Forced Convection and Natural Convection Equations

Necessary Dimensionless Numbers

	Reynolds (Re)	Grashof (Gr)	Prandtl (Pr)	Nusselt (Nu)
"Local"	$\mathrm{Re}_x = \frac{U_\infty x}{\nu_f}$	$Gr_x = \frac{g\beta x^3 (T_0 - T_\infty)}{\nu_f^2}$	$\mathrm{Pr} = \frac{\nu_f}{\alpha_f}$	$Nu_x = \frac{h_x x}{k_f}$
"Average"	$\mathrm{Re}_L = \frac{U_\infty L}{\nu_f}$	$Gr_x = \frac{g\beta L^3 (T_0 - T_\infty)}{\nu_f^2}$		$Nu_L = \frac{hL}{k_f}$

This web site contains these and a multiple array of other interesting dimensionless numbers—it even has a cool form where you can enter your data in different units and it will calculate the dimensionless number right on the website:
http://www.processassociates.com/process/dimen/dn_all.htm

Properties with a subscript "*f*" just mean that the property has been obtained at the *film temperature*, the average of the solid temperature (T_0) and the fluid stream temperature (T_∞). If the volume expansion coefficient ($\beta_f = \frac{-1}{\rho_f}\frac{d\rho}{dT}$, where the derivative is evaluated at the film temperature) isn't tabulated, it can be found in one of two ways:

1. Equation of state for your fluid (e.g. ideal gas equation). If you assume your fluid is an ideal gas, then $\beta_f = 1/T_f$. Otherwise,
2. Use whatever density vs. temperature values you have tabulated and approximate β_f with $\frac{-1}{\rho_f}\frac{\Delta\rho}{\Delta T}$.

Although convective heat transfer problems can seem incredibly confusing given the multitude of different equations available for different systems and flow regimes, it helps if you keep in mind that the whole goal of the problem is to find the overall heat transfer coefficient, *h*, from Nu_L so that we can describe the heat transfer from object in the fluid medium. Thus, finding Nu_L becomes the problem, and it does get a little ugly from time to time. Notice that in certain cases (like flat plates) one can define a local dimensionless number by using *x*, the distance down the object in the direction of the flow. Finding the local Nusselt number would allow one to then solve for h_x, the "local" heat transfer coefficient. We're generally not interested in knowing what the heat transfer coefficient at one particular point on the surface is, though—we want the average *h* (which is the one we're familiar with from all the heat transfer work we have done), and this is precisely the one we get from Nu_L.

Ok, so how do we get Nu_L? The problem is that the equations to find Nu_L are very problem-specific. Generally, however, you can identify 2 distinguishing characteristics in your problem to find the exact equation you need:

1. **Turbulent or Laminar?** For forced convection problems *Re* determines whether or not the flow is turbulent or laminar. For flow past flat plates, the transition region from laminar to turbulent is about $10^5 < Re < 10^7$. Go ahead and assume turbulence if *Re* is much higher than 10^5. For convection problems with cylinders and spheres, "*L*" (the length-scale) takes on slightly different meanings (and there isn't really an analog for the local dimensionless numbers). Nu_L is also called Nu_D in these situations sometimes.

Also, with cylinders and spheres it may not be as cut and dry as saying "laminar" or "turbulent." Certain equations have been developed to be accurate over a specific (and sometimes relatively small) range. For natural convection problems, the product *Gr Pr* is used to specify the flow regime. The laminar natural convection equations we're given hold from $10^4<Gr\,Pr<10^9$. The turbulent natural convection equations are valid from $10^9<Gr\,Pr<10^{12}$.

2. **High or Low Prandtl number?** After examining whether we have a laminar or turbulent system, check the Prandtl number. There are typically different equations for different ranges of Prandtl numbers. Unless otherwise specified, a "high" *Pr* is one > 0.5 and a "low" *Pr* is < 0.05.

That's about it for the stuff we covered in class. I've compiled a list of the equations (out of the text and notes) for *Nu* (or at least a P&G equation reference to them) for a few systems below (note: This is not an excuse not to read the text! It is simply to help you solve problems now and in the future without an overabundance of book and note flipping):

Flat Plate, Forced Convection

Laminar, Low Pr	Laminar, High Pr	Turbulent, High Pr
$Nu_x = (1/\sqrt{\pi})\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{0.5}$ $Nu_L = (2/\sqrt{\pi})\,\mathrm{Re}_L^{0.5}\,\mathrm{Pr}^{0.5}$	$Nu_x = 0.332\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{0.343}$ $Nu_L = 0.664\,\mathrm{Re}_L^{0.5}\,\mathrm{Pr}^{0.343}$	$Nu_x = 0.026\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3}$ $Nu_L = 0.037\,\mathrm{Re}_L^{0.8}\,\mathrm{Pr}^{1/3}$

Flat Plate (Vertical), Natural Convection (also, can use Fig. 8.8 for 0.5<Pr<10)

$10^4<Gr\,Pr<10^9$ $0.00835<Pr<1000$ but if $0.6<Pr<10$ use (8.18) also, check special cases (7.45b,c)	$10^9<Gr\,Pr<10^{12}$ works for most Pr but if $0.6<Pr<10$ use *(8.21)*
$\dfrac{Nu_L}{\sqrt[4]{0.25\,Gr_L}} = \dfrac{0.902\,\mathrm{Pr}^{0.5}}{(0.861+\mathrm{Pr})^{0.25}}$	$Nu_L = 0.246\,Gr_L^{2/5}\,\mathrm{Pr}^{7/15}\left(1+0.494\,\mathrm{Pr}^{2/3}\right)^{-2/5}$

Cylinder Oriented Normal to Flow (like Fig. 8.5), Forced Convection (L=length of cylinder)

$1<Re<100$, High Pr	$100<Re<10^7$, $Re\,Pr>0.2$
(8.8)	(8.10) (note special conditions)

Horizontal Cylinder, Natural Convection (L=πD/2) (also, can use Fig. 8.8 for 0.5<Pr<10)

(Use Vertical Flat Plate Equations with L=πD/2) or (8.32) for $10^{-5}<Gr_L\,Pr<10^{12}$

Sphere, Forced Convection (also, see Fig. 8.6)

(8.12 – Note validity conditions and special notation!)

Sphere, Natural Convection (L=D)

$Gr_L^{0.25}\,Pr^{1/3}<200$	$Pr\geq 0.7$ and $Gr_L\,Pr\leq 10^{11}$
$Nu_L = 2+0.060\,Gr_L^{0.25}\,\mathrm{Pr}^{1/3}$	(8.24)