

Recitation 9

1. For each night, the probability of a robbery attempt at the local warehouse is $\frac{1}{5}$. A robbery attempt is successful with probability $\frac{3}{4}$, independent of the night. After any particular **SUCCESSFUL** robbery, the robber celebrates by taking off either the next 2 or 4 nights (with equal probability), during which time there will be no robbery attempts. After that, the robber returns to his original routine.

- (a) Let K be the number of robbery attempts up to (and including) the first successful robbery. Find the PMF of K .
- (b) Let D be the number of days until (and including) the second successful robbery, including the days of celebration after the first robbery. Find the PMF of D , or its transform (whichever you find more convenient).

During a successful robbery, the robber steals a random number of candy bars, which is 1, 2, or 3, with equal probabilities. This number is independent for each successful robbery and independent of everything else (no candy bars are stolen in unsuccessful robberies).

- (c) Let W be the number of candy bars collected in two successful robberies. Plot the PMF of W .
- (d) Let T be the number of candy bars collected in ten robbery attempts (whether successful or not). Find the PMF of T , or its transform, whichever is easier. Find the expectation and the variance of T .

2. Each of n packages is loaded independently into either a red truck (with probability p) or into a green truck (with probability $1 - p$). Let R be the total number of items selected for the red truck and let G be the total number of items selected for the green truck.

- (a) Determine the PMF, expected value, and variance for random variable R .
- (b) Evaluate $P(A)$, the probability that the first item to be loaded ends up being the only one on its truck.
- (c) Evaluate $P(B)$, the probability that at least one truck ends up with a total of exactly one package.
- (d) Evaluate the expectation and the variance for the difference, $D = R - G$.
- (e) Assume $n \geq 2$. Given that both of the first two packages to be loaded go onto the red truck, find the conditional expectation, variance and probability law for random variable R .

3. **Practice Problem:** You are visiting the rainforest, but unfortunately your insect repellent has run out. As a result, at each second, a mosquito lands on your neck with probability 0.5. If one lands, with probability 0.2 it bites you, and with probability 0.8 it never bothers you, independently of other mosquitoes. What is the expected time between successive bites? What is the variance of the time between successive bites?