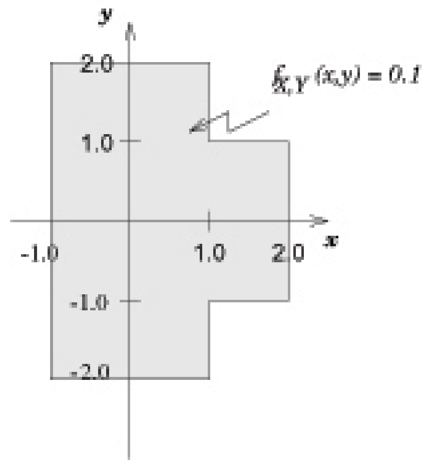
Massachusetts Institute of Technology
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Fall 2002)

Recitation 8

1. Random variables X and Y have the joint PDF shown below:



   (a) Prepare neat, fully labeled sketches of $f_{X|Y}(x|y)$.

   (b) Find $\mathbf{E}[X|Y = y]$ and $\text{var}(X|Y = y)$.

   (c) Find $\mathbf{E}[X]$.

   (d) Find $\text{var}(X)$ using the law of conditional variances.

2. Let J denote the number of crates of bulbs which arrive on any particular day, K denote the number of bulbs in any particular crate, and Y denote the lifetime of any particular bulb. Random variables J, K and Y are independent. The PMFs for J and K are given by:

$$p_J(q) = \begin{cases} (\frac{1}{2})^q & q = 1, 2, 3, \ldots \\ 0 & \text{otherwise} \end{cases} \qquad p_K(q) = \begin{cases} \frac{2^q e^{-2}}{q!} & q = 0, 1, 2, \ldots \\ 0 & \text{otherwise} \end{cases}$$

   The PDF for Y is given by:

$$f_Y(q) = \begin{cases} 2e^{-2q} & q > 0 \\ 0 & \text{elsewhere} \end{cases}$$

   Assume also that all experimental values of each of the three random variables are independent.

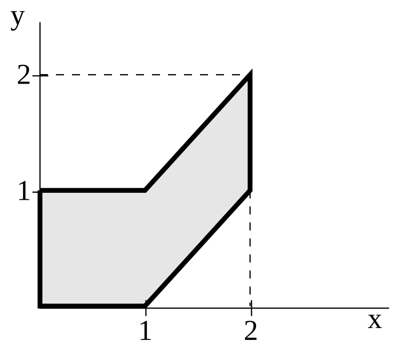
   (a) Find the probability that the number of bulbs, U, received during a day is an even number.

   (b) Find the expectation and variance for T, the total lifetime of *all the bulbs in any particular crate.*

   (c) Let R denote the total lifetime of *all bulbs that arrive on any particular day*, and evaluate $P(R = 0)$.

3. **Practice Problem:** Continuous random variables X and Y have a joint pdf given by

$$f_{X,Y}(x,y) = \begin{cases} C & \text{if } (x,y) \text{ belongs to the closed shaded region} \\ 0 & \text{otherwise} \end{cases}$$



The experimental value of X will be revealed to us; we have to design an estimator $g(X)$ of Y that minimizes the conditional expectation $E[(Y - g(X))^2 | X = x]$, for all x, over all possible estimators. Provide a plot of the optimal estimator as a function of its argument.