## Massachusetts Institute of Technology

Department of Electrical Engineering & Computer Science

## 6.041/6.431: Probabilistic Systems Analysis (Fall 2002)

## **Tutorial 11 Answers**

## 1. The High-Minded Professor.

Answers:

- (a) (proof)
- (b) For  $1 \le i, j \le n$ , we have  $p_{ij} = \begin{cases} 0 & i > j \\ \frac{i}{n} & i = j \\ \frac{1}{n} & i < j \end{cases}$
- (c) Let  $S_k$  denote the state {Professor recalls rank k}. Then  $S_n$  is recurrent, and  $S_1$  through  $S_{n-1}$  are transient.
- 2. (a)  $\frac{1}{3} \left(\frac{1}{4}\right)^{k-1}$  for  $k = 2, 3, \dots$ 
  - (b)  $\frac{7}{9}$
  - (c)  $\frac{1}{18}$
  - (d)  $\frac{1}{24}$
  - (e)  $\frac{1}{3} \left(\frac{1}{4}\right)^{n-1}$  for  $n = 1, 2, 3, \dots$

3.

- (a)  $r_{11}(6) = \frac{182}{1125} \approx 0.162$
- (b)  $\mathbf{P}(X_{1000} \neq X_{999} \text{ and } X_{1000} \neq X_{1001}) = \frac{98}{155} \approx 0.632$