14.126 Game Theory Problem Set 3

Due after Class #17

1. Given any finite event E and two agents 1 and 2, show that there exists some integer $\bar{n}$ such that for any chain $i_0 \neq i_1 \neq i_2 \neq \cdots \neq i_{n-1} \neq i_n$ with $n \geq \bar{n}$, the event
$$F = K_{i_0} K_{i_1} K_{i_2} \cdots K_{i_{n-1}} K_{i_n} E$$
is a public event between 1 and 2, i.e., $CK_{1,2}F = F$.

2. There are three players, 1,2, and 3, and three states, a, b, and c. The information partitions of players 1,2, and 3 are $\{\{a\}, \{b\}, \{c\}\}$, $\{\{a,b\}, \{c\}\}$, and $\{\{a\}, \{b,c\}\}$, respectively. There is a (random) variable v such that $v(a) = v(b) = -1$, and $v(c) = 2$. Is it common knowledge at state a that v is -1? Assume that players have a common prior, according to which each state is equally likely. Under this information structure, players play the following game. First 1 chooses between Left and Right. If he chooses Left, the game ends, yielding the payoff vector (1,1,1). If he chooses Right, then 2 is to choose between Left and Right. If 2 chooses Left, the game ends, when the payoff vector is (0,2,0). If 2 chooses Right, then 3 is to choose between Left and Right, ending the game. If 3 chooses Left, the payoff vector is $(2, 0, v)$; if he chooses Right the payoff vector is (3,3,0). Describe all sequential equilibria in pure strategies.

3. Consider a Cournot duopoly where the inverse-demand function is given by
$$P = 1 - Q$$
where P is the price of a good and $Q = q_1 + q_2$ where q_i is the supply of firm $i \in N = \{1, 2\}$. The marginal cost of firm i is denoted by c_i, so that its payoff function is
$$u_i(q_1, q_2) = q_i(1 - q_1 - q_2 - c_i).$$
The inverse demand and payoff functions are common knowledge. The marginal costs are privately known by the firms themselves. Construct a general infinite hierarchy of beliefs about the marginal costs such that a player's beliefs are independent from the other players' beliefs and from their own beliefs at other orders. Write t_i for the generic type of player i.

   (a) Define a strategy.

(b) Define a Nash equilibrium. For the remainder of the question fix a Nash equilibrium q^* of this game.

(c) For each i and t_i, write the first order condition that $q_i^*(t_i)$ must satisfy. (Make sure that $q_i^*(t_i)$ is written as a function of c_i and the expectation of q_j^* according to i.)

(d) Now recognize that q_j^* must satisfy a similar equation. Substituting this into the previous one, write $q_i^*(t_i)$ in terms of c_i, i's expectation of c_j, and i's expectation of j's expectation of q_i^*.

(e) Generalizing the procedure above, compute $q_i^*(t_i)$ (in terms of the costs and higher order expectations about these costs).

(f) How can you generalize this to all two-person games with quadratic utility functions, where $u_i(s_i, s_j, a_i) = -(s_i - a_i - b_i s_j)^2$ for some real numbers a_i and b_i where b_i is common knowledge. What happens if the equilibrium is unstable?

4. For two players, find (i) a common utility function u for some underlying uncertainty θ and (ii) an incomplete information model, such that it is common knowledge that (1) the players are rational, and (2) they play different strategies. [Here, if a player plays x, then he gets $u(x, \theta)$.]