Recitation 2

1. Suppose that two fair 6-sided dice are rolled.

   (a) Find the probability that doubles were rolled.

   (b) Given that the roll resulted in a sum of 4 or less, find the conditional probability that doubles were rolled.

   (c) Find the probability that at least one die is a 6.

   (d) Given that the two dice land on different numbers, find the conditional probability that at least one die is a 6.

2. **Communication through a noisy channel.** A binary (0 or 1) message transmitted through a noisy communication channel is received incorrectly with probability ϵ_0 and ϵ_1, respectively (see the figure). Errors in different symbol transmissions are independent.

   (a) Suppose that the channel source transmits a 0 with probability p and transmits a 1 with probability $1 - p$. What is the probability that a randomly chosen symbol is received correctly?

   (b) Suppose that the string of symbols 1011 is transmitted. What is the probability that all the symbols in the string are received correctly?

   (c) In an effort to improve reliability, each symbol is transmitted three times and the received symbol is decoded by majority rule. In other words, a 0 (or 1) is transmitted as 000 (or 111, respectively), and it is decoded at the receiver as a 0 (or 1) if and only if the received three-symbol string contains at least two 0s (or 1s, respectively). What is the probability that a transmitted 0 is correctly decoded?

   (d) Suppose that the channel source transmits a 0 with probability p and transmits a 1 with probability $1 - p$, and that the scheme of part (c) is used. What is the probability that a 0 was transmitted given that the received string is 101?

3. **Practice Problem:** An electrical system consists of identical components that are operational with probability p independently of other components. The components are connected in three subsystems, as shown in the figure. The system is operational if there is a path that starts at point A, ends at point B, and consists of operational components. This is the same as requiring that all three subsystems are operational. What are the probabilities that the three subsystems, as well as the entire system, are operational?