

Tutorial 11 Answers

1. The High-Minded Professor.

Answers:

(a) (proof)

(b) For $1 \leq i, j \leq n$, we have $p_{ij} = \begin{cases} 0 & i > j \\ \frac{i}{n} & i = j \\ \frac{1}{n} & i < j \end{cases}$

(c) Let S_k denote the state $\{\text{Professor recalls rank } k\}$. Then S_n is recurrent, and S_1 through S_{n-1} are transient.

2. (a) $\frac{1}{3} \left(\frac{1}{4}\right)^{k-1}$ for $k = 2, 3, \dots$

(b) $\frac{7}{9}$

(c) $\frac{1}{18}$

(d) $\frac{1}{24}$

(e) $\frac{1}{3} \left(\frac{1}{4}\right)^{n-1}$ for $n = 1, 2, 3, \dots$

3.

(a) $r_{11}(6) = \frac{182}{1125} \approx 0.162$

(b) $\mathbf{P}(X_{1000} \neq X_{999} \text{ and } X_{1000} \neq X_{1001}) = \frac{98}{155} \approx 0.632$