Problem Set 1

1. Express each of the following events in terms of the events A, B and C as well as the operations of complementation, union and intersection:

   (a) at least one of the events A, B, C occurs;

   (b) at most one of the events A, B, C occurs;

   (c) none of the events A, B, C occurs;

   (d) all three events A, B, C occur;

   (e) exactly one of the events A, B, C occurs;

   (f) events A and B occur, but not C;

   (g) either event A occurs or, if not, then B also does not occur.

   In each case draw the corresponding Venn diagrams.

2. An experiment consists of picking a student from the set of all students registered on the MIT campus this semester. It is *not* necessary to assume that all students are equally likely to be picked, but you may make this assumption if you like.

   (a) Consider the two events:
   A the student has had four years of high school science (FYS),
   B the student has had calculus in high school.
   For any student picked, if the probability that (s)he has had neither FYS nor calculus is 0.3, and the probability that (s)he has missed at least one of the two is 0.8, what is the probability that (s)he has had *exactly* one of the two?

   (b) Let C denote the event that the student is registered in 6.041 this semester, and let events A and B and their probabilities be as in part **(a)**. If students who had at most one of FYS and calculus did not register in 6.041 this semester,

      i. What is $P\left(A^c \cap B^c \cap C^c\right)$?

      ii. What is the probability that the student picked is not registered in 6.041 *and* has had exactly one of FYS or calculus in high school?

   (c) Using the data given in parts **(a)** and **(b)**, which of the following probabilities

   $$P(A \cap B \cap C), P(C), P(A^c \cap B \cap C^c), P(A), P(A^c \cap B^c \cap C), P(A \cup B \cup C^c)$$

   can you compute? It is not necessary to actually compute each probability.

3. Find $P\left(A \cup (B^c \cup C^c)^c\right)$ in each of the following cases:

   (a) A, B, C are mutually exclusive events and $P(A) = 3/7$.

   (b) $P(A) = 1/2$, $P(B \cap C) = 1/3$, $P(A \cap C) = 0$.

   (c) $P\left(A^c \cap (B^c \cup C^c)\right) = 0.65$.

4. Bob has a peculiar pair of four-sided dice. When he rolls the dice, the probability of any particular outcome is proportional to the product of the outcome of each die. All outcomes that result in a particular product are equally likely.

   (a) What is the probability of the product being even?

   (b) What is the probability of Bob rolling a 2 and a 3?

5. Let A and B be two events. Show the following inequalities. Under what conditions on A and B are these inequalities satisfied as equations? A mathematical derivation is required, but you can use Venn diagrams to guide your thinking.

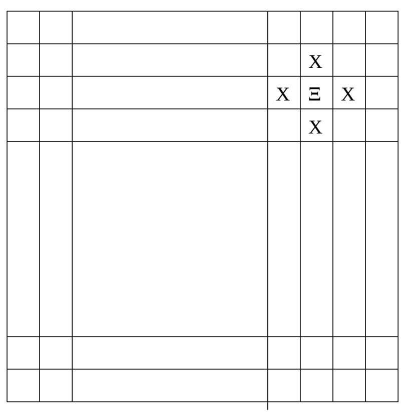
   (a) $P(A) + P(B) \geq P(A \cup B) \geq \max\{P(A), P(B)\}$.

   (b) $\min\{P(A), P(B)\} \geq P(A \cap B) \geq P(A) + P(B) - 1$.

   (c) †**Kounias's Inequality.** Show that

$$P\left(\bigcup_{r=1}^{n} A_r\right) \leq \min_k \left\{\sum_{r=1}^{n} P(A_r) - \sum_{r:r\neq k} P(A_r \cap A_k)\right\}.$$

G1†. Sonia and Norman are playing a friendly game of "Battleship" where each ship occupies only one square. Each player's grid is formed of $N \times N$ squares as shown below.



Sonia places two ships on her grid one after the other, in the following manner: for each ship, she makes an equally likely choice among all available squares in the grid. What is the probability the two are adjacent? (by adjacent we mean that the squares have a common edge, as shown in the figure.)

†Required for 6.431; optional for 6.041