately equal to the post spindle threadline tension. The approximate twist distribution at the spindle is illustrated below.

![Figure 2.8](image)

The mass continuity requirement is expressed as:

\[ V_{33}D_{33} = V'_{33}D'_{33} = V_{44}D_{44} \] (2-40)

where \( V_3 \) = Linear velocity of twisted yarn, before spindle
\( V'_{33} \) = Linear velocity of twisted yarn, at post
The friction between the yarn and spindle causes the tension in the yarn to rise as the yarn progresses downstream around the spindle surface. This tension differential and hence yarn strain differential causes the yarn denier in the last quadrant of the spindle to decrease slightly. Therefore

\[
D_3' (1 + \epsilon_{\text{yarn}}) = D_3
\]  

(2-41)

where \( \epsilon_{\text{yarn}} \) = yarn strain differential across the pin

Now

\[
V_3 D_3 = V_3' D_3'
\]  

(2-40)

Then

\[
\frac{V_3'}{V_3} = (1 + \epsilon_{\text{yarn}})
\]  

(2-42)

From (2-40) and (2-42)

\[
V_3' D_3' = V_4 D_4
\]  

(2-40)
and \[ V_3 (1 + \varepsilon_{\text{yarn}}) D'_3 = V_4 D'_4 \] (2-43)

The cooling twist contraction factor for the cooling yarn, \( C_{\text{cool}} \), can be expressed as

\[ C_{\text{cool}} = \frac{V_4}{V_3} = \frac{D'_4}{D'_3} \] (2-44)

Substituting (2-44) into (2-43) we obtain

\[ V_3 (1 + \varepsilon_{\text{yarn}}) = \frac{V_4}{C_{\text{cool}}} \] (2-45)

Again we must make the decision as to which model for cooling yarn twist contraction factor we should use in the analysis. Experimental evidence given below indicates that Hearle's Model for cooling yarn twist contraction fits the experimental data closer than does the Alternative Model.

### Cooling Yarn Twist Contraction

<table>
<thead>
<tr>
<th>Machine Twist</th>
<th>Measured</th>
<th>Hearle's Model</th>
<th>Alternative Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>1.263</td>
<td>1.305</td>
<td>1.220</td>
</tr>
<tr>
<td>60</td>
<td>1.396</td>
<td>1.438</td>
<td>1.330</td>
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<tr>
<td>65</td>
<td>1.592</td>
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<tr>
<td>80</td>
<td>1.795</td>
<td>1.926</td>
<td>1.745</td>
</tr>
</tbody>
</table>

Experimental evidence presented in Appendix 2 shows that for a range of steady state operating condition, the tensile gradient of the threadline across the spindle will result in incremental yarn strains, \( \varepsilon_Y \), of less than 1%. And since the
filament strains in the draw region of the texturing zone are of the order of 60% to 80%, it follows that for the common range of draw texturing conditions, neglect of the strain gradient across the spindle will not significantly affect the calculated value of hot threadline velocity, \( V_3 \).

### 2.3.10 Calculation of Yarn Radii

Most draw texturing feed yarn filaments have circular cross sections. When twisted in a yarn structure, the filaments generally take the spatial geometry shown in Figure 2.9. For 34 or 30 filament feed yarns (common apparel yarn structures) there is one central core filament surrounded by three radial layers of filaments. It is common practice to model the structure with 6 filaments in the first layer followed by 12 filaments in the second layer and either 15 or 11 filaments in the outer radial layer depending on the total number of filaments in the structure. The cold yarn radius is then seven times the cold filament radius. Table 2.1 compares the actual cold yarn radii with calculated values. The agreement is good.

When a filament is drawn in the draw texturing process, the ratio of drawn filament radius to the undrawn filament radius is

\[
\frac{r_{\text{drawn}}}{r_{\text{undrawn}}} = \left( \frac{\rho_{\text{undrawn}}}{\rho_{\text{drawn}}} \right)^{1/3} \frac{1}{\text{draw ratio}}
\]

where \( \rho = \) filament density.
Figure 2.9

YARN STRUCTURE FOR POY POLYESTER

245/30 DENIER
Table 2.1: Yarn Dimensions for Poly Polyester 245/30 Denier

<table>
<thead>
<tr>
<th>Machine Twist Ratio</th>
<th>Measured Cold Radius (10^-4) in</th>
<th>Calculated Cold Radius (10^-4) in</th>
<th>Measured Hot Radius (10^-4) in</th>
<th>Calculated Hot Radius (10^-4) in</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>39.1</td>
<td>40.9</td>
<td>39.9</td>
<td>40.6</td>
</tr>
<tr>
<td>60</td>
<td>39.1</td>
<td>40.9</td>
<td>39.9</td>
<td>40.0</td>
</tr>
<tr>
<td>65</td>
<td>39.1</td>
<td>40.9</td>
<td>39.9</td>
<td>40.0</td>
</tr>
<tr>
<td>70</td>
<td>39.1</td>
<td>40.9</td>
<td>39.9</td>
<td>40.0</td>
</tr>
<tr>
<td>80</td>
<td>39.1</td>
<td>40.9</td>
<td>39.9</td>
<td>40.0</td>
</tr>
<tr>
<td>1.57</td>
<td>39.1</td>
<td>40.9</td>
<td>39.9</td>
<td>40.0</td>
</tr>
</tbody>
</table>

Note: The table shows the measured and calculated dimensions for different twist ratios and temperatures.
Shealy and Kitson\(^{(10)}\) show that the ratio of densities for partially drawn polyester to drawn polyester is 0.972. Thus the ratio of filament radii for POY PET according to (2-46) is

\[
\frac{r_{\text{drawn}}}{r_{\text{undrawn}}} = 0.986 \frac{1}{\sqrt{\text{draw ratio}}}
\]

Thus we conclude that a good approximation of the hot yarn radius is:

\[
\frac{r_{\text{hot yarn}}}{r_{\text{cold}}} = 0.986 \frac{1}{\sqrt{\text{machine draw ratio}}} \tag{2-47}
\]

Using (2-47) for various threadlines we obtain the values of hot yarn radii given in Table 2.1. These predicted values are compared to measured values and manifest good agreement.

For the purpose of computation the radial distance from each filament centroid in a given layer to the yarn center is used as the radius of that yarn layer. For example, the radius of the third radial layer is 6 filament radii, rather than 6 1/2. The latter quantity represents the outer radius of that layer.

2.3.11 Summary

An analysis of steady state threadline mechanics has been presented in this section, with a view towards determining steady state levels of threadline torque and threadline tension, as well as twist distribution over the entire texturing zone. To undertake calculations based on this analysis,
one requires information concerning texturing machine settings (i.e. spindles speed, feed roller velocity, take up roller velocity, and heater temperature) and feed stock material properties. **No additional** information or measurements are necessary. The treatment of the threadline mechanics incorporates several simplifications and assumptions in accord with numerous experimental observations:

1. All filaments are strained equally in the twisted yarn of the cold zone.

2. The effect of interfiber friction and transverse stresses on threadline torque is considered negligible at the cold zone and hot zone entrances.

3. The friction between yarn and spindle does not significantly affect linear velocities of the threadline in draw texturing.

4. Hot zone threadline, cooling zone threadline, and spindle rotational velocities are equal.

5. The twist in the texturing zone has two uniform levels, one for the cold zone, the second for the hot and cooling zones.

6. Tension and torque in the texturing threadline are uniform from the entry roll to the spindle.

7. Inertial effects on threadline mechanics are considered negligible.

8. Filaments in the twisted threadline follow ideal
right circular helices.

9. Threadline torque at the entrance to the hot zone is primarily due to fiber tension.

10. In order to calculate twist contraction in the cold zone two different models are used, one based on an averaging over the entire cross section, the other based on an averaging over the yarn radius. The former model (averaged over the entire yarn section) is used to calculate twist contraction in all other zones.

In the next section computations based on these steady state analyses are compared with measured experimental results obtained in draw texturing of partially oriented polyester yarn (POY-PET). In a subsequent section we will furnish similar comparisons for texturing experiments with partially oriented nylon.

2.4 EXPERIMENTAL STUDY OF STEADY STATE DRAW TEXTURING

2.4.1 Materials and Texturing Apparatus

Polyester partially oriented yarn (POY PET) produced by the DuPont Company has been used for the experimental study. The dimensions and properties of the feed yarn are as follows:

- Undrawn Filament Denier 245/30 8.17
- Undrawn Filament Radius 5.7 x 10^-8 in
- Undrawn Filament Tensile Modulus 4.8 x 10^5 psi
- Undrawn Filament Bending Rigidity 3.97 x 10^-8 lb-in^2
Filament dimensions and tensile modulus were measured in the Fibers and Polymers Laboratories at M.I.T. The bending modulus was calculated from the tensile modulus based on the assumption of equal axial tensile and compressive moduli. The torsional rigidity \( G_I \) measured on drawn PET by Mr. J. Donovan of F.R.L. was considered applicable to the current partially drawn specimen in accordance with Ward's observation that \( G_I \) does not vary significantly with axial draw ratio.

A laboratory pin-spindle false twist texturing machine developed at MIT (MITEX False Twister) has been used for our draw texturing studies. A picture of the MITEX False Twister is presented in Appendix 4. In this apparatus the surface speed of the feed rollers is 0.29 inches per second. The take up roller speed is varied in accordance with the required draw ratio. The pin-spindle is driven by a motor-belt combination and the spindle speed is controlled by a Variac Speed Controller and monitored periodically by a General Radio Strobotac. The spindle speed is varied in accordance with the desired machine twist. The heater temperature controller unit is a standard Rosemount device which is used in commercial texturing operations. Its accuracy is within \( \pm 3^\circ \text{C} \). The threadline
torque and threadline tension are measured by the MITEK torque tension Meter. Its method of operation is described in Appendix 4.

2.4.2 Results and Discussion

Given the machine settings, i.e. machine draw ratio, basic or machine twist, and heater temperature, and material properties, one can use the computer subroutine "YARN" listed in Appendix 5 to determine the cold threadline, hot and cooling threadline twist levels and threadline torque and tension. Comparisons of the experimental and predicted results are given in Table 2.2 and Figures 2.10, 2.11 and 2.12. Model 1 includes the method of radial averaging for cold twist contraction given in section 2.3.2 and Model 2 incorporates Hearle's method of determining cold twist contraction given in Section 2.3.2.

The results show that there is good agreement over a wide range of machine twists between the experimental data and the predicted threadline behavior for both cold twist contraction models. At machine twists above 70 TPI the agreement is reduced. For example at 80 TPI machine twist, the percentage differences between the experimental and predicted cold twist, hot twist, threadline torque, and threadline tension are 24.6%, 29.6%, 2.4%, and 11.9% respectively for Model 1, and 38.8%, 29.6%, 22.8% and 11.3% respectively for Model 2. Observation of texturing threadline segments presented in Figure 2.13 in-
Table 2.2
COMPARISON OF MEASURED AND PREDICTED VALUES OF THREADLINE PARAMETERS

<table>
<thead>
<tr>
<th>Machine Twist</th>
<th>Draw Ratio</th>
<th>Heater Temp.</th>
<th>Tension, grams</th>
<th>Torque (10^{-4}) in lbs</th>
<th>Cold Twist, TPI</th>
<th>Cooling Twist, TPI</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Measured</td>
<td>Predicted</td>
<td>Model 1</td>
<td>Model 2</td>
</tr>
<tr>
<td>45</td>
<td>1.60</td>
<td>210°C</td>
<td>30.9</td>
<td>26.2</td>
<td>28.7</td>
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<tr>
<td>50</td>
<td>1.60</td>
<td>210°C</td>
<td>24.8</td>
<td>25.0</td>
<td>27.9</td>
<td>1.47</td>
</tr>
<tr>
<td>55</td>
<td>1.60</td>
<td>210°C</td>
<td>23.8</td>
<td>23.6</td>
<td>26.9</td>
<td>1.64</td>
</tr>
<tr>
<td>60</td>
<td>1.60</td>
<td>210°C</td>
<td>20.3</td>
<td>22.1</td>
<td>25.8</td>
<td>1.67</td>
</tr>
<tr>
<td>65</td>
<td>1.60</td>
<td>210°C</td>
<td>18.4</td>
<td>20.4</td>
<td>24.4</td>
<td>2.34</td>
</tr>
<tr>
<td>70</td>
<td>1.60</td>
<td>210°C</td>
<td>17.7</td>
<td>18.6</td>
<td>22.8</td>
<td>2.34</td>
</tr>
<tr>
<td>80</td>
<td>1.60</td>
<td>210°C</td>
<td>16.8</td>
<td>14.8</td>
<td>18.7</td>
<td>2.50</td>
</tr>
<tr>
<td>50</td>
<td>1.60</td>
<td>190°C</td>
<td>24.4</td>
<td>25.1</td>
<td>27.6</td>
<td>1.54</td>
</tr>
<tr>
<td>60</td>
<td>1.60</td>
<td>190°C</td>
<td>20.5</td>
<td>22.2</td>
<td>25.4</td>
<td>2.40</td>
</tr>
<tr>
<td>65</td>
<td>1.60</td>
<td>190°C</td>
<td>18.7</td>
<td>20.6</td>
<td>24.0</td>
<td>2.39</td>
</tr>
<tr>
<td>70</td>
<td>1.60</td>
<td>190°C</td>
<td>17.7</td>
<td>18.8</td>
<td>22.4</td>
<td>2.31</td>
</tr>
<tr>
<td>65</td>
<td>1.60</td>
<td>230°C</td>
<td>17.5</td>
<td>20.1</td>
<td>23.6</td>
<td>2.24</td>
</tr>
</tbody>
</table>

(note: each data point represents at least five independent measurements)
Figure 2.10

1.60 Draw Ratio 210°C Heater

- experimental
- predicted

COLD TWIST

COOLING TWIST

Tension, grams Torque, (10^-6) in lb

Basic Twist
Figure 2.11

1.60 Draw Ratio 190°C Heater

---

experimental

predicted
Figure 2.12

1.6 Draw Ratio 65 TPI Basic

- - - - experimental
- - - - predicted
DRAW TEXTURING THREADLINES

245-34 POY PET 1.60 DRAW RATIO 210°C. HEATER

<table>
<thead>
<tr>
<th>BASIC TPI</th>
<th>COLD ZONE</th>
<th>NECK</th>
<th>COOLING ZONE</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>60</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>65</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>70</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>80</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
icated when machine twist is increased to and beyond 65 TPI, twisted filaments lose their circular helical geometry. At 80 TPI the cooling zone segments are continuously torsionally buckled. Such buckling invalidates the geometric components of our assumptions so that the disagreement at 80 TPI is not unexpected.

We have calculated the root mean square of the percentage error for threadline cold twist and hot twist, threadline torque and threadline tension for each of these models under a variety of processing conditions and these values are given below:

<table>
<thead>
<tr>
<th></th>
<th>Model 1</th>
<th>Model 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tension</td>
<td>9.3%</td>
<td>23.4%</td>
</tr>
<tr>
<td>Torque</td>
<td>12.2%</td>
<td>12.0%</td>
</tr>
<tr>
<td>Hot Twist</td>
<td>12.3%</td>
<td>12.3%</td>
</tr>
<tr>
<td>Cold Twist</td>
<td>10.9%</td>
<td>16.7%</td>
</tr>
</tbody>
</table>

It is clear that Model 1 provides better agreement between theory and experiment and so it will be employed in the analysis of unsteady operation of the draw texturing process.

Figures 2.10 and 2.11 (which represent somewhat different operating conditions) show that as machine twist is increased while heater temperature and machine draw ratio are kept constant, threadline tension decreases and the threadline torque increases. This seemingly contrary behavior can be
attributed to the increase in local filament helix angles in the outer layers of the yarn, which reduces the yarn axial component of filament tension and increases its circumferential component.

As the machine twist increases, the variation of draw ratios in different filament layers in the drawing neck increases. Table 2.4 presents the draw ratios computed for each layer of filaments as well as the draw ratio RMS value for all filaments in a given yarn cross section at a given machine draw ratio.

Table 2.4

<table>
<thead>
<tr>
<th>Machine Twist</th>
<th>Core</th>
<th>1st Layer</th>
<th>2nd Layer</th>
<th>3rd Layer</th>
<th>RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>45 TPI</td>
<td>1.374</td>
<td>1.428</td>
<td>1.565</td>
<td>1.735</td>
<td>13.9%</td>
</tr>
<tr>
<td>50 TPI</td>
<td>1.323</td>
<td>1.394</td>
<td>1.564</td>
<td>1.760</td>
<td>16.0%</td>
</tr>
<tr>
<td>55 TPI</td>
<td>1.266</td>
<td>1.358</td>
<td>1.565</td>
<td>1.784</td>
<td>18.0%</td>
</tr>
<tr>
<td>60 TPI</td>
<td>1.204</td>
<td>1.322</td>
<td>1.569</td>
<td>1.809</td>
<td>20.1%</td>
</tr>
<tr>
<td>65 TPI</td>
<td>1.134</td>
<td>1.286</td>
<td>1.577</td>
<td>1.833</td>
<td>22.3%</td>
</tr>
<tr>
<td>70 TPI</td>
<td>1.057</td>
<td>1.251</td>
<td>1.589</td>
<td>1.857</td>
<td>24.8%</td>
</tr>
</tbody>
</table>

It should be noted that even though there is a radial distribution of filament draw ratios. The weighted average of the filament draw ratios must equal the machine draw ratio during steady state operations.

Figure 2.12 shows that temperature of the heater over the range tested does not greatly affect the threadline proper-
ties. This follows from the fact that the common range of
texturing temperatures has minimal effect on hot filament
drawing forces.

2.5 EFFECT OF MATERIAL PROPERTIES ON STEADY STATE THREADLINE
BEHAVIOR

If the fiber producer were to alter two material properties
of the texturing, i.e. cold shear modulus and cold extensional
modulus, such modification would affect the threadline behav-
ior during texturing. The predicted influence of changes in
either modulus, while the other is kept constant is shown in
Figures 2.14 and 2.15. These graphs indicate that if the
shear modulus or extensional modulus is increased, both thread-
line torque and tension increase, while cold zone twist level
decreases. Since the hot zone twist was previously indicated
to be a function solely of yarn throughput speed and spindle
speed, the hot zone twist remains unchanged for this case in
which the cold feed yarn properties are modified.

The effect of doubling either modulus on a given threadline
response, i.e. threadline torque, threadline tension, or cold
twist level is presented in Table 2-5. It can be concluded
that the relative changes in threadline response are minimal
considering the magnitude of either modulus change.
Figure 2.14
Threadline Parameters vs. Young's Modulus
Threadline Parameter vs. Torsional Rigidity

Figure 2.15

- Torque, \((10^{-4})\) in-lbs
- Tension, grams
- TPI

Torsional Rigidity \((10^{-8})\) in

Graph showing the relationship between threadline parameters and torsional rigidity for different TPI values (50 TPI and 60 TPI for tension and torque, respectively).
Table 2.5

<table>
<thead>
<tr>
<th>Machine TPI</th>
<th>Per Cent Torque Increase</th>
<th>Per Cent Tension Increase</th>
<th>Per Cent Cold Twist Increase</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>50 60</td>
<td>50 60</td>
<td>50 60</td>
</tr>
<tr>
<td>Shear Modulus,</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+ 100%</td>
<td>15% 15%</td>
<td>5% 5%</td>
<td>-23% -23%</td>
</tr>
<tr>
<td>Extensional</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Modulus</td>
<td>+ 100% 6% 7%</td>
<td>2% 2%</td>
<td>-10% -10%</td>
</tr>
</tbody>
</table>

As has been often pointed out in the literature, the cold tensile modulus is dependent on the draw ratio of the feed yarn, but the cold shear modulus according to Ward (11) is not. Therefore changes in cold tensile modulus are easily accomplished via the fiber producers process control but shear modulus changes require more fundamental alterations in polymer composition or fine structure. (We will discuss the effect of material changes in a later section dealing with experiments on partially drawn nylon).

Obviously, the simplest modification which can be brought about in feed yarn properties is accomplished by changing filament denier for a given yarn denier. One would expect that lower filament deniers in such a case, would increase the tensile contribution to yarn torque in the cold zone. This direction of change is being developed in the fiber industry. Further changes for a given fiber and yarn denier are easily brought about by altering the filament cross sectional profile, and this method is also under industrial development.
3.1 INTRODUCTION

It was observed in earlier studies that when texturing spindle speeds were suddenly altered, the first signs of twist change in the threadline occurred at the entry to the heater. It was at this location, during steady state texturing of PET pre-drawn yarns, that the yarn softened and the twist rose rapidly from a relatively low cold zone value to a relatively high hot zone value. Transient twist changes introduced at the heater entrance during non-steady spindle rotation appeared to lock in, remaining essentially unchanged as they moved with the threadline down towards the spindle. And a short time after the disturbance one observed that the transient twist distribution beyond the heater was tilted downward in the downstream direction for a spindle speed increase and tilted up in the downstream direction for a spindle speed decrease. Seemingly, the threadline achieved unusually high torsional rigidity after it passed the heater and locally at the heater entry, it appeared to be torsionally soft.

An experimental study was undertaken to confirm the presence of a torsionally "soft" segment in the threadline at the heater entrance and of a highly rigid segment at the
heater exit and reaching beyond to the spindle. The results of these experiments clearly confirm the presence of a soft segment and a rigid segment in the threadline beyond the heater. Further they indicate the presence of an additional soft segment located at the entry to the cold zone and a rigid segment consisting of the remainder of the cold zone.

A complementary study was undertaken to determine to what extent this mode of threadline behavior occurs in the draw texturing system. Experimental results indicate that in the draw texturing threadline there are also regions of torsionally soft material at the cold zone and heater entrance and rigid segments in the remainder of the cold zone and cooling zone.

This twist formation behavior during non-steady operation is significant since the twist level at the spindle at a given moment is not necessarily the same as the twist level at the heater entry. Rather, the twist level at the spindle at a given moment is equal to that which formed at the heater entrance at an earlier time.

This chapter is divided into three parts. First, we present the experimental observations which are subsequently used to model transient operation of the draw texturing system. In the second part we develop two theoretical models of the transient operation of the draw texturing system along with experimental verification of their validity.
Finally the models used to predict threadline response to unsteady machine operation resulting from such causes as unsteady material delivery or material take up, oscillating spindle speed, etc.

3.2 OBSERVATIONS OF TORSIONAL BEHAVIOR OF THE TEXTURING THREADLINE

3.2.1 Experiments Using Pre-Drawn PET Yarns

The interactions between processing parameters and threadline tension of the textured yarn were measured on the MITEX False Twist Texturing Device, which permitted monitoring of threadline tension through strain gage detection of the in-line forces acting on the feed roll assembly \(^6\). For these particular tests the machine conditions were, set as follows, unless otherwise specified:

- **Feed Speed**: 1.1 in/sec
- **Overfeed**: 1.8%
- **Temperature**: 210 °C
- **Heater Length**: 1.5 in
- **Cold Zone Length**: 5 in
- **Cooling Zone Length**: 5 in
- **Twist Direction**: S
- **Pretension**: 0 g.f.

The machine parameter varied was machine (or basic) twist, defined as the RPM of the spindle divided by surface velocity of the take-up roller. Four levels of twist were used: 30, 45, 56, and 60 TPI (basic). The texturing machine was stopped after operating in steady state at each of the above conditions and yarn segments from the cold zone and
from the cooling zone were affixed to a temporary cardboard-clamp system to prevent retraction and/or untwisting. The yarn segment was then transferred to a torsion wire instrument, clamped at its top end to the base of the vertical calibrated wire, and attached to a T-bar clamp at its bottom end. This mounting procedure was completed without releasing the twist and without causing further extension of the yarn segment being tested. Suitable weights were then added to the T-bar so as to apply to the yarn that level of tension which had been recorded during texturing, but without permitting the T-bar to rotate. Thus, the yarn segment in the instrument was kept under a tension equal to that of the texturing zone. At this moment, the angular rotation of the torsion wire was observed and the yarn torque was calculated. In all, it was a matter of 30 minutes between the stopping of the texturing machine and the measurement of yarn segment torque.

There was no simple way to determine the extent of torque relaxation which may have taken place during this interval. But it was observed that, except in one case, the threadline torque measurements on cold zone segments were within 20% of the measurements of segments taken from the cooling zone, with the cold zone values being the lower. This was not unexpected since it was likely that the yarn which had been up-twisted over the heater and cooled at constant torque for a
period of about 30 seconds, would have encountered sufficient torsional creep as to minimize subsequent torsional relaxation. On the other hand, the yarn in the cold zone which had shorter exposure to the given torque during texturing and which was not subjected to hot creep should manifest greater subsequent torque relaxation.

The yarn segment was then uptwisted slightly at the same tension to permit measurement of incremental torques. Similar experiments were run with incremental downtwisting. In this manner, it was possible to calculate the torsional rigidity of the yarn for four different twist levels and for two different zones of the threadline.

In addition to experiments involving yarn uptwisting at constant tension, experiments were also run at constant twist with tension increments added or removed. And the resulting torque increments were measured. Finally, fresh threadline yarn samples were untwisted completely and a full torque twist curve was plotted. This last test procedure also allowed for measurement of yarn extensions which accompanied untwisting and these data were employed to calculate twist contraction factors.

The structural data for the threadline observed in these tests are listed in Table 3.1 and the torsional data in Table 3.2. The data show an increase in cold zone twist as well as in cooling zone twist with machine twist (basic),
### Table 3.1
STRUCTURE OF THREADLINE

<table>
<thead>
<tr>
<th>Machine* Twist (TPI)</th>
<th>Twist/in of Threadline Zones</th>
<th>Retraction</th>
<th>Tangent Q**</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cold</td>
<td>Cooling</td>
<td>Cold</td>
</tr>
<tr>
<td>40</td>
<td>13.0</td>
<td>34.4</td>
<td>2%</td>
</tr>
<tr>
<td>45</td>
<td>30.8</td>
<td>57.1</td>
<td>8%</td>
</tr>
<tr>
<td>56</td>
<td>48.8</td>
<td>75.4</td>
<td>18%</td>
</tr>
<tr>
<td>60</td>
<td>56.2</td>
<td>91.5</td>
<td>26%</td>
</tr>
</tbody>
</table>

* Basic twist, turns/in of untwisted yarn

**Surface helix angle calculated on the basis of a packing factor or 0.85.

### Table 3.2
TORSIONAL PROPERTIES OF THREADLINE ZONES

<table>
<thead>
<tr>
<th>Machine Twist (TPI)</th>
<th>Tangent Q**</th>
<th>Steady State Tension</th>
<th>Steady State Torque</th>
<th>Torsional Stiffness</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cold</td>
<td>Cooling</td>
<td>gf</td>
<td>in-lbs x 10^-4</td>
</tr>
<tr>
<td>30</td>
<td>.22</td>
<td>.55</td>
<td>16.1</td>
<td>.33</td>
</tr>
<tr>
<td>45</td>
<td>.53</td>
<td>.97</td>
<td>16.7</td>
<td>.79</td>
</tr>
<tr>
<td>56</td>
<td>.89</td>
<td>1.24</td>
<td>15.0</td>
<td>1.48</td>
</tr>
<tr>
<td>60</td>
<td>1.06</td>
<td>1.75</td>
<td>14.1</td>
<td>1.65</td>
</tr>
</tbody>
</table>

* Basic Twist

** Surface helix angle, calculated
although the ratio of twist in the two zones goes from 0.38 to 0.61. In other words, a higher proportion of twist takes place in the cold zone at the higher spindle speeds.

Table 3.2 shows the steady state torque values for yarn segments taken from the cold zone and from the cooling zone. The latter values are somewhat higher for each condition of texturing. As was stated above, torsional relaxation of the cooling zone specimens is more likely to have occurred during processing, hence it is logical to select the cooling zone values as more representative of actual threadline torques.

It is noted that over the machine twist range of 30 to 60 TPI, the torque values increase by about a factor of five, that is, from 0.4 to 1.9 times $10^{-4}$ in/lbs. The cooling zone twist increases by a factor of 2.7, but the cold zone twist increases by a factor of 4.3. Threadline tensions are seen to rise slightly from 30 to 45 TPI machine twist, then decrease steadily (about 16%) as machine twist increases from 45 to 60 TPI. This is consistent with the behavior noted by Backer and Yang$^{(4)}$. The level of cold zone twist manifests a linear reflection of increases in threadline torque as determined from measurements on yarn segments from the cooling zone. This holds up to 56 TPI machine twist (basic).

If threadline tension is relatively constant at machine twists of 60 TPI the threadline torque remains the same as at 56 TPI, though the cold twist does increase somewhat.
The reason for this flattening of the torque-twist curve becomes evident when one views photographs of threadline configurations(4) showing the onset of torsional buckling at 60 TPI basic twist for texturing overfeed conditions of 0%. For the corresponding current tests at 1.8% overfeed, the torsional buckling at machine twists of 60 TPI basic is even greater than for 0% O/F.

Figure 3.1, 3.2, 3.3, 3.4, show results of uptwisting tests on a single yarn segment taken from the cold and cooling zones for the four test conditions listed in Tables 3.1 and 3.2. The torque plotted in each case for a twist increment of zero, corresponds to the offline measurement of threadline torque reading for both cold zone and cooling zone actual twist values as reported in Tables 3.1 and 3.2. Again, it is noted that torques for the cold zone yarn segments are consistently below those measured for the cooling zone segments. The uptwist-torque values for the most part vary linearly with the uptwist increments (plotted in radians-per-inch, rather than in TPI). Upon removal of twist increments, the relationship is still quite linear, with a slight hysteresis for the cooling zone segments, but with considerably greater hysteresis for cold zone segments, again reflecting the history of more earlier relaxation in the cooling zone segments in actual processing.

In view of the conclusion that the cooling zone yarn tor-
Figure 3.1

150/34 POLYESTER TEXTURED & 210°C 1.3% OVERFEED
30 TPI BASIC TWIST
16.1 GRAMS TENSION

COOLING ZONE RIGIDITY = .12 x 10^{-4} \text{ in}^2 \text{LBS}

TORSIONAL STIFFNESS RATIO = 7.1

COLD ZONE RIGIDITY = .17 x 10^{-5} \text{ in}^2 \text{LBS}
Figure 3.2

150/34 POLYESTER TEXTURED @ 210°C 1.8% OVERFEED
45 TPI BASIC TWIST
16.7 GRAMS TENSION

Cooling Zone Rigidity = \(0.24 \times 10^{-4}\) in²LBS

Torsional Stiffness Ratio = 5.5

Cold Zone Rigidity = \(0.044 \times 10^{-4}\) in²LBS
Figure 3.3

150/34 POLYESTER TEXTURED @ 210°C 1.8% OVERFEED
56 TPI BASIC TWIST
15 GRAMS TENSION

Cooling Zone GI_p = 0.32 x 10^{-4} in^2 lbs

Torsional Stiffness Ratio = 1.9

Cold Zone Rigidity = 0.17 x 10^{-4} in^2 lbs

Torque (10^{-3}) in lbs

0.0 1.0 2.0 3.0 4.0

RAD/IN

78
Figure 3.4

150/34 POLYESTER TEXTURED at 210°C 1.8% OVERFEED
60 TPI BASIC TWIST
14.1 Grams Tension

Cooling Zone Rigidity = \(0.36 \times 10^{-4}\) in²lbs
Cold Zone Rigidity = \(0.27 \times 10^{-4}\) in²lbs
ques were the more accurate values, additional incremental uptwist tests were run on a number of cooling zone segments. The results of these tests for 30, 45, and 60 TPI machine twists are furnished in Fig. 3.5 and indicate the spread of the experimental values observed, as well as the typical curve selected for Figs. 3.1, 3.2, 3.3, and 3.4. The slope of the incremental torque-twist curve is obviously greater for the 60 TPI machine twist segment than for the 45 or 30 TPI cases, and this is reflected in Table 3.2 as a three-fold increase in torsional stiffness over the test range.

The torsional data are summarized in Fig. 3.5, which highlights the tremendous difference between the slope of the forming torque-twist curve (shown as the solid bars), and the incremental torque-twist curves (shown as the open bars). It is also seen that the incremental rigidities vary over the test range by about 15 times for the cold zone, and only 3 times for the cooling zone materials. Thus, the data fully confirm the hypothesis that was put forth on the basis of observations of tilting in transient twist distributions after changing of spindle speeds, i.e. that threadline twist increases imparted at the entry to hot zone are in effect locked in by the development of high torsional rigidities to incremental uptwist over the heater and beyond. But the data also reveal that a similar high torsional rigidity to incremental twist develops right after initial formation of
Figure 3.5

TORSIONAL BEHAVIOR OF TEXTURING THREADLINE

$1.0 \times 10^{-5} \text{ in}^2 \text{ LBS}$

INCREMENTAL TORSIONAL RIGIDITY

TORSIONAL RIGIDITY OF YARN FORMATION

COLD ZONE

56 TPI

60 TPI

MACHINE TWIST (BASIC)

30 TPI

45 TPI

LOCAL TWIST TPI (NORMAL)
Figure 3.6

Torsional Stiffness of Cooling Zone at Different Levels of Nominal Twist

- Experimental Bounds
- Typical Curve

Torque (10^{-4}) Inch LB

As Processed

00 1.0 2.0 3.0 4.0 5.0 6.0 7.0

RAD/IN
the yarn in the cold zone. Although cold zone rigidities are somewhat less than the cooling zone rigidities, they are nonetheless orders of magnitude greater than the cold forming "rigidity". Likewise, the cooling zone incremental rigidity is orders of magnitude greater than the hot uptwisting "rigidity". Stated another way, the entries to the cold zone and to the heater zone are, in effect, dynamic torsional soft zones in the threadline and any sudden changes which may occur in threadline torque, for whatever reason, will be reflected in large local twist changes at these entry zones and negligible changes elsewhere. And it is to be expected that twist changes introduced at these points, will be propagated along the threadline with translational movement of the yarn.

The second series of tests were run on the yarn segments removed from the threadline. In these experiments, as was indicated, the twist of the threadline segment was kept constant while the tension was varied incrementally around the value recorded as the operating threadline tension. Incremental changes in yarn torque resulting from these tension changes were recorded as are reported in Figs. 3.7, 3.8, 3.9, and 3.10. The slopes of the curves plotted in these figures indicate the influence of tension on torque in the cold zone and cooling zone yarn segments. There is some increase in the tension-torque effect as the machine twist is increased.
Figure 3.7

150/34 POLYESTER TEXTURED @ 210°C 1.3% OVERFEED
30 TPI BASIC TWIST

![Graph showing torque vs. GPD tension with zones labeled Cold Zone, Cooling Zone, and Processing Tension.]
150/34 POLYESTER TEXTURED @ 210°C 1.3% OVERFEED
45 TPI BASIC TWIST

Figure 3.8
150/34 POLYESTER TEXTURED @ 210°C 1.8% OVERFEED
56 TPI BASIC TWIST

Figure 3.9
150/34 POLYESTER TEXTURED 210°C 1.3% OVERFEED
60 TPI BASIC TWIST

Figure 3.10
which, in turn, increases both cold zone twist and cooling zone twist as reported in Table 3.1. But, above machine twists of 45 TPI (basic), the changes in the tension-torque effect are not consistent. However, it can be noted that the tension-torque effect for 30 TPI basic machine twist is greater for the cooling zone segment than for the cold zone segment. And this condition reverses as the machine twist is increased to 56 and 60 TPI (basic).

One expects the tension-torque effect, or the slope of the torque-tension curve (at constant twist) to depend in the actual twist in the local yarn segment. Thus, the larger slope observed at 30 TPI basic machine twist in the more highly twisted cooling segment vs. the less twisted cold segment is according to expectation. The reversal which occurs at higher machine twists in the comparative levels of cooling and of cold zone tension-torque effects suggests the possibility of interfilament interactions in the cooling zones which serve to suppress the torsional response to changes in yarn tension.

The torque-tension slopes differ significantly from the cold zone to the cooling zone in a given threadline. This fact combined with the confirmed existence of torsional soft zones at the cold entry and the heater entry suggests that sudden (and large) changes in threadline tension for whatever reason will result in immediate (slight) redistribution
of twists between threadline zones to suppress the torque differences followed by considerable twist changes at the soft zones in response to the new uniform (along the threadline) level of torque. Thus, tensile transients in the threadline can be expected to cause torque transients as well, and these will introduce significant changes in soft zone twisting levels, which will then propagate along the threadline.

Finally, the torque-twist relationship of the cooling zone segments was determined for the range of untwisting down to zero torque. Typical data from such experiments are plotted in Figure 3.11 where it is seen for the 60 TPI machine basic twist case (with 91.5 TPI local twist) that the torque-un-twist curve (note, for a 5-inch specimen) is fairly linear from a torque value of about $1.75 \times 10^{-4}$ in/lbs down to about $0.25 \times 10^{-4}$ in/lbs, at which point only one TPI out of ninety has been untwisted. The curve then flattens significantly as the small residual torque is reduced with further twisting. The significance of this behavior will be discussed in Chapter 4 as it relates to product uniformity.
Figure 3.11

TORQUE - DETWIST
60 TPI BASIC TEXTURED 3 210°C 1.8% OVERFEED
LOCAL TWIST 91.5 TPI

As Processed

TORQUE (10^-4) IN LBS

DETWIST/5 INCH
3.2.2 Experiments Concerning Transient Twist, Using Predrawn PET Yarns

In their photographic study of twist levels during transient operation of the false twist process Backer and Yang did not include twist measurements at the entrance to the cold twist formation zone\(^4\). In the current study a photographic analysis was accordingly undertaken to determine how twist changed at the cold formation zone, cold zone, hot formation zone, hot zone, cooling zone, and post spindle zone during transient operation of the conventional false twist texturing process. In this case a Bolex 16 mm motion picture camera equipped with a telephoto lens was used. The feed yarns used for photographic twist measurement consisted of black and white filaments supplied by the DuPont Company. The yarn (predrawn polyester) was 140 denier with 68 filaments.

To measure local twist at the cold formation zone, a glass slide was pressed against a relatively friction free roller to trap threadline twist and prevent it from propagating behind the defined formation point at the slide -- roller 'nip'. This twist trap assembly was placed slightly downstream of the feed rollers. A schematic diagram of the experimental set up is shown in Figure 3.12.

The machine twist was set either at 30 TPI or 40 TPI and the system was allowed to reach steady state operating condi-
Figure 3.12

MODE I

TWIST TRAP

10.56" 3.0" 13.5"

1.25"

FEED ROLL

YARN

TWIST TRAP ASSEMBLY

GLASS SLIDE

ROLLER

MODE II

TRAP ASSEMBLY

6.13" 7.44" 13.5"

1.25"

FEED ROLL

TAKE UP
tions. The machine twist was then step changed by 10 TPI to either 40 TPI or 30 TPI depending on the previous twist level. Motion pictures of the threadline were taken at different positions so that twist levels could be determined as functions of time (after disturbance) and position. These pictures were taken at the following positions:

**IN COLD ZONE:**
- Cold Zone Entrance
- 3.50 Inches Past Cold Zone
- 6.88 Inches Past Cold Zone

**IN HOT ZONE:**
- Heater Entrance
- Midway Through Heater
- Heater Exit

**IN COOLING ZONE:**
- Immediately After Heater
- .50 Inches Past Heater Exit
- 3.00 Inches Past Heater Exit
- 7.37 Inches Past Heater Exit

**IN POST SPINDLE ZONE:**
- 7.00 Inches Past Spindle

Graphs of twist versus time at different positions of the threadline after a step change in spindle speed are given in Figures 3.13, 3.14, 3.15, 3.16, 3.17, 3.18, and 3.19.

Figures 3.13 and 3.14 show that there is an apparent time lag in the cold twist redistribution during transient operation. Twist levels at the entrance to the cold zone manifest immediate change after the spindle speed is changed. Twist
Figure 3.13

- At Cold Zone Entrance
- × 3 1/2" From Entrance
- ▲ 6 7/8" From Entrance

Nominal Twist Step Down From 40 - 30 Twist Per Inch
Nominal Twist Step up from 30-40 TPI

- All at Mode I
  - At Cold Zone Entrance
  - 3½" from Entrance
  - 6½" from Entrance

Figure 3.14
Nominal Twist Step down from 40-30 TPI

All at Mode I
- Heater Entrance
x 1/2" (Midway) from Heater Entrance
△ At Heater Exit
■ Immediately after Heater Exit

Figure 3.15
Figure 3.16
Nominal Twist Step down from 40-30 TPI

- x 1/2" from Heater Entrance (Mode I)
- ▲ 3" from Heater Exit (Mode II)
- ● 7 5/16" from Heater Exit (Mode III)

Figure 3.17
Nominal Twist Step up from 30-40 TPI

x ½" from Heater Entrance (Mode I)
△ 3" from Heater Exit (Mode II)
○ 7½" from Heater Exit (Mode II)

Figure 3.18
Mode I
Twist at 7\" Post Spindle

Nominal Twist Step up from 30-40 TPI
Nominal Twist Step from 40-30 TPI

Figure 3.19
levels downstream of the entrance to the cold zone appear relatively unchanged at first. About 6 seconds later the twist level 3.5 inches downstream of the entrance changes; the twist level 6.88 inches downstream of the entrance changes soon thereafter. The time for a sudden twist change at a given threadline position in the cold zone is plotted against the downstream distance of that position in Figure 3.20. The slope of the line connecting the experimental points approximates the value of the nominal yarn throughput speed. Figures 3.17 and 3.18 show this same type of twist behavior downstream of the heater.

Earlier observations of tilting threadline twist distributions following upon sudden changes in spindle speed provided the basis for the hypothesis that torsional rigidity of the conventional texturing threadline is significantly lower at the heater than beyond in the cooling zone. The existence of the "soft" torsional zone at the entry to the heater has been confirmed but it has been also shown that the entry to the cold zone in conventional texturing is likewise a torsional soft zone. Torsional disturbances in the threadline are evidenced primarily as twist changes at the soft zones and these changes then propagate downstream with yarn movement.

The formation rigidities of the texturing threadline essentially reflect the rigidity of the soft forming zones under
Figure 3.20

A

\[ v = 0.635 \text{ in/sec} \]

0.0 2.0 4.0 6.0

B

\[ v = 0.753 \text{ in/sec} \]

0.0 2.0 4.0 6.0

C

\[ v = 0.707 \text{ in/sec} \]

0.0 2.0 4.0 6.0

D

\[ v = 0.606 \text{ in/sec} \]

0.0 2.0 4.0 6.0

Position, inches

Position, inches

Time for a sudden change in twist at a given machine position after a step input

A. Step change from 40-30 nominal TPI observed in cold zone
B. Step change from 30-40 nominal TPI observed in cold zone
C. Step change from 30-40 nominal TPI observed in cooling zone
D. Step change from 40-30 nominal TPI observed in cooling zone

Note: Nominal throughput speed is 0.734 in/sec
the conditions of local twist insertion. The twist response of cold zone and cooling zone segments to incremental changes in spindle speed reflects the very high incremental torsional rigidity of these threadline segments.

The obvious question to be asked concerns the reasons for the sudden stiffening of the threadline a short distance beyond the two entry zones. One can attribute these rigidity differences to the variation in twisting deformation modes at each of the soft zones and in segments which lie downstream of these soft zones.

Observations of dynamic twist formation in larger models indicate that many filaments approaching the twist triangle apex are already twisted around their axes and sometimes around neighboring pairs. They are then wound onto the newly formed end of the yarn, with little additional twisting observed thereafter. After the twist is inserted at the cold zone entrance, interfilament friction and transverse stresses cause the yarn to behave like a helically anisotropic solid rod with subsequent high torsional rigidity. This yarn segment can now accommodate relatively large threadline torque variations with very small changes in threadline twist level.

The situation at the uptwist zone at the heater involves sudden stress relaxation or softening of the tensioned filaments as might be expected from thermomechanical studies. The requirement of constant threadline torque in presence of this stress relaxation results in a cranking up of the twist at the heater.
As the new hot twist level is being reached, the lateral forces due to filament tensions and local curvatures will also reach a new high level. And this increased interfilament pressure accompanied by thermal softening will result in lateral compressive creep and in growth of contact areas between filaments. Higher friction and interfilament shear stresses will then result in higher incremental torsional rigidities.

Experience thus far suggests that it will require extensive additional experimentation to verify these hypotheses. An alternative approach is to model the yarn on the basis of these various hypotheses, then to predict threadline behavior on the basis of these models.

Thwaites\(\textsuperscript{(14)}\) has taken this approach in his theoretical analysis of the torsional behavior of the false twist threadline.

3.2.3 Experiments Using POY PET Yarn

After experimental characterization of the torsional behavior of the conventional false twist threadline it is necessary to do the same for the draw texturing threadline. Accordingly, partially oriented yarns (POY PET) supplied by the Du Pont Company were used for the study. The POY PET consists of 30 filaments and is 245 denier with a prescribed draw ratio of 1.60.

The results of the torsional rigidity study are given in Table 3.3 and in Figures 3.21, 3.22, and 3.23. Figure 3.21 shows that for a fixed machine draw ratio and heater temperature
### Table 3.3

**TORSIONAL RIGIDITY OF DRAW TEXTURING THREADLINE, \(10^{-5}\) in\(^2\) lbs**

#### HEATER 210°C 1.60 DR

<table>
<thead>
<tr>
<th>MACHINE TWIST</th>
<th>50 TPI</th>
<th>60 TPI</th>
<th>65 TPI</th>
<th>70 TPI</th>
<th>80 TPI</th>
</tr>
</thead>
<tbody>
<tr>
<td>COLD YARN SEGMENT RIGIDITY (LOCAL TPI)</td>
<td>0.84 (25.2)</td>
<td>1.75 (38.0)</td>
<td>1.90 (39.9)</td>
<td>1.93 (40.6)</td>
<td>2.22 (44.3)</td>
</tr>
<tr>
<td>COOLING YARN SEGMENT RIGIDITY (LOCAL TPI)</td>
<td>2.68 (63.7)</td>
<td>3.55 (80.2)</td>
<td>3.37 (99.7)</td>
<td>3.24 (102.6)</td>
<td>2.95 (134.9)</td>
</tr>
</tbody>
</table>

#### HEATER 210°C 65 TPI MACHINE TWIST

<table>
<thead>
<tr>
<th>DRAW RATIO</th>
<th>1.57 DR</th>
<th>1.60 DR</th>
<th>1.64 DR</th>
</tr>
</thead>
<tbody>
<tr>
<td>COLD YARN SEGMENT RIGIDITY (LOCAL TPI)</td>
<td>1.62 (39.0)</td>
<td>1.90 (39.9)</td>
<td>1.56 (38.5)</td>
</tr>
<tr>
<td>COOLING YARN SEGMENT RIGIDITY (LOCAL TPI)</td>
<td>3.08 (100.8)</td>
<td>3.37 (99.7)</td>
<td>3.45 (92.9)</td>
</tr>
</tbody>
</table>

#### 1.60 DRAW RATIO 65 TPI MACHINE TWIST

<table>
<thead>
<tr>
<th>HEATER TEMPERATURE</th>
<th>190°C</th>
<th>210°C</th>
<th>230°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>COLD YARN SEGMENT RIGIDITY (LOCAL TPI)</td>
<td>1.52 (39.0)</td>
<td>1.90 (39.9)</td>
<td>1.55 (38.1)</td>
</tr>
<tr>
<td>COOLING YARN SEGMENT RIGIDITY (LOCAL TPI)</td>
<td>2.83 (90.4)</td>
<td>3.37 (99.7)</td>
<td>2.93 (99.6)</td>
</tr>
</tbody>
</table>
Figure 3.21

INCREMENTAL TORSIONAL RIGIDITY

DRAW TEXTURED YARNS

TEXTURING CONDITIONS: 245 DENIER YARN
1.60 DRAW RATIO
210° C.

MACHINE TWIST, TPI

TORSIONAL RIGIDITY \((10^{-5}) \text{ lb in}^2\)
Figure 3.22

INCREMENTAL TORSIONAL RIGIDITY

DRAW TEXTURED YARNS

TEXTURING CONDITIONS:

- 245 DENIER
- 65 TPI BASIC TWIST
- 1.6 DRAW RATIO

![Graph showing torsional rigidity vs heater temperature](image-url)
Figure 3.23

INCREMENTAL TORSIONAL RIGIDITY

DRAW TEXTURED YARNS

TEXTURING CONDITIONS: 245 DENIER
65 TPI BASIC TWIST
210°C HEATER

![Graph showing incremental torsional rigidity against machine draw ratio. The graph has two curves: one for cooling and one for cold, with data points at 1.57, 1.60, and 1.64 for machine draw ratio. The y-axis is labeled Torsional Rigidity, (10^-5) in^2 lbs, with values ranging from 0.0 to 4.0.]
increases in machine twist cause the incremental rigidity of the cold zone yarn segments to increase. However, the incremental torsional rigidity of segments taken from the cooling zone peaks at 60 TPI machine twist and then drops as machine twist is increased. The behavior of the cooling zone segments is attributed to the observed torsional buckling of the cooling zone segments at high machine twist levels (Figure 2.13) Figure 3.22 shows that as the heater temperature is changed there appears to be a maximum torsional rigidity at 210°C. This corresponds to the maximum cooling zone twist observed when the POY PET is textured at 210°C, as shown in Table 3.3.

Lastly, as the machine draw ratio increases, the torsional rigidity of the cooling zone segments increase as seen in Figure 3.23. For the cold zone segments, the torsional rigidity reaches a peak at 1.60 machine draw ratio. This corresponds to the peak in twist level for this machine draw ratio.

3.2.4 Experiments Concerning Transient Twist, Using POY PET Yarns

The cold zone twist distribution was photographed as before for the conventional false twist texturing threadline. The machine parameters were set as follows:

109
Cold Zone Length 6.00 in  Post Spindle Length 9.00 in
Cooling Zone Length 7.00 in  Heater Temperature 210° C
Hot Zone Length 1.25 in  Machine Draw Ratio 1.60

Pictures of the threadline twist levels were taken at a position one inch downstream of the cold zone entrance and five inches downstream.

Twist observations are recorded in Figure 3.24 and 3.25. The twist level one inch into the cold zone appears to undergo a rapid change, a short time after the step increase in spindle speed. (Fig. 3.24) At five inches into the cold zone there is a definite time lag after the step increase in spindle speed before the twist level begins to change significantly.

Difficulties were encountered in repeating the above experiments for the hot zone and the cooling zone. For with the texturing machine operating in the same twist level as that used in measuring cold twist, the hot and cooling twist levels were too high in the "semi-welded" yarn to permit measurement of twist. To overcome this difficulty, a lower steady state machine twist level was employed and the spindle speed was suddenly reduced. The hot and cooling zone transient twist distribution was then observed. The machine parameters in the latter experiment were:

Cold Zone Length 6.0 inches
Cooling Zone Length 15.0 inches
Figure 3.24
Experimental Cold Twist Mode I 50-60 TPI

![Graph showing experimental cold twist for Mode I with 50-60 TPI, comparing 1" into 6" cold zone and 5" into 6" cold zone.](image-url)
Figure 3.25

Experimental Cold Twist MODE I 60-50 TPI

Cold Twist, TPI

Time, seconds

5" into 6" Cold Zone
1" into 6" Cold Zone
Motion pictures of the threadline twist levels were taken at the beginning of the heater, at the end of the heater, immediately after the heater, and nine inches from the heater in the cooling zone. Figures 3.26 and 3.27 show the results from the twist measurements. Just as occurred in the cold zone the twist level at the heater entrance drops off immediately after the spindle speed is reduced while there is a definite time lag for the twist to drop off nine inches into the cooling zone. It should be noted that local twist at the downstream end of the heater and at the entrance to the cooling zone, also respond to the step drop in spindle speed, with no clear evidence of a time lag. Finally, Figure 3.27 shows that the twist level across the heater and through the cooling zone is not actually uniform at steady state. This slight amount of uptwisting is probably due to further relaxation of filament tensile forces. Also at the heater-cooling zone interface there is a slight steplike uptwist of about one turn per inch. Backer and Yang have identified this behavior for conventional texturing threadlines. Thwaites analytically describes this uptwist as that which causes a cooling zone incremental torque equal to the threadline torque.
Figure 3.26

MEASURED COOLING TWIST
245/30 POY PET
MACHINE TWIST 40-30 TPI

9" into cooling zone

Immediately after heater exit

immediately before heater exit

at heater entrance

40 TPI

45 TPI

50 TPI

55 TPI

60 TPI

time, seconds
Figure 3.27

245/30 POY PET MACHINE TWIST
CHANGED FROM 40-30
MEASURED

HEATER       COOLING ZONE
1.25"          9"
3.3 ANALYSIS OF TRANSIENT OPERATION

3.3.1 Introduction

The analysis of steady state operation as developed in Chapter 2 was based on the fact that the threadline tension and threadline torque are essentially uniform along the texturing zone and that during transient operation there are no transient material or twist buildups in any zone of the process. During steady state operation it was shown that there are essentially three levels of twist between the feed rollers and the take up rollers: the cold twist level, the hot or cooling twist level, and the post spindle level.

Transient operation generates material buildups and twist buildups with respect to time in different zones of the texturing system. However, the threadline tension and threadline torque at any given instant are considered uniform along the texturing zone during transient operation, as well as in steady state operation.

This section presents two models which are used to calculate threadline response to transient machine operation. Each model is used to predict cold, hot, cooling, and post spindle twist levels as well as threadline tension and threadline torque. The first model is based on the physical simplification that uniform twist levels persist during transient operation in the cold, hot, cooling, and post spindle zones. The second model provides for non-uniform twist distribution in the cold zone.
and in the cooling zone and it formulates these distributions according to experimental observations discussed in the previous section.

The numerical results derived from these two models will be compared to experimental data to determine the effectiveness of the models. After comparison is made, a parametric study is reported showing how unsteady machine behavior affects the transient operation of the draw texturing process.

3.3.2 Method of Solution

During steady state operation of the draw texturing process threadline tension and threadline torque are considered uniform in the zone between the feed rollers and the spindle. Since inertial effects are indicated to be negligible, threadline tension and threadline torque can also be considered uniform along the texturing zone during transient operation. We can now determine the material-system constitutive relations, yarn geometry, and system compatibility at any yarn cross-section, and use these relations to predict threadline torque and threadline tension.

During steady state operation neither the number of turns nor the amount of material in any threadline zone changes with respect to time. Therefore in each zone, i.e., the cold zone, hot zone, cooling zone, and post spindle zone, the input turn flow rate is equal to the output turn flow rate and the input
material flow rate equals the output material flow rate. During transient operation these flow equalities no longer obtain and the number of turns and amount of material in any zone generally varies with time.

Transient analysis of the draw texturing process must include material balances and turn balances for each defined zone of the texturing threadline (i.e. cold, hot, cooling, and post spindle). To complete the analysis, these balances are coupled with the requirements of threadline tension and threadline torque uniformity in the zone between the feed rollers and spindle. In this section, the relations which describe the transient threadline mechanics will be expressed in functional form. We will then show how these functional relations are used to predict threadline tension, threadline torque, and local twist levels during transient operation. In subsequent sections detailed expressions will be formulated.

A block diagram which shows how the variables in the draw texturing analysis are related to each other is presented in Figure 3.28. For each block there is an equation developed in the text which relates the variables according to threadline mechanics.
Figure 3.28

RELATIONSHIP OF VARIABLES IN DRAWM TEXTURING PROCESS

<table>
<thead>
<tr>
<th>MATERIAL BALANCE</th>
<th>TURN BALANCE</th>
<th>THREADLINE TORQUE</th>
<th>THREADLINE TENSION</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_c$ $e_c$ $P_j$</td>
<td>$W_c$ $W_j$ $W_h$</td>
<td>$P_c$ $e_c$</td>
<td>$P_c$ $e_c$</td>
</tr>
<tr>
<td>$V_0$ (3-132) $V_2$</td>
<td>$V_2$ (3-66) $V_3$</td>
<td>$V_3$ (3-144) $V_b$</td>
<td>$V_3$ (3-155) $V_b$</td>
</tr>
<tr>
<td>$P_h$ $P_s$ $P_{ps}$</td>
<td>$P_h$ $P_s$ $P_{ps}$</td>
<td>$P_h$</td>
<td>$P_h$</td>
</tr>
<tr>
<td>$V_b$ (3-75) $V_4$</td>
<td>$V_b$ (3-78) $V_4$</td>
<td>$V_3$ (3-73) $V_3$</td>
<td>$V_3$ (3-78) $V_4$</td>
</tr>
</tbody>
</table>

$P_c$ = TWIST/UNIT LENGTH AT UPSTREAM END OF COLD ZONE
$P_j$ = TWIST/UNIT LENGTH AT DOWNSTREAM END OF COLD ZONE
$P_h$ = TWIST/UNIT LENGTH OVER HEATER AND UPSTREAM END OF COOLING ZONE
$P_s$ = TWIST/UNIT LENGTH AT DOWNSTREAM END OF COOLING ZONE
$P_{ps}$ = TWIST/UNIT LENGTH IN POST SPINDLE ZONE
$V_0$ = FEED ROLLER SURFACE VELOCITY
$V_2$ = LINEAR VELOCITY OF YARN AT COLD ZONE EXIT
$V_3$ = LINEAR VELOCITY OF YARN AT HOT ZONE EXIT
$V_b$ = LINEAR VELOCITY OF TWISTED YARN AS IT ENTERS SPINDLE
$e_c$ = STRAIN OF FILAMENTS IN COLD ZONE
$D_{f_1}$ = DRAW RATIO OF FILAMENTS ON $i^{th}$ RADIAL YARN LAYER IN HOT ZONE
$W_c$ = ANGULAR VELOCITY OF YARN IN COLD ZONE
$W_h$ = ANGULAR VELOCITY OF YARN IN HOT ZONE
$W_s$ = ANGULAR VELOCITY OF SPINDLE

*The numbers in parenthesis refer to equations in text
LIST OF SYMBOLS

$A, B, = $ Hot Moduli of Drawing Coefficients

$A_{i1}, A_{i2}, A_{i3} = $ Defined Coefficients

$AN_i = $ No. of Filaments in Hot Zone $i^{th}$ Radial Layer

$BN_i = $ No. of Filaments in Cold Zone $i^{th}$ Radial Layer

$C_{rc} = $ Cold Contraction Factor (Alternative Model)

$C_{rh} = $ Hot Contraction Factor (Hearle's Model)

$C_{rj} = $ Contraction Factor at Twist Level $P_j$ (Hearle's Model)

$C_{rps} = $ Contraction Factor at Twist Level $P_{ps}$ (Hearle's Model)

$C_{reset} = $ Contraction Factor in Set Zone (Hearle's Model)

$Dr_i = $ Draw Ratio of Filament in $i^{th}$ Radial Layer

$E_c = $ Cold Young's Modulus Cross Sectional Area of Filament

$e_c = $ Average Tensile Strain of Filaments in Cold Zone.

$EI_{fiber} = $ Cold Bending Modulus of Fiber

$F_0, F_1, F_2, F_3, F_4, F_5, F_6, F_7 = $ Defined Coefficients

$GI_{fiber} = $ Cold Torsion Modulus of Fiber
\( G_1, G_2, G_3, G_4, G_5, G_6, G_7 \) = Defined Coefficients

\( L_c \) = Cold Zone Length

\( L_h \) = Hot Zone Length

\( L_{ps} \) = Post Spindle Zone Length

\( L_s \) = Length of Set Zone

\( P_c \) = Twist in Cold Zone, (Radians/Inch)

\( P_h \) = Twist in Hot Zone, (Radians/Inch)

\( P_j \) = Twist at Downstream End of Cold Zone (Radians/Inch)

\( P_{ps} \) = Twist in Post Spindle Zone (Radians/Inch)

\( P_s \) = Twist at Downstream End of Set Zone (Radians/Inch)

\( Q_1, Q_2, Q_3 \) = Defined Coefficients

\( R_{ic} \) = Radius of \( i^{th} \) Radial Layer in Cold Zone

\( R_{ih} \) = Radius of \( i^{th} \) Radial Layer in Hot Zone

\( \text{Sec} \theta_{3i} \) = Secant of Hot Zone Helix Angle at \( i^{th} \) Radial Layer

\( \text{Sec} \theta_{2i} \) = Secant of Cold Zone Helix Angle at \( i^{th} \) Radial Layer

\( t \) = Time, Seconds

\( V_b \) = Velocity of Hot Twisted Yarn at Spindle Entrance
$V_0 =$ Feed Roller Linear Velocity

$V_2 =$ Linear Velocity of Cold Twisted Yarn

$V_3 =$ Linear Velocity of Hot Twisted Yarn

$V_4 =$ Take Up Roller Linear Velocity

$W_c =$ Angular Velocity of Cold Zone Segments

$W_h =$ Angular Velocity of Hot Zone Segments

$W_s =$ Spindle Speed and Angular Velocity of Cooling Zone Segments

$X_1 , X_2 , X_3 , X_4 , X_5 , X_6 , X_7 , X_8 =$ Defined Coefficients

$Y_1 , Y_2 , Y_3 , Y_4 =$ Defined Coefficients

$Z, Z_1 , Z_2 =$ Defined Coefficients
In Chapter 2 it was shown that the yarn tension at the cold zone entrance can be expressed as

\[ T_c = F_1 (e_c, P_c) \]  \hspace{1cm} (2-1)

Differentiating (2-1) with respect to time we obtain

\[ \frac{dT_c}{dt} = G_1 \left( \frac{de_c}{dt}, \frac{dP_c}{dt} \right) \]  \hspace{1cm} (3-1)

Yarn torque at the cold zone entrance was expressed in Chapter 2 as

\[ M_{tc} = F_4 (e_c, P_c) \]  \hspace{1cm} (2-4)

Differentiating (2-4) with respect to time we obtain

\[ \frac{dM_{tc}}{dt} = G_4 \left( \frac{de_c}{dt}, \frac{dP_c}{dt} \right) \]  \hspace{1cm} (3-2)

At the entrance to the hot drawing zone it was shown in Chapter 2 that

\[ T_h = F_5 (V_2, V_3, P_c, P_h) \]  \hspace{1cm} (2-5)

Taking the variation of filament draw ratio in the yarn cross section into account, we can express (2-5) more simply as

\[ T_h = F'_5 (D_{ri}, P_h) \]  \hspace{1cm} (3-3)

* The symbols \( F, G, X, Y, Z \) indicate various functional expressions which are developed later.
And likewise yarn torque at the hot zone entrance is expressed as:

\[ M_{th} = F'(D_{ri}, P_h) \]  

(3-4)

Both (3-3) and (3-4) are differentiated with respect to time to obtain:

\[ \frac{dT}{dt} = G_5 \left( \frac{dD_{ri}}{dt}, \frac{dP_h}{dt} \right) \]  

(3-5)

\[ \frac{dM_t}{dt} = G_6 \left( \frac{dD_{ri}}{dt}, \frac{dP_h}{dt} \right) \]  

(3-6)

Satisfying the requirements of threadline torque and threadline tension uniformity, we can equate (3-1) and (3-5) as well as (3-2) and (3-6) to obtain:

\[ 0 = G_1 \left( \frac{de_c}{dt}, \frac{dP_c}{dt} \right) - G_5 \left( \frac{dD_{ri}}{dt}, \frac{dP_h}{dt} \right) \]  

(3-7)

\[ 0 = G_2 \left( \frac{de_c}{dt}, \frac{dP_c}{dt} \right) - G_6 \left( \frac{dD_{ri}}{dt}, \frac{dP_h}{dt} \right) \]  

(3-8)

Equations (3-7) and (3-8) represent generalized threadline coupled force differential equations. They are developed in Section 3.3.3.1. They are 2 equations with 4 unknowns. The analytical relationships required to complete the analysis are developed from the material balance relation and turn balance relation for each zone in the texturing threadline.
In Section 3.2.4 it was reported that the twist distributions observed in both the cold and cooling zones of the threadline operation were not uniform with respect to position. One of our models for transient operation is based on these observations of non-uniform twist distributions. Another model is based on the simplification that the twist levels are uniform throughout zones during transient operation. Common to both models is threadline behavior both in the hot (i.e. over the heater) and the post spindle zones. For it has been observed that twist distribution during transient operation is the same as for steady state operation across the heater and post spindle zones.

The material balance for the hot zone will be fully developed in Section 3.3.3.2 but it can be functionally expressed as

\[ V_2 = X_1 (P_h, P_c, \frac{dP_h}{dt}, D_c', V_3, \frac{dD_c}{dt}) \]  

Likewise, the turn balance for the hot zone which is to be developed in Section 3.3.3.2 can be functionally expressed as

\[ \frac{dP_h}{dt} = X_2 (W_h, W_c, P_h, P_c, V_2, P_h, V_c) \]  

The post spindle analysis given in Section 3.3.3.2 demonstrates that

\[ \frac{dP_{ps}}{dt} = X_3 (P_{ps}, P_s, W_{ps}, V_b) \]
and

\[ V_b = X_4 \left( p_{ps}, W_h, P_s, \frac{dp_{ps}}{dt} \right) \]  \hspace{1cm} (3-12)

where \( V_b \) = linear velocity of twisted yarn as it enters spindle. Since we assume that the cooling and hot zone segments rotate at the same angular speed as the pin spindle \( (W_h = W_c = W_s) \) equations (3-7), (3-8), (3-9), (3-10), (3-11), and (3-12) can be combined to yield 5 differential equations with 10 unknowns: \( P_c, P_h, Dr_1, e_c, V_b, V_3, V_2, P_{ps}, P_s, \) and \( W_c \). The analysis can be completed considering the material balances and turn balances in the cold and cooling zones. To deal with the turn balances, we resort to the two models for twist distribution as mentioned above.

3.3.2.1 MODEL 1: Flat Twist Distribution

Model 1 is based on the simplifying assumption that the twist distribution is uniform flat in the cold zone and in the hot and cooling zones during transient operation. The latter twists are equal at all times. The material balance for the cold zone developed in Section 3.3.3.3 shows that

\[ V_z = X_5 \left( e_c, P_c, \frac{dp_c}{dt}, \frac{de_c}{dt} \right) \]  \hspace{1cm} (3-13)

The turn balance for the cold zone developed in Section 3.3.3.3 shows that
\[
\frac{dP}{dt} = X_6 (W_c, P_c, V_2) \quad (3-14)
\]

The material balance for the cooling zone developed in Section 3.3.3.3 shows that
\[
\frac{dP_s}{dt} = X_7 (P_s, V_3, V_b, P_h) \quad (3-15)
\]

where \( P_s \) = turns/twisted length in cooling zone.

The turn balance for the cooling zone is expressed as
\[
\frac{dP_s}{dt} = X_8 (P_h, V_3, V_b, P_s) \quad (3-16)
\]

Lastly, by definition
\[
P_s = P_h \quad (3-17)
\]

In summary, equations (3-7), (3-8), (3-9), (3-10), (3-11), (3-12), (3-13), (3-14), (3-15), (3-16) and (3-17) represent 11 expressions of which 10 are differential equations with 10 unknowns; \( P_c, P_h, P_s, P_{ps}, W_c, V_2, V_3, V_b, e_c, D_{Dr} \).

After considerable manipulation these equations can be reduced to the following:
\[
\frac{dP_c}{dt} = z_1 \left[ P_c', P_h', e_c, D_{Dr}', P_{ps} \right] \quad (3-18)
\]
This system of non-linear ordinary differential equations is solved numerically by a digital computer. The solution technique is described in Section 3.3.4.

After obtaining \( P_c, P_h, e_c, \) and \( D_{r1} \) as functions of time, we substituted these values into (2-1), (2-4), (3-3) and (3-4) to determine threadline tension and threadline torque at any instant during transient operation.

3.3.2.2 Model 2: Tilted Twist Distribution

Model 2 is based on the assumption that the twist is not uniform, along the cold zone nor along the cooling zone. An approximative kinematic relation (twist transport equation) is formulated which relates the twist level at the downstream end of either zone to the corresponding twist level at the upstream portion of that zone. The
upstream twist levels are determined by the appropriate force, material and turn balances.

The twist transport equations for the cold zone and cooling zone are developed in Section 3.3.3.4 and are expressed functionally as

\[
\frac{d P_j}{dt} = X_g[P_c', P_j, V_2] \quad (3-23)
\]

and

\[
\frac{d P'_s}{dt} = X_{10}[P'_s, P'_h, V_b] \quad (3-24)
\]

where \( P_j \) = twist/unit length at downstream end of cold zone

\( P'_s \) = twist/unit length at downstream end of cooling zone

The material balance for the cold zone developed in Section 3.3.3.4 shows that

\[
V_2 = X_{11}[e_c', P_j, P_c', \frac{d P_j}{dt}, \frac{d P_c}{dt}, \frac{d e_c}{dt}] \quad (3-25)
\]

Likewise, the turn balance for the cold zone developed in Section 3.3.3.4 shows that

\[
\frac{d P_c}{dt} = X_{12}[W_c', P_c', P_j', V_2', \frac{d P_j}{dt}] \quad (3-26)
\]

The material balance for the cooling zone shows that

\[
\frac{d P'_s}{dt} = X_{13}[\frac{d P'_h}{dt}, P'_h, P'_s, V_3, V_b] \quad (3-27)
\]
Finally, the turn balance for the cooling zone can be expressed as

$$\frac{d P_s}{dt} = X_{14} \left[ \frac{d P_h}{dt}, P_s, P_h', V_3, V_b \right]$$  \hspace{1cm} (3-28)

In summary equations (3-7), (3-8), (3-9), (3-10), (3-11), (3-12), (3-23), (3-24), (3-25), (3-26), (3-27) and (3-28) represent a system of 12 differential equations with 11 unknowns; $P_c', P_j', P_h', P_s', P_{ps}', W_c', V_2', V_3, V_b', e_c'$, and $Dr_i$. After considerable manipulation these can be reduced to the following:

$$\frac{d P_c}{dt} = Y_1 [P_c', P_j', P_h', P_s', P_{ps}', e_c', Dr_i]$$  \hspace{1cm} (3-29)

$$\frac{d P_h}{dt} = Y_2 [P_c', P_j', P_h', P_s', P_{ps}', e_c', Dr_i]$$  \hspace{1cm} (3-30)

$$\frac{d e_c}{dt} = Y_3 [P_c', P_j', P_h', P_s', P_{ps}', e_c', Dr_i]$$  \hspace{1cm} (3-31)

$$\frac{d Dr_i}{dt} = Y_4 [P_c', P_j', P_h', P_s', P_{ps}', e_c', Dr_i]$$  \hspace{1cm} (3-32)

$$\frac{d P_j}{dt} = Y_5 [P_c', P_j', P_h', P_s', P_{ps}', e_c', Dr_i]$$  \hspace{1cm} (3-33)

$$\frac{d P_s'}{dt} = Y_6 [P_c', P_j', P_h', P_s', P_{ps}', e_c', Dr_i]$$  \hspace{1cm} (3-34)
This system of non-linear ordinary differential equations is solved numerically by digital computer. The solution technique is described in Section 3.3.4. After obtaining $P_c$, $P_h$, $e_c$, and $Dr_1$ as functions of time we substitute these values into (2-1), (2-4), (3-3) and (3-4) to determine threadline torque and threadline tensions at any instant during transient operation.
3.3.3 Transient Threadline Mechanics

3.3.3.1 General Tension & Torque Relationships

In Chapter 2 the yarn tension at the hot zone entrance was derived as *

\[ T_h = A N_0 \cdot (A \cdot \frac{D r^2}{D r_0} + B \cdot \frac{D r_3}{D r_0}) \]

\[ + A N_1 \cdot (A \cdot \frac{D r^2}{D r_0} + B \cdot \frac{D r_3}{D r_1}) \cdot \frac{1}{\sqrt{1+P^2 R^2_{h1h}}} \]

\[ + A N_2 \cdot (A \cdot \frac{D r^2}{D r_2} + B \cdot \frac{D r_3}{D r_2}) \cdot \frac{1}{\sqrt{1+P^2 R^2_{h2h}}} \]

\[ + A N_3 \cdot (A \cdot \frac{D r^2}{D r_3} + B \cdot \frac{D r_3}{D r_3}) \cdot \frac{1}{\sqrt{1+P^2 R^2_{h3h}}} \]

Equation (3-35) is now differentiated with respect to time to obtain.

*Note: Since most yarns which are draw textured are 3 layer multifilament yarns and the yarns used for experimental study are such, the analysis will be formulated to accommodate this type of yarn. Simple arithmetical changes could be made if this were not the case.
\[
\frac{dT_h}{dt} = AN_0 \cdot (2A \cdot Dr_0 + 3B \cdot Dr_0^2) \frac{dDr_0}{dt} + AN_1 \cdot \frac{(2A \cdot Dr_1 + 3B \cdot Dr_1^2)}{(1 + \frac{R_1^2}{h^2})^{1/2}} \frac{dDr_1}{dt} + AN_2 \cdot \frac{(2A \cdot Dr_2 + 3B \cdot Dr_2^2)}{(1 + \frac{R_2^2}{h^2})^{1/2}} \frac{dDr_2}{dt} + AN_3 \cdot \frac{(2A \cdot Dr_3 + 3B \cdot Dr_3^2)}{(1 + \frac{R_3^2}{h^2})^{1/2}} \frac{dDr_3}{dt} - Ph \left[ AN_1 (A \cdot Dr_1^2 + B \cdot Dr_1^3) \frac{R_1^2}{(1 + \frac{R_1^2}{h^2})^{3/2}} + AN_2 (A \cdot Dr_2^2 + B \cdot Dr_2^3) \frac{R_2^2}{(1 + \frac{R_2^2}{h^2})^{3/2}} + AN_3 (A \cdot Dr_3^2 + B \cdot Dr_3^3) \frac{R_3^2}{(1 + \frac{R_3^2}{h^2})^{3/2}} \right] \frac{dPh}{dt} \quad (3-36)
\]

To simplify equation (3-36) we define the following

\[
F_0 \equiv AN_0 (2ADr_0 + 3BDr_0^2) \quad (3-37)
\]

\[
F_1 \equiv AN_1 (2ADr_1 + 3BDr_1^2) \quad (3-38)
\]

\[
F_2 \equiv AN_2 (2ADr_2 + 3BDr_2^2) \quad (3-39)
\]

\[
F_3 \equiv AN_3 (2ADr_3 + 3BDr_3^2) \quad (3-40)
\]
\[ F_5 \equiv -P_h \left\{ A\frac{R_1^2}{(1+P_h^2R_1^2)^{3/2}} + B\frac{R_1^2}{(1+P_h^2R_1^2)^{3/2}} + AN_2(A\frac{R_2^2}{(1+P_h^2R_2^2)^{3/2}} + B\frac{R_2^2}{(1+P_h^2R_2^2)^{3/2}}) + AN_3(A\frac{R_3^2}{(1+P_h^2R_3^2)^{3/2}} + B\frac{R_3^2}{(1+P_h^2R_3^2)^{3/2}}) \right\} \] (3-41)

Substitution of (3-37), (3-38), (3-39), (3-40) and (3-41) into (3-36) gives

\[
\frac{dT_h}{dt} = F_0 \frac{dDr_0}{dt} + F_1 \frac{dDr_1}{dt} + F_2 \frac{dDr_2}{dt} + F_3 \frac{dDr_3}{dt} + F_5 \frac{dPh}{dt} \quad (3-42)
\]

In Chapter 2 yarn tension in the cold zone entrance was derived as

\[
T_c = E_c \frac{BN_1}{(1+P_{c1}^2R_{c1}^2)^{1/2}} + \frac{BN_2}{(1+P_{c2}^2R_{c2}^2)^{1/2}} + \frac{BN_3}{(1+P_{c3}^2R_{c3}^2)^{1/2}} \quad (3-43)
\]

where \( E_c = \text{Cold tensile modulus} \times \text{Area of filament} \)

Equation (3-43) applies during both transient and steady state operation. Therefore if we know the cold filament...
strain and the cold twist at any time, the threadline tension can be determined therefrom. Equation (3-43) is differentiated with respect to time to obtain

\[
\frac{d(T_c)}{dt} = E_c \left[ \frac{BN_1}{(1+P^2R^2 c)_{1c}}^{1/2} + \frac{BN_2}{(1+P^2R^2 c)_{2c}}^{1/2} \right.
\]

\[
+ \frac{BN_3}{(1+P^2R^2 c)_{3c}}^{1/2} \left( \frac{de_c}{dt} \right)
\]

\[
- \frac{dP_c}{dt} = E_c e_c \left[ \frac{BN_1 \cdot P_c \cdot R^2_{1c}}{(1+P^2R^2 c)_{1c}}^{3/2} + \frac{BN_2 \cdot P_c \cdot R^2_{2c}}{(1+P^2R^2 c)_{2c}}^{3/2} \right.
\]

\[
+ \frac{BN_3 \cdot P_c \cdot R^2_{3c}}{(1+P^2R^2 c)_{3c}}^{1/2} \left( \frac{de_c}{dt} \right)
\]

(3-44)

To simplify equation (3-44) we define the following

\[
F_6 = E_c \frac{BN_1}{(1+P^2R^2 c)_{1c}}^{1/2} + \frac{BN_2}{(1+P^2R^2 c)_{2c}}^{1/2}
\]

\[
+ \frac{BN_3}{(1+P^2R^2 c)_{3c}}^{1/2}
\]

(3-45)

\[
F_7 = E_c e_c P_c \left[ \frac{BN_1 \cdot R^2_{1c}}{(1+P^2R^2 c)_{1c}}^{3/2} + \frac{BN_2 \cdot R^2_{2c}}{(1+P^2R^2 c)_{2c}}^{3/2} \right.
\]

\[
+ \frac{BN_3 \cdot R^2_{3c}}{(1+P^2R^2 c)_{3c}}^{3/2} \left( \frac{de_c}{dt} \right)
\]

(3-46)

where \( BN_i \) = Number of filament in \( i \)th radial layer in the cold zone
To obtain:

\[
\frac{dT_c}{dt} = F_6 \frac{de_c}{dt} + F_7 \frac{dP_c}{dt} \quad (3-47)
\]

In Chapter 2, yarn torque at the hot zone entrance was derived as:

\[
M_{th} = AN_1 (A \cdot Dr_1^2 + B \cdot Dr_1^3) \cdot \frac{P_h R_{1h}^2}{(1+P_{2h}^2 R_{2h}^2)^{1/2}} + AN_2 (A \cdot Dr_2^2 + B \cdot Dr_2^3) \cdot \frac{P_h R_{2h}^2}{(1+P_{2h}^2 R_{2h}^2)^{1/2}} + AN_3 (A \cdot Dr_3^2 + B \cdot Dr_3^3) \cdot \frac{P_h R_{3h}^2}{(1+P_{2h}^2 R_{2h}^2)^{1/2}} \quad (3-48)
\]

Equation (3-48) applies during transient and during steady state operation. Therefore, given the individual draw ratios of the filaments and the hot twist at any time, we can determine the corresponding threadline torque. Equation (3-48) is now differentiated with respect to time to obtain:

\[
\frac{d(M_{th})}{dt} = \left[ AN_1 \cdot (2 \cdot A \cdot Dr_1 + 3 \cdot B \cdot Dr_1^2) \cdot \frac{P_h R_{1h}^2}{(1+P_{2h}^2 R_{2h}^2)^{1/2}} \right] \frac{dDr_1}{dt}
\]
\[ + AN_1 \cdot \left[ (ADr_1^2 + DBr_1^3) R_{1h}^2 \right] \left[ \frac{1}{(1 + P_{h1h}^2 R_{1h}^2)^{1/2}} \right] \]
\[ - \frac{P_{h1h}^2 R_{1h}^2}{(1 + P_{h1h}^2 R_{1h}^2)^{3/2}} \frac{dP_h}{dt} \]
\[ + AN_2 \left( 2 \cdot A \cdot Dr_2^2 + 3 \cdot B \cdot Dr_2^3 \right) \frac{P_h R_{2h}^2}{(1 + P_{h2h}^2 R_{2h}^2)^{1/2}} \frac{dDr_2}{dt} \]
\[ + AN_2 \left[ (A \cdot Dr_2^2 + B \cdot Dr_3^3) R_{2h}^2 \right] \left[ \frac{1}{(1 + P_{h2h}^2 R_{2h}^2)^{1/2}} \right] \]
\[ - \frac{\dot{p}_{h}^{2} R_{2h}^{2}}{(1 + \dot{p}_{h}^{2} R_{2h}^{2})^{3/2}} \frac{dP_{h}}{dt} + AN_{3} \left[ 2 \cdot A \cdot Dr_{3} + 3 \cdot B \cdot Dr_{3}^{2} \right] \]

\[ \frac{P_{h}^{2} R_{3h}^{2}}{(1 + P_{h}^{2} R_{3h}^{2})^{1/2}} \frac{dDr_{3}}{dt} + AN_{3} \left[ A \cdot Dr_{3}^{2} + B \cdot Dr_{3}^{3} \right] \]

\[ R_{3h}^{2} \left[ \frac{1}{(1 + P_{h}^{2} R_{3h}^{2})^{1/2}} - \frac{P_{h}^{2} R_{3h}^{2}}{(1 + P_{h}^{2} R_{3h}^{2})^{3/2}} \right] \frac{dP_{h}}{dt} \quad (3-49) \]

To simply Equation (3-49) we define the following:

\[ G_{1} = AN_{1} (2 \cdot A \cdot Dr_{1} + 3 \cdot B \cdot Dr_{1}^{2}) \frac{P_{h}^{2} R_{1h}^{2}}{(1 + P_{h}^{2} R_{1h}^{2})^{1/2}} \quad (3-50) \]

\[ G_{2} = AN_{2} (2 \cdot A \cdot Dr_{2} + 3 \cdot B \cdot Dr_{2}^{2}) \frac{P_{h}^{2} R_{2h}^{2}}{(1 + P_{h}^{2} R_{2h}^{2})^{1/2}} \quad (3-51) \]

\[ G_{3} = AN_{3} (2 \cdot A \cdot Dr_{3} + 3 \cdot B \cdot Dr_{3}^{2}) \frac{P_{h}^{2} R_{3h}^{2}}{(1 + P_{h}^{2} R_{3h}^{2})^{1/2}} \quad (3-52) \]

\[ G_{5} = AN_{1} (A \cdot Dr_{1}^{2} + B \cdot Dr_{1}^{3}) \frac{R_{1h}^{2}}{(1 + P_{h}^{2} R_{1h}^{2})^{1/2}} \]

\[ - \frac{P_{h}^{2} R_{1h}^{2}}{(1 + P_{h}^{2} R_{1h}^{2})^{3/2}} \right] + AN_{2} \left[ A \cdot Dr_{2}^{2} + B \cdot Dr_{2}^{3} \right] \]

\[ R_{2h}^{2} \left[ \frac{1}{(1 + P_{h}^{2} R_{2h}^{2})^{1/2}} - \frac{P_{h}^{2} R_{2h}^{2}}{(1 + P_{h}^{2} R_{2h}^{2})^{3/2}} \right] \]
\[ + AN_3 \begin{pmatrix} (A \cdot Dr^3 + B \cdot Dr^3) \\ R^3h \end{pmatrix} \begin{pmatrix} 1 \\ (1+P^2_{h}^2)^{1/2} \end{pmatrix} \]
\[ - \frac{P^2_{h}^2 R^2}{(1+P^2_{h}^2)^{3/2}} \]

Therefore

\[ \frac{dM_{th}}{dt} = G_1 \frac{dDr_1}{dt} + G_2 \frac{dDr_2}{dt} + G_3 \frac{dDr_3}{dt} + G_5 \frac{dp_h}{dt} \]

(3-54)

In Chapter 2 yarn torque at the cold zone entrance was derived as

\[ Mtc = \text{Torque}_{\text{fiber tension}} + \text{Torque}_{\text{fiber bending}} + \text{Torque}_{\text{fiber torsion}} \]

(3-55)

\[ Mtc = P_{c} E_{c} e_{c} \begin{pmatrix} BN_1 \cdot R^2_{1c} \\ (1+P^2_{c}^2)^{1/2} \\ BN_2 \cdot R^2_{2c} \\ (1+P^2_{c}^2)^{1/2} \end{pmatrix} \]

\[ + \frac{BN_3 \cdot R^2_{3c}}{(1+P^2_{c}^2)^{1/2}} \]

\[ + (EI)_{\text{fiber}} \begin{pmatrix} BN_1 \cdot P^3_{c} R^2_{1c} \\ (1+P^2_{c}^2)^{3/2} \\ BN_2 \cdot P^3_{c} R^2_{2c} \\ (1+P^2_{c}^2)^{3/2} \end{pmatrix} \]

\[ + \frac{BN_3 \cdot P^3_{c} R^2_{3c}}{(1+P^2_{c}^2)^{3/2}} \]

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\((G_{Ip})_{\text{fiber}}\) \[ \begin{align*} & = \left\{ BN_0 \cdot P_c \cdot + \frac{BN_1 \cdot P_c}{(1+P^2_{R_c^2})^{3/2}} \right. \\
& + \frac{BN_2 \cdot P_c}{(1+P^2_{R_c^2})^{3/2}} + \frac{BN_3 \cdot P_c}{(1+P^2_{R_c^3})^{3/2}} \right\} \tag{3-56} \]

Equation (3-56) is differentiated with respect to time to obtain

\[ \frac{dM_t}{dt} = \frac{de_c}{dt} \cdot P_c \cdot E_c \left\{ \begin{align*} & + \frac{BN_1 \cdot R^2_{1c}}{(1+P^2_{R_c^2})^{1/2}} \\
& + \frac{BN_2 \cdot R^2_{2c}}{(1+P^2_{R_c^2})^{1/2}} + \frac{BN_3 \cdot R^2_{3c}}{(1+P^2_{R_c^3})^{1/2}} \right\} \\
& + e_c \cdot E_c \left[ \begin{align*} & + \left( BN_1 \cdot \left[ \frac{R^2_{1c}}{(1+P^2_{R_c^2})^{1/2}} \right] - \frac{P^2_{R^4_{1c}}}{(1+P^2_{R_c^2})^{3/2}} \right) \\
& + \left( BN_2 \cdot \left[ \frac{R^2_{2c}}{(1+P^2_{R_c^2})^{1/2}} \right] - \frac{P^2_{R^4_{2c}}}{(1+P^2_{R_c^2})^{3/2}} \right) \\
& + \left( BN_3 \cdot \left[ \frac{R^2_{3c}}{(1+P^2_{R_c^3})^{1/2}} \right] - \frac{P^2_{R^4_{3c}}}{(1+P^2_{R_c^3})^{3/2}} \right) \right] \frac{dP_c}{dt} \\
& + 3 \cdot (EI)_{\text{fiber}} \left[ BN_1 \cdot \left[ \frac{P^2_{R^2_{1c}}}{(1+P^2_{R_c^2})^{3/2}} \right] - \frac{P^4_{R^4_{1c}}}{(1+P^2_{R_c^2})^{5/2}} \right] \]
\[ + \text{BN}_2 \left\{ \frac{p^2 R^2_{c c}}{(1 + p^2 R^2_{c c})^{3/2}} - \frac{p^4 R^4_{c c}}{(1 + p^2 R^2_{c c})^{5/2}} \right\} \]

\[ + \text{BN}_3 \left\{ \frac{p^2 R^2_{c 3c}}{(1 + p^2 R^2_{c 3c})^{3/2}} - \frac{p^4 R^4_{c 3c}}{(1 + p^2 R^2_{c 3c})^{5/2}} \right\} \frac{dP_c}{dt} \]

\[ + (G_{1p})_{\text{fiber}} \left[ \text{BN}_0 + \text{BN}_1 \left\{ \frac{1}{(1 + p^2 R^2_{c 1c})^{3/2}} \right. \right. \]

\[ - \frac{3p^2 R^2_{c 1c}}{(1 + p^2 R^2_{c 1c})^{5/2}} \left. \right] \]

\[ + \text{BN}_2 \left\{ \frac{1}{(1 + p^2 R^2_{c 2c})^{3/2}} - \frac{3p^2 R^2_{c 2c}}{(1 + p^2 R^2_{c 2c})^{5/2}} \right\} \]

\[ + \text{BN}_3 \left\{ \frac{1}{(1 + p^2 R^2_{c 3c})^{3/2}} - \frac{3p^2 R^2_{c 3c}}{(1 + p^2 R^2_{c 3c})^{5/2}} \right\} \frac{dP_c}{dt} \]

\[ (3-57) \]

In order to simplify equation (3-57) we define the first term of (3-57) which is multiplied by \( \frac{dC}{dt} \) as \( G_6 \) and the remainder of the terms which are multiplied by \( \frac{dP_c}{dt} \) as \( G_7 \) so that

\[ \frac{dM_{tc}}{dt} = G_6 \frac{dC}{dt} + G_7 \frac{dP_c}{dt} \]

\[ (3-58) \]
Since tension at the cold zone entrance is equal to tension at the hot zone entrance

\[ \text{Tension}_{\text{cold}} = \text{Tension}_{\text{hot}} \]

and torque at the cold zone entrance is equal to torque at the hot zone entrance

\[ \text{Torque}_{\text{cold}} = \text{Torque}_{\text{hot}} \]

the time derivatives of these relations must also be equal

\[ \frac{d\text{Tension}}{dt}_{\text{cold}} = \frac{d\text{Tension}}{dt}_{\text{hot}} \]

\[ \frac{d\text{Torque}}{dt}_{\text{cold}} = \frac{d\text{Torque}}{dt}_{\text{hot}} \]

Now substituting (3-42), (3-47), (3-54), and (3-58) into (3-59) and (3-60) we obtain

\[ 0 = F_0 \frac{d\text{Dr}_0}{dt} + F_1 \frac{d\text{Dr}_1}{dt} + F_2 \frac{d\text{Dr}_2}{dt} + F_3 \frac{d\text{Dr}_3}{dt} \]

\[ + F_5 \frac{d\text{Ph}}{dt} - F_6 \frac{d\text{ec}}{dt} - F_7 \frac{d\text{Pc}}{dt} \]

\[ 0 = G_1 \frac{d\text{Dr}_1}{dt} + G_2 \frac{d\text{Dr}_2}{dt} + G_3 \frac{d\text{Dr}_3}{dt} + G_5 \frac{d\text{Ph}}{dt} \]

\[ - G_6 \frac{d\text{ec}}{dt} - G_7 \frac{d\text{Pc}}{dt} \]
Equations (3-61) and (3-62) are rearranged to obtain

\[
\frac{\partial e_c}{\partial t} = \frac{F_0}{F_6} \frac{dD_0}{dt} + \frac{F_1}{F_6} \frac{dD_1}{dt} + \frac{F_2}{F_6} \frac{dD_2}{dt} + \frac{F_3}{F_6} \frac{dD_3}{dt} + \frac{F_5}{F_6} \frac{dP_h}{dt} - \frac{F_7}{F_6} \frac{dP_c}{dt}
\]

(3-63)

\[
\frac{\partial e_c}{\partial t} = \frac{G_1}{G_6} \frac{dD_1}{dt} + \frac{G_2}{G_6} \frac{dD_2}{dt} + \frac{G_3}{G_6} \frac{dD_3}{dt} + \frac{G_5}{G_6} \frac{dP_h}{dt} - \frac{G_7}{G_6} \frac{dP_c}{dt}
\]

(3-64)

Equations (3-63) and (3-64) represent generalized threadline force coupled differential equations. Seven dependent variables (unknowns) are present in these two coupled equations. To determine threadline torque and threadline tension during transient operation it is necessary to consider in addition, appropriate material balance and turn balance relations for each zone in the texturing threadline. As stated in the introduction to this section two models will be used for determining local material balances and turn balances. Both models will be applied to equations (3-63) and (3-64).
3.3.3.2 General Material and Turn Balances for the Hot Zone and the Post Spindle Zone

Common to both analytical models of transient behavior are the material balances and turn balances for the hot zone and post spindle zone. Along these zones the twist level is observed to be essentially uniform for both steady state and transient operation of the texturing system.

Material Balance for Hot Zone

In Chapter 2 it was pointed out that each radial layer of filaments in the hot drawing zone has a unique draw ratio, therefore it is necessary to formulate a separate material balance for each layer, i.e.:

\[ \Delta (\text{Material})_{\text{hot}} = \text{Material}_{\text{in}} - \text{Material}_{\text{out}} \]

The material balance for each radial layer can be quantitatively formulated as

\[ \frac{\Delta}{\Delta t} \left( L_i \frac{\sec \theta_{3i}}{D_r_i} \right) = V_2 \sec \theta_{2i} - V_3 \frac{\sec \theta_{3i}}{D_r_i} \]  \hspace{1cm} (3-65)
where \( L_h \) = Hot zone length

and as \( \Delta t \to 0 \)

\[
\frac{P_{h_{ih}}^2}{(1 + P_{h_{ih}}^2)^{1/2}} \frac{1}{Dr_i} \frac{dP_h}{dt} - \frac{1}{Dr_i} \frac{dDr_i}{dt} \left( \frac{(1 + P_{h_{ih}}^2)^{1/2}}{Dr_i} \right)
\]

\[
= V_2 \frac{(1 + P_{h_{ih}}^2)^{1/2}}{L_h} - V_3 \frac{(1 + P_{h_{ih}}^2)^{1/2}}{L_h Dr_i}
\]

Equation (3-66) can be rearranged:

\[
V_2 = \frac{L_h \cdot P_{h \cdot R_{ih}}^2}{(1 + P_{h_{ih}}^2)^{1/2} (1 + P_{c \cdot R_{ic}}^2)^{1/2}} \frac{1}{Dr_i} \frac{dP_h}{dt}
\]

\[
+ V_3 \frac{(1 + P_{h_{ih}}^2)^{1/2}}{Dr_i (1 + P_{c \cdot R_{ic}}^2)^{1/2}} \left( \frac{L_h \cdot (1 + P_{h_{ih}}^2)^{1/2}}{Dr_i (1 + P_{c \cdot R_{ic}}^2)^{1/2}} \right) \frac{dDr_i}{dt}
\]

Equation (3-67) represents independent equations depending on how many radial layers of filaments exist in the hot zone.

This equation can be simplified by defining

\[
A_{i1} = \frac{L_h \cdot P_{h \cdot R_{ih}}^2}{(1 + P_{h_{ih}}^2)^{1/2} (1 + P_{c \cdot R_{ic}}^2)^{1/2}} \cdot \frac{1}{Dr_i}
\]

\[
A_{i2} = \frac{V_3 (1 + P_{h_{ih}}^2)^{1/2}}{Dr_i (1 + P_{c \cdot R_{ic}}^2)^{1/2}}
\]

\[
A_{i3} = \frac{L_h (1 + P_{h_{ih}}^2)^{1/2}}{Dr_i^2 (1 + P_{c \cdot R_{ic}}^2)^{1/2}}
\]
Then,

$$V_2 = A_{i1} \frac{dP_h}{dt} + A_{i2} - A_{i3} \frac{dD_{ri}}{dt} \quad (3-71)$$

**Turn Balance for Hot Zone**

The quantitative basis for the turn balance is simply that the rate of change of the number of turns of twist in a given yarn zone equals the difference in the angular velocities at the ends of the zone, plus the net rate of transport of twist to and from the zone. Following this a turn balance for the hot zone can be expressed as

$$\frac{\Delta}{\Delta t} (L_h P_h) = W_h - W_c + P_c V_2 - P_h V_3 \quad (3-72)$$

and as \( \Delta t \to 0 \)

$$\frac{dP_h}{dt} = \frac{W_h - W_c + P_c V_2 - P_h V_3}{L_h} \quad (3-73)$$

**Material Balance in the Post Spindle Zone**

During steady state operation, twist generally is not present in the post spindle region if the feed yarn is initially twist free. However, during transient operation twist does occur in the post spindle region and this twist may affect the linear speed of the yarn as it moves downstream through the spindle. The material balance in the zone can be formulated as

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\[ \frac{\Delta}{\Delta t} (L_{ps} C_{ps}) = V_{b} C_{ps} - V_{4} C_{ps} \quad (3-74) \]

where \( C_{ps} \) = Twist contraction factor in post spindle

\( L_{ps} \) = Post spindle zone length.

as \( \Delta t \to 0 \) (3-74) becomes

\[ \frac{L_{ps}}{2} \frac{P_{ps} R_{3h}^2}{(1 + P_{ps}^2 R_{3h}^2)^{1/2}} \frac{dP_{ps}}{dt} = \frac{V_{b}}{2} \frac{(1 + (1 + P_{ps}^2 R_{3h}^2)^{1/2})}{2} \]

\[ - \frac{V_{4}}{2} \frac{(1 + (1 + P_{ps}^2 R_{3h}^2)^{1/2})}{2} \quad (3-75) \]

**Turn Balance in the Post Spindle**

The post spindle turn balance is formulated as

\[ \frac{\Delta}{\Delta t} (L_{ps} P_{ps}) = (V_{b} P_{s} - V_{4} P_{ps} - W_{s}) \quad (3-76) \]

and as \( \Delta t \to 0 \)

\[ \frac{L_{ps}}{2} \frac{dP_{ps}}{dt} = V_{b} P_{s} - V_{4} P_{ps} - W_{s} \quad (3-77) \]

Combine (3-75) and (3-77) to obtain

\[ \frac{dP_{ps}}{dt} = \left\{ \frac{V_{4} P_{ps} + W_{s}}{P_{ps}} \frac{(1 + (1 + P_{ps}^2 R_{3h}^2)^{1/2})}{P_{s}} - \frac{V_{4}}{P_{ps}} (1 + (1 + P_{ps}^2 R_{3h}^2)^{1/2}) \right\} \]

\[ \frac{L_{ps}}{2} \frac{P_{ps} R_{3h}^2}{(1 + P_{ps}^2 R_{3h}^2)^{1/2}} \frac{1}{P_{s}} - \frac{(1 + (1 + P_{ps}^2 R_{3h}^2)^{1/2})}{P_{s}} \quad (3-78) \]
Equation (3-78) can be substituted into (3-77) to obtain

\[ V_b = \frac{V_4 P_s}{P_s} + \frac{W_s}{P_s} + \frac{L_s}{P_s} \frac{d P_s}{d t} \]  

(3-79)

where \( V_b \) = The linear velocity of twisted yarn as it enters the spindle.

### 3.3.3.3 Material and Turn Balances for the Cold and Cooling Zones: Model 1 - Flat Twist Distribution

#### Material Balance in Cold Zone

In Chapter 2 it was assumed that all of the filaments in the twisted yarn of the cold zone are under equal tension and therefore equal axial strain. With this consideration it is only necessary to have one material balance equation that represents all cold filaments. The material balance in the cold zone is quantitatively expressed as

\[ \frac{\Delta}{\Delta t} \left( \frac{L_c C_r}{1+e_c} \right) = V_0 - \frac{V_2 C_r}{1+e_c} \]  

(3-80)

as \( \Delta t \to 0 \)

\[ \frac{1}{1+e_c} \frac{d C_r}{d t} - \frac{C_r}{(1+e_c)^2} \frac{d e_c}{d t} = \frac{V_0 - V_2}{L_c} \frac{C_r}{L_c} (1+e_c) \]  

(3-81)

where

\[ C_r^* = \frac{1}{2} \left( 1 + P_c^2 R_c^2 \right)^{1/2} + \ln \left( P_c R_c^3 + (1 + P_c^2 R_c^2)^{1/2} \right) \]  

(3-82)

* Note: Cold twist contraction factor obtained by radial averaging method.
\[
Z = \frac{P R^2_{c3c}}{2(1+P^2 R^2_{c3c})^{1/2}} + \frac{1}{2(P_c R^3_{3c} + (1+P^2 R^2_{c3c})^{1/2})}
\]
\[
+ \frac{R^3_{3c}}{2(1+P^2 R^2_{c3c})^{1/2}} \frac{1}{(P_c R^3 c + (1+P^2 R^2_{c3c})^{1/2})}
\]
\[- \ln \left(\frac{P_c R^3_{3c} + (1+P^2 R^2_{c3c})^{1/2}}{2P^2 R^2_{c3c}}\right)
\]

(3-83)

and

\[
\frac{dCr_c}{dt} = Z \frac{dP_c}{dt}
\]

(3-84)

Substitution of (3-84) into (3-81) gives

\[
V_2 = \frac{1+e_c}{Cr_c} V_0 - \frac{ZL_c}{Cr_c} \frac{dP_c}{dt} + \frac{L_c}{1+e_c} \frac{de_c}{dt}
\]

(3-85)

Equation (3-85) can be simplified by defining

\[
Q_1 = \frac{L_c}{(1+e_c)}
\]

(3-86)

\[
Q_2 = \frac{1+e_c}{Cr_c} \cdot V_0
\]

(3-87)

\[
Q_3 = \frac{ZL_c}{Cr_c}
\]

(3-88)

Then

\[
V_2 = Q_1 \frac{de_c}{dt} + Q_2 - Q_3 \frac{dP_c}{dt}
\]

(3-89)

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Equations (3-71) and (3-89) can be coupled to obtain

\[
\frac{dD_{ri}}{dt} = \frac{A_{i1}}{A_{i3}} \frac{dP_h}{dt} + \frac{A_{i2}}{A_{i3}} - \frac{Q_1}{A_{i3}} \frac{dc}{dt} - \frac{Q_2}{A_{i3}} + \frac{Q_3}{A_{i3}} \frac{dP_c}{dt}
\]

(3-90)

Equation (3-90) represents as many equations as there are radial layers of filaments plus the central layer at the hot zone entrance. As stated earlier, most yarns which are presently being draw textured consist of one core filament and three radial layers. Accordingly, from this point on it will be assumed that the yarns have but one core and three layers. If larger yarns are encountered appropriate arithmetical changes can be made, still, the threadline physics would remain unchanged. Equation (3-90) is substituted into (3-63) to obtain

\[
\frac{dc}{dt} = \frac{F_0}{F_6} \cdot \frac{A_{01}}{A_{03}} \frac{dP_h}{dt} + \frac{A_{02}}{A_{03}} - \frac{Q_2}{A_{03}} \frac{dc}{dt} + \frac{Q_3}{A_{03}} \frac{dP_c}{dt} + \frac{F_1}{F_6} \cdot \frac{A_{11}}{A_{13}} \frac{dP_h}{dt} + \frac{A_{12}}{A_{13}} - \frac{Q_2}{A_{13}} \frac{dc}{dt} + \frac{Q_3}{A_{13}} \frac{dP_c}{dt}
\]

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\[ + \frac{F_2 \cdot A_{21}}{F_6} \frac{dP_h}{dt} + \frac{A_{22} - Q_2}{A_{23}} - \frac{Q_1 \cdot \Delta c}{A_{23}} + \frac{Q_3 \cdot \Delta c}{A_{23}} \frac{dP_c}{dt} \]

\[ + \frac{F_3 \cdot A_{31}}{F_6} \frac{dP_h}{dt} + \frac{A_{32} - Q_2}{A_{33}} - \frac{Q_1 \cdot \Delta c}{A_{33}} + \frac{Q_3 \cdot \Delta c}{A_{33}} \frac{dP_c}{dt} \]

\[ + \frac{F_5 \cdot \Delta p}{F_6} \frac{dt}{dt} - \frac{F_7 \cdot \Delta c}{F_6} \frac{dt}{dt} \]

Equation (3-91) can be simplified by introducing abbreviated symbols

\[ X_1 = \frac{F_0 \cdot A_{01}}{F_6} \frac{A_{03}}{A_{03}} + \frac{F_1 \cdot A_{11}}{F_6} \frac{A_{13}}{A_{13}} + \frac{F_2 \cdot A_{21}}{F_6} \frac{A_{23}}{A_{23}} + \frac{F_3 \cdot A_{31}}{F_6} \frac{A_{33}}{A_{33}} \]

(3-92)

\[ X_2 = \frac{F_0 \cdot Q_1}{F_6} \frac{A_{03}}{A_{03}} + \frac{F_1 \cdot Q_1}{F_6} \frac{A_{13}}{A_{13}} + \frac{F_2 \cdot Q_1}{F_6} \frac{A_{23}}{A_{23}} + \frac{F_3 \cdot Q_1}{F_6} \frac{A_{33}}{A_{33}} \]

(3-93)

\[ X_3 = \frac{F_0 Q_3}{F_6} \frac{A_{03}}{A_{03}} + \frac{F_1 Q_3}{F_6} \frac{A_{13}}{A_{13}} + \frac{F_2 Q_3}{F_6} \frac{A_{23}}{A_{23}} + \frac{F_3 \cdot Q_3}{F_6} \frac{A_{33}}{A_{33}} - \frac{F_7}{F_6} \]

(3-94)

\[ X_4 = \frac{F_0 \left( A_{02} - Q_2 \right)}{F_6} \frac{A_{03}}{A_{03}} + \frac{F_1 \left( A_{12} - Q_2 \right)}{F_6} \frac{A_{13}}{A_{13}} + \frac{F_2 \left( A_{22} - Q_2 \right)}{F_6} \frac{A_{23}}{A_{23}} \]

\[ + \frac{F_3 \left( A_{32} - Q_2 \right)}{F_6} \frac{A_{33}}{A_{33}} \]

(3-95)

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Equations (3-92), (3-93), (3-94), (3-95) are substituted into (3-91) to obtain

\[ \frac{dc}{dt} = \frac{X_1}{1-X_2} \frac{dp_h}{dt} + \frac{X_3}{1-X_2} \frac{dp_c}{dt} + \frac{X_4}{1-X_2} \] (3-96)

Likewise the following symbols can be defined

\[ Y_1 = \frac{G_1}{G_6} \frac{A_{11}}{A_{13}} + \frac{G_2}{G_6} \frac{A_{21}}{A_{23}} + \frac{G_3}{G_6} \frac{A_{31}}{A_{33}} + \frac{G_5}{G_6} \] (3-97)

\[ Y_2 = \frac{G_1}{G_6} \frac{Q_1}{A_{13}} + \frac{G_2}{G_6} \frac{Q_1}{A_{23}} + \frac{G_3}{G_6} \frac{Q_1}{A_{33}} \] (3-98)

\[ Y_3 = \frac{G_1}{G_6} \frac{Q_3}{A_{13}} + \frac{G_2}{G_6} \frac{Q_3}{A_{23}} + \frac{G_3}{G_6} \frac{Q_3}{A_{33}} - \frac{G_7}{G_5} \] (3-99)

\[ Y_4 = \frac{G_1}{G_6} \frac{A_{12} - Q_2}{A_{13}} + \frac{G_2}{G_6} \frac{A_{22} - Q_2}{A_{23}} + \frac{G_3}{G_6} \frac{A_{32} - Q_2}{A_{33}} \] (3-100)

Finally we substitute (3-97), (3-98), (3-99), (3-100) into (3-62) to obtain

\[ \frac{dc}{dt} = \frac{Y_1}{1-Y_2} \frac{dp_h}{dt} + \frac{Y_3}{1-Y_2} \frac{dp_c}{dt} + \frac{Y_4}{1-Y_2} \] (3-101)

Equation (3-101) and Equation (3-100) represent two simultaneous first order differential equations with three unknown time derivatives and seven unknown variables.
Turn Balance in the Cold Zone

In Chapter 2 it was shown that during steady state operation of the draw texturing process the cold zone twist level is a fraction of the hot zone twist level. Since we assume that the hot zone rotates at the same angular velocity of the pin spindle, the angular velocity of the cold zone must be somewhat less.

The turn balance for the cold zone can be quantified as

\[ \frac{\Delta}{\Delta t} (L_c P_c) = W_c - P_c V^2 \]  \hspace{1cm} (3-102)

and as \( \Delta t \to 0 \)

\[ \frac{dP_c}{dt} = \frac{W_c - P_c V^2}{L_c} \]  \hspace{1cm} (3-103)

where \( W_c = \) The unknown rotational velocity of the cold zone

The cold zone turn balance (3-104) can be added to the hot zone turn balance (3-73) to obtain

\[ L_h \frac{dP_h}{dt} = W_h - L_c \frac{dP_c}{dt} - P_h V^3 \]  \hspace{1cm} (3-104)

To simplify equations (3-104) the following symbols are defined as

\[ X_5 = \frac{W_h - P_h V^3}{L_h} \]  \hspace{1cm} (3-105)

\[ X_6 = \frac{L_c}{L_h} \frac{dP_c}{dt} \]  \hspace{1cm} (3-106)
so that (3-104) becomes

\[
\frac{dP_h}{dt} = x_5 - x_6 \frac{dP_h}{dt}
\]  

(3-107)

**Material Balance in the Cooling Zone**

During steady state operation, the linear speed of the twist-ed yarn as it leaves the hot zone is assumed equal to the linear speed of the yarn as it approaches the rotating pin spindle, i.e., there is no material buildup in the cooling zone. During transient operation this is not the case and a material balance over the zone is necessary.

\[
\frac{\Delta}{\Delta t} (L_s C_{r\text{set}}) = V_3 C_{r\text{hot}} - V_b C_{r\text{set}}
\]  

(3-108)

where \( C_{r\text{set}} \) = Twist contraction factor all along the cooling zone

and as \( \Delta t \to 0 \)

\[
\frac{L_s}{2} \frac{P_s R_{3h}^2}{(1+P_s R_{3h}^2)^{1/2}} \frac{dP_s}{dt} = \frac{V_3}{2} \left(1 + \frac{(1+P_h R_{3h}^2)^{1/2}}{1+P_s R_{3h}^2}\right)
\]  

\[
- \frac{V_b}{2} \left(1 + \frac{(1+P_h R_{3h}^2)^{1/2}}{1+P_s R_{3h}^2}\right)
\]  

(3-109)

For Model 1 the assumption is made that the twist in the set zone is equal to the twist in the hot zone at all times.

Therefore

\[
P_s = P_h
\]  

(3-110)
Then (3-109) becomes

\[ V_3 = \left\{ \frac{L_s P h^2 R^2 h^3}{(1+P^2 R^2 h^3)^{1/2} + 1+P^2 R^2 h^3} \right\} \frac{dP_h}{dt} + V_b \quad (3-111) \]

Equation (3-111) is now substituted into (3-69) so that

\[ A_{i2} = \left\{ \frac{L_s P h R^2 h^3}{(1+P^2 R^2 h^3)^{1/2} + 1+P^2 R^2 h^3} \right\} \frac{dP_h}{dt} + V_b \]

\[ \left[ \frac{(1+P^2 R^2 h^2)^{1/2}}{h_{ih}} \right] \frac{dP_h}{dt} + \frac{V_b}{(1+P^2 R^2 c^2_{ic})^{1/2}} \quad (3-112) \]

\[ A_{i2} \] is now substituted into (3-90). \[ A_{i1} \] and \[ A_{i2} \] are re-defined as

\[ A_{i1} = \frac{L_s P h R^2 h_{ih}}{\left(1+P^2 R^2 h_{ih}^2\right)^{1/2} \left(1+P^2 R^2 c^2_{ic}\right)^{1/2} \frac{dP_h}{dt} + V_b} \]

\[ + \left\{ \frac{L_s P h R^2 h^3}{(1+P^2 R^2 h^3)^{1/2} + 1+P^2 R^2 h^3} \right\} \left\{ \frac{(1+P^2 R^2 h^2)^{1/2}}{h_{ih}} \right\} \frac{dP_h}{dt} + \frac{V_b}{(1+P^2 R^2 c^2_{ic})^{1/2}} \quad (3-113) \]

\[ A_{i2} = \frac{V_b (1+P^2 R^2 h_{ih}^2)^{1/2}}{D_r (1+P^2 R^2 c^2_{ic})^{1/2}} \quad (3-114) \]

**Turn Balance in the Cooling Zone**

A turn balance is formulated for the cooling zone.

\[ \frac{\Delta}{\Delta t} \left( L_s P_s \right) = V_3 P_h - V_b P_s \quad (3-115) \]
where \( L_s \) = Length of cooling zone

and with \( P_h = P_s \) (assumed for model 1)

We also assume that there is no difference in the net rates of rotation at the ends of the zone.

\[
L_s \frac{dP_h}{dt} = V_3 P_h - V_b P_h \tag{3-116}
\]

Adding (3-116) to (3-107) the following turn balance is derived for the full texturing zone.

\[
(L_h + L_s) \frac{dP_h}{dt} = W_h - L_c \frac{dP_c}{dt} - P_h V_b \tag{3-117}
\]

Defining

\[
X_5 = \frac{W_h - P_h V_b}{(L_h + L_s)} \tag{3-118}
\]

\[
X_6 = \frac{L_c}{L_h + L_s} \tag{3-119}
\]

To get

\[
\frac{dP_h}{dt} = X_5 - X_6 \frac{dP_c}{dt} \tag{3-120}
\]

Equation (3-120) is now substituted into Equations (3-96), and (3-101) to obtain

\[
\frac{d\epsilon_c}{dt} = \frac{Y_1 X_5 + Y_4}{1 - Y_2} + \frac{Y_3 Y_6}{1 - Y_2} \frac{dP_c}{dt} \tag{3-121}
\]

\[
\frac{d\epsilon_c}{dt} = \frac{X_1 X_5 + X_4}{1 - X_2} + \frac{X_3 X_6}{1 - X_2} \frac{dP_c}{dt} \tag{3-122}
\]
Equations (3-121) and (3-122) are coupled to obtain

\[
\frac{dP}{dt} = \left\{ \begin{array}{c}
\frac{Y_1 X_5 + Y_4}{1-Y_2} - \frac{X_1 X_5 + X_4}{1-Y_2} \\
\frac{X_3 - X_1 X_6}{1-X_2} - \frac{Y_3 - Y_1 X_6}{1-Y_2}
\end{array} \right\} 
\] (3-123)

Equation (3-123) can be functionally expressed as

\[
\frac{dP}{dt} = Z_1 (P_c, P_h, e_c, D_{ri}, P_{ps}) 
\] (3-124)

Equation (3-123) is substituted into (3-120) to obtain

\[
\frac{dP_h}{dt} = Z_2 (P_c, P_h, e_c, D_{ri}, P_{ps}) 
\] (3-125)

Equation (3-125) is then substituted into (3-121) to obtain

\[
\frac{dE_c}{dt} = Z_3 (P_c, P_h, e_c, D_{ri}, P_{ps}) 
\] (3-126)

Equations (3-124), (3-125) and (3-126) are now substituted into (3-90) to obtain

\[
\frac{dD_{ri}}{dt} = Z_4 (P_c, P_h, e_c, D_{ri}, P_{ps}) 
\] (3-127)

Finally

\[
\frac{dP_{ps}}{dt} = Z_5 (P_c, P_h, e_c, D_{ri}, P_{ps}) 
\] (3-128)

Threadline tension is obtained by solving the preceding set of five differential equations and substituting values of $e_c$, $e_c'$.
$P_c$ into (3-43). Finally, these values of $e_c$, and $P_c$ are also substituted into (3-56) to obtain threadline torque.

3.3.3.4 Twist Transport, Material Balances and Turn Balances in the Cold and Cooling Zones: Model 2 - Tilted Twist Distribution

Twist Transport Formula of the Tilted Twist Model 2

The previous model (MODEL 1) for transient operation was based on the physical simplification that uniform twist levels during transient operation persist in each of the four zones of the threadline (cold, hot, cooling, and post spindle). Actual photographic measurements of twist as a function of time and position after a step change in spindle speed, indicate that the twist level in both the cold and cooling zones is not uniform with position during transient operation. Experimental evidence presented earlier, in Section 3.2.3, indicates that twist changes incurred at the entrance to the cold or cooling zones propagate downstream with the movement of the threadline. As a first approximation the twist level in either the cold zone or the cooling zone can be considered to vary linearly with position along that zone. This distribution is illustrated on the following page.
This model of the twist distribution operates as follows. At position C, (which is the downstream limit of the zone in question) the twist level at anytime, $t$, is $P_1$. At the same time, $t$, the twist level at position A, (the upstream end of the zone), is $P_2$. $P_2$ is dictated by threadline mechanics. At position B, located a distance $V\Delta t$ upstream of C, the twist for time, $t$, is $P_1 + \Delta P_1$. During the time interval $\Delta t$ the twist at position B migrates downstream to position C establishing the new twist level there as $P_1 + \Delta P_1$. It follows that:

$$\frac{P_2 - P_1}{L} = \frac{\Delta P_1}{V\Delta t} \quad (3-129)$$
where $V$ is the linear yarn velocity at $C$. Now, letting $\Delta t \to 0$

\[
\frac{dP_1}{dt} = \frac{(P_2 - P_1)}{L} V \quad (3-130)
\]

This twist transport relation is now inserted in subsequent treatments of material and turn balances for the cold and cooling zones.

Material Balance in the Cold Zone for the Tilted Twist Model 2

We have assumed that the twist distribution in the cold zone varies linearly along the threadline during transient operation. Therefore the length of material in the cold zone is

\[
\frac{[Cr_c + Cr_j]}{1+e_c} \frac{L_c}{2}
\]

Further, we assume that the constant filament strain which obtains at the entrance to the cold zone at a given time, $t$, is uniform along the cold zone. Since filament strains at the cold zone entrance are of the order of 1% or less, the error introduced by this assumption in the following expressions is seen to be negligible. The material balance in the cold zone is

\[
\frac{\Delta L_c}{\Delta t} \left[ \frac{Cr_c + Cr_j}{2(1+e_c)} \right] = V_0 - \frac{V_2 Cr_j}{(1+e_c)} \quad (3-131)
\]

where $Cr_j = $ Cold twist contraction factor at downstream end of the cold zone
as \( \Delta t \to 0 \)

\[
V_2 = \frac{(1+e_c)V_0}{Cr_j} - \frac{L_c}{2Cr_j} \frac{dCr_c}{dt} - \frac{L_c}{2Cr_j} \frac{dCr_j}{dt}
+ \frac{L_c}{2} \frac{Cr_c + Cr_j}{Cr_j(1+e)} \frac{de}{dt} \tag{3-132}
\]

let \( \frac{dCr_c}{dt} = Z_1 \frac{dP_c}{dt} \) \hspace{1cm} (3-133)

where \( Z_1 = \frac{P_c R^2}{(1+P^2 R^2)^{3c}} + \frac{1}{(P_c R^3 c + (1+P^2 R^2)^{3c})^{1/2}} \) \hspace{1cm} (3-134)

and \( \frac{dCr_j}{dt} = Z_2 \frac{dP_j}{dt} \) \hspace{1cm} (3-135)

where \( Z_2 = \frac{P_j R^2}{(1+P^2 R^2)^{3c}} + \frac{1}{(P_j R^3 c + (1+P^2 R^2)^{3c})^{1/2}} \) \hspace{1cm} (3-136)

where \( P_j = \) Cold twist per twisted length at cold zone exit.

Then equation (3-132) can be expressed as

\[
V_2 = \frac{(1+e_c)}{Cr_j} \cdot V_0 - \frac{L_c}{2Cr_j} \cdot Z_1 \frac{dP_c}{dt} - \frac{L_c}{2Cr_j} \cdot Z_2 \frac{dP_j}{dt}
+ \frac{L_c}{2} \frac{Cr_c + Cr_j}{Cr_j(1+e)} \frac{de}{dt} \tag{3-137}
\]

The twist transport relation for the cold zone is

\[
\frac{dP_j}{dt} = \frac{P_c - P_j}{L_c} \cdot V_2 \tag{3-138}
\]
Equation (3-138) is now substituted into (3-137) to obtain

\[
V_2 \cdot \left[ 1 + \frac{(P_c - P_j)Z_2}{2Cr_j} \right] = \frac{1 + \epsilon_c}{Cr_j} \cdot V_0 - \frac{L_c}{2Cr_j} \cdot Z_1 \cdot \frac{dP_c}{dt}
\]

\[
+ \frac{L_c \cdot Cr_c + Cr_j}{2Cr_j(1 + \epsilon_c)} \cdot \frac{d\epsilon}{dt}
\]

(3-139)

Equation (3-139) can be simplified by defining

\[
Q_1 \equiv \frac{L_c \left[ \frac{Cr_c + Cr_j}{Cr_j(1 + \epsilon_c)} \right]}{1 + \frac{(P_c - P_j)Z_2}{2Cr_j}}
\]

(3-140)

\[
Q_2 \equiv \frac{\left[ \frac{(1 + \epsilon_c)}{Cr_j} \right] V_0}{1 + \frac{(P_c - P_j)Z_2}{2Cr_j}}
\]

(3-141)

\[
Q_3 \equiv \frac{L_c Z_1}{\left[ 1 + \frac{(P_c - P_j)Z_2}{2Cr_j} \right] (2Cr_j)}
\]

(3-142)

Equations (3-140), (3-141), and (3-142) are now substituted into (3-89) in place of (3-86), (3-87), and (3-88), so that \(V_2\) is still expressed as

\[
V_2 = Q_1 \frac{d\epsilon}{dt} + Q_2 - Q_3 \frac{dP_c}{dt}
\]

(3-89)
Material Balance in the Cooling Zone for the Tilted Twist Model 2.

If we assume that the twist distribution in the cooling zone is linear as seen in the twist sketch the length of the cooling zone segment in its untwisted state is

\[ \frac{L_s}{2} (Cr_h + Cr_s) \]

The mass balance in the cooling zone is then

\[ \frac{\Delta}{\Delta t} L_s \frac{(Cr_h + Cr_s)}{2} = V_3 Cr_h - V_b Cr_s \]  \( (3-143) \)

where

- \( Cr_h = \) Twist contraction factor at upstream end of set zone
- \( Cr_s = \) Twist contraction factor at downstream end of set zone.

Using Hearle's expression for twist contraction we convert (3-143) to

\[ \frac{L_s}{2} \frac{P_h R^2}{2(1+P_h^2 R^2)^{1/2}} \frac{dP_h}{dt} + \frac{P_s R^2}{2(1+P_s^2 R^2)^{1/2}} \frac{dP_s}{dt} \]

\[ = \frac{V_3}{2} (1+(1+P_h^2 R^2)^{1/2}) \]  \[ - \frac{V_b}{2} (1+(1+P_s^2 R^2)^{1/2}) \]  \( (3-144) \)

The twist transport equation for the cooling zone
\[
\frac{dP_s}{dt} = \frac{(P_h - P_s)}{L_s} V_b \tag{3-145}
\]

is substituted in (3-144) to obtain

\[
V_3 = \frac{L_s}{2} \frac{P_h R^2}{(1 + P_h^2 R^2) \left( 1 + \left( 1 + \frac{P_h^2 R^2}{1/2} \right) \right)} \frac{dP_h}{dt} + \frac{P_s R^2 (P_h - P_s) V_b}{2 \left( 1 + P_h^2 R^2 \right)^{1/2} \left( 1 + \left( 1 + \frac{P_h^2 R^2}{1/2} \right) \right)} \tag{3-146}
\]

\[
+ V_b \frac{(1 + \left( 1 + \frac{P_h^2 R^2}{1/2} \right))}{(1 + \left( 1 + \frac{P_h^2 R^2}{1/2} \right))} \tag{3-146}
\]

Equation (3-146) is now substituted into (3-69) with \( A_{i1} \), \( A_{i2} \), \( A_{i3} \), defined as

\[
A_{i1} = \frac{L_h P_i R^2}{(1 + P_h^2 R^2)^{1/2} \left( 1 + \frac{P_h^2 R^2}{1/2} \right) \frac{dR}{d_i}} \tag{3-147}
\]

\[
+ \frac{1/2 L_s P_i R^2 (1 + P_h^2 R^2)^{1/2}}{(1 + P_h^2 R^2)^{1/2} \left( 1 + \left( 1 + \frac{P_h^2 R^2}{1/2} \right) \right) \left( 1 + \frac{P_h^2 R^2}{1/2} \right) \frac{dR}{d_i}} \tag{3-147}
\]
\[ A_{i2} = \frac{(1+(1+P_{h}^{2}R_{3h}^{2})^{1/2})^{1/2}(1+P_{h}^{2}R_{3h}^{2})^{1/2}}{(1+(1+P_{h}^{2}R_{3h}^{2})^{1/2})^{1/2} Dr_{i} (1+P_{j}^{2}R_{i}^{2})^{1/2}} \]

\[ + \frac{P_{s}^{2}R_{3h}^{2} (P_{h}-P_{s})(1+P_{h}^{2}R_{3h}^{2})^{1/2}}{2 (1+P_{s}^{2}R_{3h}^{2})^{1/2} (1+(1+P_{h}^{2}R_{3h}^{2})^{1/2}) Dr_{i} (1+P_{j}^{2}R_{i}^{2})^{1/2}} \]

\[ A_{i3} = \frac{L_{h}^{2} (1+P_{h}^{2}R_{3h}^{2})^{1/2}}{Dr_{i}^{2} (1+P_{j}^{2}R_{i}^{2})^{1/2}} \]

which leads to:

\[ V_{2} = A_{i1} \frac{dp_{h}}{dt} + A_{i2} - A_{i3} \frac{dDr_{i}}{dt} \]

Equation (3-150) can be added to (3-89) and an expression is obtained for the time derivative of the draw ratio for each filament layer in the yarn. This expression is identical to equation (3-90) for the time derivative of draw ratio for the flat twist distribution Model #1. However, the expressions for \( A_{i1}, A_{i2}, A_{i3}, Q_{1}, Q_{2}, \) and \( Q_{3} \) are now defined differently, i.e., as shown in (3-147), (3-148), (3-149), (3-140), (3-141) and (3-142) for the case of the tilted twist Model #2.

**Turn Balances**

A turn balance exists for each of the four zones (cold, hot,
cooling, and post spindle). Each turn balance requires that in a given zone, the rate of change of the number of turns in a given zone is equal to the difference in the rates of rotation of the yarns at the ends of the zone plus the net gain by transport to and from the adjacent zones. As in Model 1 we assume here that the twist levels are uniform along the hot and post spindle zones.

Turn Balance in the Cold Zone for the Tilted Twist Model 2

We assume that if the twist varies linearly in the cold zone from $P_c$ to $P_j$, then the amount of turns in the cold zone is

$$L_c \frac{(P_c + P_j)}{2}$$

Therefore the cold zone twist balance is

$$\frac{\Delta}{\Delta t} \left[ L_c \frac{(P_j + P_c)}{2} \right] = W - P_j V_2$$

(3-151)

where

$P_j$ = Cold twist per twisted length at downstream end of the cold zone

and as $\Delta t \to 0$

$$\frac{L_c \cdot dP_j}{2 \cdot dt} + \frac{L_c \cdot dP_c}{2 \cdot dt} = W - P_j V_2$$

(3-152)

Turn Balance in the Hot Zone

The hot zone turn balance for Model 2 is the same as for Model 1, i.e.:
Turn Balance in the Cooling Zone for the Tilted Twist Model 2

If we assume that the twist distribution is linear in the cooling zone, the number of turns in that zone is

\[ L_s \left( \frac{V_s}{2} \right) \]

The cooling zone twist balance in then:

\[ \frac{\Delta}{\Delta t} L_s \left( \frac{P_h + P_s}{2} \right) = P_h \frac{V_3}{2} - P_s \frac{V_b}{2} + W_s - W_h \]  

(3-154)

and as \( \Delta t \to 0 \)

\[ \frac{L_s}{2} \frac{dP_h}{dt} + \frac{L_s}{2} \frac{dP_s}{dt} = P_h \frac{V_3}{2} - P_s \frac{V_b}{2} + W_s - W_h \]  

(3-155)

Observation indicates that for practical purposes \( W_h = W_s \).

where \( W_h \) = Angular speed of hot zone segments

\( W_s \) = Angular speed of cooling zone segments

A composite turn balance for all zones upstream of the spindle is formulated by adding the cold, hot, and cooling zone turn balances to obtain

\[ \frac{L_c}{2} \frac{dP_c}{dt} + \frac{L_c}{2} \frac{dP_i}{dt} + L_h \frac{dP_h}{dt} + L_s \frac{dP_h}{dt} + \frac{L_s}{2} \frac{dP_s}{dt} + \frac{L_s}{2} \frac{dP_s}{dt} \]

\[ = W_s - P_s \frac{V_b}{2} \]  

(3-156)
The twist transport equation for \( \frac{dP_j}{dt} \) and \( \frac{dP_s}{dt} \) can be substituted in (3-156) to obtain.

\[
\frac{1}{2}\left\{L_c \frac{dP_c}{dt}\right\} + \frac{1}{2}\left\{(P_c-P_j)\right\}V_2 + \left\{L_h \frac{dP_h}{dt}\right\}
\]

\[+ \frac{1}{2}\left\{L_s \frac{dP_s}{dt}\right\} + \frac{1}{2}\left\{(P_h-P_s)\right\}V_b = W_h - P_sV_b \tag{3-157}\]

\(V_b\) is determined by the post spindle analysis. \(V_2\) is obtained from the cold material balance.

\[
V_2 = Q_1 \frac{de}{dt} + Q_2 - Q_3 \frac{dP_c}{dt} \tag{3-158}
\]

Substituting \(V_2\) into the composite turn balance we obtain

\[
\frac{dP_h}{dt} = \frac{W_h - P_sV_b}{L_h + 1/2 L_s} + \frac{1}{2}\left\{(P_c-P_j)Q_3 - \frac{L_c}{L_h + 1/2 L_s}\right\} \frac{dP_c}{dt}
\]

\[- \frac{1}{2}\left\{(P_h-P_s)V_b\right\} - \left\{\frac{Q_1(P_c-P_j)}{2L_h + L_s}\right\} \frac{de}{dt} - \frac{Q_2(P_c-P_j)}{2L_h + L_s} \tag{3-159}\]

We now define

\[
X_5 = \frac{W_h - P_sV_b}{L_h + 1/2 L_s} \tag{3-160}
\]

\[
X_6 = \frac{L_c}{2L_h + L_s} \tag{3-161}
\]

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\[ x_7 = \frac{P_c - P_j}{2L_h + L_s} \]  

\[ x_8 = \frac{(P_h - P_s)V_b}{2L_h + L_s} \]  

so that

\[ \frac{dP_h}{dt} = x_5 + (Q_3X_7 - X_6) \frac{dP_c}{dt} - Q_1X_7 \frac{de}{dt} - Q_2X_7 - X_8 \]  

(3-164)

Using the values of \( A_{11}, A_{12}, A_{13}, Q_1, Q_2, \) and \( Q_3 \) for Model 2 a new set of \( X_1, X_2, X_3, X_4, Y_1, Y_2, Y_3, \) and \( Y_4 \) can be obtained from equations (3-92), (3-93), (3-94), (3-95), (3-97), (3-98), (3-99) and (3-100). The general threadline force equations are then expressed as:

\[ (1-X_2) \frac{de}{dt} = X_1 \frac{dP_h}{dt} + X_3 \frac{dP_c}{dt} + X_4 \]  

(3-165)

and

\[ (1-Y_2) \frac{de}{dt} = Y_1 \frac{dP_h}{dt} + Y_3 \frac{dP_c}{dt} + Y_4 \]  

(3-166)

substituting the composite turn balance (3-164) into (3-165) and (3-166) we obtain

\[ (1-X_2) \frac{de}{dt} = X_1 \left[ X_5 + (Q_3X_7 - X_6) \frac{dP_c}{dt} - Q_1X_7 \frac{de}{dt} \right. \\
- Q_2X_7 - X_8 \left] + X_3 \frac{dP_c}{dt} + X_4 \right\]  

(3-167)
and \[
(1-Y_2) \frac{de}{dt} = Y_1 \left\{ X_5 + (Q_3 X_7 - X_6) \frac{dp_c}{dt} - Q_1 X_7 \frac{de}{dt} - Q_2 X_7 - X_8 \right\} + Y_3 \frac{dp_c}{dt} + Y_4
\]

Equations (3-167) and (3-168) are combined to obtain

\[
\frac{dp_c}{dt} = \begin{bmatrix}
\frac{Y_1 X_5 - Y_1 Q_2 X_7 - X_1 X_8 + Y_4}{1 - Y_2 + Y_1 Q_1 X_7} & \frac{-X_1 X_5 - X_1 Q_2 X_7 - X_1 X_8 + X_4}{1 - X_2 + X_1 Q_1 X_7} \\
\frac{X_1 Q_3 X_7 - X_1 X_6 + X_3}{1 - X_2 + X_1 Q_1 X_7} & \frac{-Y_1 Q_3 X_7 - Y_1 X_6 + Y_3}{1 - Y_2 + Y_1 Q_1 X_7}
\end{bmatrix}
\]

Equation (3-169) can be functionally expressed as

\[
\frac{dp_c}{dt} = Y_1 (P_c, P_j, P_h, P_s, P_{ps}, e_{c}, D_{ri}) \tag{3-170}
\]

Equation (3-170) is then substituted into (3-168) to obtain

\[
\frac{de}{dt} = Y_3 (P_c, P_j, P_h, P_s, P_{ps}, e_{c}, D_{ri}) \tag{3-171}
\]

Equations (3-170) and (3-171) are substituted into the composite turn balance to obtain

\[
\frac{dp_h}{dt} = Y_2 (P_c, P_j, P_h, P_s, P_{ps}, e_{c}, D_{ri}) \tag{3-172}
\]

Equations (3-170), (3-171) and (3-172) are substituted into (3-90) to obtain
Threadline tension and torque are obtained by solving the preceding set of differential equations and substituting values of $e_c, P_c$, into equations (3-43) and (3-56).

3.3.4 Solution Technique for the Transient Analysis

Each of the analytical models of transient threadline mechanics is represented by a system of non-linear ordinary differential equations. The transient solution to these systems of equations is solved by the DYSYS (DYnamic SYstem Simulation) subroutine provided in the MIT Mechanical and Civil Engineering Joint Computer Facility. DYSYS uses a fourth order Runge-Kutta integration to obtain approximate solutions to the system of equations. The form of equations which DYSYS can solve is:

$$\frac{d}{dt} y_1 = f_1(t, y_1, y_2, \ldots, y_n)$$
\[ \frac{d}{dt} y_2 = f_2(t, y_1, y_2, \ldots, y_n) \]
\[ \vdots \]
\[ \vdots \]
\[ \frac{d}{dt} y_n = f_n(t, y_1, y_2, \ldots, y_n) \]

This may be compactly written as:

\[ \frac{d}{dt} Y = F(t, Y) \]

where $Y$ is an $n$-dimensional vector made up of the current values of $y_1, \ldots, y_n$ and $F$ is a vector valued function composed of the current values of $f_1, \ldots, f_n$.

The Computer programs that use DYSYS to give transient solutions are listed in Appendix 6 for Model 1 and 2. The initial values for the transient analysis are obtained from solutions derived from the steady state analysis presented in Chapter 2.
3.4 EXPERIMENTAL STUDY OF TRANSIENT OPERATION OF THE DRAW TEXTURING PROCESS: USING POY POLYESTER

Experiments were done with POY polyester yarns supplied by the Du Pont Company so that we could compare measured threadline behavior, i.e. threadline tension, torque, and twist distribution, with predicted results during transient operation of the draw texturing process. The dimensions and properties of these yarns are given in Section 2.4.1. The yarns are textured on the MITEX False Twister. Threadline torque and tension were measured by the MITEX Torque-Tension Meter placed slightly downstream of the feed rollers. The placement of the latter device at this position causes the actual length of the threadline to be increased by the distance from the feed rollers to the twist trap on the device, but this added length has only a minimal effect on transient threadline response since no twist is included in this extra length. For it is change in twist with respect to time which affects threadline response most. Finally, threadline twist was measured optically by taking pictures of the threadline at different stations along its length and counting twists.

Unsteady machine operation was activated by changing the machine twist stepwise. Three combinations of machine dimensions were used in these experiments.
The nominal yarn throughput speed was 0.47 inches per second; the machine draw ratio was 1.60; the heater temperature was 210°C.

Two complimentary sets of experiments were carried out for each mode of machine operation. In one set the process was allowed to operate at steady state with the machine twist at 50 TPI before the machine twist was step changed to 60 TPI. For the complimentary set of experiments the machine twist was reduced stepwise from 60 TPI to 50 TPI. Table 3.4 lists the critical threadline responses during transient operation for all experiments using POY polyester.

### Twist Increased to 60 TPI from 50 TPI

Figures 3.29, 3.30, and 3.31 show predicted and measured transient tensions for each machine mode of operation. The predicted tensions using Analytical Model 2 (tilted twist distribution) compare favorably with the measured values for Machine Mode I and Machine Mode II. However, for Machine Mode III the predicted results using Analytical Model I (flat twist distribution) compares more favorably to the measured results than the predictions using Model 2. It
### Table 3.4

**Comparison of Predicted and Measured Transient Threadline Response**

<table>
<thead>
<tr>
<th>Mode of Machine Operation</th>
<th>Initial Tension (grams)</th>
<th>Maxima or Minima Tension (grams)</th>
<th>Time to Reach Maxima or Minima (seconds)</th>
<th>Final Tension (grams)</th>
</tr>
</thead>
</table>
| **Initial Torque** (in lbs) | Mode I 50 to 60 TPI | 24.8 | 25.0 | 33.3 | 34.5 | 19 | 15 | 20.3 | 22.1  
|                           | Mode I 60 to 50 TPI | 20.3 | 22.1 | 15.5 | 15.3 | 15 | 15 | 24.8 | 25.0  
|                           | Mode II 50 to 60 TPI | 24.8 | 25.0 | 33.6 | 33.0 | 20 | 20 | 20.3 | 22.1  
|                           | Mode II 60 to 50 TPI | 20.3 | 22.1 | 13.2 | 16.1 | 19 | 20 | 24.8 | 25.0  
|                           | Mode III 50 to 60 TPI | 24.8 | 25.0 | 30.9 | 33.8 | 13 | 14 | 20.3 | 22.1  
|                           | Mode III 60 to 50 TPI | 20.3 | 22.1 | 15.5 | 16.0 | 6 | 14 | 24.8 | 25.0  
| **Final Torque** (in lbs) | Mode I 50 to 60 TPI | 1.47 | 1.43 | 2.26 | 2.16 | 20 | 19 | 1.67 | 1.80  
|                           | Mode I 60 to 50 TPI | 1.67 | 1.80 | 1.14 | 1.18 | 14 | 18 | 1.47 | 1.43  
|                           | Mode II 50 to 60 TPI | 1.47 | 1.43 | 2.32 | 2.16 | 22 | 22 | 1.67 | 1.80  
|                           | Mode II 60 to 50 TPI | 1.67 | 1.80 | 1.07 | 1.19 | 19 | 26 | 1.47 | 1.43  
|                           | Mode III 50 to 60 TPI | 1.43 | 1.47 | 2.28 | 2.17 | 10 | 20 | 1.67 | 1.80  
|                           | Mode III 60 to 50 TPI | 1.67 | 1.80 | 1.29 | 1.15 | 10 | 20 | 1.47 | 1.43  

*Note: The table continues with similar formats for other modes of machine operation.*
Figure 3.29

Mode I  50-60 TPI

Tension, grams

Time, seconds
should be emphasized that Mode III machine operation does not truly represent common texturing conditions since there is no cooling zone in the threadline before the spindle.

Figures 3.32, 3.33, and 3.34 show predicted and measured threadline torque for each mode of operation. The predicted torques using Analytical Model 2 (tilted twist distribution) compare favorably with the measured values for each mode of machine operation. It should be noted that when the cooling zone length is shortened and the heater length increased (Mode II) the peak torque is higher than for Mode I. This is due to the maximum tension in the threadline occurring at a later time for Mode II than for Mode I; at a later time the twist is higher and threadline torque is governed by hot twist and threadline tension.

Twist Decreased to 50 TPI from 60 TPI

Figures 3.35, 3.36, and 3.37 show predicted and measured threadline tension for each mode of machine operation. The predicted values using Analytical Model 2 (tilted twist distribution) compare favorably for the most part with the measured values of threadline tension for all modes of machine operation. However, after the tension bottoms out the predicted and measured values for Mode III are not in good agreement.

Figures 3.38, 3.39, and 3.40 show predicted and measured
Figure 3.33

Mode II 50-60TPI

Torque ($10^{-3}$) in lbs

Experimental

1

2

Time, seconds

0.0 20.0 40.0 60.0 80.0 100.0 120.0
Figure 3.34

Mode III  50-60 TPI

Torque, (10^-4) in-lbs

Time, seconds
Figure 3.35

Mode I  60-50 TPI

Tension, grams

Time, seconds

Experimental

1

2
Figure 3.36

Mode II 60-50 TPI

Tension, grams

Time, seconds

Experimental
1
2
Figure 3.37

Mode III  60-50TPI

Tension, grams

Time, seconds

Experimental

1

2
Figure 3.38

Mode I 60-50 TPI

Torque, (lbf-t) in lbs

Time, Seconds

Experimental

1

2
Figure 3.39

Mode II  60-50 TPI

Torque, (10^-4) in-lbs

Time, seconds
Figure 3.40

MODE III  60-50 TPI
threadline torque for each mode of machine operation. The predicted values using Analytical Model 2 (tilted twist distribution) compare favorably with the measured values of threadline torque for all modes of machine operation.

**Twist Distribution in the Cold Zone**

Figures 3.41 and 3.42 show predicted and measured results using Analytical Model 2 (tilted twist distribution) for threadline twist as functions of time at two positions in the cold zone for Mode I machine operation. Note the steady-state error between predicted and measured twist values at 60 TPI machine twist. However at 50 TPI machine twist the error is negligible. In light of steady-state errors, the measured and predicted transient threadline twist levels compare favorably with each other. In Figure 3.43 we connect the measured twist levels at two points in the cold zone by a straight line at different times after a step change in machine twist and compare the slopes of these lines with the predicted slopes using Analytical Model 2. Comparison of the measured and predicted twist distribution slopes at the same time indicates good agreement.
Figure 3.41

![Graph showing cold twist TPI over time for Mode I 50-60 TPI with predicted and measured values for 1.00 and 5.00 inches into the cold zone.](image-url)
Figure 3.42

MODE I 60-50 TPI

- PREDICTED 1.00 INCHES INTO COLD ZONE
- PREDICTED 5.00 INCHES INTO COLD ZONE
- MEASURED 1.00 INCHES INTO COLD ZONE
- MEASURED 5.00 INCHES INTO COLD ZONE
Figure 3.43

COMPARISON OF PREDICTED AND MEASURED COLD ZONE TWIST

40 TPI

35 TPI

30 TPI

25 TPI

1.00 INCHES INTO COLD ZONE 5.00 INCHES

--- PREDICTED

- - - - MEASURED
3.5 EXPERIMENTS ON THE DRAW TEXTURING OF POY NYLON

Experiments were conducted on Du Pont POY nylon yarns for the purpose of comparing measured values of threadline parameters, i.e. threadline tension and threadline torque, with predicted results during transient operation of the draw texturing process. The dimensions and properties of the feed yarn used are as follows:

Undrawn Filament Denier: 90/34 - 2.65
Undrawn Filament Radius: $3.57 \times 10^{-4}$ inches
Undrawn Filament Tensile Modulus: 268,000 psi
Undrawn Filament Bending Rigidity: $3.42 \times 10^{-9}$ lb in$^2$
Undrawn Filament Torsional Rigidity: $2.47 \times 10^{-9}$ lb in$^2$

Filament dimensions and tensile modulus were measured in the Fibers and Polymers Laboratories at MIT. The bending modulus was calculated from the tensile modulus based on the assumption of equal axial and compressive moduli. The torsional rigidity ($G_{Ip}$) was obtained from Ward$^{11}$.

The yarns were textured on the MITEX False Twister. Threadline tension and torque were measured by the MITEX Torque-Tension Meter. The machine dimensions were set as follows:

<table>
<thead>
<tr>
<th>Zone</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cold Zone</td>
<td>6.00 inches</td>
</tr>
<tr>
<td>Hot Zone</td>
<td>1.25 inches</td>
</tr>
<tr>
<td>Cooling Zone</td>
<td>7.00 inches</td>
</tr>
</tbody>
</table>
The unsteady machine operation was activated by introducing a step change in the machine twist. The nominal yarn throughput speed was .38 inches per second; the draw ratio was 1.29; the heater temperature was 210°C.

**Steady State Threadline Tension and Threadline Torque**

For this commercial textured nylon yarn the general level of machine twist is between 60 TPI and 70 TPI, and so threadline tension and threadline torque were measured and predicted for these levels as indicated below:

<table>
<thead>
<tr>
<th>Machine Twist</th>
<th>Tension, grams</th>
<th>Torque($10^{-4}$) in lbs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Predicted</td>
<td>Measured</td>
</tr>
<tr>
<td>60 TPI</td>
<td>23.3</td>
<td>26.5</td>
</tr>
<tr>
<td>70 TPI</td>
<td>20.9</td>
<td>23.0</td>
</tr>
</tbody>
</table>

|               | Predicted      | Measured                   |
| 60 TPI        | .78            | .86                        |
| 70 TPI        | .89            | 1.01                       |

It is seen that the predicted values of threadline torque and tension are about 10% less than the measured values. No effort has been made to experimentally characterize in full threadline behavior with this POY nylon. The above data were obtained to reinforce our confidence in the utility of the steady state analysis of texturing threadline behavior, which up to now was directed towards the POY PET yarn-machine system. It should be emphasized that the POY nylon feed yarn differs significantly in dimensions and properties from the
POY PET yarns generally used in this study. In particular the nylon differs considerably in its rigidity ratio, i.e. \( \frac{EI}{G I_p} \) (1.38 for nylon versus 3.05 for PET), a quantity which can have considerable influence on the determination of cold zone twist and torque.

**Transient Threadline Tension and Torque**

Using POY nylon described above, we also undertook limited study of transient behavior during the draw texturing process on the MITEX False Twister. Threadline torque and threadline tension were measured by the MITEX Torque-Tension Meter. Unsteady machine operation was activated by changing the machine twist stepwise. The texturing machine dimensions were the same as those used for the steady state study described above.

Figures 3.44 and 3.45 compare predicted threadline tension and torque using Analytical Model 2 (tilted twist distribution) with measured values when the machine twist was changed from 60 TPI to 70 TPI. It is noted that the time response for tensions and torque maxima and minima are consistent for experimental and calculated values. There appears to be considerable difference in transient tension and torque levels but if we correct for the before and after steady state error between the predicted and measured threadline tension and torque, we conclude that the transient changes in thread-
Figure 3.44

NYLON 90/34 POY
MODE I 60-70 TPI

TENSION, grams

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TIME, seconds
line tension and torque compares favorably with the measured changes. Likewise, Figures 3.46 and 3.47 compare the predicted threadline tension and torque using Analytical Model 2 (tilted twist distribution) with the measured values when the machine twist is dropped from 70 TPI to 60 TPI. Again, with the suggested steady state correction, predicted and measured changes in threadline tension and torque compare favorably, although one cannot claim the extent of agreements observed in the POY PET experimental study.

One point should be noted for the nylon yarn, i.e. that the measured torque levels during transient operation approach their new steady state levels virtually without overshoot. This behavior may play a role in preventing the cycling of tension and torque during unsteady operation as discussed in Chapter 4.
Figure 3.46

NYLON 90/34 POY
MODE I 70-60 TPI
Figure 3.47

NYLON 90/34 POY
MODE 1 70-60 TPI

EXPERIMENTAL

PREDICTED
3.6 PREDICTED EFFECT OF OSCILLATORY OPERATION ON THREADLINE RESPONSE

A commercial texturing machine running twenty four hours a day is subject to all kinds of wear. Critical machine parts subjected to wear, such as feed rollers, take up rollers, and spindle drive, may run unsteadily due to changes in critical dimensions with use. This section considers how machine tolerances affect threadline response. The effect of three such operating defects will be examined including:

1. Oscillating Spindle Speed
2. Oscillating Feed Roller Surface Speed
3. Oscillating Take Up Roller Surface Speed

3.6.1 Oscillating Spindle Speed

Under normal texturing conditions the spindle speed is assumed to be constant although wear in the spindle drive could cause the spindle speed to oscillate slightly above and below the nominal speed. To provide some understanding of how oscillatory changes affect the threadline forces, we execute the transient analysis given in Section 3.3 and program the spindle speed to oscillate sinusoidally 1% above and below the nominal spindle speed of 177.33 radians per second (60 TPI Machine Twist). The machine draw ratio was programmed as 1.60; the throughput speed was programmed as 0.47 inches per second. The frequencies of spindle speed disturbances and corresponding peak to peak variation ratios are:
The predicted peak to peak variation ratios given above show that as the frequency of the spindle speed disturbance increases, the effect on threadline behavior decreases. Accordingly, since the nominal speed is 177. radians per second we can conclude that spindle speed disturbances near that frequency do not cause significant variations in threadline behavior. However if the disturbance frequency is several orders of magnitude smaller than the nominal spindle speed, the effect on threadline behavior is more pronounced.

This finding is significant since one way of driving the spindle is to use a magnetic roller (in contact with the spindle) with a radius several orders larger than that of the spindle radius. If such a magnetic roller were to become eccentric due to wear, the disturbing frequency would be low enough to create substantial threadline force variations.

3.6.2 Oscillating Feed Roller Surface Speed

Wear on the feed roller surface can cause an oscillating rate of material delivery to the texturing zone. As the rate
of material delivery changes, so do threadline forces. To help understand how the frequency of oscillation of the material delivery rate affects threadline forces, we execute the transient analysis given in Section 3.3 and program the feed roller velocity to oscillate sinusoidally 1% above and below the nominal throughput speed of 0.294 inches per second. The frequency of forced disturbances are:

- $0.0393$ Radians per Second ($0.1 \times$ Feed Roller Speed)
- $0.393$ Radians per Second ($1 \times$ Feed Roller Speed)
- $3.93$ Radians per Second ($10 \times$ Feed Roller Speed)

The machine dimensions used in the analysis are the ones for Mode I described in Section 3.4. The programmed machine twist is $60$ TPI. Below we present the peak to peak variation ratio for Feed Roller Speed and for the corresponding threadline tension and threadline torque:

<table>
<thead>
<tr>
<th>FREQUENCY OF FORCED DISTURBANCE</th>
<th>PEAK TO PEAK VARIATION RATIO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ROLLER</td>
</tr>
<tr>
<td></td>
<td>SPEED</td>
</tr>
<tr>
<td>$0.0393$ Radians/sec</td>
<td>$0.02$</td>
</tr>
<tr>
<td>$0.393$ Radians/sec</td>
<td>$0.02$</td>
</tr>
<tr>
<td>$3.93$ Radians/sec</td>
<td>$0.02$</td>
</tr>
</tbody>
</table>

For the analytical model using Mode I machine dimensions, the entire threadline length is $23.25$ inches. Commercial machines generally have a threadline length an order of magnitude greater than this length. If the feed roller was
reduced to 1/10 its present radius in our theoretical model (so that the ratio of roller radius to threadline length were about that of a commercial machine) its angular velocity would have to be increased by tenfold to maintain the original material delivery rate. Accordingly, the effect of a worn eccentric feed roller on threadline behavior is represented by the conditions where the frequency of disturbance is 3.93 radians per second. Thus, we can conclude that the effects of small amplitude transients in the material delivery rate are negligible.

3.6.3 Oscillating Take Up Roller Surface Speed

An analysis identical to the previous analysis in Section 3.6.2 can be undertaken for the case of oscillating take up roller surface speed. The results are given below:

<table>
<thead>
<tr>
<th>FREQUENCY OF FORCED DISTURBANCE</th>
<th>PEAK TO PEAK VARIATION RATIO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TAKE UP ROLLER SPEED</td>
</tr>
<tr>
<td>0.0629</td>
<td>.02</td>
</tr>
<tr>
<td>0.629</td>
<td>.02</td>
</tr>
<tr>
<td>6.29</td>
<td>.02</td>
</tr>
</tbody>
</table>

Comparing the results above with the results given in Section 3.6.2 we see that the steadiness of take up rate is not as critical a parameter as material delivery rate with regard to variation of threadline forces. This can be attributed to the significantly higher cold fiber tensile modulus than the hot fiber "modulus".

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3.7 EFFECT OF INCREASING YARN WRAP ANGLE ON SPINDLE

In Section 2.3.9 we showed that the linear velocity of the twisted yarn, $V_3$, as it enters the spindle is minimally affected by the range of yarn-spindle contact frictional forces which generally obtain in the texturing process. However if the contact friction is significantly increased, $V_3$ must accordingly decrease. This decrease of $V_3$ will reduce the texturing zone tensions and the filaments in the hot zone will be drawn to a lesser degree. Likewise the threadline torque will be reduced if the texturing tension decreases.

A simple way to increase the frictional forces acting against threadline translational movement is to increase the yarn wrap angle around the spindle pin. Experiments were conducted with the Du Pont POY PET used earlier for the transient study to examine the effect of double wrapping the yarn around the spindle pin on threadline behavior.

Figures 3.48, 3.49, 3.50, and 3.51 show significant differences in threadline behavior depending on whether the yarn is double wrapped or single wrapped around the spindle pin. When the yarn is double wrapped both steady state and transient values of threadline torque and tension are about 15% lower than the corresponding values when the yarn is only single wrapped. Note however that the time response is relatively independent with regard to wrap angle.
Mode I  50-60 TPI

---

Figure 3.48

- Single wrap
- Double wrap

Tension, grams

Time, seconds
Figure 3.49

Mode I  50-60 TPI

Torque (10^4 in-lbs)

--- single wrap
--- double wrap

Time, seconds
Figure 3.50

Mode I  60 - 50 TPI

Torque, (10^4) in lbs

- - - single wrap
- - - double wrap

Time, seconds
Figure 3.51

Mode I 60-50 TPI
Chapter 4

PERIODIC TWIST SLIPPAGE

The translation of spindle rotation into texturing threadline rotation is clearly dependent on friction forces. The generation of these forces is due to the external contact between the yarn and spindle surfaces as well as the internal friction deformation of the yarn as it traverses the spindle\(^{(15)}\). The contact friction forces are dependent on surface conditions (yarn and spindle), threadline tension, and relative velocity between the yarn and spindle\(^{(16)}\); the internal forces are generated by rotation of the yarn in a bent configuration around the pin\(^{(15)}\). Rotation under such flexural conditions will cause cycling of tensile and compressive stresses within the cross section of the yarn which leads to interfiber slippage resulting in energy losses in the yarn due to its viscoelastic behavior and inherent loss modulus. Both of these internal forces act in opposition to the relative motion between the yarn and spindle pin\(^{(15)}\).

Thus, internal and external contact activated friction torque join together to balance the combined threadline torque and post spindle torque according to the following relation:

\[ M_e + M_i = M_{\text{threadline}} + M_{\text{post spindle}} \quad (4-1) \]

where \( M_e \) = external contact friction activated torque
More to the point, the potential internal and external (contact) frictionally activated torque should be equal or exceed the combined threadline torques of equation (4-1).

**Yarn-Pin Contact Friction Induced Torque**

The standard capstan equation for yarn friction on a capstan is:

\[
T_2 = T_1 e^{u\theta}
\]  

(4-2)

where

- \( T_2 \) = post capstan tension
- \( T_1 \) = pre capstan tension
- \( u \) = coefficient of contact friction between yarn and pin
- \( \theta \) = wrap angle of yarn around pin perimeter

Sasaki and Kuroda\(^{(17)}\) show that for textile yarns this simple relation is not entirely accurate and that there is an extra increment of tension resulting from yarn bending. However as a first approximation we will assume that the capstan equation may be used to determine the coefficient of friction between the yarn and spindle.

Experimental work was carried out to determine the values of \( T_1 \) and \( T_2 \) (pre and post spindle tension) for a range of machine conditions. The yarn tested was the POY PET used for steady state and transient experiments described earlier. The results are listed in Table 4.1.
### Table 4.1

**THREADLINE TORQUE**

<table>
<thead>
<tr>
<th>MACHINE TWIST</th>
<th>DRAW RATIO</th>
<th>HEATER TEMP.</th>
<th>COEFFICIENT OF FRICTION (YARN-PIN SPINDLE)</th>
<th>EXTERNAL CONTACT SUPPLIED TORQUE ($10^{-4}$) in lbs</th>
<th>THREADLINE TORQUE ($10^{-4}$) in lbs</th>
</tr>
</thead>
<tbody>
<tr>
<td>TPI</td>
<td>1.60</td>
<td>190°C</td>
<td>.069</td>
<td>.90</td>
<td>1.54</td>
</tr>
<tr>
<td>50</td>
<td>1.60</td>
<td>190°C</td>
<td>.060</td>
<td>.64</td>
<td>2.40</td>
</tr>
<tr>
<td>60</td>
<td>1.60</td>
<td>190°C</td>
<td>.063</td>
<td>.63</td>
<td>2.39</td>
</tr>
<tr>
<td>65</td>
<td>1.60</td>
<td>190°C</td>
<td>.062</td>
<td>.58</td>
<td>2.31</td>
</tr>
<tr>
<td>70</td>
<td>1.60</td>
<td>190°C</td>
<td>.062</td>
<td>.55</td>
<td>2.23</td>
</tr>
<tr>
<td>80</td>
<td>1.60</td>
<td>210°C</td>
<td>.051</td>
<td>.65</td>
<td>1.47</td>
</tr>
<tr>
<td>50</td>
<td>1.60</td>
<td>210°C</td>
<td>.071</td>
<td>.78</td>
<td>1.67</td>
</tr>
<tr>
<td>60</td>
<td>1.60</td>
<td>210°C</td>
<td>.067</td>
<td>.66</td>
<td>2.34</td>
</tr>
<tr>
<td>65</td>
<td>1.60</td>
<td>210°C</td>
<td>.057</td>
<td>.52</td>
<td>2.34</td>
</tr>
<tr>
<td>70</td>
<td>1.60</td>
<td>210°C</td>
<td>.037</td>
<td>.30</td>
<td>2.54</td>
</tr>
<tr>
<td>80</td>
<td>1.60</td>
<td>210°C</td>
<td>.086</td>
<td>1.17</td>
<td>1.45</td>
</tr>
<tr>
<td>50</td>
<td>1.60</td>
<td>230°C</td>
<td>.057</td>
<td>.60</td>
<td>1.58</td>
</tr>
<tr>
<td>60</td>
<td>1.60</td>
<td>230°C</td>
<td>.066</td>
<td>.60</td>
<td>2.24</td>
</tr>
<tr>
<td>65</td>
<td>1.60</td>
<td>230°C</td>
<td>.055</td>
<td>.48</td>
<td>2.19</td>
</tr>
<tr>
<td>70</td>
<td>1.60</td>
<td>230°C</td>
<td>.073</td>
<td>.60</td>
<td>2.19</td>
</tr>
</tbody>
</table>
To determine the yarn-pin friction induced torque, the normal force between the yarn and spindle pin per unit length must be multiplied by the yarn radius and the coefficient of yarn-spindle friction and the product integrated around the full spindle wrap to obtain:

\[ M_e = T_{\text{Threadline}} R_{\text{yarn}} (e^{2\pi\mu} - 1) \]  

(4-3)

Applying the measured coefficients of friction given in Table 4.1 to equation (4-3), we can thus determine yarn-pin frictionally induced torque which acts to rotate the texturing threadline. Table 4.1 lists these externally developed torques and the measured threadline torques.

Comparisons of the values of measured threadline torque and external torque supplied by the spindle shows that the torque thus developed falls short of the threadline torque required for steady state operation. It would appear that the deficit in the required torque is made up by the internal torque developed when the yarn also rotates in the bent state (around its own axis) as it traverses the spindle pin. From the diagram 4.1 (where twist \( T \) is plotted against wrap angle \( \theta \)) it can be seen that there is constant slippage \( (T_{\text{machine}} - T_\theta) \) over the entire yarn loop on the spindle surface. The difference \( (T_{\text{machine}} - T_\theta) \) is caused by the untwisting of the yarn as it proceeds on the pin surface. Observations of the twist around the pin indicates that most of the untwisting occurs in the last quadrant of the spindle surface perimeter. (4)
Figure 4.1

TYPICAL TWIST DISTRIBUTIONS ON A PIN-SPINDLE

$T_M =$ MACHINE TWIST
$T_\theta =$ TWIST AT WRAP ANGLE $\theta$
$M_{I,E} =$ FRICTION TORQUE (INTERNAL AND EXTERNAL)

1 TWIST DISTRIBUTION
2 TWIST DISTRIBUTION
This condition is shown as Twist Distribution 1. For this case the sum of $M_e + M_i$ would be relatively large. If the sum of $M_e + M_i$ is considerably less than that for Twist Distribution 1 twist can be pictured as Twist Distribution 2 over the spindle surface. The efficiency of this yarn-spindle system is now less than 100%, i.e. the yarn in the cooling zone rotates less than one time for each spindle rotation. During steady state operation with the slippage envisioned for Twist Distribution 2 no real twist will manifest itself in the post spindle region since one pitch length of twisted yarn enters the spindle for each rotation of the cooling yarn segment. This continual rotational slippage need not affect product quality but obviously affects process productivity.

**Tight Spots**

Rotational slippage at the spindle pin may take place on a steady state basis as has been pointed out. This need not promote product irregularity, only loss in process efficiency. But if rotational slippage takes place periodically or in an irregular manner, an inequality will occur between the time period of twist-pitch translation and the rotation time period of the cooling yarn as it enters the spindle. When this condition occurs, real twist will appear in the post spindle zone.

Real twist can also appear downstream of the spindle without rotational slippage,— but rather in presence of translational
spurting or acceleration. For this condition also develops an inequality between the period of twist-pitch translation and the period of cooling yarn rotation. Such spurting can cause a tight spot in the existing yarn, i.e. a short yarn segment of high twist amidst long segments of virtually untwisted yarn. The tight(twist) spot prevents local crimp development and causes the appearance of a "pin hole" in the fabric.

A picture of a tight spot is shown in Figure 4.2. This tight spot was formed as a result of the spindle speed being suddenly reduced on the MITEX False Twister after the machine had been operating at steady state. The operating conditions were:

- Cold Zone Length 6.00 inches
- Heater Length 1.25 inches
- Cooling Zone Length 7.00 inches
- Post Spindle Length 9.00 inches
- Machine Draw Ratio 1.60
- Heater Temperature 210°C
- Yarn 245/30 POY PET
- Machine Twist 60 TPI+50

In the first instant that the spindle speed is reduced, the translational movement of the twist pitch continues as before, and now the rotational period of the pin is greater than the time for the advance of one twist pitch. As a result each turn advanced into the pin is no longer fully untwisted and some real twist starts to build up in the post spindle zone.
## TIGHT SPOT

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cold Length</td>
<td>6&quot;</td>
</tr>
<tr>
<td>Heater</td>
<td>1.25&quot;</td>
</tr>
<tr>
<td>Cooling</td>
<td>7&quot;</td>
</tr>
<tr>
<td>Post Spindle</td>
<td>9&quot;</td>
</tr>
<tr>
<td>Machine Draw Ratio</td>
<td>1.60&quot;</td>
</tr>
<tr>
<td>Heater Temperature</td>
<td>210 °</td>
</tr>
<tr>
<td>Yarn</td>
<td>245/30 POY PET</td>
</tr>
<tr>
<td>Spindle Speed</td>
<td>60-50</td>
</tr>
</tbody>
</table>
The reduced rotational speed of the pin results in reduced rotational speed of the cooling zone, and, in turn in a reduced twisting rate at cold and hot entrances. With this step reduction in twisting rate, the threadline tension and torque in the texturing zone drop precipitously. This marked drop in tension upstream of the spindle pin and the slight increase of tension downstream (which is expected because of the real twist occurring in that zone) cause a tension gradient greater than that which can be accommodated by the translational frictional constraint around the pin. Translational spurting will result, carrying segments of high level real twist into the post spindle zone.

This small section of real twist which is of the same sense as the threadline twist does not spread out uniformly in the post spindle zone threadline. Instead it remains concentrated and can easily be observed.

The persistence of this concentration of real twist lies in the nature of the torsional behavior of the texturing threadline. As steady state, the texturing threadline is entirely untwisted as it leaves the spindle and enters the post spindle zone at a small negative torque, (see level C in Figure 4.3). If for any reason, translational spurting takes place in the threadline moving about the spindle pin or sudden rotational slippage takes place, a yarn segment with real twist
YARN SEGMENT FROM POYNET TEXTURING THREADLINE 65 TPI BASIC TWIST

TENSION 25.7 GRAMS
TEMPERATURE 210°C
DRAWRATIO 1.57

UPTWIST FROM 0

DETWIST

LOCAL TPI

TORQUE (10^-4) IN.-LB

Figure 4.3
at torque level A will enter the post spindle zone. Untwisted yarn further downstream in the post spindle zone at torque level C will then uptwist slightly and the tight spot will downtwist further until the torque in both segments are equal at level B where the twist level in the tight spot is observed to be 50 TPI (about half the threadline twist). The persistence of the tight spot in the post spindle zone is thus attributable to the presence of two different mechanically connected portions of threadline having equal torque but entirely unequal twist geometry.

When the spindle speed is suddenly increased after the process is operating at steady state, real twist of opposite sense to that of the threadline twist appears in the post spindle threadline, spread out uniformly and no tight spots are observed.

Observation of transient texturing zone tension during twist increases reflects marked increase in pre-spindle tension and slight increase in post spindle tension. This condition precludes translational spurting and hence the appearance of segments of concentrated twist.
Continual Occurrence of Tight Spots Along with Threadline Tension and Torque Oscillation

For steady state operation in draw texturing, the torque equality of equation (4.1) obtains. If for some reason the torque generating capacity of the yarn-pin system suddenly fails or if the torque requirement of the texturing threadline suddenly increases, then rotational slippage will occur over the entire contact perimeter of the pin. As a result the rotational speed of the cooling zone will drop precipitously and following this, the threadline tension and torque will rapidly decline as was seen in Section 3.4. Translational surging and the appearance of tight spots may accompany this drop in texturing tension.

When sudden reduction in rotational velocity of the cooling threadline occurs as a result of a stepwise reduction in spindle speed the tensile and torque drop observed will recover after a period of time due to propagation of the new twist level through the machine (and elimination of the effect of transient twist contraction). In the case at hand the tension and torque decline may be checked for other reasons, e.g. because of re-establishment of the torque generating capacity of the yarn-pin system so as to satisfy equation (4.1).

The component of torque generation at the pin which is due to yarn-pin frictional contact can be shown to depend approximately on the level of threadline tension according to
Figure 4.4

PREDICTED THREADLINE TORQUE AND TENSION CYCLING

MACHINE LENGTH 23.25 "  THROUGHPUT SPEED .4704 "/sec

![Graph showing predicted threadline torque and tension cycling](image-url)
equation (4.3). Likewise one can show that the internally generated torque in a yarn which is rotated in a bent configuration (around the pin) will depend on the level of thread-line tension. Thus the sum of components of torque generation at the pin can be related to texturing tension.

For rotational slippage to persist, the total torque generated at the pin must be less than the torque required for non-slip texturing. Or conversely, if the texturing line torque falls below the yarn-pin generated torque, slippage will cease.

Figure 4.4 shows the reduction in line tension and torque which occurs in dotted lines as a result of an initial (unspecified) rotational slippage. The tension value of Figure 4.4 calculated from our transient analysis is used to calculate a generated torque by multiplying by a constant (corresponding to the steady ratio of torque to tension). The indicated torque value in Figure 4.4 is also calculated from our transient analysis. The system is forced to bottom out when the above calculated transient torque falls below the generated torque. At this instant the slippage ceases and the rotational speed of the cooling zone assumes the speed of the spindle. This is now equivalent to a step increase in cooling zone rotational speed and yarn tension and torque will rise accordingly as calculated from the transient analysis and plotted in solid lines in Figure 4.4.
The increasing tension value, will according to the transient analysis, overshoot its steady state value, even while the torque is still rising. Here again we find a situation when the total yarn pin generated torque falls below the instant threadline torque and the onset of rotational slippage at the pin occurs, -- again being designated in Figure 4.4 by the dotted lines. This type of cycling will persist so long as the material and machine parameters are set so as to enhance system sensitivity to cycling. Experience has shown that system sensitivity to cycling is suppressed by proper choice of yarn lubricant and by increasing the ratio of post spindle tension to pre-spindle tension.

Finally it should be pointed out that the internally generated torque in a bent, rotating yarn, will depend on twist level as well as on tension, the lower the twist the greater the internal fiber slippage hence the greater the torque generated. This dependence suggests that twist changes occuring at the heater entrance because of changing threadline rotational speeds will affect internal frictional torque generation once the twist change reaches the spindle pin.
Steady State Operation of the Draw Texturing Process

During steady state operation of the draw texturing process there are essentially two levels of twist in the threadline between the feed rollers and spindle. In one segment of the threadline (over the heater and beyond) the twist level is determined simply by the spindle speed and the yarn throughput speed. Upstream of this segment lies the threadline cold zone where the twist level is somewhat less. In absence of frictional contact between yarn and heater threadline tension and torque will be constant from position to position between the feedrolls and the spindle. The introduction of twist takes place in essence, at two zones in the system: at the entrance to the cold zone and at the entrance to the hot zone. The equality of tension and torque at these zones suggests that their local twist levels will be inversely proportional to their effective torsional rigidities and this is what we predict analytically and observe experimentally. The torsional resistance at the entry to the cold zone is seen to depend on fiber bending and torsional rigidity as well as on filament tension; at the heater entry, the bending and torsional stresses are thermally relaxed with only the tensile component of yarn torsional rigidity remaining. Thus the twist level in the hot
zone far exceeds that of the cold zone. The threadline tension is developed as an aggregate of the filament tensile components along the yarn axis. The threadline torque is developed as an aggregate of the components of filament bending moments and twisting moments, to the degree that they exist, and of the moments due to filament tension. The prediction of threadline tension and torque therefore requires the determination of the filament-system constitutive relations, the local yarn geometry, and the system compatibility at the entrance to the cold and hot zones. Our analysis predicts that as the operating level of machine twist is increased, with machine draw ratio and heater temperature kept constant, threadline tension decreases, while threadline torque increases. This seemingly contrary behavior can be attributed to the increase in local filament helix angles in outer layers of the yarn, which reduces the yarn axial component of filament tension and increases the circumferential component. As the twist increases, the variation of draw ratios in different filament layers in the yarn drawing neck also increases. It is predicted that the temperature of the heater (over the range of interest) should not affect threadline behavior significantly. This follows from the fact that over the range of common texturing conditions, temperatures have minimal effect on (hot) drawing forces.

Lastly it is predicted that if the cold filament bending
rigidity or torsional rigidity is increased, both threadline torque and tension will increase while cold zone twist level will decrease. Hot zone twist should remain unchanged since this twist level is indicated to be a function solely of yarn throughput speed and spindle speed.

Numerous experiments were conducted on DuPont POY polyester yarns for the purpose of comparing measured values of threadline parameters, i.e. threadline tension, torque and twist levels with predicted results during steady state operation of the draw texturing process. The agreement between predicted and measured values is within 9.3% for tension, 12.2% for torque, 12.3% for hot twist and 10.9% for cold twist.
Transient Operation of the Draw Texturing Process

It has been observed during texturing that when spindle speeds are suddenly altered, the first signs of twist change in the threadline occur at the cold zone entrance and at the heater. These observations provided the basis for the hypothesis that torsional rigidity of the texturing threadline is significantly lower at the cold zone entrance and heater than for the rest of the threadline existing between the feed rollers and spindle. Torsional disturbances in the threadline are evidenced primarily as twist changes at the 'soft' zones and these changes then propagate downstream with yarn movement.

Thus, the formation rigidities of the texturing threadline essentially reflect the rigidity of the soft forming zones under the conditions of local twist insertion. On the other hand, the transient twist response of cold zone and cooling zone segments to incremental changes in spindle speed reflects the very high incremental torsional rigidity of these threadline segments.

To test these hypotheses we have measured the torsional rigidity of cold and cooling zone yarn segments and these were seen to be several orders of magnitude greater than the cold forming 'rigidity' and the hot uptwisting 'rigidity'. One can attribute these rigidity differences to the variation in twisting deformation modes at each of the soft zones and in segments which lie downstream of these zones.
Observations of dynamic twist formation in larger models indicate that many filaments approaching the twist triangle apex are already twisted around their axes and sometimes around neighboring pairs. They are then wound onto the newly formed end of the yarn, with little additional twisting observed thereafter. After the twist is inserted at the cold zone entrance, interfilament friction and transverse stresses cause the yarn to behave like a helically anisotropic solid rod with subsequent high torsional rigidity.

The situation at the uptwist zone at the heater involves sudden stress relaxation or softening of the filaments. The requirement of constant threadline torque in the presence of this stress relaxation results in a cranking up of twist at the heater. As the new hot twist level is reached lateral forces due to filament tensions and local curvatures will also reach a new high level. This increased interfilament pressure accompanied by thermal softening will result in lateral compressive creep and in growth in contact areas between filaments. Higher friction and interfilament shear stresses will then result in higher incremental torsional rigidities.

In our analysis of transient operation of the draw texturing process we have considered the response of the threadline to torsional disturbances, i.e. to changes in spindle speed. We reason that transient operation generates material build-
ups and twist buildups with respect to time in different zones of the texturing system and 'bookkeeping' of these buildups must be based on how the buildups distribute within each zone of the threadline. Further, the material and turn balances for each zone are coupled with the requirement of threadline tension and threadline torque uniformity throughout the texturing zone.

The transient analysis predicts that if the spindle speed is increased stepwise the threadline torque responds by rising sharply with time and then slowly settling to a new value higher than the steady state value before the spindle speed was increased while the threadline tension increases sharply and then settles to a value which was less than the previous value. Correspondingly, if the spindle speed is decreased stepwise the threadline tension and torque should decrease precipitously before the threadline tension settles to a new value, higher than the previous value while the torque settles to a new value, lower than the previous value. Under certain conditions, this type of transient threadline response increases the prospect of threadline tension and torque cycling. For example if rotational slippage of the yarn on the spindle occurs the system reacts as if there were a step change in spindle speed, and recurrence of such slippage will lead to tension, torque and twist cycling in a texturing process nominally operating at steady state.
Numerous experiments were conducted using Du Pont POY polyester yarns for the purpose of comparing measured values of threadline parameters, i.e. threadline tension, torque, and twist levels with predicted results during transient operation of the draw texturing process. We used Analytical Model 2 (tilted twist distribution) and the agreement with respect to time response and absolute levels of threadline parameters was good. Additional experiments were conducted using Du Pont POY nylon yarns to reinforce our confidence in the utility of the analysis of texturing threadline behavior. There was a persistent 10% difference in the predicted and measured values of threadline tension and threadline torque. However the predicted and measured time response of the system was indeed favorable.
Chapter 6

SUGGESTIONS FOR CONTINUING RESEARCH

A number of suggestions for continuing research are presented below. The experimental results and mathematical analyses here presented should provide the direction for continuing research in this rapidly developing segment of industrial technology.

1. Characterization of the Torsional Rigidity of Threadline Segments Over the Heater: The torsional rigidity of yarn segments taken from the cold zone and cooling zone has been determined. Likewise, we have determined the "formation rigidity" of yarn segments at the cold zone entrance and the hot zone entrance. Further, we reasoned that the yarn over the entire heater acts as a soft zone since observation of transient twist over the heater did not indicate any change in its distribution with time. With the use of very short heaters in our experimental machine, the spreading of the "soft zone" over the heater does not disturb the physics of the situation unduly. But further understanding of this important distribution is necessary because of the significantly longer heater lengths which are used in commercial machines and because of the difference in heat transfer characteristics.
2. **Effect of Sinusoidally Imposed Machine Disturbances on Threadline Response:** Our experiments conducted to study transient operation of draw texturing processes utilized step changes in spindle speeds to disturb the texturing system. More realistic to commercial application would be a sinusoidal machine disturbance such as could be implemented by use of programmable drive motors designed to deliver shaft speeds which vary with time. Results from such experiments could then be compared with the transient results predicted in our analysis.

3. **Tension Ratio Over the Spindle During Transient Operation:** As a basis for understanding the nature of twist slippage past the spindle it is important that threadline tensions before and after the spindle be measured simultaneously with spurting of the threadline. The pattern of changing tension gradients could then be compared with the lengthwise distribution of tight spots along the threadline.

4. **Twist Geometry Around the Spindle Pin:** It has been proposed that whenever there is a sudden increase in the tension gradient around the spindle pin, a threadline translational surge takes place, accompanied by a tensile jerk upstream of the spindle. This tension increase acting on the yarn in the hot zone causes a proportional increase in line torque. The latter is accompanied by a twist increase.
at the cold formation zone in order to keep threadline torque uniform in the texturing zone. Such a sudden twist increase can cause local buckling which may persist in the threadline. Upon traversing the spindle pin surface such a local buckle might cause another translation surge which would result in still another buckling, etc. Photographic study is necessary to confirm this behavior.
BIBLIOGRAPHY


APPENDIX 1

INERTIA EFFECTS ON THREADLINE

TRANSLATIONAL MOMENTUM FORCE

\[ DV^2 \]

where \( D \) = Yarn Denier
\( V \) = Translational Velocity of Yarn

For the yarns processed (70 to 250 denier) at speeds up to 1 inch per second the translational momentum force is \( 1.23 \times 10^{-8} \) GPD compared to .1 GPD tensions.

ROTATIONAL MOMENTUM FORCE

\[ I \omega^2 \]

where \( I \) = Polar Moment of Inertia
\( \omega^2 \) = Angular Velocity of Yarn

For the yarns processed (70 to 250 denier) at angular speeds up to 250 radians per second the angular momentum torque is \( 1.12 \times 10^{-9} \) in lbs compared to typical threadline torques of \( 1.0 \times 10^{-4} \) in lbs.
APPENDIX 2

EXPERIMENTAL VALUES OF THREADLINE TENSION AND TORQUE

(each data point represents the average of at least 20 points)
EXPERIMENTAL THREADLINE TENSION AND TORQUE

190°C 1.57 DR

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<tr>
<th>Basic Twist</th>
<th>50 TPI</th>
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<th>65 TPI</th>
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190°C 1.60 DR

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### Experimental Threadline Tension and Torque

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## EXPERIMENTAL THREADLINE TENSION AND TORQUE

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Hot Draw Curves  POY PET

Tension, grams

Draw Ratio
APPENDIX 4

MITEX TORQUE- TENSION METER

Threadline torque and threadline tension are measured on-line during texturing by the MITEX Torque-Tension Meter (IDR Needham, MA.). The meter sensitivity is on the order of 1 millivolt output per gram of threadline tension and 1 millivolt per $1 \times 10^{-6}$ in lbs of threadline torque.

The principles of operation are as follows. Untwisted feeder yarn which is to be textured enters the device and is wrapped ninety degrees around a near frictionless pulley. The yarn line of action upstream of action is directed through the center of a flexural pivot (pivot A). This pivot twists a calibrated amount when threadline tension imposes a net torque on it. After the yarn travels past the pulley it proceeds through the threadline torque pivot and is then wrapped around a smooth shaft ($360^\circ$) which traps twist from the texturing threadline and prevents it from bleeding past the defined cold zone entrance. Threadline torque twists the torque pivot (pivot B) a calibrated amount proportional to the threadline torque.

The displacement angle of the torque pivot and the tension pivot is detected by metal flags connected to these pivots which are part of variable push-pull capacitor systems.
MITEX TORQUE TENSION METER

YARN INPUT

YARN

TWIST TRAP

CAPACITORS

PIVOT B

PIVOT A

CAPACITOR
Laboratory False Twist Machine

Torque-Tension Meter
APPENDIX 5

COMPUTER PROGRAM FOR STEADY STATE ANALYSIS
DISCLAIMER OF QUALITY

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Thank you.

Some pages in the original document contain pictures, graphics, or text that is illegible.
THIS MODEL DETERMINES THE STEADY STATE LEVELS OF TORSION, ANGLE, AND TORQUE IN THE DRAW TEXTURING THREADLINE.

**INPUTS:**
- $\lambda_1 = 1.0 \times 10^{-15}$
- $\beta = 1.0\text{deg}$
- $C_{\text{ textile}} = 5.0$
- $P_M = 2.0 \times 10^{-3}$
- $V_4 = 1.0 \times 10^{-3}$
- $S_{\text{ in }} = 0.0$

**OUTPUTS:**
- $S_{\text{ out }}$

**FORMAT:**
1212 FORMAT (HEATER TEMP=1, F1=1.1, DRAW RATIO=1, F1A=1, CLAIN LIST=1, F1=1)

**CALLS:**
- CALL VARN (CLAIN, SPIN, V4, A, R, FY)
- CALL EXIT
1600 WRITE (5,571)
571 FORMAT(1, COI, T, TIST, 3X, TOT TIST, 4X, TURQU, 7X, TENSION)
      WRITE (5,572) TWT, TFW, TQ1, TFW
572 FORMAT(4F13.5)
      WRITE (5,714)
      VFW = VFW/VF
      WRITE (5,713) TFW, TF, TF1, TF0, TPT3, TTT, TREN, V3, V32
      EPS = V3*SECAV = 1.*
      DO 400 V3 = V3/V3
400  DRAW3.V3 = ALP3/(VP*ET3)
      DO 400 V3 = ALP2/(VP*ET2)
        DO 400 V3 = ALP1/(VP*ET1)
        CT1 = 1.0*DRAW1*DRAW2/(V4*V4*3)
        CT1 = 6.0*DRAW1*DRAW1/(V4*V4*3)
        CT2 = 9.0*DRAW2*DRAW2/(V4*V4*3)
        CT3 = 1.0*DRAW3*DRAW3/(V4*V4*3)
        RMS = SQRT(CT0+CT1+CT2+CT3-1.0*)
      WRITE (5, 571) EPS
      WRITE (5, 524) SECAV
1624 FORMAT(1, SECAV = ,F13.5)
8211 FORMAT(1, THE GEAR AVERAGE STRAIN IS ', F13.5)
713  FORMAT(9E13.5)
APPENDIX 6

COMPUTER PROGRAM FOR TRANSIENT ANALYSIS
**MIT FIGURES AND POLYMERS LAB**

**TRANSIENT ANALYSIS OF DRAW TEXTURING PROCESS**

In the following analysis, threadline torque, tension, and twist distribution are determined during unsteady operation. The following inputs are provided:

- Cold formation length
- Hot formation length
- Spindle speed, \( \omega \)
- Input roller velocity, \( v_0 \)
- Output roller velocity, \( v_1 \)
- Hot moduli, \( a \) and \( b \)
- Cold modulus
- Cold bending modulus
- Cold torsional modulus
- Yarn radial layer structure

**INITIAL CONDITIONS**

- Cold twist, \( \theta_C \) per inch
- Hot twist, \( \theta_H \) per inch
- Cold average filament strain
- Draw ratio for each radial layer
- Initial yarn tension
COLD ZONE CL, INCHES
CL=6.0
C HOT ZONE CL, INCHES
CL=1.5
C GET ZONE SL, INCHES
SL=5.
C POST SPINDLE ZONE VL, INCHES
VL=9.
C MACHING SPEEDS INPUT ROLLER SPEED V1, OUTPUT ROLLER SPEED V4,
C SPINDLE SPEED VH
V1=204
V4=47
C COLD STRUCTURE 1-6-12-11
AN1=1.
AN1=6.
AN2=12.
AN3=11.
C HOT STRUCTURE 1-6-9-14
AN1=1.
AN1=6.
AN2=9.
AN3=14.
C MATERIAL PROPERTIES
AK=490
K=91.2
A=1.046
C=222
AP=2.304E+3
RP=9.307E-3
FCOLD=222
ET=9.707E-X
V1=1.07E-X
C COMPUTED VARIABLES
USER=ROBJSKT 239 71972     JOINT COMPUTER FACILITY, MIL

C      COLD TWIST PC
C      HOT TWIST PH
C      COLD AVERAGE FILAMENT STRAIN
C      CASE FILAMENT DRAW RATIO DR0
C      FIRST LAYER DRAW RATIO DRI
C      SECOND LAYER DRAW RATIO DR2
C      THIRD LAYER DRAW RATIO DR3
C      YARN TENSION
C      TWIST AT END OF SET ZONE PR
C      POST SPOOL TWIST PR
C      TWIST AT END OF COLD ZONE PW

PR=Y(1)
P=R(Y(2)
P30=Y(3)
P4P=Y(4)
P51=Y(5)
P63=Y(6)
P73=Y(7)
PC=Y(8)
PY=Y(9)
PU=Y(10)
P1=Y(11)

C     DRAW RATIOS
D0P2=R0*DR0
D0P3=R02*DR0
DR12=R1*DR1
DR13=R1*DR12
DR22=R2*DR2
DR23=R2*DR22
DR32=R3*DR3
DR33=R3*DR32

C     YARN RADIAL DIMENSIONS
R2C=3.0*10E-4
R2C=2.5*10E-4
YAN 0.01 TRY
YAP 0.01 TRY
YAM 0.01 TRY RELATIONSHIPS
ZFTA1 = SQT (1 * P1 * PJ * PC1)
ZFTA2 = SQT (1 * P2 * PJ * PC2)
ZFTA3 = SQT (1 * P3 * PJ * PC3)
R2FT1 = SQT (1 * PC * PC * R1)
R2FT2 = SQT (1 * 0 * PC * R2)
B1ET1 = SQT (1 * PC * PC * R3)
A2LP1 = SQT (1 * 0 * PH * PH * R2)
A2LP2 = SQT (1 * 0 * PH * PH * R3)
PRC32 = Y (1) * Y (1) * PRC
PRC22 = Y (1) * Y (1) * PRC
PRC12 = Y (1) * Y (1) * 1C
PRC34 = PRC32 * PRC32
PRC24 = PRC22 * PRC22
PRC14 = PRC12 * PRC12
R2FT2 = R3FT1 * R3FT1
R2ET3 = R3ET1 * R3ET1
R3FT2 = R3ET1 * R3ET1
R2FT3 = R2F1 * R2F1
A1=A1*41 LP1/(0.12*B1*E11)  
A2=A2*42 LP1/(0.12*2*E21)  
A3=A3*43 LP1/(0.12*B3*E31)  
A1=41*1/E11/7=TA1  
A2=42*2/E21/7=TA2  
A3=43*3/E31/7=TA3  
A12=412*1/E11/7=TA1  
A22=422*2/E21/7=TA2  
A32=432*3/E31/7=TA2  
O1=(CONT+ACONT)*5+CL/(1+EPS+ACONT)  
O2=VR*(1+EPS)/CONT  
O3=CL*SAH*5/ACONT  
O4=1+(PC-PJ)*ASKAF**5/ACONT  
O1=G1/86  
O2=G2/G4  
O3=G3/G4  
T0=F0/(F*4103)  
T1=F1/(F*41A3)  
T2=F2/(F*41A2)  
T3=F3/(F*4133)  
X1=T0**A11+T1**A11+T2**A21+T3**A31+FF/F6  
X2=01*T11+T2+T3  
X3=G3*(T0+T1+T2+T3)=F7/F6  
X4=T0**A42+G2+T1**A12+G2+T2*/F2=O2+T3**A32+O2  
X5=(W1*PS*VR)/(HL+5*SL)  
X6=CL/(2*HL+SL)  
X7=(PC-PJ)/(PS*SL)  
XR=(PV*PS)*VR/(PS*HL+SL)  
S1=G1/(66*A13)  
S2=G2/(66*A23)
\[ \begin{align*}
\text{s2} &= G3/(G6*A33) \\
\text{y1} &= G1*A11+S2*A21+S3*A71+G5/C6 \\
\text{y2} &= G1*(S1+S2+S3) \\
\text{y3} &= G3*(S1+S2+S3)*G7/C6 \\
\text{y4} &= G1*(A12-QP)+S2*(A22-QP)+S3*(A32-QP) \\
\text{suf} &= (y1*Y5=y1*x7*Q5+y4+y1*x8)/(1+y2+y1*y7+y1+y1+y7+y2) \\
\text{y1}*X8 &= (1+y2+y1*x7/y1) \\
\text{AKAP} &= y3+y6+y1*y7+y1/(1+y2+y1*x7/y1) = (y5-y1+x6+y7*y1)/(1+y2+y1*y7/y1)
\end{align*} \]

**DIFFERENTIAL EQUATIONS**

\[ \begin{align*}
\text{F(1)} &= 6U/\text{AKAP} \\
\text{F(2)} &= y1*x5=y1*x7*(?+y4=y1*x8)/(1+y2+y1*y7+y1+y2+y7*y7+y5) \\
\text{F(3)} &= y1*x5=y1*x7*(?+y4=y1*x8)/(1+y2+y1*y7+y1+y2+y7*y7+y5)
\end{align*} \]

**TENS** = AK*PC*FPS*(?+R3C2/B12*F11+R2C2/P2ET1+6+*R1C2/H1F11)

**TOR** = G*K*Y(1)*(?+R3T3+12+/R2FT3+6+/R1FT3+1+)

**TRF** = F1*Y(1)+Y(1)+Y(1)*(11+R3C2/R3T3+12+R2C2/P2ET3+6+*R1C2/H1F11)

\[ \begin{align*}
\text{y1} &= \text{TENS}+\text{TOR}+\text{TRF} \\
\text{y2} &= 01*F(3)+02*G3*F(1) \\
\text{F(11)} &= \text{PC}+\text{PJ}+\text{V}/\text{CL} \\
\text{y(13)} &= y(11)/6.28 \\
\text{y(14)} &= (0.83*y(11)+.17*y(11))/6.28 \\
\text{y(15)} &= (0.1*y(11)+.83*y(11))/6.28 \\
\text{y(16)} &= y(1)/6.28 \\
\text{y(17)} &= (0.93*y(2)+1.4*y(9))/6.28
\end{align*} \]