Tutorial 4

1. Consider two random variables, X and Y, which take on only integer values, as indicated by their joint PMF:

$$p_{X,Y}(x, y) = \begin{cases} \dfrac{y}{20} & 1 \leq y \leq x \leq 4 \\ & \\ 0 & \text{elsewhere} \end{cases}$$

   (a) Find the marginal PMF $p_X(x)\ \ \forall x$, and evaluate $\mathbf{E}[X]$.

   (b) Find the PMF $p_W(w)$, where $W = \max(X, 2Y)$.

   (c) For $R = X - Y$, evaluate $\mathbf{E}[R]$ and $\mathrm{var}(R)$.

   (d) With R defined as in part (c), let A denote the event $R \geq 2$, and evaluate the conditional standard deviation $\sigma_{R|A}$.

2. You just rented a large house and the realtor gave you five keys, one for the front door and the other four for each of the four side and back doors of the house. Unfortunately, all keys look identical, so to open the front door, you are forced to try them at random.

   Find the mean and the variance of the number of trials you will need to open the door, under the following alternative assumptions:

   (a) after an unsuccessful trial, you mark the corresponding key so that you never try it again, or

   (b) at each trial, you are equally likely to choose any key.

3. **Practice Problem:** Let X and Y be discrete random variables that are also independent.

   (a) Supposing they are zero-mean, show that

$$\mathrm{var}(X + Y) = \mathrm{var}(X) + \mathrm{var}(Y) \quad .$$

   (b) Now prove that the equation holds even when the two independent random variables, X and Y, have non-zero means.