# Frequency Determination

## Outline

1. Frequency Determination
   - current practice
   - simple rules
2. Optimization formulation for independent routes
3. Heuristic for complex networks
4. Timetable Development

# Frequency Determination: Current Practice

Frequencies typically based on:

- Policy headways - vary by time of day and route type
- Maximum loads - vary by time of day and route type

These represent constraints rather than decision algorithms

# Simple Rules for Frequency Determination

A. Constant max load factor at a level below official max load factor

- may vary by time period

B. Constant average occupancy level subject to capacity constraint

- may also be subject to a max time for loads above a specified level

# Importance of Frequency Determination

- major short-range planning decision
- affects service quality through wait time and crowding
- affects transit path selection (assignment) in complex networks

Two different contexts:

- North American city:
  - ridership sensitive to service quality
  - sparse network, little transit path choice
  - maximum acceptable crowding levels specified
  - defined level of subsidy available
- Less developed country city:
  - ridership constrained by capacity
  - crowding levels very high
  - dense network, significant transit path choice

# North American Frequency Determination Problem[1]

## Decision variables:

- headway on each route for each time period

## Objective function:

- maximize: consumer surplus + social ridership benefit

  ≡ *a* • wait time savings + b • ridership

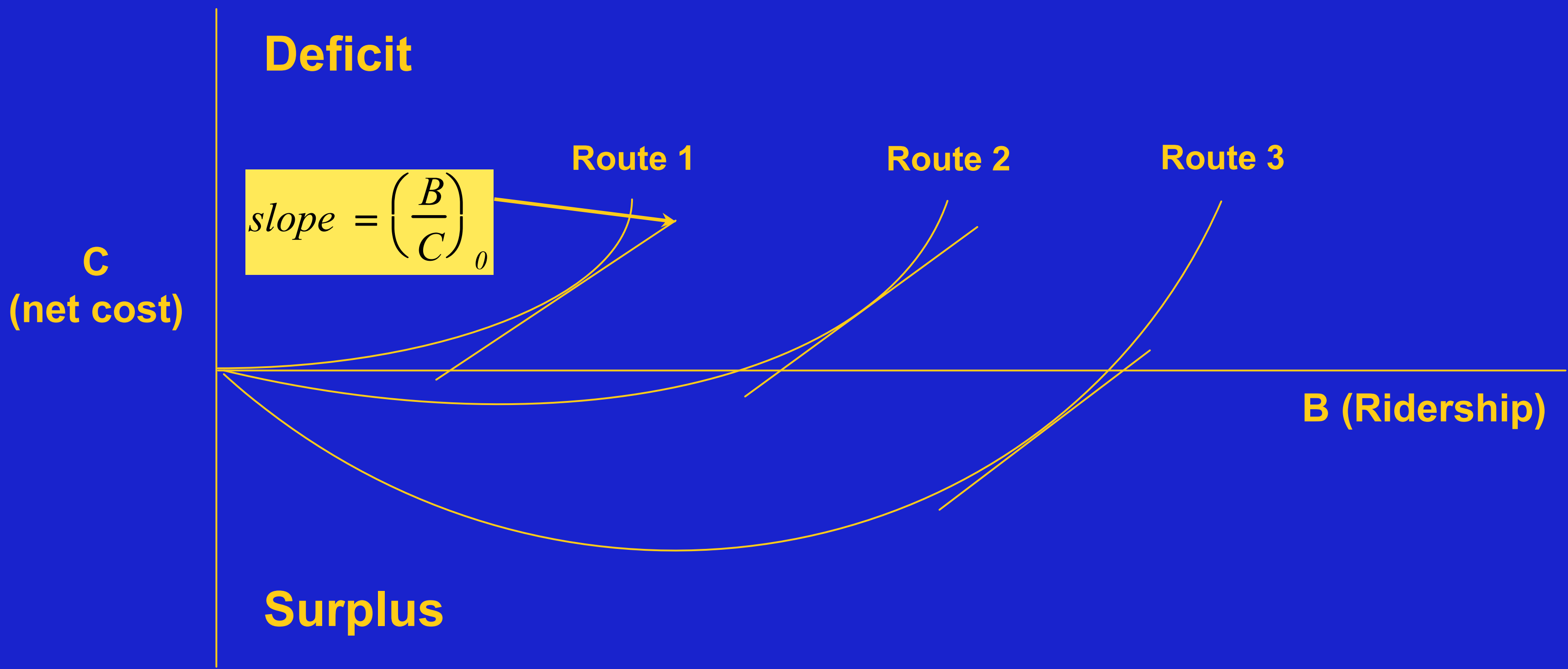
## Subject to constraints on:

- total subsidy is exhausted
- total fleet size is not exceeded
- headway meets policy maximums and loading maximums

1 Furth, P.G. and N.H.M. Wilson, "Setting Frequencies on Bus Routes: Theory and Practice," Transportation Research Record 818, 1981, pp 1-7

# Efficiency in Subsidy Allocation

This is a resource allocation problem: for optimality, allocate enough resources to each route so that Marginal Benefit/Cost Ratio is same on each route.



# Conclusions

- square root rule is valid where constraints are not binding
- problem can be solved using Lagrangian relaxation and single variable search techniques -- not very complex
- existing scheduling practice over allocates service to peak and to long, high ridership routes
- minimizing wait time assuming fixed demand gives similar solutions to more complex objective and variable demand
- best allocation of resources is quite robust with respect to objectives and parameters assumed

# Developing Country/City Frequency Determination Problem

## Objectives:

- minimize crowding levels
- minimize waiting times

## Subject to constraints on:

- loading feasibility
- passenger assignment
- total fleet size

# Passenger Assignment
# Heuristic Approach

1. **Classify flow into:**
   - "captive flow" (CF) -- any O-D pair with only one feasible path
   - "variable flow" (VF) -- O-D pairs with more than one feasible path
2. **Assign VF in proportion to frequency share on acceptable routes**
   - consistent with random bus arrival process

$$\frac{D_i}{\sum_{j \in J} D_j} = \frac{F_i}{\sum_{j \in J} F_j}$$

where $D_i$ = demand assigned to route *i* for specific O-D pair
$F_i$ = frequency offered on route *i*
$J$ = set of acceptable routes

# Models

## A. Normative Model

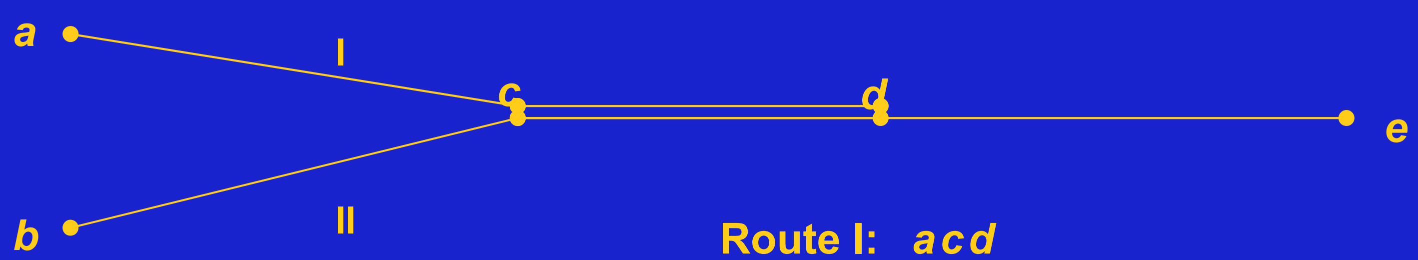
- "assign" passenger flows to routes with minimum round trip vehicle time among all acceptable paths
- compute frequency and fleet size required on this assignment basis

## B. Descriptive Model

- assign passengers to alternative acceptable paths in proportion to frequency share in an iterative process

The difference in the total fleet sizes from the normative and descriptive models indicates the extent of inefficiency resulting from the overlapping route structure.

# Simple Example of Overlapping Routes



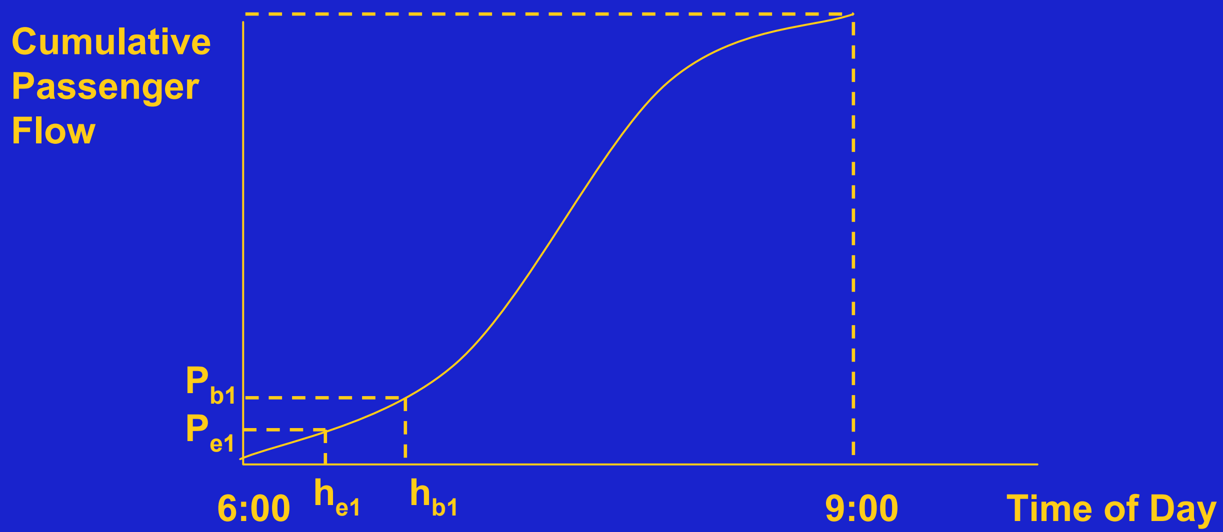

Route I: *acd*
Route II: *bcde*

- O-D pair *cd* is VF, all other pairs are CF
- ideally would like to assign *cd* flow to route I, which is shorter, but these passengers will take route I or II depending on which arrives first.
- some *ce* passengers may be forced to board route I buses, then make a transfer at *d* to route II

# Timetable Development

**Can translate frequency into timetable by specifying headways as:**

- equal -- appropriate if demand is uniformly distributed across period
- balanced load -- appropriate if there is substantial variation in demand over period
- clockface or not -- do headways repeat every hour



# Timetable Development

## If we have N departures in peak period:

- equal headway solution:

$$H = \frac{Peak\ Period}{N}$$

- balanced load solution:

$$\textit{Pass Load / Departure} = \frac{\textit{Total Passenger Flow}}{N}$$

# Fleet Size Requirement

## Salzborn's Fleet Size Theorem:

Given:

*l(k,t,s)* = # of departures from terminal *k* by time *t* following schedule *s*

*a(k,t,s)* = # of arrivals at terminal *k* by time *t* following schedule *s*

and:

*d(k,t,s)* = *l(k,t,s)* - *a(k,t,s)*, deficit function at terminal *k* at time *t* following schedule *s*

.

# Fleet Size Requirement

## Salzborn's Fleet Size Theorem:

**Then:**

***N(s),*** **the minimum size fleet to serve schedule *s*, is given by:**

$$N(s) = \sum_{k \in T} \max_t (d(k,t,s))$$

**for *T* terminals**

**Also, *N(s)* ≥ Max # of trips in simultaneous operation.**

# Fleet Size Required

The deficit function, or minimum required fleet size, may be reduced by:

- shifting departure and/or arrival times
- adding deadhead trips between terminals