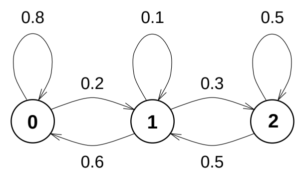
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Fall 2002)

Recitation 12

1. Consider the Markov chain shown below:



   (a) Either calculate the steady state probabilities or explain why they do not exist.

   (b) If we begin observing the process a very large number of transitions from now, evaluate:

      i. The probability there is a birth on the first transition we observe.

      ii. The probability there is a death on the first transition we observe.

      iii. The probability there is a birth on the first change of state we observe.

      iv. The probability there is a death on the first change of state we observe.

   (c) We make an equally likely choice among all changes of state over a long period of time.

      i. Evaluate the probability that the selected change of state happens to be a birth.

      ii. Evaluate the probability that the selected change of state happens to be a death.

2. Joe wishes to estimate the true fraction f of smokers in a large population without asking each and every person. He plans to select n people at random and then employ the estimator $F = S/n$, where S denotes the number of people in a size-n sample who are smokers. Joe would like to sample the minimum number of people, but also guarantee an upper bound p on the probability that the estimator F differs from the true value f by a value greater than or equal to d i.e., for a given accuracy d and given confidence p, Joe wishes to select the minimum n such that

$$\mathbf{P}(|F - f| \geq d) \leq p \quad .$$

   For $p = 0.05$ and a particular value of d, Joe uses the Chebyshev inequality to conclude that n must be at least 50,000. Determine the new minimum value for n if:

   (a) the value of d is reduced to half of its original value.

   (b) the probability p is reduced to half of its original value, or $p = 0.025$.

3. **Practice Problem:** Discrete random variable X is equal to 0 with probability 0.5 and otherwise takes on values -2 and 2 with equal probabilities. Let X_1, X_2, ... , be independent identically distributed random variables with the same distribution as X. For each of the following sequences, determine whether each converges in probability and, if so, the limit to which it converges. Justify your answer.

   (a) $Y_n = \max(X_1, X_2, ..., X_n)$

   (b) $T_n = X_1 + X_2 + ... + X_n$

   (c) $A_n = \frac{X_1+X_2+...+X_n}{n}$