Massachusetts Institute of Technology

Department of Electrical Engineering & Computer Science

6.041/6.431: Probabilistic Systems Analysis

(Fall 2002)

Recitation 14

1. We have a coin for which the probability of a head is $p = 0.1$. Consider a sequence of 15 independent flips of this coin.
   (a) Determine the exact probability of obtaining exactly 2 heads.
   (b) Determine the Poisson approximation of the probability of obtaining exactly 2 heads.
   (c) Determine the central limit theorem approximation of the probability of obtaining exactly 2 heads.

2. Consider a Poisson process, with mean arrival rate $\lambda = 1$, and let X_n be the number of arrivals between time zero and n. Give a brief explanation for your answers to the following.
   (a) Does $\frac{X_n}{n}$ converge in probability?
   (b) Does $\frac{X_n}{n}$ converge with probability 1?

3. **Practice Problem:** Consider a factory that produces $X_n \geq 0$ gadgets on day n. The X_n's are independent and identically distributed *discrete* random variables and it is known that

$$\mathbf{E}[X_n] = 5, \quad \mathbf{E}[X_n^2] = 34, \quad \mathbf{E}[X_n^3] = 412, \quad \mathbf{E}[X_n^4] < \infty \quad \text{and} \quad \mathbf{P}(X_n = 0) > 0 \quad .$$

   (a) Find an approximation to the probability that the total number of gadgets produced in 100 days is less than 440.
   (b) Find (approximately) the largest value of n such that

$$\mathbf{P}(X_1 + \cdots + X_n \geq 200 + 5n) \leq 0.05 \qquad .$$

   (c) Let N be the first day on which the total number of gadgets produced exceeds 1000. Calculate an approximation to the probability that $N \geq 220$.
   (d) For each definition of Z_n given below, state whether the sequence Z_n converges with probability 1.
      i. $Z_n = (X_1 + \cdots + X_n)/n$
      ii. $Z_n = (X_1 + \cdots + X_n - 5n)/\sqrt{n}$
      iii. $Z_n = (X_1^2 + \cdots + X_n^2)/n$
      iv. $Z_n = X_1 X_2 \cdots X_n$
      v. $Z_n = (X_1 X_2 + X_2 X_3 + \cdots + X_{n-1} X_n)/n$
