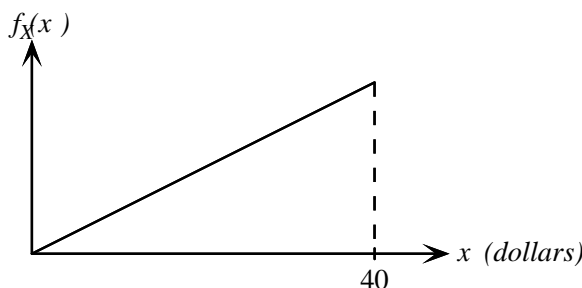


Tutorial 7

1. Paul is vacationing in Monte Carlo. The amount X (in dollars) he takes to the casino each evening is a random variable with the PDF shown in the figure. At the end of each night, the amount Y that he has on leaving the casino is uniformly distributed between zero and twice the amount he took in.



- (a) Determine the joint PDF $f_{X,Y}(x,y)$. Be sure to indicate what the sample space is.
- (b) What is the probability that on any given night Paul makes a positive profit at the casino? Justify your reasoning.
- (c) Find and sketch the probability density function of Paul's profit on any particular night, $Z = Y - X$. What is $\mathbf{E}[Z]$? Please label all axes on your sketch.
2. Suppose that

$$M_X(s) = \frac{1}{3} \cdot \frac{1}{1-s} + \frac{2}{3} \cdot \frac{3}{3-s}.$$

What is the PDF of X ?

3. **Practice Problem:** Let X and Y be independent random variables which have only non-negative integer experimental values. Each part of this question introduces a new random variable. For each part, determine the expectation and variance of the new random variable in terms of $\mathbf{E}[X]$, $\mathbf{E}[Y]$, $\text{var}(X)$, and $\text{var}(Y)$.

Notation: X_1, X_2, \dots are independent random variables with the same distribution as random variable X . Provide lucid explanations for your answers.

- (a) $W = X_1 + X_2 + X_3 + X_4$
- (b) $V = 0.25(X_1 + X_2 + X_3 + X_4)$
- (c) $U = X_1 + X_2 + X_3 + X_4 + Y$
- (d) $R = 4X - Y$
- (e) $M_Q(s) = [M_X(s)]^5$
- (f) $M_H(s) = [M_X(s)]^2 [M_Y(s)]^3$
- (g) $M_G(s) = e^{6s} M_X(s)$
- (h) $M_D(s) = M_X(7s)$