(Optional Evening) Recitation 10

1. All ships travel at the same speed through a wide canal. Eastbound ship arrivals at the canal are a Poisson process with an average arrival rate λ_E ships per day. Westbound ships arrive as an independent Poisson process with average arrival rate λ_W per day. An indicator at a point in the canal is always pointing in the direction of travel of the most recent ship to pass it. Each ship takes t days to traverse the length of the canal.
   (a) Given that the pointer is pointing west:
      i. What is the probability that the next ship to pass it will be westbound?
      ii. What is the PDF for the remaining time until the pointer changes direction?
   (b) What is the probability that an eastbound ship will see no westbound ships during its eastward journey through the canal?
   (c) We begin observing at an arbitrary time. Let V be the time we have to continue observing until we see the seventh eastbound ship. Determine the PDF for V.

2. The first-order interarrival times for cars passing a checkpoint are independent random variables with PDF
$$f_T(t) = \begin{cases} 2e^{-2t}, & \text{for } t > 0 \\ 0, & \text{otherwise.} \end{cases}$$
   where the interarrival times are measured in minutes. The successive experimental values of the durations of these first-order interarrival times are recorded on small computer cards. The recording operation occupies a negligible time periond following each arrival. Each card has space for three entries. As soon as a card is filled, it is replaced by the next card.
   (a) Determine the mean and the third moment of the first-order interarrival times.
   (b) Given that no car has arrived in the last four minutes, determine the PMF for random variable K, the number of cars to arrive in the next six minutes.
   (c) Determine the PDF, the expected value, and the transform for the total time required to use up the first dozen computer cards.
   (d) Consider the following two experiments:
      i. Pick a card at random from a group of completed cards and note the total time, Y, the card was in service. Find $\mathbf{E}[Y]$ and $\text{var}(Y)$.
      ii. Come to the corner at a random time. When the card in use at the time of your arrival is completed, note the total time it was in service (the time from the start of its service to its completion). Call this time W. Determine $\mathbf{E}[W]$ and $\text{var}(W)$.

3. **Practice Problem:** We are given the following statistics about the number of children in a typical family in a small village.

   There are 100 families.
   10 have no children;
   40 have 1;
   30 have 2;
   10 have 3;
   10 have 4.

   (a) If you pick a family at random, what is the expected number of children in that family?
   (b) If you pick a child at random (each child is equally likely), what is the expected number of children in that child's family (including the picked child)?
   (c) Generalize your approach from part (b) to the case where a fraction p_k of the families has k children, and provide a formula.