Massachusetts Institute of Technology
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Fall 2002)

Tutorial 9 Answers

1.

   (a) $p_M(m) = p_D p_G (1 - p_D p_G)^{m-1}$, $m = 1, 2, 3, ...$

   (b) $\dfrac{p_D}{p_D + p_G - p_D p_G}$

   (c) $\dfrac{p_D(1 - p_G)}{p_D + p_G - p_D p_G}$

2. (a) $\sum_{f=0}^{j-1} \binom{f+i-1}{f} p^i (1-p)^f$

   (b) Let K be the number of successes which occur before the jth failure.

$$E[K] = \frac{p}{1-p} j, \sigma_K^2 = \frac{p}{(1-p)^2} j$$

   (c) This expression is the same as saying we need at least 42 trials to get the 17th success. Therefore, it can be rephrased as having a maximum of 16 successes in the first 41 trials. Hence $b = 41$, $a = 16$.

3. **Practice Problem:** For x to be the number of failures before the rth successes, we must have $x + r$ trials. Therefore, this looks like a shifted version of a Pascal PMF, i.e. out of the $x + r - 1$ trials, we need to pick $r - 1$ successes. If p is the probability of a success,

$$p_X(x) = \binom{x + r - 1}{r - 1} p^r (1-p)^x \text{ for } x \geq 0$$