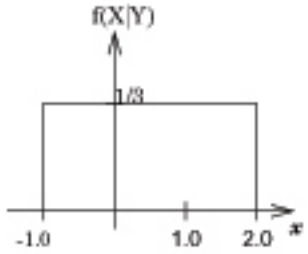
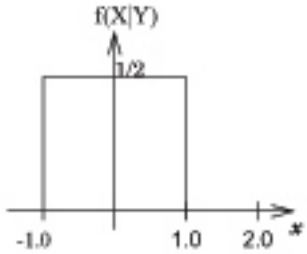
Recitation 8 Answers

1. (a)



-1<y<1



-2<=y<=-1 or 1<=y<=2

(b) $\mathbf{E}[X|Y=y] = \begin{cases} 0, & -2 \le y \le -1 \\ \frac{1}{2}, & -1 < y \le 1 \\ 0, & 1 \le y \le 2 \end{cases}$ $\quad \text{var}(X|Y=y) = \begin{cases} \frac{1}{3}, & -2 \le y \le -1 \\ \frac{1}{12}, & -1 < y \le 1 \\ \frac{1}{3}, & 1 \le y \le 2 \end{cases}$

(c) $\mathbf{E}[X] = \frac{3}{10}$

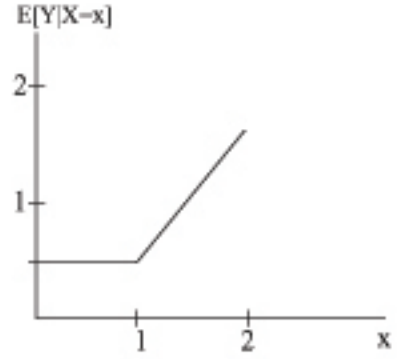
(d) $\text{var}(X) = \frac{193}{300}$

2. (a) $\mathbf{P}(\text{even number bulbs}) = \frac{1}{2-e^{-4}}$

(b) $\mathbf{E}[T] = 1$, $\text{var}(T) = 1$

(c) $\mathbf{P}(R=0) = \mathbf{P}(U=0) = \frac{1}{2e^2-1}$

3. **Practice Problem:** Here we wish to choose a $g(X)$ that minimizes the conditional mean squared error $\mathbf{E}[(Y-g(X))^2|X=x]$. As shown in section 4.6 of the course notes, this estimator is $g(X) = \mathbf{E}[Y|X=x]$.

$$\mathbf{E}[Y|X=x] = \begin{cases} \frac{1}{2} & 0 \le x < 1 \\ x - \frac{1}{2} & 1 \le x \le 2 \\ \text{Undefined} & \text{otherwise} \end{cases}$$

A plot of $g(X)$: