6.041 Fall 2002 Quiz 1
50 minutes

DO NOT TURN THIS QUIZ OVER UNTIL YOU ARE TOLD TO DO SO

- You have 50 minutes to complete the quiz.
- Write your solutions in the exam booklet. We will not consider any work not in the exam booklet.
- This quiz has 2 problems that are not necessarily in order of difficulty.
- You may give an answer in the form of an arithmetic expression (sums, products, ratios, factorials) of numbers that could be evaluated using a calculator. Expressions like $\binom{8}{3}$ or $\sum_{k=0}^{5}(1/2)^k$ are also fine.
- A correct answer does not guarantee full credit and a wrong answer does not guarantee loss of credit. You should concisely indicate your reasoning and show all relevant work. The grade on each problem is based on our judgment of your level of understanding as reflected by what you have written.
- This is a closed-book exam except for one single-sided, handwritten, 8.5 by 11 formula sheet plus a calculator.
- Be neat! If we can't read it, we can't grade it.
- At the end of the quiz, turn in your solutions along with this quiz (this piece of paper).

Write your name and your TA's name on the front of the booklet. (2 points)

Problem 1: (48 points)

Sue and Kate are roommates. Each one of them owns a car, but they both share a single spot in the building's covered garage. On any day, independently of the weather, Sue and Kate are equally likely to come back earlier than the other and hence equally likely to park their respective car in the covered garage spot, while whoever arrives late will park outside.

Sue's car is old and will not always start. If it's parked in the garage it will start with probability q. If it is parked outside, depending on whether the night was warm or cold it will start with either probability p or $p/2$ respectively. If it does not start, she walks to work. If it does start she drives.

Assume that nights are equally likely to be cold or warm independently of everything else, unless it is stated otherwise.

(a) (16 points) Sue drove to work today. What is the probability that she parked her car in the garage, given that it started ?

(b) (16 points) Judging by her luck, Sue believes the weather will conspire against her and the night will be cold 80% of the time whenever she parks outside. Based on her judgment, what is the probability that the night was cold given that her car started ?

(c) (16 points) Kate's car behavior is independent and statistically the same as Sue's. Are the events "Sue's car started" and "Kate's car started" independent ? Explain carefully.

Problem 2: (50 points)

Every day, Ramzi arrives first to work and parks his car at the edge of the parking lot that consists of n parking spaces in a row. Danielle arrives later and parks her car as close to Ramzi's as possible (Note that answers given as summations are acceptable.)

(a) (18 points) If k people (excluding Ramzi) parked their cars randomly (in an uniform manner) in the available spots before Danielle arrived, let X be the random variable representing how many spots separate Danielle's and Ramzi's cars (X is zero when the cars are next to each other.) Find the PMF of X.

(b) (16 points) Denote by p_i the PMF of X at i. If the same experiment is repeated independently every day, find the expected value of days between the times when two spots or less separate Danielle's and Ramzi's cars (Give your answer in terms of the p_i's.)

(c) (16 points) Let Y be the average number of spots separating the two cars over the period of m days, i.e.

$$Y = \frac{1}{m} \sum_{j=1}^{m} X_j,$$

where X_j is the random variable representing how many spots separate Danielle's and Ramzi's cars on day j.

Find the variance of Y (Give your answer in terms of the p_i's.)