Quiz-1 Review
October 7, 2002

1. Important Concepts

   (a) Probability Axioms
   (b) Set Theoretic Properties
   (c) Independence / Conditional Independence

   (d) DRAWING

   (e) Multiplication Rule
   (f) Total Probability Theorem
   (g) Bayes' Rule

   (h) Counting

   (i) Expectation
   (j) Variance
   (k) Independence
   (l) Joint PMF's

   (m) Uniform
   (n) Bernoulli
   (o) Binomial
   (p) Geometric
   (q) Poisson

2. Problem 1: Six horses run in a race. A horse j finishes first with probability $p_1 = 1/6$. Given that another horse, say $i \neq j$ finishes first, horse j will finish second with probability $p_2 = 1/5$, and if horses $i \neq j$ and $k \neq j$ finish first and second respectively, j will finish third with probability $p_3 = 1/4$. The same horses run the same race every day during the year.

   Alok gambles on the horse race in the following manner. He bets one dollar and chooses the horses that will finish in the first three positions. If the three horses winning the race finish in the same order as he has predicted, he will be given 15 dollars (i.e. if his wealth was one dollar before the race, it will be 15 dollars afterwards.) If the three winning horses are in a different order than he predicted, he will be given 3 dollars, otherwise he won't be given any money (and loose his dollar).

   (a) Assume that the races are independent (i.e. the same horses race every day, with the same probabilities for winning.) Alok gambles on the same combination $(1, 2, 3)$ (in this order) for n races.

      i. What is the probability of him making money on k races among those ?

      ii. Given that he won exactly k races among n, what is the expected value of his change (increase/decrease) in wealth (i.e. how much should he expect his gains (if any) would be?)

   (b) For the remainder of this problem, we consider a more realistic model where the races' outcomes are not independent. If horses $[j, k, l]$ finish in the first three positions (in any order), they are six times more likely to finish in the first three positions in the next race, (i.e. finishes like (j, k, l), (k, j, l), (l, k, j), etc ... are equally likely and 6 times more likely than any other finish involving a horse different from k, l or j). We also assume that the other finishes are equally likely.

   For the first race, Alok bets on horses $(1, 2, 3)$ as before. For the subsequent races, he will bet on the winning trio of the previous race. For the first race, assume that the horses' winning probabilities are identical to those of the previous section.

      i. What is the probability that Alok makes money on k races if he gambles on n?

      ii. Alok missed the first race. Horses $(1, 2, 3)$ won the second race. What is the probability that the horses finished in the same order on the first game ?

3. Problem 2: Let X_1 and X_2 be two independent random variables with the following PMF's

$$p_{X_1}(k) = \frac{\mu^k e^{-\mu}}{k!}, \quad k = 0, 1, 2, \ldots$$
$$p_{X_2}(k) = \frac{\lambda^k e^{-\lambda}}{k!}, \quad k = 0, 1, 2, \ldots$$

   (a) Find the PMF of the random variable $Z = X_1 + X_2$.

   (b) You roll a fair two-sided die (?), and let the random variable Y represent its outcome ($Y = 1$ with probability $1/2$ and 2 with probability $1/2$).

   Find the PMF of the random variable S defined as follows: if $Y = 1$, $S = X_1$, otherwise, $S = X_1 + X_2$.