Massachusetts Institute of Technology
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Fall 2002)

Recitation 13

1. The weight X (in ounces) of any particular banana is an independent, continuous random variable with a uniform PDF over the interval [3,7]. We classify a "light" banana as one that weighs less than four ounces and otherwise it is a "heavy" banana.

   (a) Suppose we obtain 40 bananas and separate them into sets of "light" bananas and "heavy" bananas. Determine the variance of L, the number of light bananas, and the variance of H, the number of heavy bananas. Explain how these variances compare.

   (b) Suppose we obtain 40 bananas whose total weight we denote by W.

      i. Determine the expected value and variance of W.

      ii. Repeat part (ii), only now supposing the additional information that 15 of the bananas are light (and thus 25 are heavy).

      iii. Explain how and why the additional information changes the expectation and variance of the total weight of the 40 bananas.

   (c) Bananas are to be packed into crates. Each crate will contain the same (non-random) number k of bananas. Let T be the total weight of the bananas in any particular crate. Either determine the smallest value of k, the number of bananas per crate, required to satisfy the condition

   $$\mathbf{P}\left(|T - \mathbf{E}[T]| \leq \frac{\mathbf{E}[T]}{100}\right) \geq 0.9544$$

   or explain why no such value of k exists.

2. Random variable X takes on experimental values of -8, 0, and 8 with probabilities of $\frac{1}{8}$, $\frac{6}{8}$, and $\frac{1}{8}$, respectively. Let $Y_n = nX$ and $Z_n = X_1 + X_2 + \cdots + X_n$ (e.g., Y_{100} is 100 times one experimental value of X, while Z_{100} is the sum of 100 independent experimental values of X).

   (a) Find the expectations and variances for Y_n and Z_n.

   (b) Provide an *excellent* numerical approximation for the probability

   $$\mathbf{P}\left(|Y_{100} - \mathbf{E}[Y_{100}]| \geq 32\right) \qquad .$$

   (c) Provide an *excellent* numerical approximation for the probability

   $$\mathbf{P}\left(|Z_{100} - \mathbf{E}[Z_{100}]| \geq 32\right) \qquad .$$

3. **Practice Problem:** The length in meters, X, of each section of pipe we obtain is an independent random variable with PDF

   $$f_X(x) = 2e^{-2x} \qquad x \geq 0 \qquad \Rightarrow \qquad \mathbf{E}[X] = \frac{1}{2}, \quad \mathrm{var}(X) = \frac{1}{4}$$

   (a) Suppose we obtain 400 sections of pipe. Determine the value of a bound, w, such that the total length of the sections we obtain will be greater than w with a probability of approximately 0.841.

   (b) Determine n, the number of sections of pipe needed such that the probability we obtain at least 200 meters of pipe is approximately 0.841.