Closed-loop Control of Roll Bending/Twisting:  
A Shape Control System for Beams

by

Theresa Clara Jenne

Submitted to the Department of Mechanical Engineering on June 2, 1986 in partial fulfillment of the requirements for the degree of Master of Science.

Abstract

Roll bending and roll twisting are processes for forming beams of constant cross section into desired shapes by bending and/or twisting the workpiece while it is rolled through the machine. The objective of the shaping process for beams is to impart a desired angle of twist and a desired curvature at each point along the beam. A closed-loop shape control system was developed by analyzing the mechanics of bending and twisting unsymmetrical beams, such as angles and channels. The shaping process is separated into an unsymmetrical roll bending operation and a roll twisting operation. The unsymmetrical roll bending operation bends the workpiece about a specified neutral axis to some desired unloaded curvature while the workpiece is in the loaded state. The control of the unsymmetrical roll bending operation is decoupled into two simultaneous, but separate, symmetrical roll bending controllers about each principal axis of inertia. Assuming the workpiece will spring back elastically, each principal axis controller computes the unloaded curvature from real-time measurements of the loaded curvature, the bending moment, and the bending stiffness of the beam. The roll twisting operation twists the beam to some desired unloaded angle of twist while the workpiece is rolled through the machine. The closed-loop controller computes the unloaded angle of twist from real-time measurements of the loaded angle of twist, the twisting torque, and the torsional stiffness of the beam.

Thesis Supervisor: Doctor David E. Hardt
Title: Associate Professor of Mechanical Engineering
Dedication

To my father, Otto Jenne, and stepfather, Kermit Greenberg,
Acknowledgements

I would like to thank Professor David Hardt for his support and guidance throughout this research. I would also like to thank the Aluminum Company of America for their financial support and technical assistance. In particular, I would like to thank Professor Hardt and the Aluminum Company of America for allowing me to take time off to be with my family during the recent family tragedy.

In addition, I would like to thank Mike Hale for answering my many questions and his advice concerning this project.

Finally, I thank my family and friends for their love and moral support during my stay here at MIT. For without them, I would have packed my bags a long time ago.
Table of Contents

Abstract 2
Dedication 3
Acknowledgements 4
Table of Contents 5
List of Figures 9
Notation 11

1. Introduction 15

1.1 MOTIVATION 15
   1.1.1 Background 15
   1.1.2 Motivation for Controlling the Unsymmetrical Roll Bending Process 17
   1.1.3 Motivation for Controlling the Roll Twisting Process 20
   1.1.4 Motivation for Closed-loop Control of the Shaping Process 21
      1.1.4.1 Control of the Symmetrical Roll Bending Process 22
      1.1.4.2 Control of the Unsymmetrical Roll Bending Process 24

1.2 OBJECTIVE 24

1.3 THESIS OVERVIEW 2
   1.3.1 Controlling the Unsymmetrical Roll Bending Process 25
   1.3.2 Controlling the Roll Twisting Process 27
   1.3.3 Controlling a Shaping Process for Beams 28

2. Control of the Unsymmetrical Roll Bending Process 30

2.1 INTRODUCTION 30
   2.1.1 Assumptions 31

2.2 THE SYMMETRICAL BENDING PROCESS 31
   2.2.1 Symmetrical Bending Model 32
      2.2.1.1 Plastic Moment-Curvature Relation 33
      2.2.1.2 Analysis of the Model Related to Control 34
   2.2.2 Control of the Symmetrical Roll Bending Process 36

2.3 THE UNSYMMETRICAL BENDING ROLL BENDING PROCESS 39
   2.3.1 Unsymmetrical Bending Model 39
      2.3.1.1 Principal Axes of Inertia, \( I_{XX} = 0 \) 43
      2.3.1.2 Symmetrical Bending versus Unsymmetrical Bending 44
   2.3.2 The Two Degree-of-Freedom Roll Bending Machine 45
   2.3.3 Control of the Unsymmetrical Roll Bending Process 47
      2.3.3.1 Desired Neutral Axis and Desired Curvature 47
      2.3.3.2 Decoupling the Unsymmetrical Bending Process 49
   2.3.4 Motivation for using a Two Degree-of-freedom Roll Bending Machine for the Unsymmetrical Roll Bending Process 51
3. Control of the Roll Twisting Process

3.1 INTRODUCTION

3.2 TORSION MODEL OF WORKPIECE
   3.2.1 Assumptions
   3.2.2 Elastic Torsion
   3.2.3 Torque-Unloaded Angle of Twist Relation

3.3 ROLL TWISTING APPARATUS

3.4 PRIMARY CONTROLLER

3.5 PRELIMINARY CONTROL ANALYSIS
   3.5.1 Primary Controller Applied to a Moving Beam
   3.5.2 Sequencing of the Beam
      3.5.2.1 Limiting the Length, L
   3.5.3 Incremental Sequencing of the Beam
   3.5.4 Location of Angle Measurement
      3.5.4.1 Deviation from the Desired Angle
      3.5.4.2 Drift of the Final Angle from the Desired Angle
      3.5.4.3 Influence of the Sampling Rate

3.6 ROLL TWISTING PROCESS CONTROLLER
   3.6.1 The Inner Loop
   3.6.2 The Outer Loop
   3.6.3 Position of the Angle Measurements
   3.6.4 Computing the commanded angle, \( \phi_{com}(z_{rel}) \)
   3.6.5 Forecasting the angle, \( \phi(z_{rel}) \)
   3.6.6 Summary
   3.6.7 Inner and Outer Loop Sample Rates
   3.6.8 Disturbance Rejection
   3.6.9 Potential Problems with the Roll Twisting Process Controller
      3.6.9.1 Relative Angle Measurement
   3.6.10 Error in measurements due to vibration
      3.6.10.1 Torque Measurement Location

3.7 MEASUREMENT MODEL
   3.7.1 Angle Measurement
   3.7.2 Torque Measurement
   3.7.3 Torsional Stiffness Measurement

4. Control of a Shaping Process for Beams

4.1 INTRODUCTION

4.2 DECOUPLING THE BENDING AND TWISTING PROCESSES
   4.2.1 The Shear Center
4.2.2 Experimental Location of the Shear Center
4.2.3 Twisting caused by Off-shear Center Loading During a Bending Process
   4.2.3.1 Thick-wall versus Thin-wall Cross Sections
   4.2.3.2 Twisting Restrained During the Bending Process
   4.2.4 Twisting Caused by Buckling of Thin-wall Cross Sections
   4.2.5 Minimizing Off-shear Center Loading with Roll Assembly Design
4.3 SHAPE CONTROL SYSTEM FOR BEAMS OF CONSTANT CROSS-SECTION
   4.3.1 The Combined Bending and Twisting Operation
       4.3.1.1 Thick versus thin-wall cross sections
       4.3.1.2 Moments of Inertia of the Cross Section
       4.3.1.3 The feasibility of a combined bending and twisting operation
   4.3.2 Control of the Combined Bending and Twisting Process
   4.3.3 A Shape Control System for Beams Using Separate Bending and Twisting Operations

5. Conclusions and Results
   5.1 CONTROLLING THE UNSYMMETRICAL ROLL BENDING PROCESS
   5.2 CONTROLLING THE ROLL TWISTING PROCESS
   5.3 CONTROLLING A SHAPE PROCESS FOR BEAMS
   5.4 FUTURE RESEARCH
       5.4.1 Proposed Static Experiments
       5.4.2 Future Research for Bending
       5.4.3 Future Research for Twisting
       5.4.4 Other Research

References

Appendix A. MACHINE DESIGN CONCEPTS FOR THE UNSYMMETRICAL ROLL BENDING PROCESS
   A.1 Conceptual Design for a Two Degree-of-freedom Roll Bending Machine for Straightening

Appendix B. Conceptual Design for a Roll Twisting Machine

Appendix C. Proposed Experiments and Experimental Apparatus
   C.1 Experimental Apparatus
   C.2 Experiments
       C.2.1 Unsymmetrical Bending Experiments
           C.2.1.1 T-section Experiment
           C.2.1.2 Plastic Bending of Unsymmetrical Beams
       C.2.2 Twisting Experiments
   C.3 Coupling between Bending and Twisting Experiments
   C.5 Combined Bending and Twisting Process Experiments

Appendix D. Computer Program for Twisting Simulations
D.1 COMPUTER PROGRAM FOR SIMULATING ROLL TWISTING PROCESS
## List of Figures

| Figure 1-1: | Setups for Straightening a Channel | 16 |
| Figure 1-2: | A Three-Roll Bending Arrangement | 18 |
| Figure 1-3: | Forming an Angle Section into A Ring with Form Block | 19 |
| Figure 1-4: | Parallel-roll Straightening for an Equal Leg Angle | 20 |
| Figure 2-1: | Stress-strain Relation (elastic-perfectly plastic material) | 32 |
| Figure 2-2: | Moment-Curvature Relation of a Rectangular Beam | 35 |
| Figure 2-3: | Three Roll Bending Arrangement | 37 |
| Figure 2-4: | Moment versus Position Relationship | 37 |
| Figure 2-5: | Block Diagram of the Primary Controller | 38 |
| Figure 2-6: | Bending of an Unsymmetrical Beam | 41 |
| Figure 2-7: | Cross Section of the Workpiece (z out of the paper) | 42 |
| Figure 2-8: | Schematic of the Two Degree-of-freedom Roll Bending Apparatus | 46 |
| Figure 2-9: | Control of the Unsymmetrical Roll Bending Operation | 48 |
| Figure 2-10: | Comparison of 3 Different Methods for Bending an Unsymmetrical Beam | 54 |
| Figure 2-11: | Fully Plastic Bending of an Unsymmetrical Beam | 56 |
| Figure 2-12: | Geometry of an Unequal Leg Angle | 59 |
| Figure 2-13: | A Two Degree-of-freedom Roll Bending Machine | 62 |
| Figure 2-14: | Schematic of Center Roll Assembly | 64 |
| Figure 2-15: | Rotational Transformation of Axes Systems | 65 |
| Figure 2-16: | Measurement Model for the x-z Plane | 68 |
| Figure 2-17: | Relation of Transducer Displacement to Curvature and Neutral Axis | 69 |
| Figure 3-1: | Beam subjected to Torsion | 78 |
| Figure 3-2: | Torque versus Position Relationship | 78 |
| Figure 3-3: | Angle of Twist versus Position Relationship | 78 |
| Figure 3-4: | Torque-Angle of Twist Relation for Unloading a Plastically Deformed Beam | 80 |
| Figure 3-5: | Schematic of the Roll Twisting Apparatus | 83 |
| Figure 3-6: | Block Diagram of the Primary Controller | 85 |
| Figure 3-7: | Primary Controller Step Response | 86 |
| Figure 3-8: | Shifted Torque-Angle of Twist Relation | 87 |
| Figure 3-9: | Primary Controller Straightening a Beam | 89 |
| Figure 3-10: | Primary Controller applied to Straightening | 90 |
| Figure 3-11: | Sequencing Process applied to the Beam | 92 |
| Figure 3-12: | Primary Controller used for Straightening ($z_{rel} = L/4$) | 96 |
| Figure 3-13: | Block Diagram of the Control System for the Roll Twisting Process | 100 |
| Figure 3-14: | Computation of the Commanded Angle $\phi_{com}(z_{rel})$ | 102 |
| Figure 3-15: | Updating Premasured Angles | 104 |
| Figure 3-16: | Vibration Model of each Section of the Beam | 112 |
| Figure 3-17: | Angle Measurement Device | 114 |
| Figure 3-18: | Fixed Roll Assembly Torque Measurement | 117 |
| Figure 4-1: | Effect of Applying a Load through the Shear Center | 119 |
Figure 4-2: Resolving a Torque and Force at the Centroid into an equivalent Torque and Force at the Shear Center

Figure 4-3: Cross section subjected to Off-shear Center Loading

Figure 4-4: Rectangular Cross section

Figure 4-5: Unloaded Angle of Twist for a Beam Restrained From Twisting in a Bending Process

Figure 4-6: Desired Shear Center Loading for an Angle Cross Section

Figure 4-7: Roll Design for an Angle

Figure 4-8: Off-shear Center Loading due to Non-contacting Disks

Figure 4-9: Off-shear Center Loading Caused by Roll Misalignment

Figure 4-10: Control System for the Combined Bending and Twisting Process

Figure A-1: Design for a two degree-of-freedom Roll Bending Machine

Figure B-1: Conceptual Design for a Roll Twisting Machine

Figure C-1: Experimental Apparatus

Figure C-2: Fixed End Assembly

Figure C-3: Center Assembly
Notation

\( a \): distance from outer roll A to center roll

\( A \): cross sectional area

\( b \): distance from center roll to outer roll C

\( C_{\text{mach}} \): transformation matrix, from machine axis system to workpiece axis system

\( C \): shear center

\( d, D \): maximum cross section diameter

\( e \): error

\( E \): modulus of elasticity

\( f \): feed rate of the beam (length/time)

\( f(\theta) \): coordinate transformation matrix transforming \((x, y)\) to \((X, Y)\)

\( F_R \): resultant force on workpiece at centroid

\( F_c \): the bending force at the shear center

\( g(\theta) \): coordinate transformation matrix transforming \((X, Y)\) to \((x, y)\)

\( G \): gain

\( G \): shear modulus of elasticity (modulus of rigidity)

\( I_x \): moment of inertia with respect to the x-axis

\( I_y \): moment of inertia with respect to the y-axis
I_{xy}: product of inertia with respect to the x and y axes

I_X: principal moment of inertia with respect to the X-axis

I_Y: principal moment of inertia with respect to the Y-axis

J: polar moment of inertia

K: curvature

K_L: loaded curvature

K_U: unloaded curvature

L: length of the beam in the machine over which the torque acts.

\( a = b \): the distance between the center roll and the outer rolls

M: moment

O: centroid of cross section

r: radius measured from centroid

R: radius of curvature

s: Laplace operator

t: time

T_c: torque at shear center

T_R: resultant torque on workpiece at centroid

T_{sample}: the sample time of the controller

T_{sampIL}: inner loop sample time

T_{sampOL}: outer loop sample time

x_c: x coordinate location of shear center

x_p: deflection of the beam

position of the x-axis servo
\( y_c \): y coordinate location of shear center

\( y_p \): deflection of the beam

\( \xi \): position of the y-axis servo

\((x,y)\): rectangular axis system with origin, O, at centroid of the cross section

\((x_{wp},y_{wp},z_{wp})\): workpiece axis system

\((x,y,z)\): machine axis system

\((x_{OG},y_{OG},z_{OG})\): the axis system attached to the outer gimbal at the center roll

\((X,Y)\): principal axis system with origin, O, at centroid of the cross section

\( z_{abs} \): absolute position on the moving beam relative to end of beam

\( z_{rel} \): position on the beam relative to the machine

\( z_{nrel} \): position on the workpiece where \( z_{rel} = z_{node} \)

\( z_{1rel} \): position relative to the machine where an angle is measured

\( z_{2rel} \): position relative to the machine where an angle is measured

\( \alpha \): angle defining neutral axis, CCW positive

\( \beta \): angle defining orientation of the resultant force, \( F_R \)

\( \Delta \phi \): change in the angle of twist over the interval, \( T_{sample} \)

\( \zeta \): damping ratio

\( \theta \): angle defining principal axis system

\( \kappa_T \): shape factor or torsional constant

\( \xi \): deflection of beam or center roll displacement

\( \sigma \): bending stress
\( \tau \): shear stress

\( \phi \): angle of twist

\( \phi_{\text{com}} \): commanded angle of twist

\( \phi_L \): loaded angle of twist

\( \phi_U \): unloaded angle of twist

\( \phi_{\text{Udes}}(z_{\text{abs}}) \): the desired unloaded angle of twist at each position \( z_{\text{abs}} \) on the beam

\( \Phi \): angle defining the plane of loads

\( \Psi \): angle defining rotation of the center roll

\( \omega_n \): natural frequency
Chapter 1
Introduction

1.1 MOTIVATION

A shape control system for beams is needed in the metal industry to form metal stock into a desired shape or to correct dimensional inaccuracies caused by the initial forming process. A shaping process includes straightening an initially deformed beam or forming an initially straight beam. A shape control system for beams bends and twists the beam to a desired shape. Achieving the desired shape requires bending the workpiece about a specified neutral axis for the desired curvature and twisting the workpiece for the desired angle of twist at each point along the beam. In general, the cross section is unsymmetrical (angles, channels, etc.) in contrast to the simpler case of symmetrical cross sections (squares and I-beams).

1.1.1 Background

The most demanding shaping process is straightening an initially deformed beam. One of the simpler methods for straightening a beam is press straightening. The following example illustrates the straightening process for a beam with a channel cross section (Figure 1-1).

The channel is straightened by bending the workpiece in the vertical plane to remove a constant curvature and then twisting the workpiece to remove a constant rate of twist. The channel was supported on two blocks 16 inches apart, while force was applied by a pressure ram. A force...
of 3000 pounds deflected the part about one inch for a correction of 0.020 inches. Force was applied at two-inch increments along the bar. Two 5-ton hydraulic presses, mounted on a large steel table, were used to remove the twist. One ram held one end of the part against a block on the table, while the second ram twisted the part. A slotted bar served as a lever to twist the channel 30° for a permanent correction of 0.020 inches for each 24 inches, or 0.050 inches for the entire length.

**Figure 1-1:** Setups for Straightening a Channel [2]

The previous method for straightening the channel is a slow and time-consuming process. First, the operator must measure the initially deformed channel. Then the operator must design a straightening operation to remove the twist and the curvature from the beam. The straightening operation requires computing the amount of overbend and overtwist to be applied to each particular segment of the channel.

A general straightening process for the channel, or any unsymmetrical cross section, would involve an unsymmetrical bending process and a twisting process. The unsymmetrical bending process bends the channel about an specified neutral axis to a particular curvature at each point along the beam. The goal of the
unsymmetrical bending process for straightening is to remove a continuously varying curvature at each point along the beam. The twisting process twists the channel to remove a continuously varying angle of twist at each point along the beam. Automating the straightening process is needed to increase production, and the next logical step for increasing production is to feed the workpiece through rolls while performing the bending and/or the twisting operation.

1.1.2 Motivation for Controlling the Unsymmetrical Roll Bending Process

A typical rolling arrangement is the one degree-of-freedom roll bending machine. Roll bending is often used to form cylinders from metal plates. A three-roll bending machine configuration is shown in Figure 1-2. The workpiece is subjected to three-point bending while it is rolled through the machine. The orientation of the neutral axis is determined by the relation between the axes of the rolls and the workpiece. Once the machine is set up for a part, the neutral axis is fixed and cannot be changed during the process, so the machine only has one degree of freedom and can only bend the workpiece in a fixed plane. The metal plate is fed through the machine by the driven center roll, and the center roll moves in a vertical plane between the fixed outer rolls to provide a variable bending moment. So each point along the length of the beam is subjected to a maximum moment with a corresponding plastic deformation.

A one degree-of-freedom, (DOF), roll bending machine can bend symmetrical cross sections to a desired shape, but bending an unsymmetrical section with a one DOF machine, such as an angle, does not result in the desired shape, unless special guides are used to compensate for the coupling that occurs in unsymmetrical bending. For example, one method for bending an angle into a ring bends the
workpiece in two planes simultaneously, such that when the shape is removed from the forming machine, it will springback to a ring. In figure 1-3, the angle is bent about a form block into a helical shape. The form block is smaller than the desired diameter of the ring to bend the workpiece in a circular horizontal plane, and the vertical wiper-shoe bends the workpiece in a vertical plane. When angle section is removed from the machine, the workpiece springs back to the desired shape, a circular ring with a diameter larger than the form block.

An unsymmetrical roll bending process can include straightening an initially deformed beam or forming an initially straight beam by bending the unsymmetrical beam to a desired curvature about a specified neutral axis. One current method for straightening unsymmetrical beams is parallel-roll straightening. Parallel-roll straightening is a process for straightening unsymmetrical beams with respect to bending. A parallel-roll straightening process for an angle is shown in figure 1-4. The vertical distance between the roll centers are adjusted to impose opposite and decreasing bending moments on the workpiece as it passes through the machine.
As the workpiece approaches the machine exit, the bending moments decrease such that a straight workpiece exits from the machine. The top rolls are adjusted horizontally to achieve the same effect in the horizontal plane of bending. The workpiece in figure 1-4 is an equal leg channel. The rolls are designed to bend the angle about its principal axes.

Parallel-roll straightening has several disadvantages. One disadvantage is that the workpiece is bent randomly, so the straightening process is not really controlled. The straightness of the workpiece is dependent on magnitude of the bends in the initially deformed beam and the magnitude and quantity of the bends imposed on the beam by the machine. The number of rolls in the machine and how they are adjusted directly effects the straightness of the workpiece. Another disadvantage is that special roll design is required to bend certain cross sections about their principal axes of inertia. The final disadvantage is the cost and complexity of the machine.

Automating the unsymmetrical roll bending process requires controlling the
Figure 1-4: Parallel-roll Straightening for an Equal Leg Angle

(The top rolls can be adjusted horizontally and vertically [2].)

An unsymmetrical roll bending process and a machine that can bend an unsymmetrical beam about a specified neutral axis to a desired curvature while rolling the beam through the machine.

1.1.3 Motivation for Controlling the Roll Twisting Process

To date, the roll twisting process has not yet been automated. The current method for straightening a beam, with a random angle of twist along its length, requires a skilled operator. The operator feeds a section of the beam into a fixture. The operator then applies a torque over the section of the beam to remove a constant rate of twist from that section. Each section of the beam is twisted until the entire beam has been straightened. The current method for imparting a constant rate of twist to a straight beam is by using guides in a rolling process [25], page 109-110. The guides are adjusted to overtwist the workpiece by the correct amount, such that the workpiece springs back to the desired rate of twist. This
requires a skilled operator, who adjusts the guides by a trial and error method [25]. Automating the roll twisting process would increase production and increase the quality of the finished product.

**1.1.4 Motivation for Closed-loop Control of the Shaping Process**

In the evaluation of different control schemes for a shaping process, it is important to distinguish between a defined shaping process, which forms an initially straight beam into a desired shape, and a straightening process, which straightens an initially deformed beam. Control schemes which use iterative methods can be applied to a defined shaping process. These types of control schemes rely on extensive material testing to generate data for segments of the desired shape. These types of control schemes are open-loop and cannot reject disturbances, such as random shapes or changes in the material properties of the workpiece, and they cannot be used for a straightening process.

A shaping process includes both straightening and shaping a beam to a desired shape. Controlling the straightening process requires a closed-loop control scheme with good disturbance rejection and command-following properties. In general, a *true* test of the control scheme is to control the straightening process. Once the straightening process is controlled, the control scheme can be modified for controlling a defined shaping process.

The shape control system for beams was developed by examining several forming processes: the symmetrical roll bending process, the unsymmetrical roll bending process, the roll twisting process and the combined roll bending and twisting process. The more advanced control schemes have been developed for the symmetrical roll bending process.
1.1.4.1 Control of the Symmetrical Roll Bending Process

Three-point bending takes place in the symmetrical roll bending process. The workpiece is bent in a plane about a fixed neutral axis. A skilled operator uses a trial and error method in the static case, as shown in Figure 1-1, or the process is automated by using a one degree-of-freedom roll bending machine, shown in figure 1-2. The force is applied to deflect the workpiece a certain amount, and then the force is removed and the workpiece springs back. The operator measures the unloaded curvature, and then deflects or loads the workpiece again. This process is continued until the unloaded curvature is correct. The key to controlling the symmetrical roll bending operation is measuring or predicting the workpiece springback - the change in the curvature from the loaded state to the unloaded state. The workpiece springback is dependent on the bending stiffness of the workpiece, the bending moment, and the loaded curvature. The bending stiffness of the workpiece is related to the yield stress and strain hardening characteristics of the material and the geometry of the cross section.

The early control schemes for the bending process consisted of iterative methods to sequentially bend arc segments of the beam [23, 16]. Sachs and Shanley used empirical data for springback compensation [18, 20]. Other researchers used various analytical methods to predict the residual stress distribution and springback [8, 19, 6]. Hansen and Jannerup developed a model for the roll bending process [11]. Cook, Hansen, and Jannerup used the model in a control scheme for the roll bending process which relied on knowing the material properties in advance [4]. Many of these approaches require springback curves, which require extensive material testing to generate. None of the above approaches incorporate material adaptive control. Foster developed a closed-loop controller which relied on a curvature measurement of the workpiece after it had exited from the roll bending
machine. This controller did not require prior knowledge of material properties, but it was only applicable to shapes with relatively constant curvature [7].

More recent research by Hardt, Roberts, and Stelson [12] for the roll bending process and Gossard and Stelson [9] for the brake forming process has resulted in a closed-loop controller for each process, which measures material properties during forming. Research by Hardt, Hale, and Roberts [12, 10, 17] have led to the development of a closed-loop roll bending controller, which measures the material properties of the workpiece on-line and indirectly measures the unloaded curvature. The unloaded curvature or springback is computed from real-time measurements of the bending moment, the loaded curvature, and the bending stiffness of the beam, while the workpiece is still in the loaded state. The result is a closed-loop unloaded curvature controller with good command following and disturbance rejection properties. The process is insensitive to material property variation, and can form a continuously varying and arbitrary shape in one pass through the machine. Roberts [17] implemented the control scheme in a one degree-of-freedom roll bending machine, and his experiments resulted in errors of less than 3% for bending the same material to a constant curvature shape. Tests on different materials bent to a constant curvature shape resulted in a maximum error of 4.2%. Hale [10] conducted more extensive experiments to investigate the dynamic and control characteristics of the process. Hale refined the controller and investigated different measurement alternatives, command following capability, and disturbance rejection properties of the controller. He found that the vibration of the workpiece contaminated the bending moment measurement, so bandwidth of the roll bending system is limited by the workpiece vibration. Nevertheless, the closed-loop unloaded curvature control scheme by Hardt, Hale, and Roberts is the most promising and the most versatile of the controllers for the symmetrical roll bending
1.1.4.2 Control of the Unsymmetrical Roll Bending Process

Mergler developed a control system for bending arc segments of unsymmetrical beams. The control scheme consisted of an iterative method to bend each segment sequentially. Each segment requires at least two "tries" - bending the segment, allowing it to springback, and then measuring the bend to compute the desired control action. Mergler did address the problems of twisting and out of plane deformation caused by bending unsymmetrical beams. The machine was designed to approximate a pure bending moment over the segment such that twisting caused by shear stresses was negligible. The design also applied a counter moment to compensate for out of plane bending [16].

1.2 OBJECTIVE

The objective of this research is to develop a control scheme for a shaping process for beams. The shaping process consists of bending the beam about a specified neutral axis to a desired unloaded curvature and twisting the beam to a desired angle of twist at each point along the length of the beam. So the mechanics of bending and twisting a beam need to be analyzed to develop models for the unsymmetrical roll bending process and the roll twisting process. The control scheme for the symmetrical roll bending process, developed by Hardt and Hale, is a closed-loop system and measures material properties on-line. The basic idea is to extend the control scheme of Hardt and Hale to the unsymmetrical roll bending process and the roll twisting process to achieve a shape control system for beams. The initial goal is to control the roll twisting process and the unsymmetrical roll
bending process separately. Machine design concepts for each process will be
developed, including recommendations for actuator location and obtaining the
desired measurements. The final goal is to present a control scheme for the overall
shaping process for beams.

This research is being funded by the Aluminum Company of America, and the
particular purpose of this research is to develop machine design concepts and
control schemes for straightening extrusions. After the initial extruding process,
the extrusions are severely deformed with respect to bending and twisting. The
extrusions are then subjected to stretch straightening to remove the severe bends
and twists. The focus of this research is to straighten the extrusions after the
stretch straightening process to meet quality control standards.

1.3 THESIS OVERVIEW

The thesis is divided into three major parts: the control of the unsymmetrical
roll bending process (Chapter 2), the control of the roll twisting process (Chapter
3), and the control of a shaping process for beams (Chapter 4). Finally the major
points of each process is summarized in Chapter 5. A shaping process for beams
consists of two subprocesses, the unsymmetrical roll bending process and the roll
twisting process. First, bending and twisting are assumed to be decoupled, and
separate control systems are developed for the bending process and the twisting
process.

1.3.1 Controlling the Unsymmetrical Roll Bending Process

Chapter 2 presents a control system for the unsymmetrical roll bending
process. Unsymmetrical bending is bending the workpiece about a neutral axis, not
necessarily coinciding with a principal axis. The goal of the unsymmetrical roll bending process is to bend the workpiece to a desired curvature about a specified neutral axis at every point along the beam. The model and control system for the symmetrical roll bending process, developed by Hardt, Hale, and Roberts, is briefly reviewed to establish the basic control aspects of the roll bending process. The mechanics of unsymmetrical bending are analyzed to develop a model, and the analysis reveals that unsymmetrical bending is coupled by its product of inertia, so the beam is not necessarily loaded perpendicular to the neutral axis. Unsymmetrical bending can be decoupled into symmetrical bending about each principal axis of inertia. The controller decouples the unsymmetrical bending process into two separate but simultaneous roll bending controllers about each principal axis. Each principal axis controller predicts the unloaded curvature by computing the springback of the beam from real-time measurements of the loaded curvature, the bending moment and the bending stiffness of the beam in principal coordinates. Measurements are made in convenient machine coordinates and transformed by the controller into principal axes coordinates.

Implementing the control system requires a two degree-of-freedom roll bending machine to bend the workpiece about a specified neutral axis to the desired curvature. A measurement model details alternative methods for measuring the bending moments and curvatures, and on-line methods for locating the principal axes and computing the bending stiffness for each principal axis are proposed.

The major results and conclusions of Chapter 2 are summarized in Chapter 4. Appendix A presents a machine design concept for the two DOF roll bending machine. Appendix C.2.1 outlines several proposed experiments for testing the control system and for resolving several uncertainties about the unsymmetrical roll bending process.
Chapter 3 addresses the problem of automating the process of continuously twisting a beam to a desired shape or "roll twisting". The workpiece is subjected to pure torsion by applying a torque about the shear center of the workpiece. The goal of the process is to achieve the desired unloaded angle of twist at each point along the beam. A mathematical model of the twisting process is presented and analyzed to develop a suitable control system for the process.

The roll twisting process shares many similarities with the symmetrical roll bending process, but major differences between the two prevent the direct application of the roll bending controller. The major problem in the roll twisting process is achieving point by point deformation of the workpiece since a constant torque, applied over a section of the workpiece, plastically deforms the entire section. The primary controller predicts the unloaded angle of twist by computing the springback of the beam from real-time measurements of the loaded angle of twist, the applied torque, and the torsional stiffness of the beam. To achieve point by point shape control of the beam in a straightening process, the angle measurement is made as close as possible to where the beam exits the machine. This reduces the effective length under control to much smaller length than the actual machine. Point by point deformation can be approximated by making the effective length under control very small. Computer simulations were made to investigate the effect of sensor location, feedrates and sampling times. Other possible solutions for the point by point shape control of the beam are presented and analyzed. Potential problems with the roll twisting process controller are discussed, and possible solutions are presented.

The major results and conclusions for controlling the roll twisting process are
presented in Chapter 5. Appendix B presents a conceptual design for a roll twisting machine. The continuous nature of the roll twisting process is difficult to model accurately, so Appendix C outlines experiments for testing the control system for the roll twisting process. A computer program for simulating the roll twisting process is in Appendix D.

1.3.3 Controlling a Shaping Process for Beams

Chapter 4 discusses controlling a shaping process for beams. An ideal shape control system for beams of constant cross section would simultaneously perform two-plane roll bending for the desired curvature and roll twisting for the desired angle of twist. First, the mechanics of bending and twisting are analyzed to investigate the coupling between bending and twisting. The analysis reveals that thick-wall cross sections are generally insensitive to the coupling between bending and twisting while thin-wall cross sections are sensitive. Two control schemes for the shaping process are proposed. The first control system is for a combined bending and twisting operation for thick-wall cross sections. The combined bending and twisting operation is feasible for thick-wall cross sections, but it is not recommended for thin-wall cross sections. Thin-wall cross sections are sensitive to off-shear center loading, require large angle of twists, and are sensitive to violations of cross sectional property assumptions used in the roll bending controller. The second control system divides the bending and twisting operations into two separate processes, the roll bending process and the roll twisting process, for thin-wall cross sections.

Chapter 5 contains the major results of the investigation for controlling a shaping process for beams. In Appendix C, experiments are proposed to evaluate the feasibility of a combined bending and twisting operation for thin-wall cross
sections. In addition, experiments for on-line location of the shear center and investigating the coupling between bending and twisting are presented.
Chapter 2
Control of the Unsymmetrical Roll Bending Process

2.1 INTRODUCTION

Understanding and controlling the unsymmetrical roll bending process is essential to developing a shape control system for beams. In this chapter, bending and twisting are assumed to be decoupled, so the only focus will be to control the unsymmetrical roll bending process. The goal of the unsymmetrical roll bending process is to bend the workpiece to a desired curvature about a specified neutral axis at every point along the beam. Since the symmetrical roll bending process has been successfully controlled, the first step is to review the model of the symmetrical roll bending process and the analysis which led to a successful control scheme. Next, the mechanics of unsymmetrical bending will be analyzed to develop a suitable model for the process, and then determine how the basic control concepts of the symmetrical bending process can be applied to the unsymmetrical bending process. The pros and cons of using a one degree-of-freedom (DOF) roll bending machine versus a two DOF roll bending machine for controlling the unsymmetrical bending process are discussed. The two DOF machine is selected to control the unsymmetrical roll bending process, and a closed-loop control system which measures material properties on-line and indirectly measures the unloaded curvature is presented. Finally, a model for measuring the loaded curvatures and bending moments, and on-line methods for measuring the bending stiffness and the principal axes of inertia are described.
2.1.1 Assumptions

The assumptions used in developing a model for the unsymmetrical bending process include:

- The workpiece is a beam with a constant cross section of arbitrary shape.

- A planar cross section of the beam remains a planar during bending [21].

- The workpiece is not subjected to axial or torsional loads [21].

- The workpiece is made of an elastic-perfectly plastic material with the stress-strain relation shown in Figure 2-1, and the material is homogeneous and isotropic.

- The shear stress caused by the tranverse loading is negligible if the length of the beam, \( L \), is large compared to the maximum cross section diameter, \( d \). The shearing stress is assumed negligible in comparison to the bending stress if the ratio \( L/d > 5 \) [3].

- The workpiece is subjected to three-point loading applied at the shear center of the workpiece. Loading the workpiece at its shear center allows the beam to be bent with twisting [13].

2.2 THE SYMMETRICAL BENDING PROCESS

This section is the result of research and experimental verification conducted by Hale, Hardt and Roberts [12, 10, 17] in controlling and modelling the symmetrical roll bending process using a one degree-of-freedom roll bending machine. A symmetrical bending model is developed, and the control scheme for the symmetrical roll bending process is reviewed. The result is a closed-loop system which measures material properties on-line and indirectly measures the unloaded curvature. The controller indirectly measures the
unloaded curvature while still in the loaded state, compares the measured unloaded curvature with the desired unloaded curvature, and uses the error signal to command the servo. The reader is encouraged to review Hale's thesis for a more in-depth treatment of controlling the symmetrical bending process [10].

### 2.2.1 Symmetrical Bending Model

Symmetrical bending is defined as bending a workpiece about an axis of symmetry. Based on the assumption that the shear stresses are negligible, the only non-zero stress component is the longitudinal stress distribution, $\sigma_{zz}$. Assuming the material obeys Hooke's law, the strain can be described by the curvature of the beam, $K$, and the distance from the neutral axis, $x$. 

**Figure 2-1:** Stress-strain Relation (elastic-perfectly plastic material).
where $E$ is the modulus of elasticity and is dependent on the material properties of the workpiece. For the beam to be in equilibrium, the bending stress integrated over the area must equal the bending moment, $M_y$.

$$M_y = -\int_A \sigma_{zz} x \, dA$$

Substituting equation (2.1) into equation (2.2) results in the moment-curvature relationship for an elastic material in bending.

$$M_{\text{elastic}} = E I_y K \quad \text{where } I_y = \int_A x^2 \, dA$$

and $I_y$ is the moment of inertia dependent with respect to the y-axis.

### 2.2.1.1 Plastic Moment-Curvature Relation

The following analysis derives the moment-curvature relation for a beam with a solid rectangular cross section for symmetrical bending. The workpiece is assumed to be made of an elastic-perfectly plastic material. For the portion of the cross section undergoing elastic deformation, ($x \leq x_{yld}$), the bending stress, $\sigma_{zz}$, is proportional to the distance from the neutral axis, $x$,

$$\sigma_{\text{elastic}} = -\frac{\sigma_{yld}}{x_{yld}} x$$

For the portion of the cross section undergoing plastic deformation, the bending stress is constant.

$$\sigma_{\text{plastic}} = -\sigma_{yld}$$

Equations, (2.4) and (2.5), are substituted into equation (2.2) and integrated. It can be shown that the curvature is proportional to the distance from the neutral axis,
and the result is

\[
\text{If } K \leq K_{\text{yld}}, \quad M_{\text{elastic}} = EI_y K \tag{2.6}
\]

\[
\text{If } K > K_{\text{yld}}, \quad M_{\text{plastic}} = \frac{3 M_{\text{yld}}}{2} \left(1.0 - \frac{K_{\text{yld}}^2}{3 K^2}\right) \tag{2.7}
\]

The relationship between the bending moment and the curvature, for three-point bending, is shown in Figure 2-2. If the loaded curvature, $K_L$, is less than the yield curvature, $K_{\text{yld}}$, plastic deformation will not occur, and the beam will springback to the original curvature, $K_{\text{initial}}$. Plastic deformation occurs when $K$ is loaded past $K_{\text{yld}}$. Assuming the material unloads elastically, the unloading will parallel the original moment-curvature curve in the elastic region (slope of $1/(EI_y)$), and the springback at the center roll can be computed from

\[
K_U = K_L - \frac{M_y}{EI_y} \tag{2.8}
\]

where $K_U$ is the unloaded curvature and $K_L$ is the loaded curvature at the center roll.

2.2.1.2 Analysis of the Model Related to Control

The moment-curvature relationship of Figure 2-2 is dependent on the shape of the cross section and the material properties of the beam. Material properties such as the yield stress and strain hardening can vary for different materials. Computing the plastic deformation of an arbitrary cross section requires a limit analysis, similar to the analysis in section 2.2.1.1, based on the shape of the cross section and the material properties. D. Hardt and M. Hale eliminated the problems of predicting the plastic deformation with limit analysis by simply measuring the applied moment and the loaded curvature, while the workpiece is in the loaded
If the applied moment, $M$, and the loaded curvature, $K_L$, are measured, then the predicted springback can be computed from equation (2.8). In addition, the bending stiffness, $(E I)$, is measured by elastically loading a straight beam and measuring the moment, $M$, and the loaded curvature, $K_L$. The bending stiffness, $(E I)$, is then computed from equation (2.3). Now the unloaded curvature relies only on the assumption that the beam will springback elastically; all other properties are directly measured from the workpiece.
only on the assumption that the beam will springback elastically; all other properties are directly measured from the workpiece.

The maximum loaded curvature, $K_L$, takes place at the location of the maximum moment which is at the center roll shown in Figures, 2-3 and 2-4. A fixed point on the workpiece moving through the roll bending apparatus is assumed to experience plastic deformation only at the location of the maximum moment. As the fixed point on the workpiece moves through the bending apparatus, the point experiences an increasing bending moment until it passes through the center roll. At the center roll, the point is plastically deformed. When the point moves past the center roll, the bending moment decreases and no longer plastically deforms the point. This allows the assumption of point by point deformation of the workpiece as it passes under the center roll.

2.2.2 Control of the Symmetrical Roll Bending Process

The control of the symmetrical roll bending operation is based on the primary controller of Figure 2-5. Real time measurements of the loaded curvature, $K_L$, and the applied bending moment, $M$, indirectly measures the unloaded curvature, $K_U$ (equation (2.8)). The feedback controller generates an error signal by comparing the indirectly measured unloaded curvature, $K_U$, to the desired unloaded curvature, $K_{U_{des}}$. The error signal then commands the servo to deflect the workpiece in the manner required to null the error signal. Thus, the primary controller shown in Figure 2-5, causes the workpiece unloaded curvature, $K_U$, to track the desired unloaded curvature, $K_{U_{des}}$.

M. Hale designed the primary controller for the symmetrical roll bending operation. A velocity controlled servo was chosen to deflect the beam, and the
Figure 2-3: Three Roll Bending Arrangement [10]

Figure 2-4: Moment versus Position Relationship [10]
Figure 2-5: Block Diagram of the Primary Controller [10]

The transfer function of the servo is assumed to be first order. A proportional controller with velocity feedback resulted in a second order closed loop transfer function:

\[ \frac{K_u}{K_{U_{des}}} = \frac{G}{s^2 + 2\xi\omega_n + \omega_n^2} \]  \hspace{1cm} (2.9)

where the damping ration, \(\xi\), was designed with sufficient damping to avoid overshoot. The vibration of free ends of the workpiece contaminates the bending moment measurement, so the vibration severely limits the bandwidth of the system. [10]
2.3 THE UNSYMMETRICAL BENDING ROLL BENDING PROCESS

Bending a beam about an axis of symmetry implies that the product of inertia is zero, and bending will only occur in a plane orthogonal to the axis of symmetry. Bending a beam about an axis that is not an axis of symmetry is known as unsymmetrical bending. Unsymmetrical bending implies that the product of inertia is non-zero and coupling will exist. If an unsymmetrical beam is loaded perpendicular to the desired neutral axis, as in symmetrical bending, the workpiece can bend about some other neutral axis. The following sections analyzes the mechanics of unsymmetrical bending to develop a model for the unsymmetrical roll bending process. Then a control system is developed for the unsymmetrical roll bending process. The objective is to bend the workpiece to a desired curvature about a specified neutral axis at every point along the beam.

2.3.1 Unsymmetrical Bending Model

The beam has a constant cross section of some arbitrary shape. In general, the cross section of the workpiece does not have an axis of symmetry, so the workpiece is unsymmetrical. The angle shown in Figure 2-7 is an unequal leg angle without an axis of symmetry. The beam is bent about a specified neutral axis to a desired curvature. The angle, \( \alpha \), and the centroid, \( O \), define the neutral axis of the beam, designated by the line n-n. The curvature, \( K \), is perpendicular to the neutral axis. The curvature is positive if it produces compressive stresses on the positive axis, so the center of curvature will lie on the positive side of the axis. The machine coordinate system \((x,y)\) is defined as having its origin, \( O \), at the centroid of the cross sectional area as shown in Figures 2-6 and 2-7. In Figure 2-6, this implies that a positive bending moment, \( M_y \), produces a positive curvature, \( K_x \). Similarly,
a positive $M_x$ produces a negative curvature, $K_y$.

For bending and twisting to be decoupled, the line of action of the resultant bending forces must pass through the shear center of the workpiece. (The shear center will be discussed in more detail in Chapter 4.) The axial line which passes through the shear centers of all the cross sections on the beam is the bending axis in Figure 2-6. The plane in which the bending moment, $M$, is applied is called the plane of loads. The plane of loads is defined by the angle, $\phi$, and the shear center, $C$, and is measured with respect to the $x$-axis and is positive in the CCW sense. For bending without twisting, the plane of loads, at each point along the beam, must contain the bending axis or pass through the shear center of each cross section. For bending without twisting, the centroidal axis will then parallel the bending axis.

Assuming transverse shear stresses are negligible, the only non-zero stress component is the longitudinal stress distribution, $\sigma_{zz}$. Assuming the material obeys Hooke's law, the strain can be described by the curvature of the beam, $K(x,y)$, and the distance from the neutral axis, $(x,y)$.

$$\sigma_{zz} = - E (x K_x + y K_y) \quad (2.10)$$

For the beam to be in equilibrium, the shear stress integrated over the cross sectional area must equal the bending moment, $M(x,y)$.

$$M_x = \int_A y \sigma_{zz} \, dA = - E (I_{xy} K_x + I_x K_y) \quad (2.11)$$

$$M_y = \int_A x \sigma_{zz} \, dA = E (I_y K_x + I_{xy} K_y) \quad (2.12)$$

where $I_x$ is the moment of inertia with respect to the $x$-axis,

$$I_x = \int_A y^2 \, dA \quad (2.13)$$
Figure 2-6: Bending of an Unsymmetrical Beam

and $I_y$ is the moment of inertia with respect to the y-axis,

$$I_y = \int x^2 \, dA$$  \hspace{1cm} (2.14)

and $I_{xy}$ is the product of inertia with respect to both axes.

$$I_{xy} = \int x \, y \, dA$$  \hspace{1cm} (2.15)
Substituting equation (2.10) in equations (2.11) and (2.12) and solving for $\sigma_{zz}$ results in the flexure stress distribution normal to the cross section. The bending stress, $\sigma_{zz}$, in $(x,y)$ coordinates is given by equation (2.16) [3].

$$
\sigma_{zz} = -x \frac{(M_y I_x + M_x I_{xy})}{(I_x I_y - I_{xy}^2)} + y \frac{(M_x I_y + M_y I_{xy})}{(I_x I_y - I_{xy}^2)} \tag{2.16}
$$

Rewriting equations (2.11) and (2.12) results in equations, (2.17) and (2.18). One can see that along the x-axis ($y = 0$), the curvature, $K_x$ is still dependent on both the bending moments, $M_x$ and $M_y$ due to the product of inertia, $I_{xy}$.

$$
K_x = \frac{(M_y I_x + M_x I_{xy})}{(I_x I_y - I_{xy}^2) E} \tag{2.17}
$$
Likewise, along the y-axis, \((z = 0)\),

\[
K_y = \frac{-\left( M_x I_y + M_y I_{xy} \right)}{(I_x I_y - I_{xy}^2) E}
\]  

(2.18)

In unsymmetrical bending, the bending moments, \(M_x\) and \(M_y\), and the curvatures, \(K_x\) and \(K_y\), are coupled. Applying a bending moment about the x-axis will produce curvatures, \(K_x\) and \(K_y\). On the other hand, bending the workpiece about a neutral axis, coinciding with the x-axis \((K_y = 0)\) will require bending moments, \(M_x\) and \(M_y\). In general, the plane of loads is not orthogonal to the neutral axis for unsymmetrical bending. However, for a symmetrical workpiece, \(I_{xy}\) is zero, and equations (2.17) and (2.18) are uncoupled.

**2.3.1.1 Principal Axes of Inertia, \(I_{XY} = 0\)**

Unsymmetrical bending can be decoupled into symmetrical bending about each principal axis of inertia. The principal axes of inertia are defined as the coordinate system, \((X,Y)\), for which the product of inertia, \(I_{XY}\), is zero. Let the origin of both coordinate systems coincide, and let \(\theta\) be the angle that the coordinate system, \((x,y)\), must be rotated to coincide with the coordinate system, \((X,Y)\), and be defined positive in the counter clockwise sense. The relationship for \(\theta\) can be found from equation (2.19) [3].

\[
\tan(2\theta) = -\frac{2 I_{xy}}{I_x - I_y}
\]  

(2.19)

Assuming \(I_x\) is greater than \(I_y\), the principal moments of inertia can be found from

\[
I_X = \frac{I_x + I_y}{2} \pm \sqrt{\left[\frac{(I_x - I_y)/2}2\right]^2 + (I_{xy})^2}
\]  

(2.20)

\[
I_Y = \frac{I_x + I_y}{2} - \sqrt{\left[\frac{(I_x - I_y)/2}2\right]^2 + (I_{xy})^2}
\]  

(2.21)
The relationship between the two coordinate systems is shown in Figure 2-7. Based on the coordinate systems in Figure 2-7, the coordinate transformation from the original coordinates, \((x,y)\), to the principal coordinates, \((X,Y)\), are found from

\[
\begin{bmatrix}
X \\
Y
\end{bmatrix} =
\begin{bmatrix}
\cos(\theta) & \sin(\theta) \\
-sin(\theta) & \cos(\theta)
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
\tag{2.22}
\]

and similarly,

\[
\begin{bmatrix}
x \\
y
\end{bmatrix} =
\begin{bmatrix}
\cos(\theta) & -\sin(\theta) \\
\sin(\theta) & \cos(\theta)
\end{bmatrix}
\begin{bmatrix}
X \\
Y
\end{bmatrix}
\tag{2.23}
\]

2.3.1.2 Symmetrical Bending versus Unsymmetrical Bending

In symmetrical bending the product of inertia is zero, and the following analysis illustrates that the moment-curvature relations of equations (2.17) and (2.18) for a workpiece subjected to unsymmetrical bending can be decoupled about each of its principal axis to yield a linear relationship between the moment and curvature.

Since the principal axes of inertia require \(I_{XY} = 0\), equation (2.16) simplifies to

\[
\sigma_{zz} = \frac{M_Y}{I_Y} + \frac{M_X}{I_X}
\tag{2.24}
\]

in principal axis coordinates. Along the X principal axis, \((Y = 0)\), the principal axis curvature, \(K_X\), equation (2.17) reduces to

\[
K_X = \frac{M_Y}{EI_Y}
\tag{2.25}
\]

and is only dependent on deflection in the X-z plane. Along the Y principal axis, \((X = 0)\), the principal axis curvature, \(K_Y\), equation (2.18) reduces to
and is only dependent on deflection in the Y-z plane.

The workpiece is subjected to symmetrical bending if the neutral axis coincides with one of the principal axes, or

\[ \alpha = \theta + \frac{n \pi}{2} \quad \text{where } n \text{ is an integer} \]  

For all other values of \( \alpha \), the workpiece is subjected to unsymmetrical bending. Loading the workpiece orthogonal to the neutral axis will bend the workpiece about the neutral axis for symmetrical bending. In unsymmetrical bending, loading the workpiece orthogonal to the neutral axis will not result in the workpiece bending about the same neutral axis.

2.3.2 The Two Degree-of-Freedom Roll Bending Machine

Assume the workpiece is subjected to three-point loading, as shown in Figure 2-8, and the rolls are designed to minimize twisting such that the line of action of the loads is applied through the shear center, \( C \), of the workpiece. For convenience, assume that the machine coordinate system, in Figure 2-8 coincides with the centroid of the cross section. The rolls at the origin \((z = 0)\) and at the end \((z = 2L)\) of the workpiece are fixed, and the two servos at the center \((z = L)\) of the machine position the center rolls to deflect the workpiece along the x-axis, \( x_p \), and the y-axis, \( y_p \). The deflected beam bends about a neutral axis (defined by the angle, \( \alpha \), relative to the x-axis where the bending stress, \( \sigma_{zz} \) is zero).
Figure 2-8: Schematic of the Two Degree-of-freedom Roll Bending Apparatus

The rolls at \( z = 0 \) and \( z = L \) are fixed. The x-axis and y-axis servo deflect the workpiece.
2.3.3 Control of the Unsymmetrical Roll Bending Process

The control of the roll bending operation can be decoupled into two simultaneous but separate symmetrical roll bending control systems for each principal axis. The proposed control system is shown in Figure 2-9 and utilizes the symmetrical roll bending control system, section 2.2. The curvatures, $K_x$ and $K_y$, and the bending moments, $M_x$ and $M_y$, are measured in machine coordinates, $(x,y)$, as defined in Figure 2-7. The bending moments are measured with respect to the shear center. The measurements are then transformed into principal axis coordinates, $(X,Y)$, to allow the unsymmetrical roll bending to be decoupled. The error signals, $e_X$ and $e_Y$, for each principal axis, $X$ and $Y$, are computed and transformed into $(x,y)$ coordinates, $e_x$ and $e_y$, and sent to the controller for each servo.

2.3.3.1 Desired Neutral Axis and Desired Curvature

The desired outcome of the roll bending operation is to bend the workpiece about a specified neutral axis to some desired curvature. The neutral axis (n-n in Figure 2-7) is defined as an axis on the cross section of the beam where the stress, $\sigma_{zz}$, is zero, and establishes how the workpiece is to be bent. Let the angle, $\alpha$, relative to the x-axis, define the neutral axis, n-n, passing through the centroid of the cross section. The desired curvature, $K_{U\text{des}}$, at the maximum moment is defined as

$$K_{U\text{des}} = 1 / R$$

(2.28)

where $R$ is the radius of curvature (shown in Figure 2-8) orthogonal to the neutral axis. Employing small angle approximations, it can be shown that the curvatures in the x-z plane and the y-z plane are simply the components of the curvature, $K$.
Figure 2-9: Control of the Unsymmetrical Roll Bending Operation

The coordinate transformation matrix from the \((x,y)\) coordinates to the principal axis coordinates, \((X,Y)\) is defined as \(f(\theta)\). The coordinate transformation matrix from the principal axis coordinates, \((X,Y)\), to the machine axis coordinates, \((x,y)\), is defined as \(g(\theta)\).

\[
[f(\theta)] = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} \quad [g(\theta)] = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}
\]
[5]. From the geometry of Figure 2-8, the desired unloaded curvature in the \((x,y)\) coordinates can be computed.

\[
K_{Uxdes} = K_{Udes} \sin(\alpha) \tag{2.29}
\]

\[
K_{Uydes} = -K_{Udes} \cos(\alpha) \tag{2.30}
\]

where the curvature is defined positive, if the center of curvature lies on the positive axis.

### 2.3.3.2 Decoupling the Unsymmetrical Bending Process

In principal coordinates, the unsymmetrical roll bending control system is uncoupled into two independent control systems along each principal axis (equations (2.25) and (2.26)). If the rolls of the roll bending machine were designed to align the \(X\) and \(Y\) principal axes of the workpiece with the \(x\)-axis servo and the \(y\)-axis servo of the machine, then the \(X\)-axis servo will only affect the \(X\)-axis curvature, \(K_X\), and the \(Y\)-axis servo will only affect the \(Y\)-axis curvature, \(K_Y\). However, such a roll design would be costly, so the controller performs a coordinate transformation to control in the principal axis coordinates.

The measurements from the workpiece - the loaded curvatures, \(K_{Lx}\), \(K_{Ly}\), and the bending moments, \(M_x\) and \(M_y\) in machine coordinates - are transformed into principal axes coordinates by equations (2.31) and (2.32).

\[
\begin{bmatrix}
K_{Lx} \\
K_{Ly}
\end{bmatrix} =
\begin{bmatrix}
\cos(\theta) & \sin(\theta) \\
-\sin(\theta) & \cos(\theta)
\end{bmatrix}
\begin{bmatrix}
K_{Lx} \\
K_{Ly}
\end{bmatrix}
\tag{2.31}
\]

\[
\begin{bmatrix}
M_x \\
M_y
\end{bmatrix} =
\begin{bmatrix}
\cos(\theta) & \sin(\theta) \\
-\sin(\theta) & \cos(\theta)
\end{bmatrix}
\begin{bmatrix}
M_x \\
M_y
\end{bmatrix}
\tag{2.32}
\]

The controller then computes the unloaded curvature for each principal axis,
equations (2.33) and (2.34), based on the symmetrical roll bending model.

\[
K_{UX} = K_{LX} - \frac{M_Y}{E I_Y} \quad (2.33)
\]

\[
K_{UY} = K_{LY} + \frac{M_X}{E I_X} \quad (2.34)
\]

Assume the angle, \( \alpha \), defining the neutral axis and the desired curvature, \( K_{U\text{des}} \), have been specified. The desired curvature computed in equations (2.29) and (2.30) are transformed into principal coordinates by equations (2.35).

\[
\begin{bmatrix}
K_{UX\text{des}} \\
K_{UY\text{des}}
\end{bmatrix} =
\begin{bmatrix}
\cos(\theta) & \sin(\theta) \\
-sin(\theta) & \cos(\theta)
\end{bmatrix}
\begin{bmatrix}
K_{UX\text{des}} \\
K_{UY\text{des}}
\end{bmatrix}
\]

(2.35)

The error signals are then computed in principal coordinates,

\[
e_X = K_{UX\text{des}} - K_{UX} \quad (2.36)
\]

\[
e_Y = K_{UY\text{des}} - K_{UY} \quad (2.37)
\]

It can be shown that the unloaded curvature is proportional to the deflection of the beam, or the center roll displacement, and is dependent on the the geometry of the roll bending apparatus [10].

If \( X_p < X_{pyld} \) then \( K_{UX} = 0.0 \)

If \( X_p \geq X_{pyld} \) then \( K_{UX} \approx \frac{3x_p}{a \ b} \quad (2.38) \)

where \( a \) and \( b \) are the distances between the center roll and each outer roll, respectively. In principal coordinates, the center roll displacement, \( X_p \), is proportional to the unloaded curvature.

\[
X_p \approx \frac{a \ b \ K_{UX}}{3} \quad (2.39)
\]
So the command to the servos is proportional to the error signal. The error signal is then transformed back into the original coordinates, \((x,y)\), equation (2.41) and sent to the controllers that command the x-axis servo and y-axis servo.

\[
\begin{bmatrix}
  x_p \\
  y_p
\end{bmatrix} = \left(\frac{ab}{3}\right) \begin{bmatrix}
  \cos(\theta) & -\sin(\theta) \\
  \sin(\theta) & \cos(\theta)
\end{bmatrix} \begin{bmatrix}
  e_X \\
  e_Y
\end{bmatrix}
\] (2.41)

### 2.3.4 Motivation for using a Two Degree-of-freedom Roll Bending Machine for the Unsymmetrical Roll Bending Process

The generalized unloaded curvature relations for the one DOF bending machine are

\[
K_{Ux} = K_{Lx} - \frac{I_x M_y + I_{xy} M_x}{E(I_{xy}^2 - I_{xy}^2)}
\] (2.42)

\[
K_{Uy} = K_{Ly} + \frac{I_y M_x + I_{xy} M_y}{E(I_{xy}^2 - I_{xy}^2)}
\] (2.43)

The one DOF machine determines the loaded curvatures, \(K_{Lx}\) and \(K_{Ly}\). Bending the workpiece about the x-axis requires that \(K_{Ly} = 0\), and \(K_{Lx}\) is determined by the servo. So equation (2.43) reduces to

\[
K_{Uy} = \frac{I_y M_x + I_{xy} M_y}{E(I_{xy}^2 - I_{xy}^2)}
\] (2.44)

The one DOF machine can only control \(K_{Ux}\); \(K_{Uy}\) is dependent on \(K_{Ly}\), and the coupled bending moments, \(M_x\) and \(M_y\). If \(I_{xy}\) is small, then equations, (2.42) and (2.43), reduce to
My KUX = KLx K (2.45)  

Kuy = Ky + (2.46)

which is a case of symmetrical bending.

The one degree-of-freedom roll bending machine is sufficient for cross sections with at least one-axis of symmetry, such as I-beams, T-sections, and channels. Conventional roll assemblies can be used for I-beams and T-sections, etc, since the principal axes of these cross sections coincide with the machine axes. The one degree-of-freedom roll bending machine straightens the beam in two passes by bending the workpiece about its principal axes of inertia in each pass, path 1 in Figure 2-10. If the workpiece is bent about a principal axis, the symmetrical roll bending controller described in section 2.2 can be used for the process. However, for cross sections without a convenient axis of symmetry, such as angles, the one degree-of-freedom bending machine would require special roll design to bend the workpiece about its principal axes, and the cost of such rolls may be prohibitive.

One point to note about the one DOF bending machine, it can only shape an initially straight beam about one principal axis. Shaping the beam about the x-axis in the first pass through the machine with initial zero curvature along the y-axis is not a problem. However, shaping the beam about the y-axis in the second pass through the machine will cause the beam to bend about the x-axis, since the machine requires KLx to be zero and Kux has initial curvature.

Another alternative straightening method is to use the one degree-of-freedom roll bending machine with conventional rolls for cross sections such as angles, but bending the unsymmetrical workpiece to a desired curvature requires successive passes through the roll bending machine to eventually converge on a straight
workpiece, path 2 of Figure 2-10. The first pass through the machine bends the
workpiece about the x-axis, until $K_{UX}$ is equal to zero. After the workpiece is
unloaded, the curvature along the y-axis springs back. The second pass through the
machine, the workpiece bends about the y-axis, and after unloading, the workpiece
springs back along the x-axis. The workpiece is fed through the machine in
alternate directions until the beam is straight. This method subjects the workpiece
to repeated bending and the workpiece may strain harden due to overworking of
the material.

The coupling relation between $K_{UX}$ and $K_{UY}$ can be described by

$$K_{UX} - K_{Lx} = -(K_{UY} - K_{LY}) \frac{I_{xy} - I_x \cot \Phi}{I_y - I_{xy} \cot \Phi} \tag{2.47}$$

where

$$\cot \Phi = -\frac{M_x}{M_y} \tag{2.48}$$

For the angle in Figure 2-7, let $(M_x = 0)$ and assume elastic behavior. The coupling
between $K_{Lx}$ and $K_{LY}$ for the angle is approximately one to one. Depending on the
product of inertia, the coupling in unsymmetrical bending is not trivial and cannot
be neglected.

A two degree-of-freedom bending machine can bend an unsymmetrical beam
about an arbitrary neutral axis in one pass through the bending machine, path 3 of
Figure 2-10. The machine must have two degrees of freedom to bend the workpiece
in an arbitrary plane curve. The machine will require two servos, sensors and a
machine design to bend the workpiece about an arbitrary neutral axis.

Conventional rolls can be used for the process which eliminates unique rolls for
each workpiece as discussed previously.
2.3.5 Plastic Bending of Unsymmetrical Cross Sections

When an unsymmetrical beam is bent about a fixed neutral axis, the angle of the resultant bending forces on the workpiece remains constant in the elastic region, but changes as the material begins to yield, due to a shift in the neutral axis. The control system for unsymmetrical bending is based on elastic theory, and
errors may result from assuming that the plastic plane of loads is equivalent to the elastic plane of loads.

In bending a perfectly plastic material, the neutral axis does not necessarily pass through the centroid as in elastic bending. Instead, the neutral axis shifts such that the cross sectional area, \( A \), is divided into two equal parts above and below the neutral axis. In perfectly plastic bending, the yield stress is uniform over the area, and for equilibrium, the area in compression, \( A_C \), must equal the area in tension, \( A_T \). In Figure 2-11, the resultant tensile force, \( F_T = \sigma_{yld} A/2 \) is located at the centroid, \( c_T \) of the area, \( A_T \); the resultant compressive force, \( F_C = \sigma_{yld} A/2 \) is located at the centroid, \( c_C \) of \( A_C \). For equilibrium, the compressive and tensile forces must be equal, so the plastic bending moment is a product of the force and the distance, \( d \), between the centroids.

\[
M_{\text{plastic}} = F \ d = \frac{\sigma_{yld} A \ d}{2} \tag{2.49}
\]

The plane through the centroids, \( c_C \) and \( c_T \), defines the plane of loads for the beam [3]. Determining the plane of loads requires choosing a neutral axis and finding the centroids of the tensile and compressive areas. An iterative procedure is required to determine the plane of loads corresponding to a particular neutral axis.

The proposed bending machine determines the loaded neutral axis, based on the geometry of the machine and the workpiece. The control system assumes that the bending moments and curvatures are decoupled about the principal axes (equations (2.36) and (2.37)). However, the assumption that principal axis bending is decoupled is only valid for elastic behavior. After the material has yielded, the neutral axis may shift above or below the centroid. To satisfy equilibrium conditions, the orientation of the plane of loads may change, and the bending moments may no longer be decoupled.
Figure 2-11: Fully Plastic Bending of an Unsymmetrical Beam [3]

If the neutral axis coincides with an axis of symmetry, then the orientation of the plane of loads will remain at the same angle for both elastic behavior and plastic behavior. The neutral axis may shift, but due to the symmetry of the cross section, the angle of the plane of loads will remain constant. This implies that the superposition assumptions (based on elastic theory) used in the proposed control system, will hold for plastic bending of cross sections with at least one axis of symmetry (symmetrical T-sections, channels etc.). For cross sections without an axis of symmetry (unequal leg angles, Z-sections, etc.) the orientation of the plane of loads remains constant in the elastic region, but may change as the workpiece
exhibits elastic-plastic behavior. Bending is no longer decoupled along the principal axes.

For example, the unequal leg angle, shown in Figure 2-12, does not have an axis of symmetry. To compute the plane of loads using perfectly-plastic theory requires the following properties and the dimensions in Figure 2-12.

\[ x_o = 19.74 \text{ mm} \]
\[ y_o = 39.74 \text{ mm} \]
\[ \theta = 23.78^\circ \]

First, it is assumed that the neutral axis coincides with the principal X-axis.

\[ \alpha = \theta = 23.78^\circ \]

By elastic theory, the plane of loads will parallel the principal Y-axis,

\[ \phi = 90^\circ + \alpha \]

and this implies that the bending moment, \( M_Y \), is zero. Perfectly plastic theory requires an iterative procedure. First, it is assumed that the neutral axis passes through the centroid. From the geometry of Figure 2-12, the area on one side of the neutral axis (which includes point B) \( A_B \), and the area other side of the neutral axis (which includes point A) \( A_A \), are computed.

\[ A_A = 867 \text{ mm}^2 \]
\[ A_B = 1032 \text{ mm}^2 \]

But plastic theory requires that \( A_A = A_B \), so \( A_A \) is increased, and \( A_B \) is decreased by shifting the neutral axis from the centroid. The new areas are
\[ A'_A = 867 + \frac{(1032 - 867)}{2} \]
\[ A'_B = 867 - \frac{(1032 - 867)}{2} \]

The neutral axis is shifted by a distance of 9.15 mm, perpendicular to the neutral axis, toward point A. The centroids of \( A'_A \) and \( A'_B \), \( C_A \) and \( C_B \), are then computed from the new geometry. The centroids of \( A'_A \) and \( A'_B \) are located at

\[
(x_A', y_A) = (-14.78, 32.73) \\
(x_B', y_B) = (14.78, -32.73)
\]

The line passing through both centroids has angle of 114.3°. The plane of loads from plastic theory and elastic theory, for bending about the principal X-axis, are

\[ \Phi_{X_{\text{elastic}}} = 113.8° \quad \Phi_{X_{\text{plastic}}} = 114.3° \]

The difference between the angles is only 0.5°, and the error may be caused by rounding off in the calculations. For bending the angle about the principal X-axis, the proposed control system should give satisfactory results.

A similar computation, for bending the angle about the principal Y-axis, results in the neutral axis shifting by a distance of 0.81 mm towards point C (perpendicular to the neutral axis). The centroid of the area which includes point C is located at

\[
(x_C, y_C) = (10.6, 11.2)
\]

and the centroid of the area which includes points A and B is located at

\[
(x_{AB}, y_{AB}) = (-10.6, -11.2)
\]

The line passing through both centroids has an angle of 46.78°. The angles of the
plane of loads for plastic and elastic theory, for bending the angle about the principal Y-axis, are

\[ \phi_{\text{elastic}} = 23.78^\circ \quad \phi_{\text{plastic}} = 47.78^\circ \]

The difference between the two angles is 23°, and this difference is not negligible. The control system, which is based on elastic theory, will see a maximum error of

\[ K_{UY} = M_{\text{plastic}} \sin(23^\circ)/(E I_X) \] (2.50)

and will try to compensate even though the actual value of \( K_{UY} \) is zero.

Since the intended application is straightening relatively stiff beams, the
workpiece will not reach fully plastic behavior. The error of the angle of the plane of loads will be somewhere between 0° and 23°, so the controller will see an error less than the error predicted by equation (2.50). Fully plastic theory requires an inverse method for calculating the bending moment and the plane of loads, so it does not lend itself to computational analysis. Experiments are needed to investigate the significance of this error.

2.4 MEASUREMENT MODEL

The proposed unsymmetrical roll bending controller requires measuring the loaded curvature, Kx and Ky, the bending moments, Mx and My. In addition, the location of the principal axes of the beam and the elastic moment-curvature slopes of each principal axis, 1/(EIy) and 1/(EIx), must be provided to the controller. Recommendations for measuring the loaded curvatures and the bending moments are presented, and on-line methods for locating the principal axes of inertia and computing the elastic moment-curvature slopes for each principal axis are discussed.

A two degree-of-freedom roll bending machine is shown in Figure 2-13. The machine is designed to bend the workpiece in a plane cutting through the machine z-axis. To avoid applying a pinch bending moment at the outer rolls, the outer rolls are mounted on gimbals. The gimbals allow the outer roll assembly to rotate freely in both the x-z plane and in the y-z plane. The center roll assembly is also mounted on gimbals (not shown). This allows the center roll to rotate to seek the
maximum bending moment and to avoid applying a *pinch* bending moment. To simplify the computations, each roll assembly is assumed to be adjusted such that the intersection of the gimbal axes coincide with the shear center of the workpiece. Each gimbal will act as a ball joint attached to the shear center of the workpiece at each roll. For bending without twisting, the bending axis is the line passing through the center of rotation of each set of gimbals. The centroidal axis of the workpiece will then parallel the bending axis. (The gimbals do pose a design problem. To feed the workpiece through the machine, at least one of the roll assemblies must have a drive mechanism. One solution is to apply the torque to the rolls with a flexible shaft.) Servos move the center roll assembly in the x-y plane to bend the beam about a specified neutral axis. The displacement of the center roll assembly along the x and y axes is designated by \( x_p \) and \( y_p \).

A schematic of the measurement model for the beam in the x-z plane is shown in Figure 2-16. The roll assemblies at A, B, and C are free to rotate. The loaded curvature, \( K_L \), is measured from the center roll, B. And forces, \( F_{xA} \), \( F_{yA} \), and \( F_{zA} \) are measured at the outer roll, A. Outer roll A is positioned a distance \( a \) from the center roll B, and outer roll C is positioned a distance \( b \) from the center roll B. At roll B, the workpiece is assigned a coordinate system, \( (x_{wp}, y_{wp}, z_{wp}) \). The measurement model for the y-z plane is analogous to Figure 2-16.

### 2.4.1 Coordinate Transformation From Machine Coordinates to Workpiece Coordinates

The bending moments and the curvatures should be measured with respect to the workpiece coordinate system, \( (x_{wp}, y_{wp}, z_{wp}) \). For simplicity in computations, the coordinate systems are assumed to rotate about a point. To transpose the bending moments or curvatures measured in machine coordinates, \( (x, y, z) \), to the
workpiece coordinate system, \((x_{wp}, y_{wp}, z_{wp})\), requires measuring the rotation of the gimbals at the center roll, Figure 2-14. Let \(\psi_y\) be the angle of the rotation of the outer gimbal with respect to the machine y-axis (measured positive for CCW rotation about the y-axis). Let \(\psi_{xOG}\) represent the angle of rotation of the inner gimbal (measured positive for CCW rotation about the outer gimbal \(x_{OG}\) axis). The coordinate transformation from the machine axis system, \((x,y,z)\), to the outer gimbal axis system, \((x_{OG}, y_{OG}, z_{OG})\), is described by

\[
\begin{bmatrix}
x_{OG} \\
y_{OG} \\
z_{OG}
\end{bmatrix} =
\begin{bmatrix}
\cos\psi_y & 0 & -\sin\psi_y \\
0 & 1 & 0 \\
\sin\psi_y & 0 & \cos\psi_y
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
\]

and is seen in Figure 2-15a. The coordinate transformation from the outer gimbal axis system, \((x_{OG}, y_{OG}, z_{OG})\), to the inner gimbal system or the workpiece axis system, \((x_{wp}, y_{wp}, z_{wp})\), is

\[
\begin{bmatrix}
x_{wp} \\
y_{wp} \\
z_{wp}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 \\
0 & \cos\psi_{xOG} & \sin\psi_{xOG} \\
0 & -\sin\psi_{xOG} & \cos\psi_{xOG}
\end{bmatrix}
\begin{bmatrix}
x_{OG} \\
y_{OG} \\
z_{OG}
\end{bmatrix}
\]

as seen in Figure 2-15b. Now, the coordinate transformation for the machine axis system, \((x,y,z)\), to the workpiece axis system, \((x_{wp}, y_{wp}, z_{wp})\), is

\[
\begin{bmatrix}
x_{wp} \\
y_{wp} \\
z_{wp}
\end{bmatrix} =
\begin{bmatrix}
\cos\psi_y & 0 & -\sin\psi_y \\
\sin\psi_{xOG} & \cos\psi_y & \cos\psi_{xOG} \\
\cos\psi_{xOG} & \sin\psi_{xOG} & \cos\psi_{xOG}
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
\]

where \(C_{mach_{wp}}\) is designated as the above transformation matrix.

The bending moments and curvatures measured in machine coordinates can then be transposed into the workpiece coordinates. The control would then be performed in the workpiece coordinate system. The transformation matrix, \(C_{mach_{wp}}\),
Figure 2-14: Schematic of Center Roll Assembly

The \((x,y,z)\) axis system is attached to the frame positioned by the servos and corresponds to the machine axis system. The \((x_{OG},y_{OG},z_{OG})\) axis system is attached to the outer gimbal. The \((x_{wp},y_{wp},z_{wp})\) axis system is attached to the inner gimbal and coincides with the workpiece axis system.

would then be inverted to compute the commands to the servos.

For a straightening process, \(\psi_{xOG}\) and \(\psi_{y}\), will be small, and the transformation matrix, \(C_{wp}^{\text{mach}}\), reduces to an identity matrix, and both the machine and the workpiece coordinate systems coincide. For processes involving large curvatures, \(\psi_{xOG}\) and \(\psi_{y}\), will not be negligible, and the coordinate transformation should be included in the control system.

2.4.2 Loaded Curvature Measurements

Hale [10] made a thorough analysis of several methods of measuring the loaded curvature for the symmetrical bending process, and the reader is encouraged
Figure 2-15: Rotational Transformation of Axes Systems

(a) Rotation of the Outer Gimbal Axes with respect to the Machine Axes. The angle, $\psi_y$, is the angle of the rotation of the outer gimbal with respect to the machine y-axis. (b) Rotation of the Inner Gimbal or Workpiece Axes with respect to the Outer Gimbal Axes. The angle, $\psi_{xOG}$, is the angle of rotation of the inner gimbal with respect to the outer gimbal.

to read this for greater insight. At the yield limit, the curvature of a straight beam is assumed to be constant from outer roll A to outer roll B. As the outer fibers of the beam pass beyond the elastic limit, two zones of yielded material begin to grow and extend from maximum moment at the center roll to the yield moment. In the limiting case for fully plastic behaviour, the entire cross section has yielded and forms a plastic hinge. For fully plastic behaviour, the curvature will approach infinity at the center roll and the sections of the beam between the center roll and the outer rolls can be considered as leaves on a hinge [5]. This implies that the large curvatures should be measured as close as possible to the center roll.

The loaded curvature is measured at the center roll B. The most accurate method of measuring the curvature is to measure the curvature orthogonal to the $z_{wp}$ axis of the workpiece (Figure 2-16). This requires attaching curvature sensors to the roll assembly, so that the curvature sensors rotate with the workpiece. The
curvature sensors are mounted on the inner gimbal at the center roll.) This method of measuring the loaded curvature, \( K_L \), assumes that the curvature is constant in the region of the maximum moment (near the center roll). Based on the geometry of Figure 2-17 and employing small angle approximations, the curvature, \( K_L \) can be found from

\[
K_{L2} = \frac{1}{R_2} = \frac{2.0 h_2}{s_2^2 + h_2^2}
\]

However, \( h_2 \) cannot be measured directly, so the displacements, \( h_{2xwp} \) and \( h_{2ywp} \), are measured by transducers mounted a distance, \( s_2 \), from the center roll along the \( z_{wp} \)-axis as shown in Figure 2-16. Figure 2-17 illustrates the geometrical relations needed to find \( h_2 \) and the angle of the neutral axis, \( \alpha \).

\[
h_2 = \sqrt{h_{2xwp}^2 + h_{2ywp}^2}
\]

(2.55)

\[
\tan \alpha = \frac{h_{2xwp}}{-h_{2ywp}}
\]

(2.56)

Substituting (2.55) into (2.54) and solving for \( K_x \) and \( K_y \) results in

\[
K_{Lxwp} = K_{L2} \sin \alpha = \frac{2.0 h_{2xwp}}{s_2^2 + h_{2xwp}^2 + h_{2ywp}^2}
\]

(2.57)

\[
K_{Lywp} = -K_{L2} \cos \alpha = \frac{2.0 h_{2ywp}}{s_2^2 + h_{2xwp}^2 + h_{2ywp}^2}
\]

(2.58)

The curvature at point 1 can be found in the same manner. Displacements, \( h_{1xwp} \) and \( h_{1ywp} \), are measured by transducers mounted a distance, \( s_{1xwp} \) from the center roll along the \( z_{wp} \)-axis.

\[
K_{L1xwp} = K_{L1} \sin \alpha = \frac{2.0 h_{1xwp}}{s_1^2 + h_{1xwp}^2 + h_{1ywp}^2}
\]

(2.59)

\[
K_{Ly1} = -K_{L1} \cos \alpha = \frac{2.0 h_{1ywp}}{s_1^2 + h_{1xwp}^2 + h_{1ywp}^2}
\]

(2.60)

The loaded curvature for the x-axis is the average of the two measured curvatures.
and the loaded curvature for the y-axis is an average of the two measured curvatures.

\[ K_{Ly} = \frac{K_{Ly1} + K_{Ly2}}{2.0} \]  

The resolution of the transducers will determine the range of the dimensions: \( h_{1xwp}, h_{2xwp}, h_{1ywp}, \text{ and } h_{2ywp} \). If the shaping process requires large curvatures then the dimensions, \( s_1 \) and \( s_2 \), should be as small as the resolution of the transducers will allow.

Ideally, the curvature would be measured at the neutral axis; since that is not possible, the curvature is measured somewhere on the perimeter of the cross section. The radius of curvature, \( R \), is usually very large (dependent on the process) compared to the distance from the neutral axis to the perimeter of the cross section, \( (x,y) \). Thus the error is usually negligible. If large curvatures are required in the process, then \( x \) and \( y \) should be included in equations: (2.57), (2.58), (2.59), and (2.60).

Choosing where to measure on the perimeter of the cross section is dependent on the type of cross section. For instance, the legs of an angle-section tend to bow during bending, so the curvature on an angle-section should be measured on the outer perimeter at the intersection of the legs and orthogonal to the legs. Another example is a T-section. The top of the web tends to buckle in compression, and the web tends to bow when the flange is bent (the flange bends plastically, while the web bends elastically). The curvature should be measured close to the intersection of the web and flange; for instance, measuring the curvature orthogonal to the bottom of the flange, and measuring the curvature orthogonal to the web at the
Figure 2-10: Measurement Model for the x-z Plane
Figure 2.17: Relation of Transducer Displacement to Curvature and Neutral Axis
edge of the flange. A proposed guideline is to measure the curvature orthogonal to the stiffest portion of the cross section. If the beam is subjected to twisting, the curvature should be measured close to the shear center (see Chapter 4).

For a straightening process, the curvature will be very small and should not extend appreciably beyond the yield curvature. If the beam has a high bending stiffness, the loaded curvature can be measured from the center roll displacement rather than complex curvature sensors [10]. The relation between the loaded curvature and the center roll displacement are described by equations:

\[
K_{Lx} = \frac{3_x p}{a \ b} \quad (2.63)
\]

\[
K_{Ly} = \frac{3_y p}{a \ b} \quad (2.64)
\]

where \(a\) and \(b\) are the distances between the center roll and each outer roll, respectively. For a straightening process, the gimbals at the center roll will not rotate, so the coordinate transformation matrix, \(C_{wp}^{\text{mach}}\), defined in equation (2.53) reduces to an identity matrix. Thus, the workpiece coordinate axes will coincide with the machine coordinate axes. Since the proposed application of this research is to develop a machine for straightening beams, measuring the loaded curvature from the center roll displacement should be sufficient. Static experiments experiments should be conducted to confirm this assumption.

2.4.3 Bending Moment Measurements

The most accurate method of measuring the bending moments experienced by the workpiece at the center roll is to measure the forces at one of the outer rolls, \(F_{xA}, F_{yA}, \text{ and } F_{zA}\) (point A in Figure 2-16). The bending moment experienced by the workpiece, at the center roll in machine coordinates, is
where \( x_{BA}, y_{BA}, \) and \( z_{BA} \) are the moment arms measured from the center roll (point B) to the outer roll (point A). The dominant forces in the bending moment measurements are, \( F_{xA} \) and \( F_{yA} \), and \( F_{zA} \) is significant in a shaping process. Similarly, the dominant moment arm is \( z_{BA} \), and, \( x_{BA} \) and \( y_{BA} \), are significant in a process requiring large curvatures [10].

The bending moments measured in machine coordinates, equations (2.65), (2.66), and (2.67), can then be transposed into the workpiece coordinates.

\[
\begin{bmatrix}
M_{xwp} \\
M_{ywp} \\
M_{zwp}
\end{bmatrix} = \begin{bmatrix}
C_{\text{mach}}^{\text{wp}} \\
\end{bmatrix} \begin{bmatrix}
M_x \\
M_y \\
M_z
\end{bmatrix}
\]  
(2.68)

where \( C_{\text{mach}}^{\text{wp}} \) is the transformation matrix in equation (2.53). For a straightening process, \( \psi_{xOG} \) and \( \psi_y \), will be small, and the transformation matrix, \( C_{\text{mach}}^{\text{wp}} \), reduces to an identity matrix, so both the machine and the workpiece coordinate systems coincide. For processes involving large curvatures, \( \psi_{xOG} \) and \( \psi_y \), will not be negligible, and the coordinate transformation should be included in the control system. The bending moment, \( M_{zwp} \), should go to zero, since the workpiece is loaded through the shear center.

For a straightening process, assume \( C_{\text{mach}}^{\text{wp}} \) is an identity matrix. Substituting the notation of Figure 2-16, equations, (2.65) and (2.66), can be rewritten, and the bending moments become
For a straightening process, the moment arms, $x_{BA}$ and $y_{BA}$, are relatively small, as is the force, $F_{zA}$. For a straight symmetrical beam subjected to simple three-point bending, the deflection, $x_p$, can be found from

$$x_{p_{max}} = \frac{M_{max} L^2}{3 E I}$$  \hspace{1cm} (2.70)

Integrating equation (2.2) to the outer fiber and substituting into equation (2.70) results in

$$x_p = \frac{\sigma_{zz} L^2}{3 E x}$$  \hspace{1cm} (2.71)

Substituting $\sigma_{yld} = 48 \times 10^3$, $E = 10.6 \times 10^6$, $x = d/2$, and $L = 5d$ (where $d$ is the maximum diameter of the section) results in a yield deflection of

$$x_{pyld} = 0.075 \frac{d}{d} = 0.015 L$$

If each of the moments arms, $x_{BA}$ and $y_{BA}$, are assumed to be equivalent to $x_{pyld}$, and the moment arm, $z_{BA}$, is made equivalent to $L$, equation (2.71) becomes

$$\begin{bmatrix} M_x \\ M_y \end{bmatrix} = \begin{bmatrix} -0.015L F_{zA} - L F_{yA} \\ L F_{xA} + 0.015L F_{yz} \end{bmatrix}$$  \hspace{1cm} (2.72)

Since both $x_{BA}$ and $y_{BA}$ are small compared to $z_{BA}$, and the force, $F_{zA}$ is small in a straightening process, the moments from the products of $x_{BA} F_{zA}$ and $y_{BA} F_{zA}$ can be neglected. An increase in the $L/d$ ratio will increase the contribution of these bending moments. The magnitude of the deflection is also dependent on the maximum diameter of the cross section, $d$, so stiff cross sections will contribute very little to the moments, $x_{BA} F_{zA}$ and $y_{BA} F_{zA}$. 

\[
\begin{bmatrix} M_x \\ M_y \end{bmatrix} = \begin{bmatrix} -y_p F_{zA} - a F_{yA} \\ a F_{xA} + x_p F_{zA} \end{bmatrix}
\]  \hspace{1cm} (2.69)
An alternative, but less accurate, method of measuring the bending moments, $M_x$ and $M_y$, is to sense the forces, $F_x$ and $F_y$, at the center roll; however, this method should be sufficient for a straightening process and for stiff workpieces. Let, $a$ and $b$, be the distances between the center roll and each outer roll. The relation between the bending moments and the forces at the center roll is

$$M_x = \frac{-a}{a+b} \frac{b}{a} F_y$$

(2.73)

$$M_y = \frac{a}{a+b} \frac{b}{a} F_x$$

(2.74)

**2.4.4 On-line Computation of the Principal Axes of Inertia**

If the neutral axis of the workpiece coincides with a principal axis of inertia, the plane of loads will be orthogonal to the neutral axis in the elastic region. The principal axes can be found on-line by holding the workpiece at a constant curvature, $K_x$, and varying the curvature, $K_y$, in the elastic region. The curvatures, $K_x$ and $K_y$, and bending moments, $M_x$ and $M_y$, are monitored and used to determine the angle defining the plane of loads, $\phi$,

$$\cot \phi = -\frac{M_y}{M_x}$$

(2.75)

and the angle defining the neutral axis, $\alpha$.

$$\tan \alpha = -\frac{K_{LX}}{K_{LY}}$$

(2.76)

When

$$|\alpha - \phi| = \pi/2$$

(2.77)

then the neutral axis coincides with a principal axis, so the angle defining the neutral axis is a principal axis.
2.4.5 On-line Computation of the Elastic Moment-Curvature Slope

Once the principal axes have been found, the workpiece is loaded along each of the principal axes to find the elastic moment-curvature slopes, $1/(E I_Y)$ and $1/(E I_X)$. Equations (2.25) and (2.26) define the elastic moment-curvature relation for the principal axes. Computing the slopes requires several measurements of $K_X$, $K_Y$, $M_X$ and $M_Y$ in the elastic region. Since $K_X$, $K_Y$, $M_X$ and $M_Y$ cannot be measured directly, $K_X$, $K_Y$, $M_X$ and $M_Y$ must be transformed into principal coordinates. Now the slopes can be computed on-line. Chapter 3, Section 3.7.3 discusses the range and type of data needed for computing the slopes.

2.4.6 Set-up Procedure for a Workpiece with an Arbitrary Cross-section

1. Perform the procedure for locating the principal axes of inertia. If the properties of the cross-section: $A$, $I_x$, $I_y$, and $I_{xy}$, can be easily obtained, then the theoretical location of the principal axes of inertia can be computed from equation (2.19), page 43. Knowing the theoretical principal axes of inertia will reduce the iterative procedure for finding the principal axes on-line.

2. Perform the procedure for computing the elastic moment-curvature slopes, $1/(E I_X)$ and $1/(E I_Y)$, from experimemtal data obtained by loading an initially straight workpiece.
3.1 INTRODUCTION

Understanding and controlling the roll twisting process is essential to developing a shape control system for beams. This chapter addresses the problem of automating the process of continuously twisting a beam to a desired shape or "roll twisting". The workpiece is subjected to pure torsion by applying a torque about the shear center of the workpiece. The goal of the process is to achieve the desired unloaded angle of twist at each point along the beam.

The roll twisting process shares many similarities with the symmetrical roll bending process, but major differences between the two processes prevent the direction application of the symmetrical roll bending control system (Chapter 2, Section 2.2). The major problem in controlling the roll twisting process is achieving point by point deformation of the workpiece since a constant torque, applied over a section of the workpiece, plastically deforms the entire section. A mathematical model of the roll twisting process is presented and analyzed to develop a suitable control system for the process. A preliminary design of a control system for the roll twisting process is presented. Further research and an experimental model is required to confirm this preliminary investigation.
3.2 TORSION MODEL OF WORKPIECE

3.2.1 Assumptions

The assumptions used in the model of the roll twisting process include:

- The workpiece is a beam with a constant cross section of arbitrary shape.

- The workpiece is made of an elastic-perfectly plastic material with the stress-strain relation shown in Figure 2-1.

- The workpiece has an equal and opposite torque, $T_c$, applied at the shear center (or the center of twist), $C$, over the length, $L$, of the beam as shown in Figure 3-1. Applying the twisting couple at the shear center avoids inducing bending stresses in the workpiece [13].

- For sufficiently long torsion members, Saint Venant's principle assumes that the surface of the beam is free of stress if the cross section under study is a sufficient distance from the point where the torque is applied to the workpiece. This assumption allows the contact stresses to be neglected. Advanced studies have shown that the stress-strain relations for Saint Venant's torsion are valid within one or two diameters from where the torque is applied to the external surface of the member [22].

3.2.2 Elastic Torsion

The torque-rate of twist relationship for an elastic material under torsion is given by

$$T_{celastic} = G \kappa_T \frac{d\phi}{dz_{rel}} \quad (3.1)$$

where $\kappa_T$ is a torsional constant dependent on the shape of the cross section and $G$ is the shear modulus of elasticity dependent on the material properties of the workpiece. The torsional constant (or shape factor), $\kappa_T$, is computed from
membrane analogy, and has been derived for common cross sections by Heins [13] and Boresi [3]. For a circular cross section, the shape factor, \( \kappa_T \), is equivalent to the polar moment of inertia, \( J \).

The torque is constant over the length of the beam, as shown in Figure 3-2. The rate of twist, \( \frac{d\phi}{d\zrel} \), is the relative rotation between two transverse plane cross sections. Since the torque is constant, the angle of twist, \( \phi \), is proportional to the position on the beam, \( \zrel \) (Figure 3-3). Since \( T_c \), \( G \), and \( \kappa_T \) are constant with respect to \( \zrel \), equation (3.1) can be integrated to give

\[
\phi(\zrel) = \frac{T_c \zrel}{G \kappa_T}
\]

3.2.3 Torque-Unloaded Angle of Twist Relation

For simplicity in the analysis, let the workpiece be a solid shaft subjected to an equal and opposite torque and made of an elastic-perfectly plastic material. The shear stress for a solid shaft, \( \tau \), is found from Hooke's law and symmetry arguments.

\[
\tau = G \frac{\phi(\zrel)}{\zrel}
\]

where \( r \) is the radial distance from the centroid. If the shaft is twisted in the elastic region \( (r \leq r_{\text{yld}}) \), the shear stress is proportional to the radius, \( r \),

\[
r \leq r_{\text{yld}}, \quad \tau_{\text{elastic}} = \frac{r}{r_{\text{yld}}}
\]

In the outer plastic region, the shear stress is a constant.

\[
r > r_{\text{yld}}, \quad \tau_{\text{plastic}} = \tau_{\text{yld}}
\]

Equilibrium requires
Figure 3-1: Beam subjected to Torsion

Figure 3-2: Torque versus Position Relationship

Figure 3-3: Angle of Twist versus Position Relationship


\[ T_c = \int_A r \tau \, dA \]  

\[ (3.6) \]

Substituting equations, (3.4) and (3.5), into equation (3.6), and integrating, and then solving for \( T_{cyld} \) and \( \phi_{yld} \) results in

\[ \phi \leq \phi_{yld}, \quad T_{\text{elastic}} = G \frac{J}{z_{rel}} \]  

\[ (3.7) \]

\[ \phi > \phi_{yld}, \quad T_{\text{plastic}} = \frac{4}{3} \frac{T_{cyld}}{\phi_{yld}} \left( 1 - \frac{4}{3} \phi_{yld}^3 \right) \]  

\[ (3.8) \]

Assuming the material unloads elastically, the unloading will parallel the original torque-angle slope in the elastic region. The relationship between the applied torque and the angle of twist is shown in Figure 3-4. If the applied torque, \( T_c \), is less than the yield torque, \( T_{cyld} \), plastic deformation will not occur, and the beam will springback to the original angle, \( \phi_{initial} \). Plastic deformation occurs when \( \phi \) is loaded past \( \phi_{yld} \). Assuming the material of the beam unloads elastically, the springback at some point, \( z_{rel} \), along the beam can be computed from equation (3.9).

\[ \phi_U(z_{rel}) = \phi_L(z_{rel}) - \frac{z_{rel}}{G \kappa_T} T_c \]  

\[ (3.9) \]

where \( \phi_U(z_{rel}) \) is the unloaded angle of twist at position, \( z_{rel} \), and \( \phi_L(z_{rel}) \) is the loaded angle of twist at position, \( z_{rel} \) [5].

The torque-angle of twist relationship of Figure 3-4 is dependent on the shape of the cross section and the material properties of the beam. Material properties, such as the yield stress and strain hardening, can vary for different materials. Computing the plastic deformation of the cross section involves a limit analysis based on the shape of the cross section and the material properties. Hardt [12] and Hale [10] eliminated the problems of predicting the plastic deformation with limit analysis by measuring the bending moment and loaded curvature for the
symmetrical roll bending process. The same analogy is used for the twisting process.

If the torque, $T_c$, and the loaded angle of twist, $\phi_L(z_{rel})$ are measured, then the predicted springback can be computed from equation (3.9). In addition, the torsional rigidity, $(G \kappa_T)$, can be measured by elastically loading a straight beam and measuring the torque, $T_c$, and the loaded angle of twist, $\phi_L(z_{rel})$, at some position, $z_{rel}$. The torsional rigidity, $(G \kappa_T)$, can then be computed from equation (3.2). Now the unloaded angle of twist relies only on the assumption that the beam
will springback elastically; all other properties are directly measured from the workpiece. This eliminates computing the shape factor, \( \kappa_T \), which requires relatively sophisticated mathematical techniques for unsymmetrical cross sections.

The torque-angle of twist relationship is also dependent on the past history of the workpiece. By measuring the moment and the loaded angle of twist in real time and assuming elastic springback, there is no need to account for the past history of the workpiece.

### 3.3 ROLL TWISTING APPARATUS

The beam is continuously fed through the roll twisting machine at a feedrate, \( f \), in the direction indicated in Figure 3-5. The rolls serve two functions; one function is to apply a torque about the shear center of the workpiece, and the other function is to allow the workpiece to move continuously through the apparatus. Each particular workpiece requires particular roll shapes and roll configurations. The roll assemblies are mated or attached to the roll twisting apparatus in the manner required to ensure that the torque will be applied with respect to the shear center, \( C \), of the workpiece. In Figure 3-5, a T-section is twisted. The roll assemblies are designed to have the shear center, \( C \), of the T-section coincide with the machine center of twist. The line through the shear center at each and every point on the beam is a straight line, so the beam does not bend. This means that bending and twisting are decoupled, and further discussion of the shear center is in Chapter 4, Section 4.2.1.

The torque, \( T_c \), is applied to the workpiece by holding one roll assembly fixed, while the other roll assembly rotates. The fixed roll assembly, at \( z_{rel} = 0 \), is rigidly attached to the roll twisting apparatus, while the roll assembly at \( z_{rel} = L \) is
rotated by the servo. The torque is applied over the length, L, of the beam in Figure 3-5. And the applied torque is constant over the length, L, as shown in Figure 3-2. Figure 3-3 shows the result of an initially straight beam twisted by the roll twisting apparatus.

An angle measurement device is used to measure the angles at position, \( z_{1\text{rel}} \). The device is designed to rotate with the workpiece, while remaining in the x-y plane at position, \( z_{1\text{rel}} \). The torque applied by the device is caused by friction and inertia effects, and should be negligible. A transducer is used to measure the amount the plate has rotated. The measurement model will be discussed in more detail in Section 3.7.1.

If the beam were considered stationary, then the roll twisting apparatus could be considered as a window that moves along the beam in the opposite direction of the feedrate, \( f \). A position on the moving beam is defined as \( z_{\text{abs}} \), where \( z_{\text{abs}} = 0 \) is one end of the beam and is independent of the window's location on the beam, (or the relation of the beam to the machine). The angle \( \phi(z_{\text{abs}}) \) defines the angle at position, \( z_{\text{abs}} \). The position of the beam relative to the machine, or window, is defined as \( z_{\text{rel}} \), where \( z_{\text{rel}} = 0 \) is the position on the beam where the beam passes out of the window, and \( z_{\text{rel}} > 0 \) is the portion of the beam still influenced by the machine. The angle \( \phi(z_{\text{rel}}) \) defines the angle at position \( z_{\text{rel}} \).

### 3.4 PRIMARY CONTROLLER

Assume the beam has some initial angle of twist, \( \phi_U(z_{\text{abs}}) \), along the beam, and the objective of the control system is to produce \( \phi_{U\text{des}}(z_{\text{abs}}) = 0 \) at each and every point along the beam on completion of the roll twisting operation. For the initial analysis, assume that the system is able to measure the angle relative to \( z_{\text{abs}} \).
Figure 3.5: Schematic of the Roll Twisting Apparatus
\[
\phi_U(z_{rel}) = \phi_L(z_{rel}) - \frac{z_{rel}}{G \kappa_T} \frac{T_c}{\kappa_T}
\]

(3.10)

The feedback controller generates an error signal by comparing the indirectly measured unloaded angle, \(\phi_U(z_{rel})\), to the desired unloaded angle, \(\phi_{Udes}(z_{abs})\). The error signal then commands the servo to rotate the workpiece in the manner required to null the error signal. The primary controller shown in Figure 3-6, causes the workpiece unloaded angle, \(\phi_U(z_{rel})\), to track the desired unloaded angle, \(\phi_{Udes}(z_{rel})\).

The design of the primary controller is based on the controller Hale designed for the roll bending operation [10]. Hale used a velocity servo in his experiments and found the steady state error to be negligible, so he was able to use a simple proportional controller. A proportional controller with velocity feedback results in a second order closed loop transfer function:

\[
\frac{\phi_U(z_{rel})}{\phi_{Udes}(z_{rel})} = \frac{G}{s^2 + 2 \zeta \omega_n + \omega_n^2}
\]

(3.11)

where \(\zeta\), the damping ratio, was designed to be sufficiently damped to avoid overshoot, and the bandwidth of the system as large as the servo and discrete time analysis will allow. The dynamics of the vibration of the workpiece were ignored for the preliminary analysis and will be addressed in section 3.6.10.

The primary controller response is to a step input is shown in Figure 3-7.
Figure 3-6: Block Diagram of the Primary Controller

The objective is to straighten a static, initially twisted beam, so the desired unloaded angle of twist, so $\phi_{U_{\text{des}}} = 0.0$. Initially, the beam is twisted such that the unloaded angle, $\phi_U$ at position $z_{\text{rel}}$, is $-0.3^\circ$. At time $t = 0$, the control system measures $\phi_{L}(z_{\text{rel}})$ and $T_c$ and predicts (from equation (3.9)) that the $\phi_{U}(z_{\text{rel}})$ is $-0.3^\circ$. The angle from the torque measurement, $T_c/G\kappa_T$ is a $0.3^\circ$. The desired angle is compared to the measured angle to generate an error signal of

$$e = \phi_{U_{\text{des}}}(z_{\text{rel}}) - \phi_{U}(z_{\text{rel}}) = -0.3^\circ \quad (3.12)$$

The error signal is used to command the servo to twist the beam, and $\phi_L$ increases. But at the time of 0.35 seconds, $\phi_{U}(z_{\text{rel}})$ still hasn't moved from an angle of $-0.3^\circ$.

The dead zone exhibited in Figure 3-7 is caused by the elastic behaviour of the beam. Referring to Figure 3-4, one can see that plastic deformation will not
Figure 3-7: Primary Controller Step Response

Stationary Beam, (feedrate, f = 0). The angle is measured at position, \( z_{rel} = L \).
occur until the beam is twisted beyond the elastic limit. The dead zone will exist, \( \phi_U = 0 \), until \( \phi \) is greater than \( \phi_{yld} \) (Figure 3-4). In Figure 3-7, the initial torque, \( T_c \) is not zero, so the horizontal axis in Figure 3-4 translates to correspond to \( T_c \) and \( \phi_L \) at time zero. The shifted torque-angle of twist relation is shown in Figure 3-8. The loading path begins at point A, where \( \phi_U = -0.3^\circ \). The loaded angle, \( \phi_L \), starts at 0 and then increases causing the torque to increase. The beam is still in the elastic region from point A to point B, so the unloaded angle does not change and causes the dead zone. At point B, \( t = 0.4 \) seconds, plastic deformation begins, and the unloaded angle begins to increase along with \( \phi_L \) and \( T_c \) until point C is reached. At point C, \( \phi_U \) is equal to \( \phi_{U\text{des}} \) and the control system reaches steady-state.

\[ \phi_YLD = 4.5^\circ \]

\[ \phi_L = 4.5^\circ \]

**Figure 3-8:** Shifted Torque-Angle of Twist Relation
The primary controller is applied to a stationary workpiece with an initial angle, $\phi_U(z_{\text{abs}},0)$ in Figure 3-9. The torque is applied over the portion of the beam, $L$, and at time, $t$, the beam is deformed as illustrated by the loaded angle of twist, $\phi_L(z_{\text{abs}},t)$, and unloaded angle of twist, $\phi_U(z_{\text{abs}},t)$, (Figure 3-9).

3.5 PRELIMINARY CONTROL ANALYSIS

This section investigates several methods for controlling the roll twisting process. The analysis of each approach assists in understanding the roll twisting process and eventually leads to a satisfactory controller design, Section 3.6.

3.5.1 Primary Controller Applied to a Moving Beam

The first approach applies the primary controller to a moving beam, but the simulation of the controller does not result in the anticipated angle, $\phi_{\text{Udes}}$, instead the outcome is shown in Figure 3-10. The window (machine), is 12 units long, and 48 units of the beam have passed through the window. The portion of the beam still within the window is designated by $L$.

To understand what is happening, the analysis follows a particular point on the moving beam, $z_{\text{abs}}$. Initially, the controller sees the error, $e$,

$$ e = \phi_{\text{Udes}}(z_{\text{abs}}) - \phi_U(L) \tag{3.13} $$

where $\phi_U(L)$ is the relative angle with respect to the end of the beam, where $\phi_{\text{Udes}}(z_{\text{abs}}) = 0$. The servo acts to drive the error signal to zero as shown in Figure 3-9. At this time, let ($t = t_0$) and designate the angle at point $z_{\text{abs}}$ as $\phi_U(z_{\text{abs}},t_0)$.

As the beam moves through the window, the controller continually drives the
Figure 3-9: Primary Controller Straightening a Beam.

Stationary Beam, (feedrate, f = 0). The angle is measured at position, $z_{1rel} = L$. 
The workpiece is rolled through the machine at a feedrate, $f$. The angle is measured at position, $z_{rel} = L$.

Figure 3-10: Primary Controller applied to Straightening
measured angle (located at \( z_{rel} = L \)) to the desired angle. However, the point \( z_{abs} \) is still within the window, and the angle \( \phi_{U}(z_{abs}, t > t_{0}) \) changes everytime the workpiece is plastically deformed to satisfy the current error signal. When the point \( z_{abs} \) leaves the window, the angle \( \phi_{U}(z_{abs}, t) \) will no longer be influenced by the machine. Now \( \phi_{U}(z_{abs}, t) \) will no longer change and will remain the angle it was when it left the window.

This method of controlling the roll twisting process is unsatisfactory because future inputs will plastically deform a point on the beam as long as the point is still within the machine.

3.5.2 Sequencing of the Beam

This approach sequences the beam to avoid the problems of section 3.5.1. Sequencing is feeding the beam into the system a section of length, \( L \), at a time. The beam is fed into the window, then the feeding of the beam is stopped and the system twists the beam to the desired angle. Another section of length, \( L \), is fed in and the process is repeated. A computer simulation of the sequencing process is shown in Figure 3-11. Notice the nodes at each length \( L \) in Figure 3-11. If the sequencing operation was perfect (the beam is started and stopped at the exact location) each node of the beam would be twisted to the desired angle by the system. Consider the portion of the beam between the nodes. If the original \( \phi_{U}(z_{abs}) \) was a high frequency sine wave, the controller would only see the error at the nodes. The end result of such a situation may not even improve the original \( \phi_{U}(z_{abs}) \). Sequencing is not a practical solution; timing would be crucial, and it does not have the production advantages of continuously rolling the beam through the machine.
Figure 3-11: Sequencing Process applied to the Beam
The original deformation of the beam was a half-sine wave with a maximum magnitude of 10 units. The machine twists a segment of the workpiece of length, L, at a time.
3.5.2.1 Limiting the Length, L

Some of the problems associated with sequencing could be eliminated if the length, L, of the window was made very small. But, the assumption of Saint Venant's torsion implies the surface of the beam is free of stress, so the length, L, must be finite in order to neglect the contact stresses caused by applying the torque to the beam. Advanced studies have shown that the stress-strain relationships for torsion are valid to within one or two diameters from the point at which the torque is applied to the external surface of the beam [5]. The length, L, is also limited by the diameter of the rolls, as seen in Figure 3-5.

3.5.3 Incremental Sequencing of the Beam

Based on the analysis of the previous sections, this approach attempts to sequence the beam by using the sampling periods of a discrete controller correlated with the location of the angle measurement. The objective is to approximate point by point deformation by sequencing small segments of the workpiece.

Initially, the primary controller samples a sensor, measuring the loaded angle, located a distance, $z_{1rel}$, from $z_{rel} = 0$ (Figure 3-5). Assume that the feedrate, $f$, is slow enough to allow the system to respond. The sample period, $T_{sample}$, is chosen such that at the next sample time, the point $z_{abs}$ on the moving beam has passed out of the window and is no longer influenced by the system, equation (3.14)

$$T_{sample} = \frac{f}{z_{1rel}}$$  \hspace{1cm} (3.14)

Ideally, the sensor would be located an incremental (or very small) distance, $z_{nrel}$, from $z_{rel} = 0$ such that $z_{1rel}$ would approach $z_{nrel}$. Let a point on the workpiece located at $z_{abs} = z_{nrel}$ at time $t_0$. For a very small distance, $z_{nrel}$, one
can consider the beam as being stationary during the sample period 

\((t_0 \text{ to } (t_0 + T_{\text{sample}}))\). During the sample period allow the system to drive \(\phi_U(z_{\text{abs}})\) to \(\phi_{U\text{des}}(z_{\text{abs}})\). At time, \(t_0 + T_{\text{sample}}\), the beam is then moved an incremental distance, \(z_{\text{arel}}\), and the process is repeated. Now \(\phi_U(z_{\text{abs}})\) is out of the window and is no longer influenced by the system. Taking the limit, \(z_{\text{arel}} \rightarrow 0\), this system can achieve point by point shape control of the roll twisting process. In between each node, errors will exist, but the angle at each node is driven to the desired value. Figure 3-14 illustrates the concept.

3.5.4 Location of Angle Measurement

The location of the angle measurement, \(z_{\text{arel}}\), cannot be located at the incremental position, \(z_{\text{arel}}\). A sensor located at \(z_{\text{arel}}\) would be required to measure a very small angle, and realistically such a sensor does not exist. In addition, the diameter of the rolls physically restrict how close \(z_{\text{arel}}\) can be to \(z_{\text{rel}} = 0\). Computer simulations investigated the effect of varying the location of the angle sensor and varying the sampling period of a discrete controller.

3.5.4.1 Deviation from the Desired Angle

The influence of the sensor location, \(z_{\text{arel}}\), is illustrated in Figures 3-9, 3-10, and 3-12. In Figure 3-9, angle was measured at \(z_{\text{rel}} = L\) or \(z_{\text{arel}} = L\). Note the error that develops in Figure 3-10 at \(z_{\text{abs}} = 12 = L\). The error occurs because after the controller has twisted \(\phi_U(z_{\text{abs}})\) to \(\phi_{U\text{des}}\), the beam moves within the window, and \(\phi_U(z_{\text{abs}})\) changes every time the workpiece is plastically deformed until point \(z_{\text{abs}}\) leaves the window. The magnitude of the error is dependent on the location of the angle measurement, \(z_{\text{arel}}\). As stated earlier, if \(z_{\text{arel}}\) approaches zero then a deviation cannot develop because the portion of the beam has passed out of the
machine. In Figure 3-12, \( z_{1\text{rel}} \) is measured at \( z_{\text{rel}} = L/4 \). The deviation of the angle from the desired angle is smaller, and similar simulations showed that the magnitude of the deviation is dependent on \( z_{1\text{rel}} \). The smaller the distance, \( z_{1\text{rel}} \) is, the smaller the magnitude of the error. The frequency of the wave is also related to the value of \( z_{1\text{rel}} \).

3.5.4.2 Drift of the Final Angle from the Desired Angle

In addition to the deviation of the angle from the desired angle, the average value of the final angle drifts away from the desired angle. The drift is dependent on the initial unloaded angle \( \phi_U(z_{\text{abs}}, 0) \), and the location of the angle measurement, \( z_{1\text{rel}} \). In Figure 3-12, one can see that the sensor location causes the average value of the angle, \( \phi_U(z_{\text{abs}}) \), to drift from the average value of \( \phi_{U\text{des}} \). The drift is related to the required control action, which is turn dependent on \( \phi_U(z_{\text{abs}}, 0) \) and \( z_{1\text{rel}} \). If the beam were long, one could expect the system to go unstable as \( \phi_U(z_{\text{abs}}) \) drifts away from \( \phi_{U\text{des}} \). Larger and larger control actions would be required to meet the error signal until the actuator would be bouncing between its limits. The control system is trying to control the angle relative to the end of the beam, but it cannot span that space, since it can only influence the portion of the beam within the machine.

The stability problem can be solved by measuring the angle relative to the machine rather than relative to one end of the beam. The simulations, measured the angle, \( \phi(z_{1\text{rel}}) \), relative to the angle at the end of the beam, \( \phi(z_{\text{abs}}=0) \). Measuring \( \phi(z_{1\text{rel}}) \) relative to \( \phi(z_{1\text{rel}}=0) \) would solve the stability problem at the cost of accuracy.
Figure 3-12: Primary Controller used for Straightening ($z_{rel} = L/4$)
The workpiece is rolled through the machine at a feedrate, $f$. The angle is measured at position, $z_{rel} = L/4$. 
3.5.4.3 Influence of the Sampling Rate

The sampling rate also influences the magnitudes of the deviation and the drift. Equation (3.14) illustrates that the sample period, $T_{\text{sample}} = t$, is dependent on the location of the angle measurement, $z_{1\text{rel}}$. Thus, $T_{\text{sample}}$ is proportional to the distance, $z_{1\text{rel}}$. As $z_{1\text{rel}}$ is required to be some finite distance, the sample period may have to be large, and high frequencies can be lost due to the slow sampling rate. The deviation and the drift is the effect of decreasing the sampling rate, $T_{\text{sample}}$ without decreasing $z_{1\text{rel}}$.

An alternative is to measure the angle relative to the fixed roll assembly, $z_{\text{rel}} = 0$. If the initial rate of twist along the workpiece is relatively constant, then as the segment of the beam of length, $z_{1\text{rel}}$, passes through the machine, the control system will respond to twist the segment to the desired angle. By the time the controller sees a new rate of twist, the segment will have passed out of the machine. If $\phi_L(z_{1\text{rel}})$ is measured relative to $z_{\text{rel}} = 0$ and the rate of twist is fairly constant, then the sample rate can be made independent of the position, but this will be at the cost of accuracy. However, it does solve the stability problem.

3.6 ROLL TWISTING PROCESS CONTROLLER

The objective of the roll twisting process controller is to incrementally sequence the beam with a discrete controller sample time and compensate for the non-ideal angle measurement located a realistic distance, $z_{1\text{rel}}$. The roll twisting process controller is based on the concepts of section 3.5.3, Incremental Sequencing of the Beam. Ideally the roll twisting apparatus would be able to measure an angle at position $z_{n\text{rel}}$, but physical limitations and sensor accuracy require that the sensor be located at position $z_{1\text{rel}}$, where $z_{1\text{rel}}$ is greater than $z_{n\text{rel}}$. Since the angle
at $z_{nrel}$, $\phi(z_{nrel})$, cannot actually be measured, the controller utilizes available angle measurements to forecast the angle, $\phi(z_{nrel})$. The controller then manipulates the sensor data to incrementally sequence the workpiece using the discrete controller sample time.

The roll twisting process controller consists of an inner loop and an outer loop (Figure 3-13). The outer loop provides the correct tracking commands to the inner loop which in turn commands the servo. The inner loop is the primary controller of Figure 3-4. The function of the inner loop is to measure the angle, $\phi(z_{1rel})$, located at $z_{1rel}$ and drive $\phi(z_{1rel})$ to a commanded angle, $\phi_{com}(z_{1rel})$. The outer loop provides the commanded angle, $\phi_{com}(z_{1rel})$, to the inner loop. The commanded angle, $\phi_{com}(z_{1rel})$, is computed from available measurements, $\phi(z_{1rel})$, and $\phi(z_{2rel})$, such that the angle $\phi(z_{rel}z_{nrel})$ is driven to the desired angle, $\phi_{Udes}(z_{abs})$ at the end of the sample period as discussed in Section 3.5.3.

At time $t$, assume that the following measurements are available:

- $\phi_L(t,z_{1rel})$: the loaded angle at $z_{1rel}$
- $\phi_L(t,z_{2rel})$: the loaded angle at $z_{2rel}$
- $T_c$: the torque measurement containing the disturbance
- $\phi_U(t-T_{sampOL},0<z_{rel}<z_{2rel})$: the unloaded angles on the beam in the machine

### 3.6.1 The Inner Loop

The inner loop measures the angle $\phi_L(z_{1rel})$ located at $z_{1rel}$, and the torque, $T_c$, and computes the unloaded angle of twist

$$\phi_U(z_{1rel}) = \phi_L(z_{1rel}) - \frac{T_c z_{1rel}}{G \kappa_T}$$ (3.15)
The unloaded angle, $\phi_U(z_{rel})$, is fed back and compared to the commanded angle, $\phi_{com}(z_{rel})$, to generate the error signal to drive the servo (Figure 3-13).

$$e = \phi_{com}(z_{rel}) - \phi(z_{rel})$$

(3.16)

3.6.2 The Outer Loop

The outer loop provides the correct tracking commands, $\phi_{com}(z_{rel})$, to the inner loop such that the angle, $\phi_U(z_{nrel})$, is driven to the desired angle, $\phi_{Udes}(z_{nrel})$. The analysis for computing the commanded angle begins by explaining the strategic location of the two angle measurements and their importance in monitoring the variations of an angle at some absolute position of the moving workpiece. The angle, $\phi(z_{nrel})$ is forecasted by recording the history of an angle at some absolute position on the workpiece as it moves through the window. Finally, the desired angle change at position, $z_{nrel}$, is projected in the appropriate proportion to compute the commanded angle.

3.6.3 Position of the Angle Measurements

The roll twisting process is actually continuous, but the control mechanism utilizes the discrete nature of the computer to sequence the workpiece. If the controller only has data from one angle measurement, it can only give information about a position on the workpiece relative to the window. By strategically locating a second angle measurement, $\phi_L(z_{2rel})$, upstream of the first angle measurement, $\phi_L(z_{1rel})$, and by manipulating the data from the two measurements, the controller can keep track of the changes in an angle located at an absolute position, $z_{abs}$. Since the controller is discrete, it can only monitor nodes or spaced absolute positions on the workpiece that are correlated with the sample period, $T_{sample}$, and
Figure 3.13: Block Diagram of the Control System for the Roll Twisting Process
the feedrate, \( f \). So the distance between the nodes is

\[
z_{\text{node}} = f \, T_{\text{sample}} \quad (3.17)
\]

The controller can only change tracking commands, \( \phi_{\text{com}}(z_{1\text{rel}}) \), at each sample time, and during the sample period the controller drives the angle located at \( z_{1\text{rel}} \) to the commanded angle \( \phi_{\text{com}}(z_{1\text{rel}}) \). When the roll twisting process is complete, each node will be driven to its commanded angle, but errors will exist between the nodes.

The relative location of the two angle measurements \( \phi_L(z_{1\text{rel}}) \), and \( \phi_L(z_{2\text{rel}}) \), should be spaced such that

\[
z_{2\text{rel}} - z_{1\text{rel}} = n \, z_{\text{node}} \quad (3.18)
\]

or that each angle measurement is located on a node. (Note: The sample period can also be manipulated if required). In this analysis, it will be assumed that

\[
z_{2\text{rel}} - z_{2\text{rel}} = z_{\text{node}} \quad (3.19)
\]

for simplicity in the computations. The controller needs to monitor an angle located at position, \( z_{\text{abs}} \) on the workpiece. Let, \( z_{\text{abs}} = z_{2\text{rel}} \), at time, \( t = t_0 \), as shown in Figure 3-14a, then at the next sample time, \( t_0 + T_{\text{sample}} \), the workpiece will have moved a distance, \( z_{\text{node}} \) and \( z_{\text{abs}} = z_{1\text{rel}} \). The two angle measurements allow the controller to monitor the angle, \( \phi_U(z_{\text{abs}}) \), from time, \( t_0 \), to time, \( t_0 + T_{\text{sample}} \):

\[
\phi_U(t_0, z_{\text{abs}}) = \phi_U(t_0, z_{2\text{rel}}) \quad (3.20)
\]

\[
\phi_U(t_0 + T_{\text{sample}}, z_{\text{abs}}) = \phi_U(t_0 + T_{\text{sample}}, z_{1\text{rel}}) \quad (3.21)
\]
Figure 3-14: Computation of the Commanded Angle $\phi_{com}(z_{1rel})$
The strategically located angle measurements allow the controller to feedforward the commanded angle, so that at the next sample period, the angle located at $z_{\text{abs}}$ has been driven to the desired angle. For example, let the angle measurement at $z_{\text{rel}}$, $\phi(z_{\text{rel}})$, be used for feedback to the inner loop. The outer loop must compute the commanded angle, $\phi_{\text{com}}(z_{\text{rel}})$, for the inner loop such that the angle located at $z_{\text{abs}}$ is driven to the desired angle. In Figure 3-14a, the desired control action is to drive the angle at $z_{\text{abs}}$, $\phi_U(t_0; z_{\text{abs}} = z_{\text{rel}})$, to

$$\phi(t_0 + T_{\text{sample}}, z_{\text{abs}}) = \phi(t_0, z_{\text{abs}}) + \Delta \phi_{\text{des}}(z_{\text{abs}})$$

where $\Delta \phi_{\text{des}}(z_{\text{abs}})$ is the desired change in angle.

Consider if the workpiece was stopped in the window and held stationary during the sample period as if the workpiece were actually sequenced. The desired result would be the curve shown in Figure 3-14b, where

$$\phi(t_0 + T_{\text{sample}}, z_{\text{abs}}) = \phi(t_0 + T_{\text{sample}}, z_{\text{rel}})$$

Now, if the workpiece were instantaneously moved a distance, $z_{\text{node}}$, as shown in Figure 3-14c, then

$$z_{\text{rel}} = z_{\text{abs}} = z_{\text{rel}} - z_{\text{node}}$$

So at time, $t_0 + T_{\text{sample}}$, the angle measurement at $z_{\text{rel}}$ is now feeding back the angle located at $z_{\text{abs}}$. The original angle measurement at $z_{\text{rel}}$ allows the controller to compute the desired control action and feedforward the commanded angle,

$$\phi_{\text{com}}(z_{\text{rel}}) = \phi(t_0, z_{\text{rel}}) + \Delta \phi_{\text{des}}(z_{\text{abs}})$$
Figure 3-15: Updating Premasured Angles
3.6.4 Computing the commanded angle, $\phi_{\text{com}}(z_{1\text{rel}})$

The objective of the controller is to drive the angle located at $z_{\text{rel}}$, $\phi(z_{\text{rel}})$, to some desired angle, $\phi_{U\text{des}}(z_{\text{rel}})$ by the end of the sample period. If the objective of the roll twisting process is to straighten the workpiece, then $\phi_{U\text{des}}(z_{\text{rel}}) = 0$.

The controller computes the commanded angle, $\phi_{\text{com}}(z_{1\text{rel}})$, so that at the end of the sample period $\phi(z_{\text{rel}})$ is driven to $\phi_{U\text{des}}(z_{\text{rel}}) = 0$, as shown in Figures 3-14b and 3-14c. At the end of the sample period, a section of the workpiece of length, $z_{\text{node}}$, has moved out of the window and is no longer influenced by the controller.

For now, assume $\phi(t_0, z_{\text{rel}})$ is known, and the desired change in the angle at $z_{\text{rel}}$ is

$$\Delta \phi_{\text{des}}(z_{\text{rel}}) = \phi_{U\text{des}}(z_{\text{rel}}) - \phi(t_0, z_{\text{rel}}) = -\phi(t_0, z_{\text{rel}})$$

(3.26)

The change in angle, $\Delta \phi_{\text{des}}(z_{\text{rel}})$, is projected onto $\Delta \phi_{\text{des}}(z_{2\text{rel}})$ by

$$\Delta \phi_{\text{des}}(z_{2\text{rel}}) = \Delta \phi_{\text{des}}(z_{\text{rel}}) \frac{z_{2\text{rel}}}{z_{\text{rel}}}$$

(3.27)

since the angle of twist is proportional to the relative position in the window.

Substituting equations (3.26) and (3.27) into equation (3.25) results in the commanded angle

$$\phi_{\text{com}}(z_{1\text{rel}}) = \phi(t_0, z_{2\text{rel}}) - \phi(t_0, z_{\text{rel}}) \frac{z_{2\text{rel}}}{z_{\text{rel}}}$$

(3.28)

The outer loop command mechanism is illustrated in Figure 3-13.

3.6.5 Forecasting the angle, $\phi(z_{\text{rel}})$

The angle, $\phi(z_{\text{rel}})$, is determined by monitoring the changes in an angle at an absolute position on the workpiece, $z_{\text{abs}}$, as the workpiece moves through the
window. Monitoring the angle at a point on the workpiece as it moves through the machine requires knowing the past history of the unloaded angle of twist at a position, $z_{rel}$. Consider a process which applies a constant angle of twist, $\phi_U(L)$, to the workpiece. The relation between the rate of twist along the beam and $\phi_U(L)$ is

$$\frac{d\phi_U(z_{abs})}{dt} = \frac{\phi_U(L)}{L}$$  \hspace{1cm} (3.29)

Equation (3.29) can be integrated to find the angle of twist at position, $z_{abs}$.

$$\phi_U(z_{abs}) = \phi_U(z_{abs0}) + (z_{abs} - z_{abs0}) \frac{\phi_U(L)}{L}$$  \hspace{1cm} (3.30)

In general, $\phi_U(L)$ is not constant, and the relation for $\phi_U(z_{abs})$ can be found from

$$\phi_U(z_{abs}) = \phi_U(z_{rel0}) + \frac{1}{L} \left[ \phi_U(L,t) \right]_{z_{rel0}}^{z_{rel}} dt - \int_{0}^{t} f dt$$  \hspace{1cm} (3.31)

valid for

$$z_{rel0} \leq z_{abs} < L$$

Equation (3.31) implies that to monitor an angle as it passes through the window requires a time history of the unloaded angle at position, $z_{rel}$.

The angle, $\phi(z_{nrel})$, is determined by monitoring the changes in an angle at an absolute position on the workpiece, $z_{abs}$, as the workpiece moves through the window. The controller uses the available measurements, $\phi(z_{1rel})$ and $\phi(z_{2rel})$ to project what an angle measurement device located at $\phi(z_{nrel})$ would record if such a device were available. First the controller records the angle, $\phi(t_0, z_{abs}=z_{1rel})$ at some time, $t_0$. At each sample time the controller saves the current angle, $\phi(t, z_{1rel})$, and updates the previously measured angles, $\phi(t < t_0, z_{abs})$. When $z_{nrel} = z_{abs}$, the controller retrieves the updated angle, $\phi(z_{abs}=z_{nrel})$, to compute the commanded angle to the inner loop, $\phi_{com}(z_{1rel})$, per equation (3.28). The recorded
angles are updated by computing the change in the angle at $z_{2\text{abs}}$ over the last sample period and projecting how the change has altered the recorded angles. The updating mechanism is illustrated in Figure 3-13.

The actual change in the angle at location, $z_{2\text{abs}} = z_{1\text{rel}}$, at time, $t$ is found from

$$
\Delta \phi_{\text{act}}(\Delta t, z_{2\text{abs}}) = \phi(t, z_{2\text{abs}} = z_{1\text{rel}}) - \phi(t-T_{\text{sample}}, z_{2\text{abs}} = z_{2\text{rel}}) \tag{3.32}
$$

and is illustrated in Figure 3-15b. (Note, the desired change in angle, $\Delta \phi_{\text{des}}(z_{1\text{rel}})$, is not necessarily equal to the actual change in angle, $\Delta \phi_{\text{act}}(z_{1\text{rel}})$.) Since all of the recorded angles are updated only to time, $t-T_{\text{sample}}$, assume that the workpiece has remained stationary over the sample period. Each of the recorded angles has changed, and the angle adjustment is projected from

$$
\Delta \phi(\Delta t, z_{\text{oldrel}} = z_{\text{abs}}) = \Delta \phi(\Delta t, z_{2\text{abs}}) \frac{z_{\text{oldrel}}}{z_{2\text{rel}}} \tag{3.33}
$$

where $z_{\text{oldrel}}$ refers to time, $t-T_{\text{sample}}$. Each angle is adjusted by an amount proportional to its position in the window.

$$
\phi(t, z_{\text{oldrel}} = z_{\text{abs}}) = \phi(t-T_{\text{sample}}, z_{\text{oldrel}}) + \Delta \phi(\Delta t, z_{2\text{abs}}) \frac{z_{\text{oldrel}}}{z_{2\text{rel}}} \tag{3.34}
$$

Now the workpiece is assumed to move instantaneously, so each updated angle is incremented a distance of length, $z_{\text{node}}$ (Figure 3-45c),

$$
z_{\text{rel}} = z_{\text{oldrel}} - z_{\text{node}} \tag{3.35}
$$

$$
\phi(t, z_{\text{rel}}) = \phi(t, z_{\text{oldrel}}) \tag{3.36}
$$

The recorded angles are now updated. This process is repeated every sample time so that $\phi(z_{n\text{rel}})$ is available to the controller. The forecasted angle can be
written in summation notation as shown in equation (3.37).

\[ \phi(z_{nrel}) = \phi((z_1,t_0) + \sum_{j=1}^{m} [z_1 - (j - 1) z_{node}] \frac{\Delta \phi_U(t-j T_{sampOL} z_{2abs})}{z_{1rel}} \]  \tag{3.37}

where \( m = \frac{z_{1rel}}{z_{node}} \)

Converting equation (3.31) into a discrete form results in equation (3.37).

3.6.6 Summary

This control scheme compensates for the non-ideal location of the angle measurement device. The two angle measurements allow an angle at a point on the beam to be recorded and then updated as it passes through the machine. Just before the point exits the machine, the recorded angle is retrieved and used to compute a command to the inner loop. The inner loop then drives the angle at the point to the desired angle. At the next outer loop sample time, the point has passed out of the machine. As the limit of the length, \( z_{nrel} \), approaches zero, the controller is able to achieve point by point shape control.

3.6.7 Inner and Outer Loop Sample Rates

Due to sensor accuracy and physical and computer constraints, the outer loop may require a slow sampling rate. Since the outer loop is able to feed forward the commanded angle, \( \phi_{com}(z_{1rel}) \), the inner loop sampling rate does not have to coincide with the outer loop rate. In fact, the inner loop should sample at a faster rate than the outer loop for better disturbance rejection. The controller can be programmed to perform the outer loop computations every \( n \) inner loop sample times.
With a faster inner loop, the controller is able to converge more rapidly and have better disturbance rejection.

3.6.8 Disturbance Rejection

The controller must have good disturbance rejection properties to reject disturbances in the torque measurement. As the workpiece moves through the window over one sample period, the angle located at \( z_{rel} = L \) changes some unknown amount and causes a disturbance in the torque measurement.

\[
\Delta T_c = \frac{\Delta \phi(L) G \kappa_T}{L}
\]  

3.6.9 Potential Problems with the Roll Twisting Process Controller

Potential problems include error in the model, error caused by noisy measurements, error caused by the absence of an absolute reference angle and error in the measurements due to the vibration of the beam.

The Saint Venant's torsion model may not be valid near the vicinity where the torque is applied to the beam. This may be resolved by increasing the incremental distance, \( z_{rel} \), in the controller.

Another potential problem caused by model error is in updating the predicted angles. Equation (3.37) involves summations of the difference between a measured and predicted angle. If the assumption that the loaded angle unloads elastically is incorrect, the error in the predicted angles would be magnified since equation (3.37) involves summations. One solution is to add a sensor on the system less than \( z_{rel} \approx 0 \). This sensor could be used to confirm the model.
3.6.9.1 Relative Angle Measurement

In the previous discussion, all of the angles have been measured with respect to the angle at \( z_{abs} = 0 \) at the end of the beam. This requires a sensor that will move with the beam, and this is difficult to achieve if the beam is long. The rolls at \( z_{rel} = 0 \) are fixed, so \( \phi(z_{rel} = 0) \) is a potential reference angle. Section 3.5.4.2 discussed the possible instability problem of measuring the angle relative to one end of the beam. The solution to the instability problem is to measure the angle relative to the machine. This implies that all angles would be measured relative to \( \phi(z_{rel} = 0) \). But if the system allows any errors in the beam to pass out of the window, the error will become the reference angle and all angles will be twisted to that angle. An alternative is to add an additional measuring device such that the angle can be monitored relative to the end of the beam, and then use this information to adjust the commands to the control system to compensate for accumulated error. This deserves further investigation. Experiments are required to verify the sensitivity of this error.

3.6.10 Error in measurements due to vibration

The system can excite the torsional vibration modes of the beam. The vibration of the beam will introduce error into the torque measurement, where

\[
T_{cmeas} = T_{act} + T_{dyn} \tag{3.40}
\]

The error in the torque measurement will cause the controller to respond to the erroneous error signal containing the actual unloaded angle, \( \phi_U \), and the dynamic unloaded angle, \( \phi_{U_{dyn}} \). If the excitation frequency of the system approaches the natural frequency of the beam, resonance will occur, and the system will go
unstable. This implies that the bandwidth of the control system must be restricted to avoid exciting the modes of vibration of the beam.

Assuming the servo is rigid, the acceleration of the servo at position, \( L \), excites the free end of the beam \( (z_{rel} > L) \). The first mode of vibration for the portion of the beam in the machine can be modelled as a beam with fixed ends as shown in Figure 3-16. The natural frequency, \( \omega_n \), of the section \( (0 < z_{rel} < L) \) is

\[
\omega_n(0 < z_{rel} < L) = \frac{(G / \rho)^{i} \pi}{L}
\]

where \( i = 1, 2, ... \) and \( \rho = \) density

The length, \( L \), is always fixed, so the natural frequency, \( \omega_n(0 < z_{rel} < L) \), will remain a constant. The first mode of vibration for the free end of the beam, \( (z_{rel} > L) \) can be modelled as a cantilever beam as shown in Figure 3-16.

\[
\omega_n(z_{rel} > L) = \frac{(G / \rho)^{i} \left\{2i + 1\right\} \pi}{2 \left(z_{relend} - L\right)}
\]

where \( i = 0, 1, 2, ... \) and \( \rho = \) density

Comparing the natural frequencies of equations (3.41) and (3.42) shows that \( \omega_n(0 < z_{rel} < L) \) is constant, and that \( \omega_n(z_{rel} > L) \) is dependent on \( (z_{relend} - L) \). The free end of the beam is much greater than \( L \), so assume that \( \omega_n(0 < z_{rel} < L) \) is much higher than the frequency at which the system operates and the vibration of the beam \( (0 < z_{rel} < L) \) can safely be ignored. As the length of the free end of the beam increases, the natural frequency decreases, so the system bandwidth will be determined by the maximum length of the free end of the beam. [24] Equation (3.42) that shows the natural frequency is dependent on the length \( (z_{relend} - L) \) and decreases as the length increases. This end is the portion of the beam being fed into the system and can be very long. Damping can be added to the portion of the beam \( (z_{rel} > L) \) to take the energy out of the vibrating beam.
3.6.10.1 Torque Measurement Location

The excitation of the free end, $z_{abs} < z_{rel}$, is caused by elastic unloading of the workpiece, and the vibration of the portion of the beam, ($0 < z_{rel} < L$), was assumed negligible. This implies the dynamic contribution to the torque measurement will be smaller if the torque is measured at $z_{rel} = 0$. The direction of feed would reverse for implementing the controller for use in the shaping process (twisting a straight beam to some desired shape). This is required for the assumptions of the vibration model to be valid. A straight beam would enter the system at $z_{rel} = 0$ and be twisted to $\phi_{Udes}(L)$. The controller would require modifications to achieve the desired shape. The sensitivity of the location of the torque measurement should be verified experimentally.
3.7 MEASUREMENT MODEL

3.7.1 Angle Measurement

The proposed method of measuring the angle uses an angle measurement device which rotates with the workpiece in the x-y plane at a position, $z_{rel}$. To allow for the possibility of bending occurring during the twisting process, the angle measurement device should have the capability of floating in the x-y plane. The mounting plate is restrained to float in the x-y plane at position $z_{rel}$ in its attachment to the frame of the roll twisting machine. The angle measurement device is illustrated in Figure 3-17. The angle measurement device uses four small rollers to contact the perimeter of the cross section such that the rotating plate will twist with the workpiece (Figure 3-17). Guide wheels attached to the mounting plate allow the rotating plate to rotate with the workpiece. Measurement of the angle of twist requires a transducer to measure the rotation of the rotating plate. If the length $z_{rel}$ is small, then $\phi_L(z_{rel})$ will be small. The position, $z_{rel}$, and the twisting characteristics of the workpiece will determine the required resolution of the transducer. The angle of twist can be magnified mechanically by increasing the radius, $R_H$, from the center of twist to where the transducer measures the angle of twist. The proposed method of measuring the angle of twist is to measure the vertical displacement, $H$. The angle of twist can be found from

$$\tan\phi_L(z_{rel}) = \frac{H}{R_H}$$  \hspace{1cm} (3.43)

and for small angle displacements, the angle can be approximated as

$$\phi_L(z_{rel}) \approx \frac{H}{R_H}$$  \hspace{1cm} (3.44)
Figure 3.17: Angle Measurement Device
3.7.2 Torque Measurement

The torque measurement can be measured from the servo used to twist the workpiece, or it can be measured from load cells at the fixed roll assembly.

The torque can be measured from the servo (for instance a DC motor). For a DC motor, the torque is proportional to the armature current, \( i_a \),

\[
T_m = k_T i_a
\]  

(3.45)

where \( k_T \) is the torque constant for the motor. A gear train or some other method is required to transfer power from the motor to the workpiece. The accuracy of the torque measurement is dependent on the method of transferring power. The motor torque's relation to the workpiece torque, \( T_R \), is

\[
T_m = T_R + T_f + T_I + T_k
\]  

(3.46)

where \( T_f \) is the torque from friction effects, \( T_I \) is the torque from inertia effects, and \( T_k \) is the torque from spring effects. If \( T_f, T_k, \) and \( T_I \) are negligible, then the torque on the workpiece can be measured from the servo.

An alternate method of measuring the torque is measuring the torque at the fixed roll assembly (Figure 3-18). Load cell A measures the forces in the x and y directions and is connected to the mounting plate by a pin joint. Load cell B measures the forces in the y direction only and is connected to the mounting plate by a pin joint. The roll assembly is attached to the mounting plate and is restricted to the position \( z_{rel} = 0 \). For the configuration illustrated in Figure 3-18, the torque, \( T_c \), is

\[
T_C = +x_A F_{y_A} - x_B F_{y_B}
\]  

(3.47)
This assumes that the center of twist of the machine and the shear center coincide. If this is not the case, bending and twisting will be coupled. The proposed design for the roll twisting machine uses the fixed roll station for driving the rolls to feed the workpiece through the machine. The torque to the drive rolls is orthogonal to the torque of the workpiece, but vibration of the rolls may be a problem in measuring the torque by this method.

### 3.7.3 Torsional Stiffness Measurement

The torsional stiffness measurement requires twisting the workpiece in the elastic region and measuring the angle of twist, \( \phi_\mu(z_{rel}) \), and the torque, \( T_c \). This data is then used to compute the slope of \( z_{rel}/GK_T \). For a straight workpiece, the range of the data taken should include the range of the elastic region. So an estimate of the yield angle, \( \phi_{yld} \), is needed to define the range. For a workpiece made up of thin rectangular sections, the yield angle can be found from

\[
\phi_{yld}(z_{rel}) = \frac{\tau_{yld} z_{rel}}{G h_{\text{max}}}
\]

(3.48)

where \( h_{\text{max}} \) is the maximum width of the individual rectangular sections (\( b > > h \), and \( h \) is the smallest dimension of each rectangle) [3]). For solid sections, the relation for the yield stress can be found from handbooks [15] or texts [3,13]. Only an estimate is needed, so approximations should be sufficient. The range of the data should stay within the elastic region, so the yield angle should be conservative. For instance, the range could be defined by the maximum angle, which is equivalent to \( 90^\circ \) of estimated yield angle. The accuracy is dependent of the number of data points and their distribution over the range. Once the data is gathered, a least squares method is used to fit the data to a straight line.
Figure 3-18: Fixed Roll Assembly Torque Measurement
Chapter 4
Control of a Shaping Process for Beams

4.1 INTRODUCTION

The objective of a shaping process for beams is to bend and twist the beam to some desired shape. In general, the beam has a constant cross section along the length of the beam, and the shape of the cross section is arbitrary. Study of the mechanics of bending and twisting beams show that, in general, bending and twisting are coupled. This means that during a bending process, the beam may plastically twist, and during a twisting process, the beam may plastically bend. The coupling between bending and twisting is analyzed to investigate how it will effect the bending process and the twisting process. Based on the analysis, roll assembly design guidelines are proposed to minimize twisting during the bending operation, and machine design concepts are presented to decouple bending and twisting for the twisting process.

The feasibility of a combined bending and twisting process is investigated by examining the mechanics of bending and twisting thin-wall cross sections versus thick-wall cross sections. Finally, two shape control system for beams are presented: a shape control system employing a combined bending and twisting process for thick-wall cross sections, and a shape control system which performs the bending and twisting processes separately.
4.2 DECOUPLING THE BENDING AND TWISTING PROCESSES

4.2.1 The Shear Center

If the transverse loading of a workpiece is applied such that the line of action of the loading passes through the shear center of every cross section of the beam, then the workpiece can be bent without twisting as shown in Figure 4-1. So the roll bending and twisting operations can be decoupled by loading the workpiece at or about its shear center.

Figure 4-1: Effect of Applying a Load through the Shear Center

(a) Load P applied at centroid, O, produces twisting and bending.
(b) Load P applied at shear center, C, produces bending only. [3] page 317

If the cross section has two axes of symmetry, then the shear center coincides
with the centroid of the cross sectional area. A cross section with one axis of symmetry will locate the shear center somewhere along the axis of symmetry. Finding the exact shear center for an arbitrary cross section is complex. Boresi [3] and Heins [13] have derived solutions for locating the shear center for common cross sections. However, the numerical analysis for finding the shear center is rather complex, so a proposed method of experimentally locating the shear center is presented in section 4.2.2.

When the resultant bending forces applied to the workpiece do not pass through the shear center, twisting will occur during a bending process. If the location of the shear center is known and the resultant force and torque are measured quantities, then the equivalent force and torque at the shear center can be computed, \( F_c \) and \( T_c \). The resultant force, \( F_{Rx} \) and \( F_{Ry} \), and the resultant torque, \( T_R \), are measured with respect to the centroid, \( O \), as shown in Figure 4-2. The location of the shear center is defined by the coordinates, \( x_c \) and \( y_c \), with respect to the centroid of the cross section.

**Figure 4-2:** Resolving a Torque and Force at the Centroid into an equivalent Torque and Force at the Shear Center
The force and torque at the centroid can be resolved into an equivalent force and couple at the shear center. The twisting torque $T_c$ is by definition a couple, so the torque can be measured anywhere. If $x_c$, $y_c$, $F_{Rx}$, $F_{Ry}$, and $T_R$ are known, then the shear center torque, $T_c$, and the shear center forces, $F_{cx}$ and $F_{cy}$ can be computed from

\[
\begin{bmatrix} F_{cx} \\ F_{cy} \end{bmatrix} = \begin{bmatrix} F_{Rx} \\ F_{Ry} \end{bmatrix} \]

(4.1)

\[
T_c = T_R - x_c F_{Ry} + y_c F_{Rx} \]

(4.2)

Now the effects of bending and twisting can be examined separately. The torque, $T_c$, will only produce twisting, and the forces, $F_{cx}$ and $F_{cy}$, will only produce bending.

4.2.2 Experimental Location of the Shear Center

To find the location of the shear center experimentally requires a machine that performs bending and twisting simultaneously. The forces, $F_{Rx}$ and $F_{Ry}$, and the torque, $T_R$, are measured at some point with respect to the cross section, Figure 4-2. It is assumed that the workpiece is not loaded through its shear center, and if it is, the experiment will verify the location of the shear center. Equations (4.1) and (4.2) are used to compute the shear center location, $x_c$ and $y_c$ measured from the shear center to the point on the cross section where the forces and torques are measured. (Here, the centroid, $O$, is used for convenience.) Assume that the machine is able to measure the angle of twist, $\phi_L$.

Locating the shear center experimentally requires bending the workpiece through different planes (which cut through the z-axis) until the measured angle is equal to zero. In the elastic region, $\phi_L$ is proportional to $T_c$. If $\phi_L$ is zero, then the
torque at the shear center, $T_c$, is zero, and equation (4.2) reduces to

$$T_R = x_c F_{Ry} - y_c F_{Rx} \quad (4.3)$$

Let

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} \quad (4.4)$$

and

$$\tan \beta = \frac{F_{Ry}}{F_{Rx}} \quad (4.5)$$

Substituting for $F_{Rx}$ and $F_{Ry}$, equation (4.3) can be rewritten

$$T_R = F_R [x_c \cos \beta - y_c \sin \beta] = F_R \ l \quad (4.6)$$

The workpiece is bent in the manner required to have the angle of twist, $\phi_L$, equal to zero, while still imposing bending loads on the workpiece. The moment arm, $l_1$, can be found from

$$l_1 = \frac{T_R}{F_R} \quad (4.7)$$

Assuming the workpiece will not be loaded in the same manner for all planes of bending, there should be at least one other loading for which $\phi_L$ is equal to zero. And $l_2$ can be found from

$$l_2 = \frac{T_R}{F_R} \quad (4.8)$$

The location of the shear center can then be found from

$$x_c = \frac{l_1 \cos \beta_1 - l_2 \cos \beta_2}{\sin \beta_2 - \sin \beta_1} \quad (4.9)$$

$$y_c = \frac{l_1 + x_c \sin \beta_1}{\cos \beta_1} \quad (4.10)$$
The location of the shear center can be verified by measuring the angle of twist, and measuring the torque, $T_c$, from equation (4.2). In the elastic range, the angle of twist and the torque should be proportional. If the angle of twist is zero for all planes of bending, then the workpiece is loaded through the shear center. Once the shear center is located, the torsional stiffness can be computed from the method outlined in Chapter 3, 3.7.3.

### 4.2.3 Twisting caused by Off-shear Center Loading During a Bending Process

The amount of twist caused by off-shear center loading can be examined if locations of the resultant forces, $F_{x1}$, $F_{x2}$, $F_{y1}$, and $F_{y2}$ in Figure 4-3 are known. Referring to Figure 4-3, the twisting torque, $T_c$, applied to the workpiece is

$$T_c = -y_1 F_{x1} - y_2 F_{x2} + x_1 F_{y1} + x_2 F_{y2}$$  \hspace{1cm} (4.11)

where $y_1$, $y_2$, $x_1$, and $x_2$ are the moment arms for the forces $F_{x1}$, $F_{x2}$, $F_{y1}$, and $F_{y2}$, respectively, with respect to the shear center. Twisting can be minimized by minimizing the moment arms, $y_1$, $y_2$, $x_1$, and $x_2$.

### 4.2.3.1 Thick-wall versus Thin-wall Cross Sections

Twisting caused by off-shear center loading in the bending process can be neglected if the stress due to twisting does not enter the plastic region. Thick-wall cross sections have a high torsional resistance and are not prone to plastic twisting deformation during the bending process. Thin-wall cross sections have a low torsional resistance and may plastically twist.

For example, assume the bending process bends the workpiece about the $x$-axis, but the roll assembly causes the workpiece to be loaded off the shear center.
Figure 4-3: Cross section subjected to Off-shear Center Loading

by an amount, $x_1$. The twisting torque can be found from equation (4.11).

$$T_c = x_1 F y_1$$

and the bending moment is

$$M_{cx} = \frac{L F y_1}{2}$$

where $2L$ is the distance between supports for a beam subjected to three-point bending. Usually, $L/2 \gg x_1$, and comparing equations (4.12) and (4.13), the torque caused by off-shear center loading can be ignored for thick-wall cross sections; however thin-wall cross sections have low torsional resistance and may plastically twist.

Consider the shear stress due to torsion for the rectangular cross section shown in Figure 4-4. The shear stress, $\tau$, due to torsion is compared for both thick and thin-wall rectangular cross sections [3].
\[ \tau_{\text{max}} = \frac{T_c}{k (2b)(2h)^2} \]  \hspace{1cm} (4.14)

For a thick-wall cross section, with a \( b/h \) ratio = 1 and \( k = .208 \), the shear stress is

\[ \tau_{\text{thick max}} = \frac{.600 T_c}{k b^3} \]  \hspace{1cm} (4.15)

For a thin-wall cross section, with a \( b/h \) ratio = 10 and \( k = .312 \), the shear stress is

\[ \tau_{\text{thin max}} = \frac{40.0 T_c}{k b^3} \]  \hspace{1cm} (4.16)

Let, \( L = 5\ b, \ b = 1, \ x_1 = h, \) and \( \tau_{\text{yld}} = 0.7 \ \sigma_{\text{yld}} \), and let the beam bend about the \( x \)-axis until the beam begins to yield. The shear stress for the thin and thick-wall cross sections are

\[ \tau_{\text{thick}} = 0.1596 \ \sigma_{\text{yld}} < \tau_{\text{yld}} \]  \hspace{1cm} (4.17)

\[ \tau_{\text{thin}} = 1.067 \ \sigma_{\text{yld}} > \tau_{\text{yld}} \]  \hspace{1cm} (4.18)

So the thin-wall cross section will twist during bending due to off-shear center loading, and torsion caused by off-shear center loading cannot be ignored for thin-wall cross sections. Section 4.2.5 investigates minimizing twisting by carefully designing the roll assemblies.

4.2.3.2 Twisting Restrained During the Bending Process

Thin-wall cross section can plastically twist during the bending process even though the beam is restrained from twisting. The twisting is caused by off-shear center loading of the workpiece. If twisting is restrained, then the loaded angle of twist, \( \phi_L \), must equal zero. To understand the phenomenon, first assume that the
workpiece is allowed to twist during the bending process, curve A in Figure 4-5. Assume the beam is then unloaded, and the unloaded angle of twist is $\phi_{UA}$. Since the workpiece is actually restrained from twisting during bending, this requires that $\phi_L$ is zero, so an opposite bending moment is applied to bring the loaded angle back to zero, curve B in Figure 4-5. The beam is unloaded, and the workpiece springs back to the unloaded angle of twist, $\phi_{UC}$, curve C in Figure 4-5. For elastic behavior, the torque is less than the yield torque.

$$T_c < T_{yd}, \quad \phi_{UC} = 0.0$$

(4.19)

If the torque is greater then the yield torque, then the workpiece will twist by the amount

$$T_c \geq T_{yd}, \quad \phi_{UC} = \frac{(T_c - T_{yd})^{2 \text{rel}}}{G \kappa_T}$$

(4.20)
Figure 4-5: Unloaded Angle of Twist for a Beam Restrained From Twisting in a Bending Process

Path A: Twisting unrestricted during bending. Path B: Deformed beam twisted back to $\phi_L = 0$. Path C: Beam is unloaded.

4.2.4 Twisting Caused by Buckling of Thin-wall Cross Sections

Thin-wall cross sections may buckle, and may twist as the result of buckling. A T-section with deep thin-walls is bent such that the center web is in compression due to bending stresses. $W$ the compressive stresses at the top of the cross
section are greater than an equivalent Euler buckling load, $P_{cr}$, the web will buckle. The Euler buckling load can be found from

$$P_{cr} = \frac{\pi^2 E I}{(L_{eu}/2)^2}$$

(4.21)

where $I$ is the moment of inertia about an axis which bending due to buckling occurs, and $L_{eu}$ is the length of the section between supported ends [14]. The Euler buckling load is inversely proportional to the square of the length of the section, so buckling will occur in the longest unsupported portion of the workpiece. The maximum compressive stresses occur at the top of the web of the T-section and at the center roll in the bending apparatus, so maximum buckling will take place at the top of the web and close to the center roll since the center roll is a supported end. To satisfy equilibrium conditions, the buckling causes localized twisting in the buckled portion of the beam, so twisting will be a side effect of buckling.

4.2.5 Minimizing Off-shear Center Loading with Roll Assembly Design

The function of the roll assembly is to transmit bending loads to the workpiece and restrain the workpiece from twisting. Ideally, the roll assembly would be configured to load the workpiece such that the line of action of the loads would pass through the shear center. But loading the workpiece through its shear center may not always be possible. Off-shear center loading causes bending and twisting to be coupled, so the workpiece will twist during the bending operation. Twisting of the workpiece during the bending operation can be minimized through design of the roll assemblies.

The rolls in use today are made up of several disks, of varying radii and thicknesses, and the disks are mounted on a shaft. The operator designs the roll assembly by choosing disks that conform to the shape of the workpiece cross...
section. The operator must design the roll from a limited selection of disks, so the exact sizes may not be available. Minimizing off-shear center loading through the roll assembly design process can best be explained through an example.

Consider the cross section shown in Figure 4-6. Ideally, the line of action of the resultant forces on the workpiece should pass through the shear center, C, but the disks do not conform to the rounded portion of the angle near the shear center. Since it is not possible to load the workpiece directly through its shear center, the roll assembly is designed to minimize the torque caused by the off-shear center loading.

Let disks, A, B, C, and D in Figure 4-7 be denoted as the force disks. Disks A and D are selected to load the workpiece directly through its shear center, so the moment arms, $x_2$ and $y_2$ are zero. Disks, B and C, are selected to minimize the moment arms, $x_1$ and $y_1$. Since the disks do not conform to the rounded portion of the angle, disks, B and C extend beyond the rounded portion of the angle to provide line to line contact between the disk and the perimeter of the cross section. At any one time, during the bending operation, the loading imposed on the workpiece will be a combination of $F_x$ and $F_y$. As far as bending is concerned, only two force disks will be acting on the workpiece at any one time. For the worst case, the applied torque caused by bending will be

$$ T_c = -y_1 F_{x_1} + x_1 F_{y_1} \quad (4.22) $$

where $y_1$ and $x_1$ are the moment arms for the forces $F_{x_1}$ and $F_{y_1}$, respectively, with respect to the shear center. Twisting can be minimized by minimizing the moment arms, $y_1$ and $x_1$.

Now that the force disks' location, radii and thickness are established, other disks are selected to provide the proper spacing and to restrain the workpiece from
Figure 4-6: Desired Shear Center Loading for an Angle Cross Section

twisting. To minimize off-center loading, the force disks should provide the forces
required to bend the workpiece. In Figure 4-7, disks E, F, G, and H will restrain the workpiece from twisting and provide the proper spacing for the force disks. If the workpiece twists counter clockwise, either disk B, E, or G will restrain the workpiece from twisting. If the workpiece twists clockwise, either disk C, F, or H will restrain the workpiece from twisting. The radii of the spacer disks is less than the radii of the force disks to ensure that the bending loads will be applied by the force disks.

One problem that may cause undesired twisting is a roll design that is required to contact the workpiece in two places as shown in Figure 4-8. Ideally both disks, A and B, contact the workpiece such that

\[ T_c = -x_B F_{yB} - x_A F_{yA} = 0 \]  \hspace{1cm} (4.23)

If either disc is not of the proper radii or if the some roll misalignment exists, only one disc will contact the workpiece. The original roll design intended for the twisting torque to be canceled when both discs contacted the workpiece. Now only one disc contacts the workpiece, so a twisting torque is applied to the workpiece.

\[ T_c = -x_B F_{yB} \]  \hspace{1cm} (4.24)

The preferred roll design in Figure 4-8 only contacts the workpiece with one disc and eliminates the the requirement that both discs contact the workpiece.

Roll misalignment may result in off-shear center loading of the workpiece. If both roll shafts are not parallel, the forces may not be applied to the workpiece as desired. Figure 4-9 illustrates the consequences of roll misalignment.
(a) Non-contacting disk

(b) Preferred roll design.

**Figure 4-8:** Off-shear Center Loading due to Non-contacting Disks.

**Figure 4-9:** Off-shear Center Loading Caused by Roll Misalignment
4.3 SHAPE CONTROL SYSTEM FOR BEAMS OF CONSTANT CROSS-SECTION

An ideal shape control system for beams of constant cross section would simultaneously perform unsymmetrical roll bending for the desired curvature and roll twisting for the desired angle of twist. The mechanics of combined bending and twisting of beams need to be analyzed to evaluate the feasibility of a shape control system using a combined bending and twisting operation. Then a shape control system for beams is can be designed.

4.3.1 The Combined Bending and Twisting Operation

This section investigates the feasibility of a combined bending and twisting operation. The success of the combined bending and twisting operation is dependent on the properties of the workpiece and is related to feasible machine design.

4.3.1.1 Thick versus thin-wall cross sections

Thin-wall cross sections require a large angle of twist to result in plastic twisting of the workpiece, and a large angle of twist violates the assumptions used in the roll bending controller; namely, the bending controller assumption that a planar section before bending remains a planar after bending. For example, the yield angle of twist required for the solid rectangular section in Figure 4-4 is

$$\phi_{yld} = \frac{k_2 \tau_{yld} z_{rel}}{k_1 G (2h)}$$

Increasing the L/D ratio ($z_{rel}/b$), (where L is the length of the beam over which the torque is applied) will increase the angle of twist. The yield angle for a thick-wall
cross section, \((b/h = 1.74, k_1 = 0.141, k_2 = 0.208)\), is

\[
\phi_{\text{yldthk}} = \frac{0.7375 \tau_{\text{yld}} z_{\text{rel}}}{b G} \quad (4.26)
\]

And the yield angle for a thin-wall cross section, \((b/h = 10, k_1 = k_2 = 0.312)\), is

\[
\phi_{\text{yldthin}} = \frac{5.0 \tau_{\text{yld}} z_{\text{rel}}}{b G} = 6.78 \phi_{\text{yldthk}} \quad (4.27)
\]

The thin-wall cross section requires an angle of twist 6.78 times greater than the thick-wall cross section to make the material yield [3]. Letting \(b = 1.0\), and \(z_{\text{rel}} = 10.0\), \((L/D\text{ ratio} = 5)\) for an 2024-T4 aluminum cross section. The thick-wall cross section has a small yield angle of twist, \(2.95^0\). The violation of the assumption that a planar section remain a planar is minimal. The thin-wall cross section has a large yield angle of twist, \(20.1^0\), so the assumption of a planar remaining a planar is no longer valid. If the moments of inertia were equivalent, the control system for bending might give satisfactory results when the workpiece is twisted while bending.

4.3.1.2 Moments of Inertia of the Cross Section

Controlling a combined bending and twisting operation may give satisfactory results if

\[
I_x \approx I_y \quad (4.28)
\]

Workpieces, with cross sections that violate equation (4.28), such as channels or I-beams, when twisted during a bending process may give erroneous results. Channels and I-beams are usually designed so that one moment of inertia is much greater than the other. For an I-beam, let

\[
I_x = 50 I_y \quad (I_{xy} = 0) \quad (4.29)
\]
A bending load applied at the shear center along the y-axis, without any twisting, results in a bending stress of $\sigma_{0\text{deg}}$. Applying a bending load of the same magnitude through the shear center (without any twisting) but rotated one degree from the y-axis results in a bending stress of

$$\sigma_{1\text{deg}} = 0.794 \sigma_{0\text{deg}} \tag{4.30}$$

The bending stress has decreased by 25% by rotating the bending load from its original orientation.

Consider if the I-beam was twisted during the bending process. The moment applied to the beam is illustrated in Figure 2-4. The workpiece will bend at a point and about the axis corresponding to the maximum stress or the maximum moment/stiffness ratio. For instance, if the intention is to bend the I-beam about the x-axis, and the beam is twisted, the beam will yield at some other point other than the center roll. The bending control system of Chapter 2 is no longer valid for a combined bending and twisting operation under these circumstances.

4.3.1.3 The feasibility of a combined bending and twisting operation

Using the proposed control systems for the roll bending process and the roll twisting process, the analysis shows that a combined bending and twisting operation may be feasible for thick-wall cross sections, but is not feasible for thin-wall cross sections. Thin-wall cross sections are sensitive to off-shear center loading, require large angle of twists, and violate assumptions used in the roll bending control system. Workpieces with the following properties are potential candidates for a successful combined bending and twisting operation. The workpiece should have a thick-wall cross section which is insensitive to minor off-shear center loading. In addition the moments of inertia, $I_x$ and $I_y$ should be
approximately equivalent. The shaping process should not require large angles of
twist relative to the L/D ratio of the machine. The success of the combined
bending and twisting operation is dependent of the properties of the workpiece and
is related to feasible machine design. Since the proposed application of the shaping
process is for thin-wall cross sections, the bending and twisting operations should
be performed separately to avoid the problems discussed in this section, unless
experiments can prove otherwise.

4.3.2 Control of the Combined Bending and Twisting Process

The criteria for a successful combined bending and twisting process was
outlined in the last section. Assuming the criteria is met, the control scheme for
the combined bending and twisting process includes the control system for the roll
twisting process (Chapter 3) and the control system for the unsymmetrical roll
bending process (Chapter 2). The proposed controller is illustrated in Figure 4-10.
The torque caused by off-shear center loading due to bending forces is included in
the torque measurement input to the roll twisting controller. By measuring the
torque at the shear center, equation (4.2), the roll twisting controller can treat the
torque, caused by bending, as a disturbance. The possibility of feeding forward the
torque disturbance (caused by the bending) was considered and was rejected. To
feed forward the torque disturbance, caused by bending, would require knowing the
moment-curvature curves for the workpiece in advance.

4.3.3 A Shape Control System for Beams Using Separate Bending and
Twisting Operations

The combined bending and twisting operation is feasible for thick-wall cross
sections, but it is not recommended for thin-wall cross sections. Since the proposed
Figure 4-10: Control System for the Combined Bending and Twisting Process
application of the shaping process is for thin-wall cross sections, the bending and twisting operations should be performed separately to avoid the problems discussed in this section. The proposed shape control system divides the shape control system into two separate operations: the unsymmetrical roll bending operation and the roll twisting operation.

Ideally, the twisting and bending operations are decoupled by applying the torque and bending moments about the shear center of the workpiece. This is not always possible. During the roll bending process, the workpiece may be plastically twisted due to off-shear center loading. The roll twisting machine twists the workpiece about its shear center, so no bending will occur during the twisting operation. The order in which the operations are performed is dependent on the roll bending operation. To assure a straight workpiece, first the roll bending operation is performed on the workpiece; then the roll twisting operation is performed to remove any twist introduced during the bending operation. The final result is a straight workpiece.

One alternative to decouple the bending process from the twisting process is to add twisting capabilities to the bending process to negate the effects of off-shear center loading. In this respect, the only control of the twisting controller would be to maintain the unloaded angle of twist at the center roll at zero, and point by point shape control would not be attempted. However, if large angles of twist are required to yield the material for twisting, the assumptions used in the roll bending control system are no longer valid.
Chapter 5
Conclusions and Results

A shaping process for beams consists of both bending and twisting the workpiece to a desired shape. Control systems were developed for the roll twisting process and the roll bending process, individually, by assuming bending and twisting were decoupled. The coupling between bending and twisting was then analyzed to develop a shape control system for beams.

5.1 CONTROLLING THE UNSYMMETRICAL ROLL BENDING PROCESS

Controlling the unsymmetrical bending process is essential to developing a shape control system for beams. The objective of the control system is to bend the unsymmetrical workpiece about a specified neutral axis to a desired curvature. Analysis of the mechanics of unsymmetrical bending found that, in general, unsymmetrical bending is coupled by its product of inertia, and the beam is not necessarily loaded perpendicular to the neutral axis. However, unsymmetrical bending can be decoupled into symmetrical bending about each principal axis of inertia, and the control scheme for the symmetrical roll bending process developed by Hardt, Hale, and Roberts [12, 10, 17] can be used to control bending about each principal axis of inertia.

The proposed controller decouples the bending process into two separate, but simultaneous, symmetrical roll bending controllers about each principal axis. Each
principal axis controller predicts the unloaded curvature by computing the springback of the beam from real-time measurements of the loaded curvature, the bending moment, and the bending stiffness of the beam in principal coordinates. Measurements are made in convenient machine coordinates and transformed by the controller into principal axes coordinates. The unloaded curvature error signal for each principal axis is determined by each of the principal axis controllers, and then transformed onto servo coordinates to control the servos.

The measurement model offers alternative methods of measuring the bending moments and the loaded curvature. For a process requiring large curvatures, all three roll stations need to be gimbaled, and the curvature is measured with respect to the workpiece axis system for accurate results. The bending moments are measured from load cells at one of the outer rolls. For a process requiring large curvatures, the bending moments in machine coordinates must be transposed into workpiece coordinates. For a straightening process, the workpiece coordinate system and the machine coordinate system coincide. The simplest method of measuring the loaded curvature in a straightening process is from the beam deflections. The bending moments and control can be with respect to the machine coordinate system. A less accurate method measures the bending moments at the center roll, but this method should be sufficient for straightening stiff workpieces. The loaded curvature and bending moment measurements are used in an iterative on-line method for computing the principal axes of inertia. This negates the need for knowing the geometry of the cross section in advance. The bending stiffness, or elastic moment-curvature slopes, are computed on-line, so advance knowledge of the material properties of the workpiece are not required.

The control scheme for the unsymmetrical roll bending process assumes that the beam will behave elastically. Analysis of the mechanics for fully plastic bending
of unsymmetrical beams reveal that the plane of loads changes as the beam begins to yield, and the controller will try to compensate even though an error does not actually exist. Experiments are needed to investigate the significance of this error.

The drawback of using a one degree-of-freedom roll bending machine was the need for special rolls to bend the beam about its principal axes, so that an unsymmetrical beam could be straightened in two passes through the one degree-of-freedom bending machine. The other alternative was to use conventional rolls in the one degree-of-freedom bending machine for a straightening process, but bending an unsymmetrical beam would require successive passes to eventually converge on the desired shape. A two degree-of-freedom roll bending machine can bend an unsymmetrical beam about a specified neutral axis to the desired curvature in one pass through the machine and can use conventional rolls, so the two degree-of-freedom roll bending machine was chosen.

Hardt [12] found that vibration of the workpiece contaminated the bending moment measurement, so the fundamental frequency of the vibration of the free ends of the beam limits the bandwidth of the system. Research on the vibration problem is currently being conducted.

5.2 CONTROLLING THE ROLL TWISTING PROCESS

Roll twisting is the process of continuously twisting a beam to a desired shape while rolling the workpiece through a machine. The roll twisting process has not yet been automated and is currently performed manually by skilled operators, so automating the roll twisting process is needed to achieving a shape control shape control system for beams.

The roll twisting process shares many similarities with the roll bending
process, but major differences between the two prevent the direct application of the control system for the symmetrical roll bending process. The workpiece is subjected to a pure torsion by applying a torque with respect to the shear center of the workpiece (this prevents coupling between bending and twisting). A constant torque is applied over a section of the beam, while in the three-roll bending process, the location of the maximum bending moment allows the assumption of point by point deformation. The major problem in controlling the roll twisting process is achieving point by point deformation of the workpiece since a constant torque, applied over a section of the workpiece, plastically deforms the entire section.

A primary controller for the static twisting process is analogous to the control system for the symmetrical roll bending process. The primary controller indirectly measures the unloaded angle of twist by computing the elastic springback of the workpiece from real-time measurements of the loaded angle of twist, the applied torque, and the torsional stiffness of the beam. Advance knowledge of material properties and properties of the geometry of the cross section are not required, since the torsional stiffness can be computed on-line. In a static twisting process, the primary control system provides closed-loop control of the unloaded angle of twist.

The primary twisting controller can be directly applied to a sequencing process, where segments of the workpiece are fed sequentially into the machine. Each segment is twisted to desired angle resulting in a uniform rate of twist over the entire segment. Sequencing has several drawbacks. The machine would be required to start and stop the workpiece at exact locations on the beam, and production time would be slow. In addition, the sequencing method would not see high frequency inputs.

Applying the primary control system to a rolling process results in large
errors. As a point on the beam moves through the machine, it is subjected to continuous plastic deformation until the point exits the machine. The preliminary control analysis showed that point by point deformation could be approximated if the length of the machine approached a point. Since such a machine is not realistic, the next step was to measure the angle very close to where the beam exits from the machine to achieve the same effect.

The simplest control system for a continuous rolling process measures the angle close to where the workpiece exits from the machine. The primary controller is implemented to drive the angle to the desired angle at that particular location in the machine. This allows the length of the beam between the angle measurement device and the machine exit to be considered as the effective length under control. Point by point deformation can be approximated by making this length small. If the rate of twist of the segment and the incoming segment is relatively constant, then the control system maintains the angle at the desired angle. By the time the incoming rate of twist on the workpiece changes, the old segment has passed out of the machine and is no longer under the influence of the machine. The transducer resolution and the design of the roll twisting machine determines the location of the angle measurement device.

A more complex variation of the above control system, compensates for the location of the angle measurement device. The angle of a point on the beam is measured at some convenient location, and the history of the angle at the point on the workpiece is updated as it moves through the machine. When the point approaches the machine exit, the control system retrieves the updated angle and uses it to compute a command to the primary controller to drive the angle to the desired angle. The segment of the beam under control is not dependent on the location of the angle measurement device from a machine design standpoint, but it
does require two angle measuring devices which can be located at convenient positions. The segment of the beam being controlled can be made as small as transducer resolution and computer speed will allow.

The most promising control system is the simplest, measuring the angle as close as possible to the exit of the machine. Computer simulations were made to investigate the effect of deviations from the ideal criteria with respect to sensor location, feedrates and sampling times. This method of controlling the roll twisting process does subject the workpiece to repeated twisting, and the material may strain harden.

5.3 CONTROLLING A SHAPING PROCESS FOR BEAMS

The objective of a shaping process for beams is to bend and twist the beam to some desired shape. In general, the beam has a constant cross section along the length of the beam, and the shape of the cross section is arbitrary. Study of the mechanics of bending and twisting beams show that, in general, bending and twisting are coupled. This means that during a bending process, the beam may plastically twist, and during a twisting process, the beam may plastically bend. Bending and twisting can be decoupled, if the workpiece is loaded with respect to its shear center. In a bending process, if the resultant bending forces, applied to the workpiece, do not pass through the shear center of the workpiece, the beam will twist. Thick-wall cross sections have a high torsional resistance and are not prone to twisting while bending. Thin-wall cross sections, however, have a low torsional resistance and may plastically twist as a result of off-shear center bending forces. Another cause of twisting during the bending process is due to buckling of thin-wall cross sections. Minimizing twisting while bending can be accomplished by
designing roll assemblies to apply the bending forces at the shear center of the workpiece. Preventing bending from occurring during a twisting process requires twisting the workpiece about its shear center.

Two different shape control systems are proposed. The first control system utilizes a combined bending and twisting operation, while the second separates the bending and twisting processes into two separate operations. The analysis of different sections shows that the combined bending and twisting operation may only be feasible for thick-wall cross sections with moments of inertia, $I_x$ and $I_y$, approximately equivalent. For all other sections, the assumptions used in the control system for the roll bending process and the control system for the roll twisting process are violated, and this may cause erroneous results. Appendix C outlines experiments for testing the feasibility of a combined bending and twisting operation for thin-wall cross sections.

Control of the combined bending and twisting process merely combines the control systems for the bending and twisting processes. The forces and torques are measured with respect to the shear center to achieve the desired result. One concern is that bending and twisting are, in general, coupled, so the twisting caused by bending will create a disturbance in the twisting control system and vice versa. Another possible problem is designing a machine that can perform both processes at once.

Finally, a shape control system for beams is proposed which consists of separate bending and twisting processes. Since bending is more sensitive to twisting, the bending operation is performed on the workpiece first. Any twist introduced by the bending process, is then compensated for in the second operation - the twisting process, which is less sensitive to bending. The twisting process is effectively decoupled from bending by twisting the workpiece about the shear
An ideal shape control system for beams of constant cross section would simultaneously perform two-plane roll bending for the desired curvature and roll twisting for the desired angle of twist. The combined bending and twisting operation is feasible for thick-wall cross sections, but it is not recommended for thin-wall cross sections. Thin-wall cross sections are sensitive to off-shear center loading, require large angle of twists, and are sensitive to violations of cross sectional property assumptions used in the roll bending controller. Since the proposed application of the shaping process is for thin-wall cross sections, the bending and twisting operations should be performed separately to avoid the problems discussed in this section.

5.4 FUTURE RESEARCH

5.4.1 Proposed Static Experiments

The next phase of this research will consist of static experiments which are designed to test and evaluate the control systems for the unsymmetrical bending process and the twisting process. In addition, several experiments are proposed to answer questions raised by theoretical analysis. The purpose of these experiments is to:

- test and evaluate the control system for the unsymmetrical bending process, including on-line computations of material properties and measurement alternatives.

- examine the coupling between bending and twisting. The objective of these experiments is to establish guidelines for roll assembly design to minimize twisting during a bending operation and vice versa.
- evaluate the feasibility of a combined bending and twisting process for thin wall cross sections.

- perform some preliminary experiments on the roll twisting process.

5.4.2 Future Research for Bending

The completion of the proposed static bending experiments should resolve several uncertainties about the unsymmetrical roll bending process. Based on results and evaluation of the static experiments, the next step is to test the control system under actual rolling conditions. However, Hale and Roberts performed extensive testing of the control system for the symmetrical roll bending process, and at this point in the research, it is not anticipated that the dynamic characteristics of the unsymmetrical roll bending process will differ very much from the dynamics of the symmetrical roll bending process. Hale and Roberts found that the vibration of the free end of the beam severely limited the bandwidth of the overall process, and research is needed to investigate controlling the vibration such that the feedrate can be increased above quasi-static levels [10, 12, 17].

One area of research that deserves further investigation or testing is the relation of the curvature to the position on the beam. When the beam is bent, the neutral axis shifts and the beam elongates. Roberts measured the arc length at the exit roll to correlate beam position with curvature [17]. This does not account for the elongation that occurs between the exit roll and the center roll. The accuracy of measuring the arc length at the outer roll versus the curvature should be investigated relative to the final part shape. One possibility might be to measure the arc length at both the center and exit roll to determine the elongation that has occurred, and determine the position of the point on the beam at the center roll. The objective is to have a control system which determines the
position of the point at the center roll in relation to the arc length of the finished workpiece. Such a control system would negate the need for computing the shift in neutral axis and the elongation of the beam in determining the commands of curvature versus position. The final desired shape would simply be input to the computer, and the position of the point of the beam at the center roll would then determine the appropriate curvature command.

5.4.3 Future Research for Twisting

After the completion of the static twisting experiments, roll twisting experiments should be conducted to evaluate the proposed control system. In particular, the simulations show that instability can occur if the angle is measured with respect to the end of the beam, but measuring the angle relative to the machine exit results in larger errors. For a straightening process, the governing concern is the absolute angle of twist over the entire length of the part. Further research should be conducted to investigate controlling the overall angle of twist, by measuring the absolute angle and adjusting the commands to the control system for the roll twisting process.

The proposed control system for the roll twisting process approximates point by point shape control by assuming a sequencing process which acts over a very small segment of the part. Further research should be conducted to develop a continuous model of the process and find a better method of achieving point by point shape control. Another alternative is develop a machine which can apply a maximum torque, analagous to the maximum bending moment for the bending process. However, this does not appear feasible from a machine design standpoint of view.
5.4.4 Other Research

The machine design for the combined bending and twisting process using a three-point, two DOF bending machine which incorporates a twisting servo at the center roll may be difficult to achieve. Another possibility is to incorporate a twisting process in a pinch-roll bending machine. However, such machine would require one roll assembly to be fixed. The diameter of the rolls and the curvature will then determine the location of the maximum moment. The model assumption for the roll bending controller assumes that the bending caused by shear stresses is negligible, but in a pinch bending machine the bending from shear may not be negligible. This would then require modification of the bending model and possibly the twisting model. Further research should be conducted to examine the effects of shear in a pinch bending machine or designing a pinch bending machine such that bending from shear is negligible.

Another metal forming process that can benefit from the springback control concept is the contour roll forming process. Research is need to develop a model and a control design for this process. The springback control concept might be also used in measuring defects in a cold rolling process. One problem is measuring the defects before the part exits the many stationed machine.
References

Bending Tube, Pipe, and Structural.  

Straightening of Bars, Shapes and Long Parts.  
Metals Handbook.  

Advanced Mechanics of Materials.  

General Scheme for Automatic Control of Continuous Bending of Beams.  

An Introduction to the Mechanics of Solids.  

[6] De Angelis, R. J., and Queener, C. A.  
Elastic Springback and Residual Stresses in Sheet Metal Formed by Bending.  

[7] Foster, G. B.  
Springback Compensated Continuous Roll Forming Machine.  

The Springback of Metals.  

[10] **Hale, Michael B.**  
*Dynamic Analysis and Control of a Roll Bending Process.*  


[13] **Heins, Conrad P.**  
*Bending and Torsional Design in Structural Members.*  

[14] **Higdon, Ohlsen, Stiles, Weese, and Riley.**  
*Mechanics of Materials.*  

[15] **Baumeister, T. (editor).**  
*Standard Handbook for Mechanical Engineers.*  

[16] **Mergler, H.**  
*Computer-Controlled Bender Makes Complex-Curve Beams.*  

[17] **Roberts, M. A.**  
*Experimental Investigation of Material Adaptive Springback Compensation in Roller Bending.*  

[18] **Sachs, G.**  
*Principles and Methods of Sheet Metal Fabricating.*  

[19] **Shaffer, B. W., and Ungar, E. E.**  
*Mechanics of the Sheet Bending Process.*  
[20] Shanley, F. R.
Elastic Theory in Sheet Metal Forming Problems.

[21] Shigley, J. E.
*Mechanical Engineering Design.*

[22] Timoshenko, S., and Goodier, J.
*Theory of Elasticity.*

[23] Trostmann, E.
Numerically Controlled Bending of Metal Beams.
4th IFAC/IFIP International Conference on Digital Computer Applications to Process Control, Zurich, Switzerland, Mar 19-22, 1974 p.87-98.

[24] Tse, F., Morse, I., and Hinkle, R.
*Mechanical Vibrations.*

[25] Tselikov, A. I., and Smirnov, V. V.
*Rolling Mills.*
Translated from Russian.
Appendix A

MACHINE DESIGN CONCEPTS FOR THE UNSYMMETRICAL ROLL BENDING PROCESS

A.1 Conceptual Design for a Two Degree-of-freedom Roll Bending Machine for Straightening

A conceptual design for a two degree-of-freedom roll bending machine for straightening unsymmetrical extrusions is shown in Figure A-1. A modified vertical-spindle knee-and-column milling machine has two servos to move the table along the x and y axes, while the spindle remains fixed. It is somewhat easier to think of the table as being fixed, and letting the spindle move in the x-y plane. The spindle is used to drive the center roll to roll the workpiece through the machine. The outer rolls are gimbaled to avoid applying a bending moment to the workpiece at the outer rolls.

Based on Hale's error analysis of beams with high bending stiffness, the center roll is not allowed to rotate, and the loaded curvature is measured from the beam deflection rather than complex curvature sensors. [10] The bending moments can either be measured from forces measured at an outer roll or from forces measured at the center roll. For this machine configuration, the bending moments are measured at one of the outer rolls, since the center roll is attached rigidly to the spindle. The spindle will carry some of the bending forces, so measuring the center roll forces is not recommended. At the outer roll, the simplest force measurement would be to measure $F_{xA}$ and $F_{yA}$. For more accuracy, the force, $F_{zA}$,
could also be measured. A three-axis load cell would simply be sandwiched between the gimbal support and the table.

The proposed roll assembly design is more complex for the two degree-of-freedom bending machine. Each roll assembly should consist of four sets of rolls. This is required to minimize the error in the bending moment measurements, since a conventional roll assembly can introduce errors in the bending moment measurement. The proposed design will also eliminate most of the sliding friction caused by the workpiece sliding rather than rolling against the rolls. The four-roll assembly is illustrated in Figure A-1 at the center roll. Each roll has point contact with the workpiece. Using a roll assembly with only two rolls would require line to line contact of the workpiece and the roll in one of the planes. If the process consists of large curvatures and the diameter of the rolls is large, each roll assembly would have the capability of applying a local three-point bending moment on the workpiece at each roll assembly. In addition, the line to line contact of the rolls and the workpiece at the center roll apply forces at two points on the workpiece. Since the center roll is not gimbaled, the maximum moment will be applied somewhere between the point where the workpiece first contacts the roll and where the workpiece last contacts the roll.

In the straightening process, the curvatures will be small, so the roll assembly with only two sets of rolls may be sufficient. To avoid the local pinch bending moments, the diameter of the rolls that have line to line contact with the workpiece should be as small as possible. Sliding friction will still exist.
Appendix B
Conceptual Design for a Roll Twisting Machine

A conceptual design for a roll twisting machine is illustrated in Figure B-1. The workpiece is pulled through the machine by the feed motor at the fixed roll station. The other roll station is used to twist the workpiece. The machine should have the capability to adjust the roll assemblies such that the machine center of twist will coincide with the shear center of the workpiece. To allow for adjustment of the roll assembly, the power from the feed motor can be transmitted through a flexible coupling or the motor can be mounted on the adjustment plate. The workpiece is rotated by the twist motor at the other roll station. The power from the twist motor can be transmitted with gears, Figure B-1. A better solution is to transmit the power with a timing belt to minimize backlash.

The pros and cons of a roll assembly with two sets of rolls versus four sets of rolls was discussed in Appendix A. Four rolls can achieve point to point contact between the roll and the workpiece; two rolls result in line contact between the roll and the workpiece. The length over which the torque is applied, and the position where the angles are measured are clearly defined when the rolls have point contact with the workpiece. With line contact, the geometry is no longer clearly defined, and errors will result. The alternative is to go to smaller diameter rolls.
Figure B-1: Conceptual Design for a Roll Twisting Machine
Appendix C

Proposed Experiments and Experimental Apparatus

Previous theoretical analysis has resulted in machine design concepts and on-line methods for computing material properties. This section outlines several experiments which are designed to test the control systems for the unsymmetrical bending process and the twisting process and answer several questions which arose during the theoretical analysis.

C.1 Experimental Apparatus

The experimental apparatus, in Figure C-1, is designed for static tests. The experimental apparatus can configured for unsymmetrical bending experiments, twisting experiments, and combined bending and twisting experiments. The supports at either end of the workpiece are attached to the table of the milling machine. The table can be moved either along the x-axis, or the y-axis, or both axes, to bend the workpiece in two planes. The center assembly support is attached to the quill of the milling machine and remains fixed in space. The center support assembly is designed to allow the workpiece to rotate for certain experiments, or it can be clamped to prevent workpiece rotation for bending experiments. Two load cells on the center support assembly measure the forces and torques on the workpiece. The experimental apparatus will be used for several different experiments, and dial gauges or other transducers will be used to monitor the desired measurements in each experiment.
Each support uses a keystock design to restrain the workpiece, Figure C-2. The keystock is adjusted to conform to the cross section of the workpiece and load the workpiece in the desired manner. The keystock is then clamped. The keystock design can easily be reconfigured for various cross sections and for experiments investigating off-shear center loading.

C.2 Experiments

Several static experiments are proposed to verify the controller and answer questions raised by theoretical analysis. The purpose of these experiments is to:

- test and evaluate the control system for the unsymmetrical bending process, including on-line computations of material properties and measurement alternatives.

- examine the coupling between bending and twisting. The objective of these experiments is to establish guidelines for roll assembly design to minimize twisting during a bending operation and vice versa.

- evaluate the feasibility of a combined bending and twisting process for thin wall cross sections.

- perform some preliminary experiments on the roll twisting process.

C.2.1 Unsymmetrical Bending Experiments

Completion and evaluation of these experiments will resolve the uncertainties about the unsymmetrical bending process and the proposed control system. If the results are satisfactory, recommendations can be made regarding a production machine.
Figure C-2: Fixed End Assembly
C.2.1.1 T-section Experiment

The first experiment will consist of bending a T-section which has one axis of symmetry. The principal axes of the T-section are well established by its geometry, and the shear center lies on the axis of symmetry. The keystock in the bending machine is adjusted to load the workpiece through the theoretical shear center. The experiments will be performed in the following order:

1. The principal axes of the T-section will be verified based on experimental data.

2. The moment-curvature slopes along the principal axes, $1/(E I_x)$ and $1/(E I_y)$, will be computed from experimental data.

3. The T-section is loaded in the elastic region, and the loaded curvature is measured from the beam deflection and with dial guages near the shear center. The validity of the curvature measured from the beam deflection will be compared with the dial guage curvature measurement.

4. The unsymmetrical roll bending controller will be tested by bending the workpiece beyond its yield point. The unloaded curvature predicted by the controller will be compared with the actual unloaded curvature.

C.2.1.2 Plastic Bending of Unsymmetrical Beams

Similar tests will be conducted on unsymmetrical cross sections, such as unequal leg angles without an axis of symmetry. Of particular interest, is the significance of the error predicted by fully plastic bending theory. The proposed experiment monitors the plane of loads as the workpiece is bent about a neutral axis coinciding with one of the principal axes. The bending moments, $M_x$ and $M_y$, are recorded as an initially straight beam is bent through the elastic region and into the plastic region. By elastic theory, the bending moments should be proportional
to one another in the elastic region, so the slope of $M_x$ versus $M_y$ is constant in the elastic region, and should not deviate very much in the plastic region for superposition principles to hold.

C.3 Twisting Experiments

Experiments will be performed to evaluate the twisting controller. The roll twisting controller cannot be evaluated completely, since the experimental apparatus can only perform non-rolling tests. Experiments will include:

- on-line computation of the torque-rate of twist slope from experimental data.

- a comparison between the unloaded angle of twist as predicted by the controller and the actual unloaded angle of twist.

- experiments to evaluate the coupling between bending and twisting as related to the twisting process. Guidelines for roll assembly design that will minimize twisting can be established.

- examining the validity of the twisting model as the length of the beam, over which the torque is applied, is decreased.

- examining the validity of the angle measurement as the distance between the angle sensor and the fixed support is decreased.

C.4 Coupling between Bending and Twisting Experiments

The experimental apparatus is designed to allow simultaneous bending and twisting. This allows for on-line computation of the shear center. And once the shear center has been established, the center support assembly can be clamped to investigate the coupling between bending and twisting.
The keystock design allows the workpiece to be loaded in a particular configuration, and twisting caused by off-shear center loading in the unsymmetrical bending process can be tested. Twist induced by the bending operation is undesirable, and these experiments will establish guidelines for roll assembly design to minimize twisting during the bending operation.

C.5 Combined Bending and Twisting Process Experiments

Theoretical analysis of thin wall cross sections questioned the feasibility of a combined bending and twisting operation. After the previous experiments have been completed and evaluated, a combined bending and twisting experiment will be performed on a thin wall cross section. Will the bending and twisting controllers give valid results even though assumptions made for each controller are violated? Evaluation of this experiment should establish the feasibility of a combined bending and twisting machine for thin wall cross sections.
Appendix D
Computer Program for Twisting Simulations

This program simulates the roll twisting process. Various versions of this program were used to investigate different methods of controlling the process and measurement alternatives. The position on the beam is identified by \( Z \) and an integer index \( I \). The loaded and unloaded angle of twist on the workpiece are identified by \( \phi_L \) and \( \phi_U \), respectively. A Runge-Kutta integration loop is used. The torque, \( T_{SENSOR} \), is computed from the angle \( \phi_{SYS} \). The angles, \( \phi_L \) and \( \phi_U \), are updated over the entire workpiece at the end of each outer loop, and then the workpiece is indexed by an amount \( \Delta Z \) to a new position. The inner loop controls the unloaded angle at position \( Z_{rel} \) and feeds back the angle, \( \phi_{ULM} \), and the angular velocity, \( \phi_{USYS} \). The subroutine SYSTEM simulates a velocity servo. The inner loop control system uses proportional control, \( GAIN_1 \), with velocity feedback, \( GAIN_2 \), to control the servo.

This particular program simulates a control system which monitors an angle at some point as the workpiece moves through the machine (or window). Angles are measured at positions, \( z_{1rel} \) and \( z_{2rel} \), and are recorded by the control system. The subroutine UPDATANG updates the angles on the beam at each pass, or each outer loop sample time. As the angle at a certain position on the workpiece nears the exit of the machine, the outer loop control system retrieves the updated angles, \( \phi_U(2) \) and \( \phi_U(2) \) to compute the command to the inner loop, \( COM_0 \) such that the angle at position, \( Z_{rel0} + \Delta Z \), is driven to desired angle,
D.1 COMPUTER PROGRAM FOR SIMULATING ROLL TWISTING PROCESS

c relation of variables to workpiece position
   *----------TOTALL------------------*
   c Z : 0 TOTALL
   c ANGINIT : 0 PI
   c INDEX : 1 NEND
   c
   c relative initial first second final
   c initial moment angle angle moment
   c angle applied measured measured applied
   c *--LENGTHC --------- LENGTH of window ---------*
   c
   c Z : ZM0 Zrel0 Z1rel Z2rel ZLrel
   c index : I0 Irel0 I1rel I2rel ILrel

REAL LENGTH,LENGTH0,INERTIA,MODRIG
COMMON /XYDATA/F(4),Y(4),SAVEY(4),PSI(4),M,RKU,COM,DT
1 /CONST/C1,C2,C3,C4,C5,C6,C7
2 /data/PI,XMAG,NEND,TCON,PHIUDES,TAUY,RADIUS,MODRIG,
   & GAIN1,GAIN2,TOTAL,FRACTL,FRACTR,FRACTL,FRACR,NRK,KEND,
   & NLENGTH,NTCON,FUDGE,TSAMPLE,FEED,DELTAZ,LENGTH,
   & LENGTH0,TYLDD,PHIYLD,INERTIA,SLOPANG,NRUNGE
   & /CURV/PHIL(101),PHIU(101),PHINIT(101)
   & /POS/Z,Zrel0,Z1rel,Z2rel,ZLrel,ZM0,Irel0,I1rel,
   & I2rel,ILrel,I0
   & /TORQ/PHIREL,PHYSYS,APHIREL,TSENSOR,FLAG,PHIUSYS,
   & PHIUTOT,PHIUOLD
   & /ANG/ N1REL,N2REL,PHIULM,PHI1(101),PHI2(101)

CALL DATA /initialize variables
CALL PLOT8(-1.) /initialize plotting file
CALL INITCURV /define original deformation of workpiece
CALL POSITION /identify position on workpiece
CALL PREANG /initialize premeasured angles
   PHIULM = PHINIT(Irel)
   PHIUSYS = PHINIT(Irel)
PHISYS = PHINIT(IIrel)
Y(1) = PHINIT(IIrel)

c -------------- Main Program (Outer Loop) -----------------------
DO 100 K = 1,KEND  /# of passes
   PHIUOLD = PHIULM
   PHILOLD = PHISYS
   CALL POSITION
   ! measurement for forecasting angles (outer loop)
   PHIU1(IIrel) = PHIU(IIrel) - 1 FLAG*TSENSOR*FRACTL*LENGTH/SLOPANG
   PHIU2(IIrel) = PHIU(IIrel) - 1 FLAG*TSENSOR*LENGTH/SLOPANG
   ! command to inner loop
   COMol = -(NLENGTH -1.0)*PHIU1(2) + PHIU2(2)

! ------------ servo (inner loop) ----------------
DO 150 NDT=1,NRUNGE
   N = 2  /index for RUNGE subroutine
   KS = 1  /index for RUNGE subroutine
   ! compute command for servo
   ERROR = (PHIUSES + COMol) - PHIULM - GAIN2*DPHIUSYS
   PHICOM(K) = ERROR*GAIN1/TCON
   COM = PHICOM(K)
   DO 110 M = 1,4  /runge kutte integration loop
      CALL SYSTEM  /define system equations
      CALL RUNGE(N,KS)  /runge-kutta integration
   110 CONTINUE
   ! ------------ measure servo  -----------
   PHISYS = Y(1)
   DPHISYS = (Y(2))
   CALL TORQ(FLAGOLD)  /compute torque
   ! PHIU at LENGTH measurement
   PHIULM = PHISYS - FLAG*TSENSOR*LENGTH/SLOPANG
   ! find PHIUTOT, DPHIUSYS at LENGTH (controller)
   IF (APHIREL.LE.PHIYLD) DPHIUSYS = 0.0
   IF (APHIREL.GT.PHIYLD)
      DPHIUSYS = DPHISYS*(1.0 - ((PHIYLD/APHIREL)**4))
   T = T + DT
   150 CONTINUE
   CALL UPDATANG  /update premeasured angles
   CALL UPDATE  /update angles on beam
\[ Z = Z + \Delta Z \]

100 CONTINUE

c end of main program

CALL PLOT8(1.) /plotting file

END

c SUBROUTINE DATA

initialize variables

SUBROUTINE DATA

REAL LENGTH, LENGTH0, INERTIA, MODRIG

COMMON same as main program

READ(9,*)PI,

& FUDGE, /used for converting real #’s to integers
& NEND, /number of points on beam
& TCON, /sec; time constant of servo
& TAUY, /lbs/inch2; shear stress(yield)
& RADIUS, /inch; radius of rod
& MODRIG, /(real) lbs/inch2; shear modulus
& GAIN1, /servo input gain
& GAIN2, /velocity feedback gain
& TOTALL, / inch; total length of the beam
& FRACTL, / fraction of window size where angle is to be measured
& FRACT2L, /fraction of window size where second angle is measured
& NRK, /number of DT’s each sample time
& PHIUDES, /radians; desired angle

TYPE*, 'enter KEND, NEND = 101'
READ(5,*) KEND /# of passes

TYPE*, 'enter NLENGTH, 21'
READ(5,*) NLENGTH /# of increments assigned to window

TYPE*, 'enter NTCON, number of TSAMPLE/DELTAZ,(32)'
READ(5,*) NTCON

TYPE*, 'enter XMAG (1.)'
READ (5,*) XMAG /magnitude of sine wave

c compute initial variables

TSAMPLE = .25*TCON

c sample period

FEED = (TOTALL/(TSAMPLE*(NEND - 1)))/NTCON

c FEED; inch/sec; velocity of beam passing thru window
DELTAZ = TOTALL/(NEND - 1.)
   inch; change in length/interval
LENGTH = (NLENGTH - 1.)*TOTALL/(NEND - 1.)
   ( real ) inch; length of window REAL
LENGTH0 = LENGTH
   (real) inch; length behind Zrel0 where PHI(i0) is measured
TYLD = TAUY*(RADIUS**3.0)*PI/2.0
   yield moment
PHIYLD = (TATJY*LENGTH)/(MODRIG*RADIUS)
   yield angle of twist
INERTIA = PI*(RADIUS**4.0)/2.0
   moment of inertia
SLOPANG = (MODRIG*INERTIA)
   slope of unloading line = moment/angle
C1 = 1/TCON
   constant for system equations
NRUNGE = (DELTAZ/(TSAMPLE*FEED))*NRK + FUDGE
   # of NRK*TSAMPLE's in runge-kutta loop (NTCON)
DT = TSAMPLE/NRK
   time for Runge-Kutta integration
RETURN
END

*************** SUBROUTINE PLOT8 ***************
plotting file
SUBROUTINE PLOT8(PLOT)
REAL LENGTH,LENGTH0,INERTIA,MODRIG
COMMON same as main program
OPEN(UNIT=8,FILE='TWIST',STATUS='NEW')
IF (PLOT.LT.0) THEN
   WRITE(8,*) '100' /# of points to plot
   WRITE(8,*)'4,5,6' /plotting-marking symbols
END IF
IF (PLOT.GT.0) THEN
   Z = 0
   DO 60 I2 = 1,100
      WRITE(8,*) Z, PHINIT(I2),PHIL(I2), PHIU(I2)
      Z = Z + DELTAZ
   60   CONTINUE
   WRITE(8,*)'LENGTH' /x label,y label
WRITE(8,*)'PHI (DEGREES)'
WRITE(8,*)'0,1000,0,1000' /xmin,xmax,ymin,ymax
END IF
RETURN
END

c **************************************** SUBROUTINE INITCURV ***************
c original deformation of workpiece
SUBROUTINE INITCURV
REAL LENGTH,LENGTH0,INERTIA,MODRIG
COMMON same as main program
DO 10 15 = 1,NEND
   AI5 = IS -1.0
   ANGPINc = PI/(NEND - 1.0) /radians/increment
   ANGINIT = AI5*ANGPINc /radians (0 to Pi)
   PHINIT(I5) = XMAG*SIN(ANGINIT)
c PHINIT = initial angles along the length of the beam
   PHIU(I5) = PHINIT(I5) /unloaded angles
   PHIL(I5) = PHINIT(I5) /loaded angles
10 CONTINUE
RETURN
END

c **************************************** SUBROUTINE POSITION ******************
c identify position on workpiece(relation to machine)
SUBROUTINE POSITION
REAL LENGTH,LENGTH0,INERTIA,MODRIG
COMMON same as main program
   Z rel0 = Z /where workpiece exits machine
   Z1rel = Zrel0 + LENGTH*FRACTL /angle measurement
   Z2rel = Zrel0 + LENGTH*FRAC2L /angle measurement
   ZLrel = Zrel0 + LENGTH /where beam enters machine
   Ire0 = Zrel0/DELTAZ + FUDGE + 1 /index at z = Zrel0
   I1rel = Z1rel/DELTAZ + FUDGE + 1 /index at z = Z1rel
   I2rel = Z2rel/DELTAZ + FUDGE + 1
   ILrel = ZLrel/DELTAZ + FUDGE + 1 /index at z = ZLrel
RETURN
END
c ************ SUBROUTINE PREANG ************
c initialize angles on beam for 0 < zrel < z2rel
SUBROUTINE PREANG
    REAL LENGTH, LENGTH0, INERTIA, MODRIG
    COMMON same as main program
    N1rel = I1rel
    N2rel = I2rel - I1rel + 1
    DO 15 I9 = 1, N1rel
        PHIU1(I9) = PHINIT(I9)
    15 CONTINUE
    DO 16 I11 = 1, N2rel
        PHIU2(I11) = PHINIT(I1rel - 1 + I11)
    16 CONTINUE
RETURN
END

c ************ SUBROUTINE SYSTEM ************
c define system equations
SUBROUTINE SYSTEM
    COMMON /XYDATA/F(4), Y(4), SAVEY(4), PSI(4), M, RKU, cOM, DT
1    /CONST/C1, C2, C3, C4, C5, C6, C7
    F(2) = cOM - C1*Y(2)
    F(1) = Y(2)
RETURN
END

C ***************** SUBROUTINE RUNGE ************************
c Runge Kutta Integration
SUBROUTINE RUNGE(N, K)
    COMMON /XYDATA/F(4), Y(4), SAVEY(4), PSI(4), M, RKU, cOM, DT
    GO TO (10, 30, 50, 70), M
10 CONTINUE
    DO 20 J = K, N
        SAVEY(J) = Y(J)
        PSI(J) = F(J)
        Y(J) = SAVEY(J) + 0.5*DT*F(J)
    20 CONTINUE
    GO TO 90
30 CONTINUE

c *************** SUBROUTINE TORQ ***************
c compute torque, TSENSOR at position, ZLrel
SUBROUTINE TORQ(FLAGOLD)
 REAL LENGTH, LENGTH0, INERTIA, MODRIG
 COMMON same as main program
 PHIREL = PHISIS - PHIU(ILrel + 1)
 APHIREL = ABS(PHIREL)
 IF (APHIREL.GT.PHIYLD) THEN
  TSENSOR = (4.0/3.0)*TYLD*
            (1.0 - ((PHIYLD/APHIREL)**3.0)/4.0)
 END IF
 IF (APHIREL.LE.PHIYLD) THEN
  TSENSOR = SLOPANG*APHIREL/LENGTH
 END IF
 IF (PHIREL.LT.0.0) FLAG = -1.0
 IF (PHIREL.GE.0.0) FLAG = 1.0
 IF (FLAG.NE.FLAGOLD.AND.APHIREL.GT.PHYLD)
  to prevent jumping
  FLAGOLD = FLAG
 PHIUSYS = FLAG*(APHIREL - TSENSOR*LENGTH/SLOPANG)
 PHIUTOT = PHIU(ILrel+1) + PHIUSYS
 c total = system + disturbance, real value
 RETURN
 END
SUBROUTINE UPDATANG
REAL LENGTH, LENGTH0, INERTIA, MODRIG
COMMON same as main program
SLOP1 = (PHIULM - PHIU2(2))/(LENGTH - 1.0)
DO 19 I10 = 1, N1rel
    RK10 = I10 - 1.0
    PHIU1(I10) = PHIU1(I10) + SLOP1*RK10
19 CONTINUE
DO 21 I13 = 1, N1rel
    PHIU1(I13) = PHIU1(I13 + 1)
21 CONTINUE
SLOP2 = PHIULM - PHIU2(2)
DO 20 I12 = 1, N2rel
    PHIU2(I12) = PHIU2(I12) + SLOP2
20 CONTINUE
DO 22 I14 = 1, N2rel
    PHIU2(I14) = PHIU2(I14 + 1)
22 CONTINUE
RETURN
END

SUBROUTINE UPDATE
REAL LENGTH, LENGTH0, INERTIA, MODRIG
COMMON same as main program
ZREL = Zrel0
PHIU1rel = PHIUTOT - (PHIU(Irel+1) - PHIUOLD)
DPHIU = PHIU1rel - PHIUOLD
SLOPU = DPHIU/LENGTH
PHIULM = PHISYS - FLAG*TSENSOR*LENGTH/SLOPANG
SLOPL = FLAG*TSENSOR/SLOPANG
DO 180 I1 = Irel0, NEND
    IF (ZREL.LE.ZLrel.AND.ZREL.GE.Zrel0) THEN
        PHIU(I1) = PHIU(I1) + SLOPU*(ZREL - Zrel0)
180 CONTINUE
RETURN
END
IF (I1.EQ.Irel0) PHIL(Irel0) = PHIU(Irel0)
IF (I1.GT.Irel0) THEN
   PHIL(I1) = PHIU(I1) + SLOPL*(ZREL - (Zrel0 + DELTAZ))
END IF
END IF
IF (ZREL.GT.ZLrel) THEN
   PHIU(I1) = PHIU(I1) + SLOPU*LENGTH
   PHIL(I1) = PHIU(I1) + SLOPL*LENGTH
END IF
ZREL = ZREL + DELTAZ
CONTINUE
RETURN
END