Recitation 4

1. Consider an experiment in which a fair four-sided die (with faces labeled 0, 1, 2, 3) is thrown once to determine how many times a fair coin is to be flipped. In the sample space of this experiment, random variables N and K are defined by

   - N = down-face value on the throw of the tetrahedral die
   - K = total number of heads resulting from the coin flips

   Determine and sketch each of the following functions for all values of their arguments:

   (a) $p_N(n)$
   (b) $p_{K|N}(k \mid 2)$
   (c) $p_{N|K}(n \mid 2)$
   (d) $p_K(k)$
   (e) Also determine the conditional PMF for random variable N, given that the experimental value of K is an odd number.
   (f) Finally, compute the expected value and variance for each distribution found in parts (a) - (e).

2. Suppose that X and Y are independent, indentically distributed, geometric random variables with parameter p. Show that

$$\mathbf{P}(X = i|X + Y = n) = \frac{1}{n-1}, i = 1, ..., n-1 \quad .$$

3. **Practice Problem:** A company is interviewing potential employees. Suppose that each potential employee is either qualified, or unqualified, with equal probability. The company tries to determine this by asking 20 true or false questions. A candidate gets a C for each correct answer, and an I for each incorrect answer. A qualified candidate has probability p of answering a question correctly, while an unqualified candidate has a probability p of answering incorrectly, with questions independent of each other. If the company considers anyone with at least 15 C's qualified, and everyone else unqualified, what is the probability that the 20 questions will correctly categorize the candidate?