14.12 Game Theory

Fall 2002

Homework 1

1. Consider an agent with Von Neumann and Morgenstren utility function u with $u(x) = (x-1)^2$. Check whether the following VNM utility functions can represent this agent's preferences. (Provide the details.)

   (a) $u^* : x \mapsto x - 1$;

   (b) $u^{**} : x \mapsto (x-1)^4$;

   (c) $\hat{u} : x \mapsto -(x-1)^2$;

   (d) $\tilde{u} : x \mapsto 2(x-1)^2 - 1$.

2. Compute the set of rationalizable strategies in the following game that is played in a class of n students where $n \geq 2$: Without discussing with anyone, each student i is to write down a real number $x_i \in [0, 100]$ on a paper and submit it to the TA. The TA will then compute the average

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

   of these numbers. The payoff of any student i is $100 - |x_i - 2\bar{x}/3|$ where x_i is the number student bids. Everything described above is common knowledge.

3. Consider the game depicted in Figure 1 in extensive form (where the payoff of player 1 is written on top, and the payoff of 2 is on the bottom).

   (a) Write this game in strategic form.

   (b) What are the strategies that survive the *iterative elimination of weakly-dominated strategies* in the following order: first eliminate all weakly-dominated strategies of player 1; then, eliminate all the strategies of player 2 that are weakly dominated in the remaining game; then, eliminate all the strategies of player 1 that are weakly dominated in the remaining game, and so on?

   (c) What are the Nash equilibria in pure strategies?

4. Consider the following game in normal form.

	L	M	R
S1	2,2	3,0	4,0
S2	3,3	2,0	1,0
S3	1,3	5,5	0,2
S4	1,1	1,1	2,3

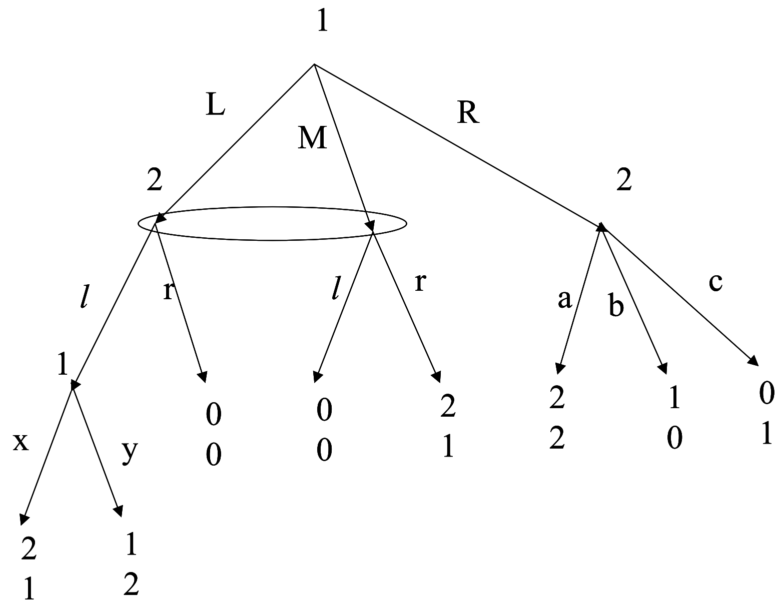

Figure 1:

(a) Iteratively eliminate all strictly dominated strategies; state the assumptions necessary for each elimination.

(b) What are the rationalizable strategies?

(c) What are the pure-strategy Nash equilibria?

5. We have an employer and a worker. The employer provides the capital K (in terms of investment in technology, etc.) and the worker provides the labor L (in terms of the investment in the human capital) to produce $f(K, L) = \sqrt{KL}$, which they share equally. The parties determine their investment level (the employer's capital K and the worker's labor L) simultaneously. The per-unit costs of capital and the labor for the employer and the worker are $r > 1/4$ and $c > 1/4$, respectively. The worker cannot invest more than $\bar{L}$ which is positive. The payoffs for the employer and the worker are

$$\frac{1}{2} f(K, L) - rK$$

and

$$\frac{1}{2} f(K, L) - cL,$$

respectively. Everything described up to here is common knowledge. Find all the rationalizable strategies.